Research Article

A New Six-Parameter Model Based on Chebyshev Polynomials for Solar Cells

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This paper presents a new current-voltage (I - V) model for solar cells. It has been proved that series resistance of a solar cell is related to temperature. However, the existing five-parameter model ignores the temperature dependence of series resistance and then only accurately predicts the performance of monocrystalline silicon solar cells. Therefore, this paper uses Chebyshev polynomials to describe the relationship between series resistance and temperature. This makes a new parameter called temperature coefficient for series resistance introduced into the single-diode model. Then, a new six-parameter model for solar cells is established in this paper. This new model can improve the accuracy of the traditional single-diode model and reflect the temperature dependence of series resistance. To validate the accuracy of the six-parameter model in this paper, five kinds of silicon solar cells with different technology types, that is, monocrystalline silicon, polycrystalline silicon, thin film silicon, and tripe-junction amorphous silicon, are tested at different irradiance and temperature conditions. Experiment results show that the six-parameter model proposed in this paper is an I - V model with moderate computational complexity and high precision.

1. Introduction

Photovoltaic (PV) power generation system directly converts solar energy into electrical energy by using PV arrays. To obtain higher energy efficiency, PV power generation systems need to establish their simulation models to get optimized parameters. PV power generation system is mainly composed of PV arrays, controller [1–3], and inverter [4–6]. PV arrays, the core devices of PV power generation system, usually consist of solar cells in series and/or in parallel. There are two kinds of popular simulation models for solar cells, that is, the single-diode model and the double-diode model [7–10]. Because of having fewer parameters, the single-diode model is simpler than the double-diode model. This makes the single-diode model widely used. Some efforts have been made to improve the accuracy of the single-diode model. According to the number of parameters, there are mainly three kinds of models, that is, the four-parameter model, the five-parameter model, and the seven-parameter model.

The four-parameter model in [11] includes ideality factor, diode reverse saturation current, light-generated current,

series resistance. The four-parameter model has the fewest cell parameters, and then its expression is the simplest. However, the predicted accuracy of the four-parameter model is very limited. And the four-parameter model is validated only for monocrystalline silicon modules. On the basis of the four-parameter model, the five-parameter models are obtained by introducing shunt resistance [11–21]. The five-parameter models are more precise than the four-parameter model in [11]. Compromising the number of parameters and the approximate accuracy, the five-parameter models are the most commonly used. The five-parameter models proposed in [11, 19–21] utilize the reciprocals of the slopes at the open-circuit point and short-circuit point to calculate the cell parameters. The two slope values are not usually provided by the manufacturers' datasheet. And they can be obtained by using enough data pairs of experimental I - V characteristic under the certain condition. This makes it very complicated to obtain the slope values. Therefore, these five-parameter models actually have seven parameters rather than five parameters. The five-parameter model in [12] does not utilize the above-mentioned two slope parameters

FIGURE 1: Single diode model.

and can accurately predict the performance of monocrystalline and polycrystalline silicon modules. However, the five-parameter model in [12] exists large modeling errors for amorphous silicon solar cells. This may be due to the facts that series resistance in [12] is assumed to be a constant and the temperature dependence of series resistance is ignored. Here, temperature refers to the temperature of solar cells. The seven-parameter models proposed in [13, 22] add two additional parameters on the basis of the five-parameter model. For example, in [13], the two additional parameters are temperature coefficient for series resistance and the diode reverse saturation current radiation dependence, respectively. Temperature coefficient for series resistance describes the temperature dependence of series resistance. The value of series resistance changes exponentially with the cell temperature. The diode reverse saturation current radiation dependence describes the influence of irradiance on diode reverse saturation current, which is obtained by using the current and the voltage at maximum power point under the irradiance of 200 W/m^2 and the temperature of 25°C . The extraction process for this parameter is very complicated. Although the seven-parameter model in [13] provides a more accurate I - V characteristic model, it is not widely used because of the complexity of parameter extraction. Therefore, we propose a new six-parameter model.

In this paper, Chebyshev polynomials are developed to describe the relationship between series resistance and temperature. This makes a new parameter called temperature coefficient for series resistance introduced into the single-diode model. Therefore, the single-diode model has six cell parameters in this paper, that is, ideality factor, light-generated current, diode reverse saturation current, series resistance, shunt resistance, and temperature coefficient for series resistance. The new six-parameter model provides a simpler and more reasonable expression for series resistance and accurately predicts I - V characteristic for silicon solar cells. Five kinds of silicon solar cells with four different technology types, that is, monocrystalline silicon, polycrystalline silicon, thin film silicon and triple-junction amorphous silicon, are tested to validate the six-parameter model in this paper.

2. The Single-Diode Model of Solar Cells

A solar cell is essentially a very large area p-n junction diode. Under illumination, a single-diode model of a solar cell can

be described by an equivalent circuit, shown in Figure 1. The I - V equation of a single-diode model is shown as follows:

$$I = I_L - I_o \left[\exp \left(\frac{V + IR_s}{a} \right) - 1 \right] - \frac{V + IR_s}{R_{sh}}, \quad (1)$$

where I_L (A) is light-generated current, I_o (A) is diode reverse saturation current, R_{sh} (Ω) is shunt resistance, and R_s (Ω) is series resistance, a is ideality factor which is defined as follows:

$$a = N_s n_i V_t, \quad (2)$$

where N_s is the number of solar cells in series in a PV module, n_i is diode ideality factor, and V_t is thermal voltage which is defined as follows:

$$V_t = \frac{kT}{q}, \quad (3)$$

where k is Boltzmann's constant ($1.38066E - 23 \text{ J/K}$), T (K) is cell temperature, and q is electron charge ($1.60218E - 19 \text{ C}$).

3. Chebyshev Polynomials Are Used to Model Series Resistance in the Six-Parameter Model

Temperature dependence of series resistance has been quantitatively analyzed and various expressions are obtained [18, 21, 23–27]. Most of the expressions are nonlinear and implicit formulations about temperature. In [18, 21, 24], the relationship between series resistance and temperature is expressed as the exponential function of temperature. In [25], series resistance is literally expressed as the linear function of temperature. In fact, the expression also contains other electrical parameters at the same time, such as the short-circuit current and the voltage and the current at maximum power point, which are closely related to temperature. Therefore, the expression of series resistance in [25] is actually complex nonlinear. In addition, [13, 27] have given explicit representation of series resistance about temperature. Reference [27] has proved that series resistance of a solar cell belongs to positive temperature coefficient type. However, some of the series resistance of solar cells monotonically increases with the increase of temperature [23], but some of the series resistance of solar cells monotonically decreases with the increase of temperature [28, 29]. Therefore, for different kinds of solar

cells, the expression in [27] for temperature dependence of series resistance is limited. As is known, the expression of I - V characteristic is a transcendental equation and series resistance is an exponential function of temperature. When series resistance with the exponential function is substituted into I - V characteristic model, the processes of I - V characteristic modeling and parameter extraction will become complex and have computational burden. Therefore, in this paper, we follow the relationship between series resistance and temperature in [13] to develop a new expression of series resistance about temperature. Chebyshev polynomials are used to describe temperature dependence of series resistance. It is worth mentioning that the new expression of series resistance about temperature is an explicit form in this paper.

3.1. Chebyshev Polynomials Is Used to Model Series Resistance.

In this paper, we assume the range of temperature T is from 233 K to 353 K and the magnitude order of δ is 10^{-3} . Here, δ denotes temperature coefficient for series resistance. To satisfy the need of accuracy and simplification, we select cubic Chebyshev polynomials [30, 31] to establish the expression for temperature dependence of series resistance. The remainder of this cubic Chebyshev polynomials is shown as follows in this section.

Since the magnitude order of δ is 10^{-3} , we have

$$|\delta| \in [1 \times 10^{-3}, 9 \times 10^{-3}]. \quad (4)$$

Let

$$\Delta_T = \delta (T - T_{\text{ref}}), \quad (5)$$

where T_{ref} denotes the temperature of 25°C at Standard Reference Condition (the irradiance is 1000 W/m², the temperature is 298 K, and air mass is 1.5, SRC). Then, we have $\Delta_T \in [-0.585, 0.495]$. Since $T - T_{\text{ref}} \in [-65, 55]$, the following formula holds:

$$|M_n| = \max_{-65\delta \leq x \leq 55\delta} \{|e^{\Delta_T}|\} = e^{55\delta} \approx 1. \quad (6)$$

According to (6), we have

$$|R_3(x)| = \frac{(120\delta)^3 |M_n|}{2^7 \times 3!} = \frac{(120\delta)^3 e^{55\delta}}{2^7 \times 3!} \approx \frac{(120\delta)^3}{2^7 \times 3!}. \quad (7)$$

Substituting (4) into (7), we can obtain $|R_3(x)| \in [6.75 \times 10^{-8}, 4.43 \times 10^{-4}]$.

Now, we use cubic Chebyshev polynomials [30, 31] to describe the expression for temperature dependence of series resistance as follows:

$$R_s = R_{s,\text{ref}} (W_3 \Delta_T^3 - W_2 \Delta_T^2 + W_1 \Delta_T - W_0), \quad (8)$$

where R_s is the series resistance, $R_{s,\text{ref}}$ is the series resistance at SRC, $W_3 = \sum_{i=1}^4 A_i$, $W_2 = \sum_{i=1}^4 (A_i \sum_{j=1}^4 x_j)$, $W_1 = \sum_{j \neq i, k \neq j \neq i}^{n \neq k \neq j \neq i} A_i [(x_j + x_k)x_n + x_j x_k]$, $W_0 = \sum_{i=1}^4 A_i (\prod_{j=1}^{j \neq i} x_j)$, ($i, j, k, n = 1, 2, 3, 4$), and A_i is defined as follows:

$$A_i = \frac{e^{x_i}}{\prod_{j=2, j \neq i}^4 (x_i - x_j)}, \quad (9)$$

where x_i is defined as follows:

$$x_i = -5\delta + 60\delta \cdot \cos\left(\frac{2i-1}{8}\pi\right), \quad (i = 1, 2, 3, 4). \quad (10)$$

3.2. Six-Parameter Model. Manufacturers usually provide some data at SRC, such as short-circuit current I_{sc} , open-circuit voltage V_{oc} , current and voltage at maximum power point, I_{mp} , V_{mp} , and their temperature coefficient $\alpha_{I_{sc}}$, $\beta_{V_{oc}}$, $\alpha_{I_{mp}}$, and $\beta_{V_{mp}}$.

To obtain current corresponding to different voltage at various conditions, a , I_L , I_o , R_s , and R_{sh} must be known. Firstly, we calculate these parameters at SRC, that is, a_{ref} , $I_{L,\text{ref}}$, $I_{o,\text{ref}}$, $R_{s,\text{ref}}$, $R_{sh,\text{ref}}$. Here, the parameters with subscript "ref" denote the values at SRC. According to (8), we know that series resistance is related to the temperature coefficient for series resistance δ . Therefore, δ also needs to be calculated. Thus, we need six pieces of independent information to calculate the six parameters. In this paper, we make full use of the definitions of I_{sc} , V_{oc} , I_{mp} , V_{mp} , $\alpha_{I_{sc}}$, $\beta_{V_{oc}}$, $\alpha_{I_{mp}}$, $\beta_{V_{mp}}$, and manufacturers' data at SRC to obtain the above-mentioned six parameters.

According to the definition of short-circuit current at SRC, we have $I = I_{sc,\text{ref}}$ when $V = 0$. Equation (1) is written as follows:

$$I_{sc,\text{ref}} = I_{L,\text{ref}} - I_{o,\text{ref}} \left[e^{I_{sc,\text{ref}} R_{s,\text{ref}} / a_{\text{ref}}} - 1 \right] - \frac{I_{sc,\text{ref}} R_{s,\text{ref}}}{R_{sh,\text{ref}}}. \quad (11)$$

According to the definition of open-circuit voltage at SRC, we have $V = V_{oc,\text{ref}}$ when $I = 0$. Equation (1) is written as follows:

$$0 = I_{L,\text{ref}} - I_{o,\text{ref}} \left[e^{V_{oc,\text{ref}} / a_{\text{ref}}} - 1 \right] - \frac{V_{oc,\text{ref}}}{R_{sh,\text{ref}}}. \quad (12)$$

According to the definition of maximum-power point at SRC, we have $V = V_{mp,\text{ref}}$ when $I = I_{mp,\text{ref}}$. Equation (1) is written as follows:

$$I_{mp,\text{ref}} = I_{L,\text{ref}} - I_{o,\text{ref}} \left[\frac{V_{mp,\text{ref}} + I_{mp,\text{ref}} R_{s,\text{ref}}}{a_{\text{ref}}} - 1 \right] - \frac{V_{mp,\text{ref}} + I_{mp,\text{ref}} R_{s,\text{ref}}}{R_{sh,\text{ref}}}. \quad (13)$$

At maximum-power point, the derivation of power is zero, we can obtain

$$0 = I_{mp,\text{ref}} + V_{mp,\text{ref}} \cdot \frac{(-I_{o,\text{ref}}/a_{\text{ref}}) e^{(V_{mp,\text{ref}} + I_{mp,\text{ref}} R_{s,\text{ref}})/a_{\text{ref}}} - 1/R_{sh,\text{ref}}}{1 + (I_{o,\text{ref}} R_{s,\text{ref}}/a_{\text{ref}}) e^{(V_{mp,\text{ref}} + I_{mp,\text{ref}} R_{s,\text{ref}})/a_{\text{ref}}} + R_{s,\text{ref}}/R_{sh,\text{ref}}}. \quad (14)$$

For now, we have four pieces of independent information with six unknown parameters. We need to add two pieces of information. $\beta_{V_{oc}}$ and $\gamma_{P_{mp}}$ (the temperature coefficient for maximum power) are used as the added information. It is worth mentioning that the units of $\beta_{V_{oc}}$ and $\gamma_{P_{mp}}$ are V/K and 1/K, respectively.

TABLE 1: Values of different types of solar cells provided by NIST and SNL.

PV module specifications	SP-75	SM-55	MSX-64	APX-90	US-21
Material	Monocrystalline	Monocrystalline	Polycrystalline	Thin film	3-jun amorphous
$I_{sc,ref}$ (A)	4.37	3.45	4.25	5.11	1.59
$V_{oc,ref}$ (V)	42.93	21.7	41.5	29.61	23.8
$I_{mp,ref}$ (A)	3.96	3.15	3.82	4.49	1.27
$V_{mp,ref}$ (V)	33.68	17.4	32.94	23.17	16.5
$\alpha_{I_{sc}}$ (A/K)	0.00175	0.0019	0.00238	0.00468	0.00135
$\beta_{V_{oc}}$ (V/K)	-0.15237	-0.087	-0.15280	-0.12995	-0.098
$\alpha_{I_{mp}}$ (A/K)	-0.00154	-0.000254	0.00018	0.00160	0.0015
$\beta_{V_{mp}}$ (V/K)	-0.15358	-0.089	-0.15912	-0.13039	-0.052
$E_{g,ref}$ (eV)	1.12	1.12	1.14	1.12	1.6
N_s	72	36	72	56	11

The temperature coefficient for open-circuit voltage ($\beta_{V_{oc}}$) can be defined as follows:

$$\beta_{V_{oc}} \approx \frac{V_{oc,ref} - V_{oc,T}}{T_{ref} - T}. \quad (15)$$

The temperature coefficient for maximum-power point ($\gamma_{P_{mp}}$) can be defined as follows:

$$\gamma_{P_{mp}} \approx \frac{P_{mp,ref} - P_{mp,T}}{P_{mp,ref} (T_{ref} - T)}. \quad (16)$$

In (15) and (16), temperature T can change from 288 K to 308 K. $\beta_{V_{oc}}$ can be given by manufacturers, $\gamma_{P_{mp}}$ is also given by manufacturers or calculated by using the values of $\alpha_{I_{mp}}$ and $\beta_{V_{mp}}$. But $V_{oc,T}$ and $P_{mp,T}$ are unknown in (15) and (16). To compute $V_{oc,T}$ and $P_{mp,T}$, we have to obtain the values of a , I_L , I_o , and R_{sh} at any temperature and irradiance conditions. Therefore, (17) show the expressions of a , I_L , I_o , and R_{sh} at any temperature and irradiance conditions.

At any conditions, a , I_o , I_L , and R_{sh} can be expressed as follows [7]:

$$\begin{aligned}
 a &= a_{ref} \frac{T}{T_{ref}}, \\
 I_o &= I_{o,ref} \left(\frac{T}{T_{ref}} \right)^3 \\
 &\quad \cdot \exp \left[\frac{1.60207 \times 10^{-19}}{k} \left(\frac{E_{g,ref}}{T_{ref}} - \frac{E_{g,T}}{T} \right) \right], \\
 I_L &= \frac{G}{G_{ref}} [I_{L,ref} + \alpha_{I_{sc}} (T - T_{ref})], \\
 R_{sh} &= R_{sh,ref} \frac{G_{ref}}{G},
 \end{aligned} \quad (17)$$

where k is Boltzmann's constant and E_g is the material band gap. $E_{g,ref}$ is the material band gap at SRC shown in Table 1, which depends on different technology types of solar cells. In addition, the value of E_g at any conditions is shown as follows:

$$E_g = E_{g,ref} [1 - 0.0002677 (T - T_{ref})]. \quad (18)$$

Form (5) and (8), the expression of series resistance can be rewritten as follows:

$$\begin{aligned}
 R_s &= R_{s,ref} \{ W_3 [\delta (T - T_{ref})]^3 - W_2 [\delta (T - T_{ref})]^2 \\
 &\quad + W_1 [\delta (T - T_{ref})] - W_0 \}.
 \end{aligned} \quad (19)$$

According to the data provided by manufacturers at SRC, we can obtain a_{ref} , $I_{L,ref}$, $I_{o,ref}$, $R_{sh,ref}$, $R_{s,ref}$, and δ by using (11)–(16). Then, (17)–(19) are used to determine a , I_L , I_o , R_{sh} , and R_s of I - V characteristics for different solar cells at any temperature and irradiance conditions. Once the five parameters are obtained at certain temperature and irradiance condition, we can easily obtain the current corresponding to different voltage according to (1). Therefore, (11)–(19) are called the six-parameter model. V_{mp} , I_{mp} , V_{oc} , and I_{sc} at any conditions are significant and have practical application. However, (11)–(14) are special representation of (1) at SRC for three important points: (V_{mp}, I_{mp}) , $(0, I_{sc})$, and $(V_{oc}, 0)$. That is to say, (11)–(14) can directly obtain V_{mp} , I_{mp} , V_{oc} , and I_{sc} at any conditions when subscript “ref” is deleted.

4. Model Validation and Discussion

In this section, we select five kinds of PV modules with four technology types to validate the six-parameter model proposed in this paper. They are Siemens SP75 and Siemens SM-55 (monocrystalline silicon), Solarex MSX-64 (polycrystalline silicon), Anstropower APX-90 (thin film silicon), and USSC US-21 (tripe-junction amorphous silicon), respectively. These five PV modules are composed of different number of solar cells in series, shown in Table 1. The reference values of SP75, MSX-64, APX-90 are provided by NIST (Nation Institute of Standards and Technology). The reference values of US21, SM-55 are provided by SNL (Sandia National Laboratory), shown in Table 1. To analyse the accuracy of the six-parameter model proposed in this paper, the absolute

TABLE 2: The parameter values of different models at SRC.

	Parameters	SP-75	SM-55	MSX-64	APX-90	US-21
Six-parameter model	a_{ref}	1.76895	0.95447	1.71640	1.37500	0.76136
	$I_{L,\text{ref}}$ (A)	4.39492	3.46140	4.27627	5.15344	1.69385
	$I_{o,\text{ref}}$ (A)	$1.200098E - 10$	$4.432196E - 10$	$1.262302E - 10$	$2.092911E - 9$	$3.467982E - 14$
	$R_{s,\text{ref}}$ (Ω)	1.03711	0.48750	0.91975	0.56864	4.01381
	$R_{sh,\text{ref}}$ (Ω)	181.86447	147.52754	148.79240	66.89309	61.45488
	δ	0.0038193	0.0042272	0.0039503	0.0036350	-0.0090814
Five-parameter model in [14]	a_{ref}	1.76895	0.95447	1.71640	1.37500	0.76135
	$I_{L,\text{ref}}$ (A)	4.39492	3.46140	4.27627	5.15344	1.69385
	$I_{o,\text{ref}}$ (A)	$1.200098E - 10$	$4.432190E - 10$	$1.262314E - 10$	$2.092908E - 9$	$3.467660E - 14$
	$R_{s,\text{ref}}$ (Ω)	1.03711	0.48750	0.91974	0.56864	4.01382
	$R_{sh,\text{ref}}$ (Ω)	181.86449	147.52752	148.79407	66.89302	61.45483
Seven-parameter model in [13]	a_{ref}	1.76895	0.95446	1.71640	1.37500	0.76136
	$I_{L,\text{ref}}$ (A)	4.39492	3.46140	4.27627	5.15344	1.69385
	$I_{o,\text{ref}}$ (A)	$1.200098E - 10$	$4.431875E - 10$	$1.262312E - 10$	$2.092909E - 9$	$3.467982E - 14$
	$R_{s,\text{ref}}$ (Ω)	1.03711	0.48750	0.91974	0.56864	4.01381
	$R_{sh,\text{ref}}$ (Ω)	181.86445	147.52677	148.79405	66.89309	61.45484
	δ	0.0038200	0.0042278	0.0039510	0.0036360	-0.0090720
	m	$5.248E - 3$	$-8.527E - 5$	$5.040E - 6$	$-1.602E - 6$	$1.665E - 4$

error (AE), the relative error (RE), and the root mean squared error (RMSE) are used which are defined as follows:

$$\begin{aligned}
 \text{AE} &= |I_{\text{exi}} - I_i|, \\
 \text{RE} &= \frac{|I_{\text{exi}} - I_i|}{I_{\text{exi}}} \times 100\%, \\
 \text{RMSE} &= \sqrt{\frac{1}{n} \sum_{i=1}^n (I_i - I_{\text{exi}})^2},
 \end{aligned} \tag{20}$$

where n is the number of experimental values. I_i is the current of tested models. I_{exi} is the current of experimental values.

To validate the effectiveness of the six-parameter model proposed in this paper, we compare the new six-parameter model with the five-parameter model in [14] and the seven-parameter model in [13]. We compute the parameter values by Engineering Equation Solver (EES) software. The parameter values of the six-parameter model in this paper, the five-parameter model in [14], and the seven-parameter model [13] at SRC are shown in Table 2.

We extract the experimental data in [12] by the software called Get Data. GetData software can make every data point amplified 15 times the original size. Thus, we can control that the abscissa of data point (voltage) has an absolute error of less than 0.04 V and the ordinate of data point (current) has an absolute error of less than 0.006 A. The experimental data obtained by GetData software has enough number of significant digit to achieve enough precision. Therefore, the experimental data obtained by GetData software can be taken as real experimental data provided in [14]. Then, we use the software Matlab to plot the figures of I - V characteristic curves and the absolute errors between compared models

FIGURE 2: I - V characteristic for SP75.

and experimental data. The I - V characteristic curves of SP-75 module, MSX-64 module, and APX-90 module at different temperature and irradiance conditions are shown in Figures 2–4, Figures 5–7, and Figures 8–10, respectively. The corresponding absolute errors of I - V characteristic curves for these three modules are shown in Figures 11–19. AEs of the six-parameter model in this paper is very close to the seven-parameter model in [13] at every point. For higher irradiance, the absolute errors of the six-parameter model in this paper are less than that of the five-parameter model in [14] within

TABLE 3: RMSEs of different models for SP75, MSX-64, and APX-90 module.

Module	RMSE		
	Five-parameter model in [14]	Seven-parameter model in [13]	Six-parameter model
SP75	0.06293749	0.056315982	0.056864662
MSX-64	0.130276895	0.125862026	0.125859806
APX-90	0.154802955	0.147627147	0.147625261

FIGURE 3: I - V characteristic for SP75. FIGURE 5: I - V characteristic for MSX-64. FIGURE 4: I - V characteristic for SP75. FIGURE 6: I - V characteristic for MSX-64.

the range of maximum-power point to open-circuit voltage point, shown in Figures 11-12, 14-15, and 17-18. For lower irradiance, AEs of the six-parameter model in this paper and the five-parameter model in [14] are almost the same. In addition, We use the maximum absolute error and RMSE to further analyze the effectiveness of the six-parameter model

in this paper. RMSE shows overall error. We select 97 data sets for SP-75 module, 101 data sets for MSX-64 module and 108 data sets for APX-90 module to compute RMSEs, see Table 3. From Table 3, we can find that that RMSEs of the six-parameter model in this paper are almost the same as that of the seven-parameter model in [13] and are far less than that

FIGURE 7: I - V characteristic for MSX-64. FIGURE 9: I - V characteristic for APX-90. FIGURE 8: I - V characteristic for APX-90. FIGURE 10: I - V characteristic for APX-90.

of the five-parameter model in [14]. Therefore, in estimating of I - V characteristic, the six-parameter model in this paper is obviously superior to the five-parameter model in [14], but slightly less than seven-parameter model in [13].

I - V characteristic of PV modules are widely used to estimate the maximum power point. Next, we take the data of SM-55 module and US-21 module given in [32] as experimental data to validate the accuracy of the new six-parameter model in estimating the maximum power point. Tables 4 and 5 show P_{mp} and V_{mp} for the two PV modules at the temperature of 50°C and the irradiance of 1000 W/m^2 to 100 W/m^2 . For SM-55 module and US-21 module, the relative errors of the six-parameter model in this paper are almost the

same as that of the seven-parameter model in [13]. For SM-55 module, the relative errors of the six-parameter model in this paper are far less than that of five-parameter model in [14] except for irradiance of 1000 W/m^2 and 100 W/m^2 . For US-21 module, the relative errors of the six-parameter model in this paper are far less than that of five-parameter model in [14] except for irradiance of 1000 W/m^2 . This shows that the six-parameter model in this paper can give a better estimation of maximum power point at lower irradiance for tripe-junction amorphous silicon module.

Therefore, the six-parameter model in this paper, used to estimate I - V characteristic and the maximum power point, is obviously superior to the five-parameter model in

TABLE 4: Voltage, power, and RE at maximum power point of different models for SM-55 at $T = 50^\circ\text{C}$.

G (W/m^2)	Experimental data	Five-parameter model	Seven-parameter model	Six-parameter model	RE(5-para)%	RE(7-para)%	RE(6-para)%
1000	$V_{\text{mp}} = 15.175$	$V_{\text{mp}} = 15.20995$	$V_{\text{mp}} = 15.06347$	$V_{\text{mp}} = 15.06359$	0.230 at V_{mp}	0.735 at V_{mp}	0.734 at V_{mp}
	$P_{\text{mp}} = 47.706$	$P_{\text{mp}} = 48.04719$	$P_{\text{mp}} = 47.56305$	$P_{\text{mp}} = 47.56339$	0.715 at P_{mp}	0.300 at P_{mp}	0.299 at P_{mp}
900	$V_{\text{mp}} = 15.162$	$V_{\text{mp}} = 15.24041$	$V_{\text{mp}} = 15.10802$	$V_{\text{mp}} = 15.10812$	0.517 at V_{mp}	0.356 at V_{mp}	0.355 at V_{mp}
	$P_{\text{mp}} = 42.984$	$P_{\text{mp}} = 43.37989$	$P_{\text{mp}} = 42.99262$	$P_{\text{mp}} = 42.99290$	0.921 at P_{mp}	0.020 at P_{mp}	0.021 at P_{mp}
800	$V_{\text{mp}} = 15.140$	$V_{\text{mp}} = 15.25940$	$V_{\text{mp}} = 15.14129$	$V_{\text{mp}} = 15.14138$	0.789 at V_{mp}	0.009 at V_{mp}	0.009 at V_{mp}
	$P_{\text{mp}} = 38.228$	$P_{\text{mp}} = 38.65117$	$P_{\text{mp}} = 38.35024$	$P_{\text{mp}} = 38.35045$	1.107 at P_{mp}	0.320 at P_{mp}	0.320 at P_{mp}
700	$V_{\text{mp}} = 15.105$	$V_{\text{mp}} = 15.26368$	$V_{\text{mp}} = 15.16002$	$V_{\text{mp}} = 15.16010$	1.051 at V_{mp}	0.364 at V_{mp}	0.365 at V_{mp}
	$P_{\text{mp}} = 33.440$	$P_{\text{mp}} = 33.86514$	$P_{\text{mp}} = 33.63989$	$P_{\text{mp}} = 33.64005$	1.271 at P_{mp}	0.598 at P_{mp}	0.598 at P_{mp}
600	$V_{\text{mp}} = 15.053$	$V_{\text{mp}} = 15.24850$	$V_{\text{mp}} = 15.15945$	$V_{\text{mp}} = 15.15952$	1.299 at V_{mp}	0.707 at V_{mp}	0.708 at V_{mp}
	$P_{\text{mp}} = 28.621$	$P_{\text{mp}} = 29.02721$	$P_{\text{mp}} = 28.86691$	$P_{\text{mp}} = 28.86702$	1.419 at P_{mp}	0.859 at P_{mp}	0.860 at P_{mp}
500	$V_{\text{mp}} = 14.975$	$V_{\text{mp}} = 15.20650$	$V_{\text{mp}} = 15.13224$	$V_{\text{mp}} = 15.13228$	1.546 at V_{mp}	1.050 at V_{mp}	1.050 at V_{mp}
	$P_{\text{mp}} = 23.774$	$P_{\text{mp}} = 24.14476$	$P_{\text{mp}} = 24.03857$	$P_{\text{mp}} = 24.03863$	1.560 at P_{mp}	1.113 at P_{mp}	1.113 at P_{mp}
400	$V_{\text{mp}} = 14.854$	$V_{\text{mp}} = 15.12542$	$V_{\text{mp}} = 15.06608$	$V_{\text{mp}} = 15.06610$	1.827 at V_{mp}	1.428 at V_{mp}	1.428 at V_{mp}
	$P_{\text{mp}} = 18.903$	$P_{\text{mp}} = 19.22832$	$P_{\text{mp}} = 19.16532$	$P_{\text{mp}} = 19.16534$	1.721 at P_{mp}	1.388 at P_{mp}	1.388 at P_{mp}
300	$V_{\text{mp}} = 14.657$	$V_{\text{mp}} = 14.98228$	$V_{\text{mp}} = 14.93800$	$V_{\text{mp}} = 14.93799$	2.219 at V_{mp}	1.917 at V_{mp}	1.917 at V_{mp}
	$P_{\text{mp}} = 14.017$	$P_{\text{mp}} = 14.29398$	$P_{\text{mp}} = 14.26321$	$P_{\text{mp}} = 14.26319$	1.976 at P_{mp}	1.757 at P_{mp}	1.756 at P_{mp}
200	$V_{\text{mp}} = 14.301$	$V_{\text{mp}} = 14.72561$	$V_{\text{mp}} = 14.69652$	$V_{\text{mp}} = 14.69648$	2.969 at V_{mp}	2.766 at V_{mp}	2.765 at V_{mp}
	$P_{\text{mp}} = 9.136$	$P_{\text{mp}} = 9.36934$	$P_{\text{mp}} = 9.35976$	$P_{\text{mp}} = 9.35973$	2.554 at P_{mp}	2.449 at P_{mp}	2.449 at P_{mp}
100	$V_{\text{mp}} = 13.482$	$V_{\text{mp}} = 14.19163$	$V_{\text{mp}} = 14.17782$	$V_{\text{mp}} = 14.17771$	5.264 at V_{mp}	5.161 at V_{mp}	5.160 at V_{mp}
	$P_{\text{mp}} = 4.315$	$P_{\text{mp}} = 4.51282$	$P_{\text{mp}} = 4.51341$	$P_{\text{mp}} = 4.51337$	4.585 at P_{mp}	4.598 at P_{mp}	4.597 at P_{mp}

TABLE 5: Voltage, power, and RE at maximum power point of different models for US-21 at $T = 50^\circ\text{C}$.

G (W/m^2)	Experimental data	Five-parameter model	Seven-parameter model	Six-parameter model	RE(5-para)%	RE(7-para)%	RE(6-para)%
1000	$V_{\text{mp}} = 15.2$	$V_{\text{mp}} = 14.01544$	$V_{\text{mp}} = 14.84689$	$V_{\text{mp}} = 14.84671$	7.793 at V_{mp}	2.323 at V_{mp}	2.324 at V_{mp}
	$P_{\text{mp}} = 19.883$	$P_{\text{mp}} = 18.29121$	$P_{\text{mp}} = 19.70321$	$P_{\text{mp}} = 19.70291$	8.006 at P_{mp}	0.904 at P_{mp}	0.906 at P_{mp}
900	$V_{\text{mp}} = 15.335$	$V_{\text{mp}} = 14.33650$	$V_{\text{mp}} = 15.10970$	$V_{\text{mp}} = 15.10955$	6.511 at V_{mp}	1.469 at V_{mp}	1.470 at V_{mp}
	$P_{\text{mp}} = 18.227$	$P_{\text{mp}} = 16.99216$	$P_{\text{mp}} = 18.15277$	$P_{\text{mp}} = 18.15254$	6.775 at P_{mp}	0.407 at P_{mp}	0.409 at P_{mp}
800	$V_{\text{mp}} = 15.477$	$V_{\text{mp}} = 14.66351$	$V_{\text{mp}} = 15.37056$	$V_{\text{mp}} = 15.37044$	5.256 at V_{mp}	0.688 at V_{mp}	0.689 at V_{mp}
	$P_{\text{mp}} = 16.507$	$P_{\text{mp}} = 15.57550$	$P_{\text{mp}} = 16.50499$	$P_{\text{mp}} = 16.50483$	5.643 at P_{mp}	0.012 at P_{mp}	0.013 at P_{mp}
700	$V_{\text{mp}} = 15.626$	$V_{\text{mp}} = 14.99208$	$V_{\text{mp}} = 15.62622$	$V_{\text{mp}} = 15.62613$	4.057 at V_{mp}	0.001 at V_{mp}	0.001 at V_{mp}
	$P_{\text{mp}} = 14.720$	$P_{\text{mp}} = 14.03842$	$P_{\text{mp}} = 14.75907$	$P_{\text{mp}} = 14.75896$	4.630 at P_{mp}	0.265 at P_{mp}	0.265 at P_{mp}
600	$V_{\text{mp}} = 15.782$	$V_{\text{mp}} = 15.31685$	$V_{\text{mp}} = 15.87229$	$V_{\text{mp}} = 15.87224$	2.947 at V_{mp}	0.572 at V_{mp}	0.572 at V_{mp}
	$P_{\text{mp}} = 12.862$	$P_{\text{mp}} = 12.37893$	$P_{\text{mp}} = 12.91469$	$P_{\text{mp}} = 12.91463$	3.756 at P_{mp}	0.410 at P_{mp}	0.409 at P_{mp}
500	$V_{\text{mp}} = 15.945$	$V_{\text{mp}} = 15.63061$	$V_{\text{mp}} = 16.10244$	$V_{\text{mp}} = 16.10243$	1.972 at V_{mp}	0.987 at V_{mp}	0.987 at V_{mp}
	$P_{\text{mp}} = 10.930$	$P_{\text{mp}} = 10.59594$	$P_{\text{mp}} = 10.97225$	$P_{\text{mp}} = 10.97222$	3.056 at P_{mp}	0.387 at P_{mp}	0.386 at P_{mp}
400	$V_{\text{mp}} = 16.114$	$V_{\text{mp}} = 15.9248$	$V_{\text{mp}} = 16.30647$	$V_{\text{mp}} = 16.30651$	1.189 at V_{mp}	1.194 at V_{mp}	1.195 at V_{mp}
	$P_{\text{mp}} = 8.917$	$P_{\text{mp}} = 8.68954$	$P_{\text{mp}} = 8.93308$	$P_{\text{mp}} = 8.93309$	2.551 at P_{mp}	0.180 at P_{mp}	0.180 at P_{mp}
300	$V_{\text{mp}} = 16.279$	$V_{\text{mp}} = 16.17323$	$V_{\text{mp}} = 16.46573$	$V_{\text{mp}} = 16.46582$	0.650 at V_{mp}	1.147 at V_{mp}	1.148 at V_{mp}
	$P_{\text{mp}} = 6.818$	$P_{\text{mp}} = 6.66162$	$P_{\text{mp}} = 6.80017$	$P_{\text{mp}} = 6.80020$	2.294 at P_{mp}	0.262 at P_{mp}	0.261 at P_{mp}
200	$V_{\text{mp}} = 16.414$	$V_{\text{mp}} = 16.34112$	$V_{\text{mp}} = 16.53891$	$V_{\text{mp}} = 16.53908$	0.444 at V_{mp}	0.761 at V_{mp}	0.762 at V_{mp}
	$P_{\text{mp}} = 4.624$	$P_{\text{mp}} = 4.51744$	$P_{\text{mp}} = 4.57978$	$P_{\text{mp}} = 4.57981$	2.305 at P_{mp}	0.956 at P_{mp}	0.956 at P_{mp}
100	$V_{\text{mp}} = 16.377$	$V_{\text{mp}} = 16.29485$	$V_{\text{mp}} = 16.39503$	$V_{\text{mp}} = 16.39531$	0.502 at V_{mp}	0.110 at V_{mp}	0.112 at V_{mp}
	$P_{\text{mp}} = 2.327$	$P_{\text{mp}} = 2.27106$	$P_{\text{mp}} = 2.28688$	$P_{\text{mp}} = 2.28691$	2.404 at P_{mp}	1.724 at P_{mp}	1.723 at P_{mp}

FIGURE 11: Absolute error for SP75.

FIGURE 13: Absolute error for SP75.

FIGURE 12: Absolute error for SP75.

FIGURE 14: Absolute error for MSX-64.

[14] under most weather conditions, and is close to seven-parameter model in [13]. In fact, the six-parameter model in this paper makes a good compromise between the five-parameter model in [14] and the seven-parameter model in [13] at accuracy and computational complexity. Therefore, the six-parameter model is an I - V model with moderate computational complexity and high precision.

5. Conclusions

We propose a new six-parameter model for solar cells in this paper. At calculation accuracy, the six-parameter model is better than the five-parameter model. The prediction of

I - V characteristic of six-parameter model is very close to the seven-parameter model, but less than seven-parameter model. However, at the computational complexity, the six-parameter model is superior to the seven-parameter model. Experiment results show that the new six-parameter model can provide a good prediction for silicon solar cells with different technology types, that is, monocrystalline silicon, polycrystalline silicon, thin film silicon, and tripe-junction amorphous silicon. That is to say, the model in this paper is more universal. It is worth mentioning that the relationship between the series resistance and temperature is described by using Chebyshev polynomials in this paper. Therefore, the six-parameter model proposed in this paper is an I - V model with moderate computational complexity and high precision.

FIGURE 15: Absolute error for MSX-64.

FIGURE 17: Absolute error for APX-90.

FIGURE 16: Absolute error for MSX-64.

FIGURE 18: Absolute error for APX-90.

Nomenclature

a : Ideality factor parameter which is defined as $a \equiv N_s n_i V_t$
 a_{ref} : Ideality factor parameter at SRC
 E_g : Energy bandgap (eV)
 $E_{g,\text{ref}}$: Energy bandgap at SRC (eV)
 G : Solar irradiance (W/m^2)
 G_{ref} : Solar irradiance at SRC (W/m^2)
 I : Current (A)
 I_L : Light-generated current (A)
 $I_{L,\text{ref}}$: Light-generated current at SRC (A)
 I_{mp} : The current at maximum power point (A)

$I_{\text{mp,ref}}$: The current at maximum power point at SRC (A)
 I_o : Diode reverse saturation current (A)
 $I_{o,\text{ref}}$: Diode reverse saturation current at SRC (A)
 I_{sc} : Short-circuit current (A)
 $I_{\text{sc,ref}}$: Short-circuit current at SRC (A)
 k : Boltzmann's constant ($1.38066 \times 10^{-23} \text{ J/K}$)
 n_i : Diode ideality factor
 N_s : Number of solar cells in series in one module
 P_{mp} : Power at maximum power point (W)
 $P_{\text{mp,ref}}$: Power at maximum power point at SRC (W)
 q : Electron charge ($1.60218 \times 10^{-19} \text{ C}$)

FIGURE 19: Absolute error for APX-90.

δ :	Temperature coefficient for series resistance
R_s :	Series resistance (Ω)
$R_{s,\text{ref}}$:	Series resistance at SRC (Ω)
R_{sh} :	Shunt resistance (Ω)
$R_{sh,\text{ref}}$:	Shunt resistance at SRC (Ω)
T :	Cell temperature (K)
T_{ref} :	Cell temperature at SRC (K)
V :	Voltage (V)
V_{mp} :	The voltage at maximum power point (V)
$V_{\text{mp,ref}}$:	The voltage at maximum power point at SRC (V)
V_{oc} :	Open-circuit voltage (V)
$V_{\text{oc,ref}}$:	Open-circuit voltage at SRC (V)
V_t :	Thermal voltage (V)
$\alpha_{I_{\text{mp}}}$:	Temperature coefficient for maximum power current (A/K)
$\alpha_{I_{\text{sc}}}$:	Temperature coefficient for short-circuit current (A/K)
$\beta_{V_{\text{mp}}}$:	Temperature coefficient for maximum power voltage (V/K)
$\beta_{V_{\text{oc}}}$:	Temperature coefficient for open-circuit voltage (V/K)
$\gamma_{P_{\text{mp}}}$:	Temperature coefficient for maximum power (1/K)
ΔT :	Temperature changing factor.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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