Research Article

Decentralized Finite-Time H_∞ Connective Control for a Class of Large-Scale Systems with Different Structural Forms

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Decentralized finite-time H_∞ connective control problem for a class of large-scale interconnected systems is studied in this paper. The research aims at two structural forms, namely, the interconnected structure and the one with expanding construction. A new method is proposed to design a decentralized state feedback control law for a large-scale interconnected system so that the closed-loop system is finite-time H_∞ connectively bounded. The sufficient conditions for the existence of such a decentralized control law are deduced by using LMI method. Another method is presented for a large-scale interconnected system with expanding construction which can be used without changing the decentralized state feedback control law of the original system to design a controller for the newly added subsystem so that both the new subsystem and the resulting expanded system are finite-time H_∞ connectively bounded. The feasibility and effectiveness of the proposed method are verified by some simulation results.

1. Introduction

In many practical applications, the operating time of a given dynamic system is finite. It is natural to study finite-time stability rather than the classical Lyapunov asymptotic stability. The concept of finite-time stability was first presented in [1], which was extended to finite-time boundedness in [2]. Nowadays, finite-time control problem has been paid close attention by many scholars and a large number of results have been reported in the literature [3–6].

Due to the development of H_∞ control theory, finite-time H_∞ control problem has been well investigated in [7–12]. Therein, the finite-time H_∞ control problems of a variety of systems have been studied, such as linear continuous systems, linear stochastic systems, uncertain discrete-time singular systems, switched systems with time-varying delay, linear time-invariant and time-varying systems, and a class of nonlinear time-delay Hamiltonian systems. But all the above results are based on the single system. In [13], decentralized finite-time H_∞ control method is applied to interconnected linear systems. However, so far, there is no research report on decentralized finite-time H_∞ connective control problem

of large-scale interconnected systems and the one with expanding construction.

In this paper, connective boundedness and finite-time H_∞ performance index are applied to a class of large-scale interconnected systems and the concepts of finite-time connective stability, finite-time connective boundedness, and finite-time H_∞ connective boundedness are given. A decentralized state feedback control law is designed for a large-scale interconnected system so that the corresponding closed-loop system is finite-time H_∞ connectively bounded. Furthermore, without changing the decentralized state feedback control law of the original system, a control law is constructed for the newly added subsystem so that both the new subsystem and the resulting expanded system are finite-time H_∞ connectively bounded.

The paper is organized as follows. In Section 2, some definitions and preliminary results are provided and the mathematical models of large-scale interconnected systems and the one with expanding construction are precisely stated. Sections 3 and 4 present the sufficient conditions for the existence of decentralized state feedback control laws for finite-time H_∞ connective boundedness of large-scale interconnected systems and the expanded systems. To

FIGURE 1: The basic structure of an expanded system.

illustrate the feasibility of the proposed method, one example is provided in Section 5.

2. Mathematical Description and Preliminary Results

Consider a large-scale interconnected system and the one with expanding construction as in [14]. The basic structure of these systems is shown in Figure 1.

In Figure 1, $\bar{S}_{N-1}$ is the original system composed of $(N - 1)$ subsystems, which are described by

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + \Gamma_i \omega_i(t) + G_i v_i(t), \\ z_i(t) &= C_i x_i(t), \\ w_i(t) &= H_i x_i(t), \\ v_i(t) &= \sum_{j=1, j \neq i}^{N-1} E_{ij} w_j(t),\end{aligned}\quad (1)$$

where $i = 1, 2, \dots, N - 1$, $x_i \in R^{n_i}$, $u_i \in R^{p_i}$, $z_i \in R^{p_i}$, and $\omega_i \in R^{l_i}$ are the state, control input, output, and external disturbance vectors of the i th subsystem, respectively; v_i represents the impact on the i th subsystem from the other subsystems; w_i denotes the impact on the other subsystems by the i th subsystem. A_i , B_i , C_i , Γ_i , G_i , and H_i are the constant matrices with appropriate dimensions; E_{ij} represents the interconnection term from the j th subsystem to the i th subsystem. It is clear that $E_{ij} = 1$ represents the fact that there is an interconnection and $E_{ij} = 0$ means that there is no interconnection. For simplification, system (1) can be rewritten as

$$\begin{aligned}\dot{\bar{X}}_{N-1}(t) &= \bar{A}_{N-1} \bar{X}_{N-1}(t) + \bar{B}_{N-1} \bar{u}_{N-1}(t) \\ &\quad + \bar{\Gamma}_{N-1} \bar{\omega}_{N-1}(t) + \bar{G}_{N-1} \bar{v}_{N-1}(t), \\ \bar{z}_{N-1}(t) &= \bar{C}_{N-1} \bar{X}_{N-1}(t), \\ \bar{w}_{N-1}(t) &= \bar{H}_{N-1} \bar{X}_{N-1}(t), \\ \bar{v}_{N-1}(t) &= \bar{E}_{N-1} \bar{w}_{N-1}(t),\end{aligned}\quad (2)$$

where

$$\begin{aligned}\bar{X}_{N-1} &= (x_1^T, x_2^T, \dots, x_{N-1}^T)^T, \\ \bar{u}_{N-1} &= (u_1^T, u_2^T, \dots, u_{N-1}^T)^T,\end{aligned}$$

$$\bar{\omega}_{N-1} = (\omega_1^T, \omega_2^T, \dots, \omega_{N-1}^T)^T,$$

$$\bar{z}_{N-1} = (z_1^T, z_2^T, \dots, z_{N-1}^T)^T,$$

$$\bar{w}_{N-1} = (w_1, w_2, \dots, w_{N-1})^T,$$

$$\bar{v}_{N-1} = (v_1, v_2, \dots, v_{N-1})^T,$$

$$\bar{A}_{N-1} = \text{diag}(A_1, A_2, \dots, A_{N-1}),$$

$$\bar{B}_{N-1} = \text{diag}(B_1, B_2, \dots, B_{N-1}),$$

$$\bar{C}_{N-1} = \text{diag}(C_1, C_2, \dots, C_{N-1}),$$

$$\bar{\Gamma}_{N-1} = \text{diag}(\Gamma_1, \Gamma_2, \dots, \Gamma_{N-1}),$$

$$\bar{G}_{N-1} = \text{diag}(G_1, G_2, \dots, G_{N-1}),$$

$$\bar{H}_{N-1} = \text{diag}(H_1, H_2, \dots, H_{N-1}),$$

$$\bar{E}_{N-1} = \begin{bmatrix} \mathbf{0} & E_{12} & \cdots & E_{1,N-1} \\ E_{21} & \mathbf{0} & \cdots & E_{2,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ E_{N-1,1} & E_{N-1,2} & \cdots & \mathbf{0} \end{bmatrix}.$$

In Figure 1, S_N is the N th subsystem newly added to the original system, which is modeled by

$$\begin{aligned}\dot{x}_N(t) &= A_N x_N(t) + B_N u_N(t) + \Gamma_N \omega_N(t) \\ &\quad + G_N v_N(t), \\ z_N(t) &= C_N x_N(t), \\ w_N(t) &= H_N x_N(t),\end{aligned}\quad (4)$$

where $v_N(t) = \bar{E}_{N,N-1} \bar{w}_{N-1}(t)$, which represents the impact on the N th subsystem by the original $(N - 1)$ subsystems. After the N th subsystem is added, the impact on the original $(N - 1)$ subsystems by the N th subsystem is $\bar{v}_{N-1} = \bar{E}_{N-1,N} w_N$. $\bar{E}_{N-1,N} = [E_{1N}, E_{2N}, \dots, E_{N-1,N}]^T$ and $\bar{E}_{N,N-1} = [E_{N1}, E_{N2}, \dots, E_{N,N-1}]$. The large-scale system with N subsystems becomes

$$\begin{aligned}\dot{\bar{X}}_{N-1}(t) &= \bar{A}_{N-1} \bar{X}_{N-1}(t) + \bar{B}_{N-1} \bar{u}_{N-1}(t) \\ &\quad + \bar{\Gamma}_{N-1} \bar{\omega}_{N-1}(t) + \bar{h}(t, \bar{X}_{N-1}, x_N, E), \\ \dot{x}_N(t) &= A_N x_N(t) + B_N u_N(t) + \Gamma_N \omega_N(t) \\ &\quad + h(t, \bar{X}_{N-1}, x_N, E),\end{aligned}\quad (5)$$

where

$$\begin{aligned}\bar{h}(t, \bar{X}_{N-1}, x_N, E) &= \bar{G}_{N-1} \bar{E}_{N-1} \bar{H}_{N-1} \bar{X}_{N-1}(t) \\ &\quad + \bar{G}_{N-1} \bar{E}_{N-1,N} H_N x_N(t),\end{aligned}\quad (6)$$

$$h(t, \bar{X}_{N-1}, x_N, E) = G_N \bar{E}_{N,N-1} \bar{H}_{N-1} \bar{X}_{N-1}(t).$$

Equation (5) can be written as the following compact form:

$$\begin{aligned}\dot{\hat{x}}(t) &= \widehat{A}\hat{x}(t) + \widehat{B}\hat{u}(t) + \widehat{\Gamma}\hat{\omega}(t) + \widehat{h}(t, \hat{x}(t), E), \\ \widehat{z}(t) &= \widehat{C}\hat{x}(t),\end{aligned}\quad (7)$$

where

$$\begin{aligned}\widehat{A} &= \begin{bmatrix} \overline{A}_{N-1} & \mathbf{0} \\ \mathbf{0} & A_N \end{bmatrix}, \\ \widehat{B} &= \begin{bmatrix} \overline{B}_{N-1} & \mathbf{0} \\ \mathbf{0} & B_N \end{bmatrix}, \\ \widehat{\Gamma} &= \begin{bmatrix} \overline{\Gamma}_{N-1} & \mathbf{0} \\ \mathbf{0} & \Gamma_N \end{bmatrix}, \\ \widehat{C} &= \begin{bmatrix} \overline{C}_{N-1} & \mathbf{0} \\ \mathbf{0} & C_N \end{bmatrix}, \\ \widehat{u}(t) &= \begin{bmatrix} \overline{u}_{N-1}(t) & \mathbf{0} \\ \mathbf{0} & u_N(t) \end{bmatrix}, \\ \widehat{z}(t) &= \begin{bmatrix} \overline{z}_{N-1}(t) & \mathbf{0} \\ \mathbf{0} & z_N(t) \end{bmatrix}, \\ \widehat{\omega}(t) &= \begin{bmatrix} \overline{\omega}_{N-1}(t) & \mathbf{0} \\ \mathbf{0} & \omega_N(t) \end{bmatrix}, \\ \widehat{h}(t, \hat{x}(t), E) &= \left[\overline{h}^T(t, \overline{X}_{N-1}, x_N, E), h^T(t, \overline{X}_{N-1}, x_N, E) \right]^T, \\ \hat{x}(t) &= \begin{bmatrix} \overline{X}_{N-1}(t) \\ x_N(t) \end{bmatrix} \\ &= \left[x_1^T(t), x_2^T(t), \dots, x_{N-1}^T(t), x_N^T(t) \right]^T.\end{aligned}\quad (8)$$

It is assumed that $\widehat{\omega}(t)$ satisfies

$$\int_0^\infty \widehat{\omega}^T(t) \widehat{\omega}(t) dt < d^2. \quad (9)$$

The following definitions for the large-scale systems with expanding construction are given.

Definition 1 (finite-time connective stability). The large-scale system described by (7) with $\widehat{u}(t) = \widehat{\omega}(t) = 0$ is said to be finite-time connectively stable with respect to (c_1, c_2, T, R) with $0 < c_1 < c_2$ and $R > 0$ if $\widehat{x}_0^T R \widehat{x}_0 \leq c_1$; then

$$\widehat{x}^T(t) R \widehat{x}(t) < c_2, \quad \forall t \in [0, T], \quad (10)$$

for any $E_{ij}, i = 1, \dots, N, j = 1, \dots, N$, and $\widehat{x}(0) = \widehat{x}_0$.

Definition 2 (finite-time connective boundedness). The large-scale system described by (7) with $\widehat{u}(t) = 0$ is said to be

finite-time connectively bounded with respect to (c_1, c_2, T, R) with $0 < c_1 < c_2$ and $R > 0$ if $\widehat{x}_0^T R \widehat{x}_0 \leq c_1$; then

$$\widehat{x}^T(t) R \widehat{x}(t) < c_2, \quad \forall t \in [0, T], \quad (11)$$

for any $E_{ij}, i = 1, \dots, N, j = 1, \dots, N$, and any nonzero $\widehat{\omega}(t)$ satisfying (9).

Definition 3 (finite-time H_∞ connective boundedness). The large-scale system described by (7) with $\widehat{u}(t) = 0$ is said to be finite-time H_∞ connectively bounded if

- (1) equation (7) with $\widehat{u}(t) = 0$ is finite-time connectively bounded, and
- (2) with zero initial condition

$$\int_0^t \|\widehat{z}(s)\|^2 ds < e^{\alpha T} \gamma^2 \int_0^t \|\widehat{\omega}(s)\|^2 ds \quad (12)$$

for any $E_{ij}, i = 1, \dots, N, j = 1, \dots, N$, and any nonzero $\widehat{\omega}(t)$ satisfying (9), where γ is a positive constant and $t \in [0, T]$.

The decentralized state feedback control law of the form

$$\widehat{u}(t) = \widehat{K}\widehat{x}(t) \quad (13)$$

with $\widehat{K} = \text{diag}(K_1, \dots, K_N)$ is introduced to control system (7). Then the closed-loop system becomes

$$\begin{aligned}\dot{\hat{x}}(t) &= (\widehat{A} + \widehat{B}\widehat{K})\widehat{x}(t) + \widehat{\Gamma}\widehat{\omega}(t) + \widehat{h}(t, \widehat{x}(t), E) \\ &= \widehat{A}\widehat{x}(t) + \widehat{\Gamma}\widehat{\omega}(t) + \widehat{h}(t, \widehat{x}(t), E), \\ \widehat{z}(t) &= \widehat{C}\widehat{x}(t),\end{aligned}\quad (14)$$

where

$$\begin{aligned}\widehat{A} &= \widehat{A} + \widehat{B}\widehat{K} = \begin{bmatrix} \overline{A}_{N-1} + \overline{B}_{N-1}\overline{K}_{N-1} & \mathbf{0} \\ \mathbf{0} & A_N + B_N K_N \end{bmatrix} \\ &= \begin{bmatrix} \widehat{A}_{N-1} & \mathbf{0} \\ \mathbf{0} & \widehat{A}_N \end{bmatrix}.\end{aligned}\quad (15)$$

Definition 4. The decentralized finite-time H_∞ connective control problem is said to be solvable if there exists a decentralized state feedback control law (13) so that the closed-loop system (14) is finite-time H_∞ connectively bounded.

In order to obtain the main results in this paper, the following assumption and lemmas are proposed.

Assumption 5. The interconnection item $\widehat{h}(t, \widehat{x}(t), E)$ is uncertain and satisfies the following inequality:

$$\widehat{h}^T(t, \widehat{x}(t), E) \widehat{h}(t, \widehat{x}(t), E) \leq \widehat{\beta}^2 \widehat{x}^T(t) \widehat{Q}^T \widehat{Q} \widehat{x}(t), \quad (16)$$

where $\widehat{\beta} > 0$ is bounding parameter and $\widehat{Q}$ is the interconnected constraint matrix.

Lemma 6. System (7) with $\hat{u}(t) = 0$ is finite-time connectively bounded if there exist two scalars, $\alpha \geq 0$ and $\gamma > 0$, and a symmetric positive definite matrix P such that

$$\begin{bmatrix} \hat{A}^T \hat{P} + \hat{P} \hat{A} - \alpha \hat{P} + \hat{\beta}^2 \hat{Q}^T \hat{Q} & \hat{P} \hat{\Gamma} & \hat{P} \\ \hat{\Gamma}^T \hat{P} & -\gamma^2 I & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I \end{bmatrix} < 0, \quad (17)$$

$$\frac{c_1}{\lambda_{\min}(P)} + d^2 \gamma^2 < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(P)} \quad (18)$$

with $\hat{P} = R^{1/2} P^{-1} R^{1/2}$ and $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ being the minimum and maximum eigenvalues of P , respectively.

Proof. Let $V(\hat{x}(t)) = \hat{x}^T(t) \hat{P} \hat{x}(t)$; then

$$\begin{aligned} \dot{V}(\hat{x}(t)) &= \hat{x}^T(t) (\hat{A}^T \hat{P} + \hat{P} \hat{A}) \hat{x}(t) + \hat{\omega}^T(t) \hat{\Gamma}^T \hat{P} \hat{x}(t) \\ &+ \hat{x}^T(t) \hat{P} \hat{\Gamma} \hat{\omega}(t) + \hat{h}^T(t, \hat{x}(t), E) \hat{P} \hat{x}(t) + \hat{x}^T(t) \\ &\cdot \hat{P} \hat{h}(t, \hat{x}(t), E) \leq \hat{x}^T(t) (\hat{A}^T \hat{P} + \hat{P} \hat{A}) \hat{x}(t) \\ &+ \hat{\omega}^T(t) \hat{\Gamma}^T \hat{P} \hat{x}(t) + \hat{x}^T(t) \hat{P} \hat{\Gamma} \hat{\omega}(t) + \hat{h}^T(t, \hat{x}(t), E) \\ &\cdot \hat{P} \hat{x}(t) + \hat{x}^T(t) \hat{P} \hat{h}(t, \hat{x}(t), E) - \hat{h}^T(t, \hat{x}(t), E) \\ &\cdot \hat{h}(t, \hat{x}(t), E) + \hat{\beta}^2 \hat{x}^T(t) \hat{Q}^T \hat{Q} \hat{x}(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \\ \hat{h}(t) \end{bmatrix}^T \end{aligned} \quad (19)$$

$$\cdot \begin{bmatrix} \hat{A}^T \hat{P} + \hat{P} \hat{A} + \hat{\beta}^2 \hat{Q}^T \hat{Q} & \hat{P} \hat{\Gamma} & \hat{P} \\ \hat{\Gamma}^T \hat{P} & \mathbf{0} & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \\ \hat{h}(t) \end{bmatrix}.$$

From (17) and (19), we have

$$\dot{V}(\hat{x}(t)) < \alpha V(\hat{x}(t)) + \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t). \quad (20)$$

Multiplying (20) by $e^{-\alpha t}$, we can obtain

$$e^{-\alpha t} \dot{V}(\hat{x}(t)) - e^{-\alpha t} \alpha V(\hat{x}(t)) < e^{-\alpha t} \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t), \quad (21)$$

which is equivalent to

$$\frac{d}{dt} (e^{-\alpha t} V(\hat{x}(t))) < e^{-\alpha t} \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t). \quad (22)$$

Integrating (22) from 0 to t gives

$$e^{-\alpha t} V(\hat{x}(t)) - V(\hat{x}(0)) < \int_0^t e^{-\alpha s} \gamma^2 \hat{\omega}^T(s) \hat{\omega}(s) ds. \quad (23)$$

Noting $\alpha \geq 0$, then

$$\begin{aligned} V(\hat{x}(t)) &< e^{\alpha t} V(\hat{x}(0)) + e^{\alpha t} \int_0^t e^{-\alpha s} \gamma^2 \hat{\omega}^T(s) \hat{\omega}(s) ds \\ &< e^{\alpha t} V(\hat{x}(0)) + e^{\alpha t} \gamma^2 \int_0^t \hat{\omega}^T(s) \hat{\omega}(s) ds \\ &< e^{\alpha t} [V(\hat{x}(0)) + d^2 \gamma^2]. \end{aligned} \quad (24)$$

Due to $\hat{P} = R^{1/2} P^{-1} R^{1/2}$, it follows from (24) that

$$\begin{aligned} \hat{x}^T(t) R^{1/2} P^{-1} R^{1/2} \hat{x}(t) \\ < e^{\alpha t} [\hat{x}^T(0) R^{1/2} P^{-1} R^{1/2} \hat{x}(0) + d^2 \gamma^2]. \end{aligned} \quad (25)$$

Now, we have

$$\begin{aligned} \hat{x}^T(t) R^{1/2} P^{-1} R^{1/2} \hat{x}(t) &\geq \lambda_{\min}(P^{-1}) \hat{x}^T(t) R \hat{x}(t), \\ e^{\alpha t} [\hat{x}^T(0) R^{1/2} P^{-1} R^{1/2} \hat{x}(0) + d^2 \gamma^2] \\ &\leq e^{\alpha t} [\lambda_{\max}(P^{-1}) \hat{x}^T(0) R \hat{x}(0) + d^2 \gamma^2]. \end{aligned} \quad (26)$$

Putting (25) and (26) together, we have

$$\begin{aligned} \hat{x}^T(t) R \hat{x}(t) &< \frac{e^{\alpha t} [\lambda_{\max}(P^{-1}) \hat{x}^T(0) R \hat{x}(0) + d^2 \gamma^2]}{\lambda_{\min}(P^{-1})} \\ &= \frac{e^{\alpha t} [\lambda_{\max}(P^{-1}) c_1 + d^2 \gamma^2]}{\lambda_{\min}(P^{-1})}. \end{aligned} \quad (27)$$

Condition (18) implies that, for all $t \in [0, T]$, $\hat{x}^T(t) R \hat{x}(t) < c_2$. Because (16) holds for any E_{ij} , then system (7) with $\hat{u}(t) = 0$ is connectively bounded. The proof is completed. $\square$

Lemma 7. System (14) is finite-time H_∞ connectively bounded if there exist two scalars, $\alpha \geq 0$ and $\gamma > 0$, and a symmetric positive definite matrix P such that

$$\begin{bmatrix} \hat{A}^T \hat{P} + \hat{P} \hat{A} - \alpha \hat{P} + \hat{C}^T \hat{C} + \hat{\beta}^2 \hat{Q}^T \hat{Q} & \hat{P} \hat{\Gamma} & \hat{P} \\ \hat{\Gamma}^T \hat{P} & -\gamma^2 I & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I \end{bmatrix} < 0, \quad (28)$$

$$\frac{c_1}{\lambda_{\min}(P)} + d^2 \gamma^2 < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(P)} \quad (29)$$

with $\hat{P} = R^{1/2} P^{-1} R^{1/2}$.

Proof. Note that

$$\begin{bmatrix} \hat{C}^T \hat{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \hat{C}^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} [\hat{C} \quad \mathbf{0} \quad \mathbf{0}] \geq 0. \quad (30)$$

Therefore, condition (28) implies that

$$\begin{bmatrix} \hat{A}^T \hat{P} + \hat{P} \hat{A} - \alpha \hat{P} + \hat{\beta}^2 \hat{Q}^T \hat{Q} & \hat{P} \hat{\Gamma} & \hat{P} \\ \hat{\Gamma}^T \hat{P} & -\gamma^2 I & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I \end{bmatrix} < 0. \quad (31)$$

From Lemma 6, conditions (29) and (31) guarantee that system (14) is finite-time connectively bounded. Now, we need to proof that (12) holds.

Let $V(\hat{x}(t)) = \hat{x}^T(t) \hat{P} \hat{x}(t)$; then

$$\begin{aligned} \dot{V}(\hat{x}(t)) &= \hat{x}^T(t) \left(\hat{A}^T \hat{P} + \hat{P} \hat{A} \right) \hat{x}(t) + \hat{\omega}^T(t) \hat{\Gamma}^T \hat{P} \hat{x}(t) \\ &+ \hat{x}^T(t) \hat{P} \hat{\Gamma} \hat{\omega}(t) + \hat{h}^T(t, \hat{x}(t), E) \hat{P} \hat{x}(t) + \hat{x}^T(t) \\ &\cdot \hat{P} \hat{h}(t, \hat{x}(t), E) \leq \hat{x}^T(t) \left(\hat{A}^T \hat{P} + \hat{P} \hat{A} \right) \hat{x}(t) \\ &+ \hat{\omega}^T(t) \hat{\Gamma}^T \hat{P} \hat{x}(t) + \hat{x}^T(t) \hat{P} \hat{\Gamma} \hat{\omega}(t) + \hat{h}^T(t, \hat{x}(t), E) \\ &\cdot \hat{P} \hat{x}(t) + \hat{x}^T(t) \hat{P} \hat{h}(t, \hat{x}(t), E) - \hat{h}^T(t, \hat{x}(t), E) \\ &\cdot \hat{h}(t, \hat{x}(t), E) + \hat{\beta}^2 \hat{x}^T(t) \hat{Q}^T \hat{Q} \hat{x}(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \\ \hat{h}(t) \end{bmatrix}^T \\ &\cdot \begin{bmatrix} \hat{A}^T \hat{P} + \hat{P} \hat{A} + \hat{\beta}^2 \hat{Q}^T \hat{Q} & \hat{P} \hat{\Gamma} & \hat{P} \\ \hat{\Gamma}^T \hat{P} & \mathbf{0} & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \\ \hat{h}(t) \end{bmatrix}. \end{aligned} \quad (32)$$

It follows from (28) and (32) that

$$\dot{V}(\hat{x}(t)) < \alpha V(\hat{x}(t)) + \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t) - \hat{z}^T(t) \hat{z}(t) \quad (33)$$

which implies that

$$\begin{aligned} e^{-\alpha t} \dot{V}(\hat{x}(t)) - \alpha e^{-\alpha t} V(\hat{x}(t)) \\ < e^{-\alpha t} [\gamma^2 \hat{\omega}^T(t) \hat{\omega}(t) - \hat{z}^T(t) \hat{z}(t)] \end{aligned} \quad (34)$$

that is,

$$\frac{d}{dt} (e^{-\alpha t} V) < e^{-\alpha t} [\gamma^2 \hat{\omega}^T(t) \hat{\omega}(t) - \hat{z}^T(t) \hat{z}(t)]. \quad (35)$$

Taking $\hat{x}(0) = 0$ into consideration, integrating (35) from 0 to t produces

$$\begin{aligned} e^{-\alpha t} V(\hat{x}(t)) \\ < \int_0^t e^{-\alpha s} [\gamma^2 \hat{\omega}^T(s) \hat{\omega}(s) - \hat{z}^T(s) \hat{z}(s)] ds, \end{aligned} \quad (36)$$

which is equivalent to

$$\int_0^t e^{-\alpha s} \hat{z}^T(s) \hat{z}(s) ds < \gamma^2 \int_0^t e^{-\alpha s} \hat{\omega}^T(s) \hat{\omega}(s) ds. \quad (37)$$

Due to $e^{-\alpha T} < e^{-\alpha s}$ for $s \in [0, T]$ and $e^{-\alpha s} < 1$ for $s \in (0, T)$, it follows that

$$\begin{aligned} e^{-\alpha T} \int_0^t \hat{z}^T(s) \hat{z}(s) ds &< \int_0^t e^{-\alpha s} \hat{z}^T(s) \hat{z}(s) ds, \\ \gamma^2 \int_0^t e^{-\alpha s} \hat{\omega}^T(s) \hat{\omega}(s) ds &< \gamma^2 \int_0^t \hat{\omega}^T(s) \hat{\omega}(s) ds. \end{aligned} \quad (38)$$

Equation (12) can be obtained from (37)-(38). Because (16) holds for any E_{ij} , then system (14) with $\hat{u}(t) = 0$ is connectively bounded. The proof is completed. $\square$

3. Decentralized Finite-Time H_∞ Connective Control

The main result is given in Theorem 8 which is based on Lemmas 6 and 7 in Section 2.

Theorem 8. *The decentralized finite-time H_∞ connective control problem for (14) including N subsystems is solvable with feedback control laws $\hat{u}(t) = \hat{K} \hat{x}(t)$ if there exist two scalars, $\alpha \geq 0$ and $\gamma > 0$, matrix $\hat{M}$, and the interconnected constraint matrix, $\hat{Q}$, as well as symmetric positive definite matrices $\hat{Y}$ and P such that*

$$\min(\gamma^2 + \hat{\varepsilon} + c_2), \quad (39)$$

$$\begin{bmatrix} \hat{\Psi} & \hat{\Gamma} & I & \hat{Y} \hat{Q}^T & \hat{Y} \hat{C}^T \\ \hat{\Gamma}^T & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ I & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} \\ \hat{Q} \hat{Y} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} \\ \hat{C} \hat{Y} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I \end{bmatrix} < 0, \quad (40)$$

$$\frac{c_1}{\lambda_{\min}(P)} + d^2 \gamma^2 < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(P)}, \quad (41)$$

where

$$\begin{aligned} \hat{\Psi} &= \hat{Y} \hat{A}^T + \hat{A} \hat{Y} + \hat{M}^T \hat{B}^T + \hat{B} \hat{M} - \alpha \hat{Y}, \\ \hat{\varepsilon} &= \frac{1}{\hat{\beta}^2}, \\ \hat{Y} &= \hat{P}^{-1}, \\ P &= R^{1/2} \hat{P}^{-1} R^{1/2} \end{aligned} \quad (42)$$

with $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ being the minimum and maximum eigenvalues of P , respectively. And the control law can be determined by $\hat{K} = \hat{M} \hat{Y}^{-1}$.

Proof. According to Lemmas 6 and 7, let $\hat{M} = \hat{K} \hat{Y}$. Then (40) is equivalent to the following inequality:

$$\begin{bmatrix} \hat{\Psi} & \hat{\Gamma} & I & \hat{Y} \hat{Q}^T & \hat{Y} \hat{C}^T \\ \hat{\Gamma}^T & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ I & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} \\ \hat{Q} \hat{Y} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} \\ \hat{C} \hat{Y} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I \end{bmatrix} < 0 \quad (43)$$

with $\hat{\Psi} = \hat{Y} \hat{A}^T + \hat{A} \hat{Y} + \hat{Y} \hat{K}^T \hat{B}^T + \hat{B} \hat{K} \hat{Y} - \alpha \hat{Y}$.

Using Schur complement lemma, then (43) can be changed to

$$\begin{bmatrix} \hat{\Psi} & \hat{\Gamma} & I & \hat{Y} \hat{Q}^T \\ \hat{\Gamma}^T & -\gamma^2 I & \mathbf{0} & \mathbf{0} \\ I & \mathbf{0} & -I & \mathbf{0} \\ \hat{Q} \hat{Y} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I \end{bmatrix} < 0 \quad (44)$$

with $\hat{\Psi} = \hat{Y} \hat{A}^T + \hat{A} \hat{Y} + \hat{Y} \hat{K}^T \hat{B}^T + \hat{B} \hat{K} \hat{Y} - \alpha \hat{Y} + \hat{Y} \hat{C}^T \hat{C} \hat{Y}$.

Set $\Theta = \text{diag}(\hat{Y}^{-1}, I, I, I)$ and $\hat{Y}^{-1} = \hat{P}$. Then, pre- and postmultiplying (44) by Θ yields

$$\begin{bmatrix} \bar{\Psi} & \hat{P}\hat{\Gamma} & \hat{P} & \hat{Q}^T \\ \hat{\Gamma}^T\hat{P} & -\gamma^2 I & \mathbf{0} & \mathbf{0} \\ \hat{P} & \mathbf{0} & -I & \mathbf{0} \\ \hat{Q} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon}I \end{bmatrix} < 0 \quad (45)$$

with $\bar{\Psi} = \hat{A}^T\hat{P} + \hat{P}\hat{A} + \hat{K}^T\hat{B}^T\hat{P} + \hat{P}\hat{B}\hat{K} - \alpha\hat{P} + \hat{C}^T\hat{C} = \hat{A}^T\hat{P} + \hat{P}\hat{A} - \alpha\hat{P} + \hat{C}^T\hat{C}$ due to $\hat{A} = \hat{A} + \hat{B}\hat{K}$.

Using Schur complement lemma again, it can be proved that (45) is equivalent to (28), which completes the proof. When E_{ij} varies from 1 to 0 or from 0 to 1, the change of interconnection is still within limited range. So the control law $\hat{K}$ can make the system stable and robustly connectively stable. $\square$

It is obvious that (41) is not an LMI. However, it is guaranteed by imposing the conditions

$$\lambda_1 I < P < \lambda_2 I, \quad (46)$$

$$\frac{c_1}{\lambda_1} + d^2\gamma^2 < \frac{c_2 e^{-\alpha T}}{\lambda_2} \quad (47)$$

for two positive numbers λ_1 and λ_2 . Using Schur complements, inequality (47) can be converted to the following LMI:

$$\begin{bmatrix} I & \sqrt{c_1} & d\gamma & \sqrt{c_2}e^{-\alpha T/2} \\ \sqrt{c_1} & \lambda_1 & \mathbf{0} & \mathbf{0} \\ d\gamma & \mathbf{0} & I & \mathbf{0} \\ \sqrt{c_2}e^{-\alpha T/2} & \mathbf{0} & \mathbf{0} & -\lambda_2 \end{bmatrix} > 0. \quad (48)$$

Therefore, (41) holds if LMIs (46) and (48) are true. From a computational point of view, it is important to notice that, for

a given α , the feasibility of the conditions stated in Theorem 8 can be turned into the following feasibility problem of LMIs.

LMI Feasibility Problem 1 (from Theorem 8). For system (14), given c_1, T, R , and $\alpha > 0$, let $\hat{P} = R^{1/2}P^{-1}R^{1/2}$; find a symmetric positive definite matrix $\hat{Y}$ and two positive scalars, λ_1 and λ_2 , satisfying LMIs (39), (40), (46), and (48). If the problem is feasible, the decentralized finite-time H_∞ connective control problem can be solved by $\hat{K} = \hat{M}\hat{Y}^{-1}$.

4. Decentralized Finite-Time H_∞ Connective Control for Large-Scale Systems with Expanding Construction

Section 3 has introduced a method for constructing a decentralized state feedback control law to solve the finite-time H_∞ connective control problem for large-scale systems. This section will propose a control design method for the finite-time H_∞ connective control problem of large-scale systems with expanding construction. The technical difficulty of dealing with this kind of systems is that a control law is constructed for the newly added subsystem so that both the new subsystem and the resulting expanded system are finite-time H_∞ connectively bounded without changing the decentralized state feedback control law of the original system. To this end, the following assumption is made.

Assumption 9. The decentralized finite-time H_∞ connective control problem for the original system (1) can be solved by the control law $\bar{u}_{N-1}(t) = \bar{K}_{N-1}\bar{X}_{N-1}(t)$.

The main result is given in Theorem 10.

Theorem 10. The decentralized finite-time H_∞ connective control problem for the N th subsystem in (14) is solvable with feedback control law $u_N(t) = K_N x_N(t)$, if there are positive constants, α and γ , as well as symmetrical positive definite matrices, Y_N and P_N , and matrix M_N , so that the following inequalities are feasible:

$$\min(\gamma^2 + \hat{\varepsilon} + c_2), \quad (49)$$

$$\begin{bmatrix} \bar{\Lambda}_{N-1} & \mathbf{0} & \bar{P}_{N-1}\bar{\Gamma}_{N-1} & \mathbf{0} & \bar{P}_{N-1} & \mathbf{0} & \hat{q}_1^T & \hat{q}_3^T & \mathbf{0} \\ \mathbf{0} & \bar{\Lambda}_N & \mathbf{0} & \bar{\Gamma}_N & \mathbf{0} & I & Y_N\hat{q}_2^T & \mathbf{0} & Y_N C_N^T \\ \bar{\Gamma}_{N-1}^T \bar{P}_{N-1} & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{\Gamma}_N^T & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{P}_{N-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hat{q}_1 & \hat{q}_2 Y_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon}I & \mathbf{0} & \mathbf{0} \\ \hat{q}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon}I & \mathbf{0} \\ \mathbf{0} & C_N Y_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I \end{bmatrix} < 0, \quad (50)$$

$$\frac{c_1}{\lambda_{\min}(P_N)} + d^2\gamma^2 < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(P_N)}, \quad (51)$$

where $\hat{q}_1$, $\hat{q}_2$, and $\hat{q}_3$ are the elements of the interconnected constraint matrix $\hat{Q}$ and

$$\begin{aligned}\bar{\Lambda}_{N-1} &= \hat{A}_{N-1}^T \bar{P}_{N-1} + \bar{P}_{N-1} \hat{A}_{N-1} - \alpha \bar{P}_{N-1} \\ &\quad + \bar{C}_{N-1}^T \bar{C}_{N-1}, \\ \tilde{\Lambda}_N &= Y_N A_N^T + A_N Y_N + M_N^T B_N^T + B_N M_N - \alpha Y_N, \\ \hat{\varepsilon} &= \frac{1}{\hat{\beta}^2}, \\ Y_N &= \hat{P}_N^{-1},\end{aligned}$$

$$P_N = R^{1/2} \hat{P}_N^{-1} R^{1/2},$$

$$\bar{P}_{N-1} = \text{diag}(P_1, P_2, \dots, P_{N-1}),$$

$$\hat{Q} = \begin{bmatrix} \hat{q}_1 & \hat{q}_2 \\ \hat{q}_3 & \mathbf{0} \end{bmatrix}$$

(52)

with $\lambda_{\min}(P_N)$ and $\lambda_{\max}(P_N)$ being the minimum and maximum eigenvalues of P_N , respectively. And the control law can be determined by $K_N = M_N Y_N^{-1}$.

Proof. The result of Theorem 10 is based on Lemmas 6 and 7 and Theorem 8. Let $M_N = K_N Y_N$. Then, (50) is equivalent to

$$\begin{bmatrix} \bar{\Lambda}_{N-1} & \mathbf{0} & \bar{P}_{N-1} \bar{\Gamma}_{N-1} & \mathbf{0} & \bar{P}_{N-1} & \mathbf{0} & \hat{q}_1^T & \hat{q}_3^T & \mathbf{0} \\ \mathbf{0} & \tilde{\Lambda}_N & \mathbf{0} & \Gamma_N & \mathbf{0} & I & Y_N \hat{q}_2^T & \mathbf{0} & Y_N C_N^T \\ \bar{\Gamma}_{N-1}^T \bar{P}_{N-1} & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Gamma_N^T & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{P}_{N-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hat{q}_1 & \hat{q}_2 Y_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} & \mathbf{0} \\ \hat{q}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} \\ \mathbf{0} & C_N Y_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I \end{bmatrix} < 0, \quad (53)$$

where

$$\begin{aligned}\tilde{\Lambda}_N &= Y_N A_N^T + A_N Y_N + Y_N K_N^T B_N^T + B_N K_N Y_N \\ &\quad - \alpha Y_N.\end{aligned} \quad (54)$$

Using Schur complement lemma results in

$$\begin{bmatrix} \bar{\Lambda}_{N-1} & \mathbf{0} & \bar{P}_{N-1} \bar{\Gamma}_{N-1} & \mathbf{0} & \bar{P}_{N-1} & \mathbf{0} & \hat{q}_1^T & \hat{q}_3^T \\ \mathbf{0} & \tilde{\Lambda}_N & \mathbf{0} & \Gamma_N & \mathbf{0} & I & Y_N \hat{q}_2^T & \mathbf{0} \\ \bar{\Gamma}_{N-1}^T \bar{P}_{N-1} & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Gamma_N^T & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{P}_{N-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} \\ \hat{q}_1 & \hat{q}_2 Y_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} \\ \hat{q}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I \end{bmatrix} < 0 \quad (55)$$

with $\tilde{\Lambda}_N = Y_N A_N^T + A_N Y_N + Y_N K_N^T B_N^T + B_N K_N Y_N - \alpha Y_N + Y_N C_N^T C_N Y_N$.

Set $\Xi = \text{diag}(I, Y_N^{-1}, I, I, I, I, I, I)$ and $Y_N^{-1} = \hat{P}_N$. Then, the following inequality follows from pre- and postmultiplying (55) by Ξ :

$$\begin{bmatrix} \bar{\Lambda}_{N-1} & \mathbf{0} & \bar{P}_{N-1} \bar{\Gamma}_{N-1} & \mathbf{0} & \bar{P}_{N-1} & \mathbf{0} & \hat{q}_1^T & \hat{q}_3^T \\ \mathbf{0} & \Lambda_N & \mathbf{0} & \hat{P}_N \Gamma_N & \mathbf{0} & \hat{P}_N & \hat{q}_2^T & \mathbf{0} \\ \bar{\Gamma}_{N-1}^T \bar{P}_{N-1} & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Gamma_N^T \hat{P}_N & \mathbf{0} & -\gamma^2 I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{P}_{N-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{P}_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & -I & \mathbf{0} & \mathbf{0} \\ \hat{q}_1 & \hat{q}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I & \mathbf{0} \\ \hat{q}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{\varepsilon} I \end{bmatrix} < 0, \quad (56)$$

where

$$\begin{aligned}\Lambda_N &= A_N^T \hat{P}_N + \hat{P}_N A_N + K_N^T B_N^T \hat{P}_N + \hat{P}_N B_N K_N \\ &\quad - \alpha \hat{P}_N + C_N^T C_N.\end{aligned} \quad (57)$$

Set $\hat{P} = \begin{bmatrix} \bar{P}_{N-1} & \mathbf{0} \\ \mathbf{0} & \hat{P}_N \end{bmatrix}$. Due to

$$\hat{A}^T \hat{P} + \hat{P} \hat{A} - \alpha \hat{P} + \hat{C}^T \hat{C} = \begin{bmatrix} \bar{\Lambda}_{N-1} & \mathbf{0} \\ \mathbf{0} & \Lambda_N \end{bmatrix}, \quad (58)$$

then (56) is equivalent to

$$\begin{bmatrix} \widehat{A}^T \widehat{P} + \widehat{P} \widehat{A} - \alpha \widehat{P} + \widehat{C}^T \widehat{C} & \widehat{P} \widehat{\Gamma} & \widehat{P} & \widehat{Q}^T \\ \widehat{\Gamma}^T \widehat{P} & -\gamma^2 I & \mathbf{0} & \mathbf{0} \\ \widehat{P} & \mathbf{0} & -I & \mathbf{0} \\ \widehat{Q} & \mathbf{0} & \mathbf{0} & -\widehat{\varepsilon} I \end{bmatrix} < 0. \quad (59)$$

Applying Schur complement lemma to (59) shows that (28) is true. It follows from Lemma 7 that Theorem 10 holds. Similar to Theorem 8, the control law K_N can make the system stable and robustly connectively stable and the following feasibility problem of LMIs is given. $\square$

LMIs Feasibility Problem 2 (from Theorem 10). For system (14), given c_1, T, R , and $\alpha > 0$, let $\widehat{P}_N = R^{1/2} P_N^{-1} R^{1/2}$; find a symmetric positive definite matrix, Y_N , matrix M_N , and two positive scalars, λ_1 and λ_2 , satisfying LMIs (46), (48), (49), and (50). If the problem is feasible, the decentralized finite-time H_∞ connective control problem for the expanded subsystem can be solved by $K_N = M_N Y_N^{-1}$.

5. Simulation Example

Consider a class of multiarea interconnected power systems, in which each area includes a hydroelectric power unit and a thermal power unit. The mathematical model, state variables, and output variables can be found in [15, 16]. This is a deviation model of automatic generation control (AGC). The i th area-subsystem model S_i can be described as

$$\dot{x}_i = A_{ii} x_i + B_i u_i + \Gamma_i \omega_i + \sum_{j=1, j \neq i}^N G_i E_{ij} H_j x_j, \quad (60)$$

$$y_i = C_i x_i,$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $y_i \in R^{l_i}$, and $\omega_i \in R^{m_i}$ are the state, control input, output, and uncertain disturbance input of subsystems, respectively. And the matrices in (60) are, respectively,

$$A_{ii} = \begin{bmatrix} A_i & \mathbf{0} & a_{ii} \\ d_i^T & \mathbf{0} & 1 \\ \sigma_{1i} \sum_{j=1, j \neq i}^N m_{ij}^T & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$B_i = \begin{bmatrix} b_i \\ 0 \\ 0 \end{bmatrix},$$

$$\Gamma_i = \begin{bmatrix} f_i \\ 0 \\ 0 \end{bmatrix},$$

$$C_i = \begin{bmatrix} c_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & 1 \end{bmatrix},$$

$$G_i = [0 \ 0 \ m_{ij}^T]^T,$$

$$H_i = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0].$$

(61)

The specific parameters of (61) can be found in [15, 16]. And other parameters are given as follows:

$$\Gamma_1 = \Gamma_2 = \Gamma_3$$

$$= [0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T,$$

$$R_1 = R_2 = R_3 = I_{10},$$

$$G_1 = G_2 = G_3$$

$$= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 22.2144]^T,$$

$$H_1 = H_2 = H_3 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0], \quad (62)$$

$$\omega_1 = \omega_2 = \omega_3 = 0.1,$$

$$\alpha = 20,$$

$$T = 6,$$

$$d = 1,$$

$$c_1 = 1.$$

Suppose that the original system consists of two subsystems. The initial values are $\widehat{x}_{10} = \widehat{x}_{20} = [0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T$. The connective relations are $\overline{E} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. According to Theorem 8, the corresponding control gain matrices K_1 , and K_2 are obtained as

$$K_1 = [-0.2372 \ -0.0634 \ 0.0042 \ 0.0061 \ -0.6209 \ -0.5948 \ 0.1474 \ 0.0028 \ -0.0098 \ -0.0699],$$

$$K_2 = [-0.3613 \ -0.0974 \ 0.0052 \ 0.0076 \ -0.6277 \ -0.6034 \ 0.1416 \ 0.0041 \ -0.0091 \ -0.0664]$$

(63)

and $\min(c_2) = 37.6856$. $\theta = \sum_{i=1}^2 \widehat{x}_i^T(t) R_i \widehat{x}_i(t)$ is plotted as shown in Figure 2. It can be easily seen from Figure 2 that $\theta <$

$\min(c_2)$ and $t \in [0, 6]$, which means that the decentralized finite-time H_∞ connective control problem is solvable for

FIGURE 2: The simulation curve of the original system.

FIGURE 3: The simulation curve of the original system when E_{12} varies from 1 to 0.

the original system. In order to check connective stability of the system, we cut off the interconnection E_{12} between two subsystems. The result is shown in Figure 3, which shows that the closed-loop system is finite-time H_∞ connectively bounded.

Now the third subsystem is added to the original system with

$$\hat{x}_{30} = [0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T. \quad (64)$$

The connective relations are given by

$$E = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \quad (65)$$

According to Theorem 10, the control gain matrix K_3 is obtained as

$$K_3 = [-0.3093 \ -0.0954 \ 0.0048 \ 0.0071 \ -0.5086 \ -0.5855 \ 0.1406 \ 0.0113 \ 0.0254 \ 0.0479] \quad (66)$$

FIGURE 4: The simulation curve of the expanded system.

FIGURE 5: The simulation curve of the expanded system when E_{23} varies from 1 to 0.

and $\min(c_2) = 36.7686$. $\theta_1 = \sum_{i=1}^3 \hat{x}_i^T(t) R_i \hat{x}_i(t)$ is shown in Figure 4. It is obvious that $\theta_1 < \min(c_2)$ and $t \in [0, 6]$, which implies that the decentralized finite-time H_∞ connective control problem is solvable for the expanded system. In order to check the connective stability of the expanded system, the connection E_{23} varies from 1 to 0 and the result is shown in Figure 5.

From the figures, it can be known that both the original system and the resulting expanded system are finite-time H_∞ connectively bounded.

6. Conclusion

Decentralized finite-time H_∞ connective control problem for a class of large-scale systems is studied in this paper. The large-scale systems include the original structural interconnected system and the systems with expanding construction. Based on state feedback, the sufficient conditions of decentralized finite-time H_∞ connective boundedness

for large-scale systems are deduced by using LMI method. The design methods for the decentralized finite-time H_∞ connective control problem are given. The simulation examples verify the feasibility and effectiveness of the proposed method. In particular, the paper proposes a method for the structure expansion of large-scale systems. A controller can be designed for the newly added subsystem on the basis of keeping the decentralized state feedback control laws of the original construction systems unchanged so that both the new subsystem and the resulting expanded system are finite-time H_∞ connectively bounded. Therefore, this paper can be used as the theoretical basis for expansion of large-scale interconnected system online.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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