Research Article

Prediction Method of Bottom Water Coning Profile and Water Breakthrough Time in Bottom Water Reservoir without Barrier

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During the exploitation of bottom water oil reservoir, bottom water coning influences the breakthrough of bottom water significantly. Because water cut rises quickly after the breakthrough of bottom water, measures should be taken before the breakthrough to postpone production period without water, thus improving oil recovery. So accurate prediction of water coning profile and breakthrough time is very essential. Through mathematical derivation, this paper proposed a prediction method of bottom water coning profile and bottom water breakthrough time in bottom water reservoir without barrier. Based on theory of fluids flow in porous media, this paper assumes that the flow models are plane radial flow in opened intervals and spherical radial flow in unopened intervals. Further, factors of fluid viscosity, irreducible water saturation, residual oil saturation, and oil-water contact (OWC) movement are also taken into account. Compared with other prediction equations, this method achieves more precise bottom water breakthrough time, and the relative deviation is only 1.14 percent.

1. Introduction

Sandstone reservoir with bottom water is a common kind of oil reservoir. During the recovery of this kind of reservoir, bottom water provides energy to drive oil. However, the breakthrough of bottom water, which is influenced by bottom water coning, will lead to the quick increase of water cut, thus decreasing recovery rate. Therefore, it is essential to obtain bottom water coning profile and control the breakthrough time.

For bottom water reservoir without barrier, a number of researches have explored the recovery theory and the breakthrough of bottom water. Zhu [1] developed his prediction method using a conic bottom water body with straight generatrix; however the generatrix should in fact be curved. Tang [2] concluded the prediction equations for breakthrough time and geometrical function of water cone height, postulating that there are plane radial flow and spherical flow but disregarding the impact of residual oil saturation, irreducible water saturation, and oil/water viscosity. Providing curved generatrix of the bottom water cone, Li [3] and Xiong et al. [4] presented their prediction equations of breakthrough time, respectively. Moreover, Li

believes bottom water right below the well hole goes up vertically and reaches well bottom first. However, neither of them took account of the OWC movement and nor do they consider the bottom water coning profile.

On condition that the shot density of the oil-bearing zone in bottom water reservoir is relatively low or open hole completion is performed, this paper proposed prediction method of the bottom water profile and the breakthrough time. In this paper, two flow models, that is, plane radial flow and spherical radial flow, are applied, so these models yield more accurate prediction results than the previous simple model [5–7]. Meanwhile, oil/water viscosity, initial irreducible water saturation, residual oil saturation, and the OWC movement which are often ignored are also considered. What is more, the water coning profile is considered and presented, which will give visual assistance for the coning process.

2. Model and Equations

During the development of bottom water reservoir, formation pressure will shape like a funnel, causing bottom water

FIGURE 1: Bottom water coning body.

to rise like a cone and OWC to rise. So bottom water breakthrough is the result of bottom water coning and OWC increase. Moreover, bottom water begins to break through when bottom water cones to the highest point, namely, the well bottom.

Figure 1 describes the condition where oil-bearing zone is partly opened and completion intervals are fully perforated. To simplify the mathematical analysis, assumptions are put forward as follows:

- (1) Reservoir is homogenous.
- (2) Effect of oil-water transition zone is ignored.
- (3) Capillary force and gravity are ignored.
- (4) Flow is governed by Darcy's law.
- (5) The opened reservoir sections are planar radial flow; the unopened reservoir sections are spherical radial flow.

According to the assumptions above, prediction method is developed as follows.

If the impact of oil-water transition zone is not considered, the vertical seepage velocity of oil and water is [8]

$$\begin{aligned} v_o &= -\frac{KK_{owi}}{\mu_o} \frac{dP_o}{dz} \\ v_w &= -\frac{KK_{wor}}{\mu_w} \frac{dP_w}{dz}, \end{aligned} \quad (1)$$

where z is the height covered by water from OWC, P_o is the vertical pressure of oil phase, K is the average permeability, K_{owi} is the oil relative permeability at initial irreducible water saturation, μ_w is the water viscosity, P_w is the vertical pressure of water phase, and K_{wor} is the water relative permeability at residual oil saturation.

Oil and water share the same pressure gradient on the oil-water level [4] due to the ignorance of capillary force, which generates the following equation:

$$\frac{v_o}{v_w} = \frac{-\alpha KK_{owi}/\mu_o}{-\alpha KK_{wor}/\mu_w}. \quad (2)$$

So the following equation can be obtained by (2):

$$v_w = \frac{\mu_o K_{wor}}{\mu_w K_{owi}} v_o. \quad (3)$$

The relationship between the real and the upper seepage velocity yields the real seepage velocity:

$$\begin{aligned} v_o &= \frac{v_o}{\phi}, \\ v_w &= \frac{v_w}{\phi}, \end{aligned} \quad (4)$$

where ϕ is the porosity.

Concerning the influence of initial irreducible water saturation S_{wi} and the residual oil saturation S_{or} after water flooding, (4) can be corrected as

$$v_o = \frac{v_o}{\phi(1 - S_{or} - S_{wi})}, \quad (5)$$

$$v_w = \frac{v_w}{\phi(1 - S_{or} - S_{wi})}, \quad (6)$$

where S_{or} is the residual oil saturation and S_{wi} is the initial irreducible water saturation.

From (5), (3) can be

$$v_w = \frac{\mu_o K_{wor}}{\mu_w K_{owi}} \frac{v_o}{\phi(1 - S_{or} - S_{wi})}. \quad (7)$$

In the perforated interval, oil recovery can be determined from the plane radial flow as [9]

$$Q_1 = \frac{2\pi KK_{owi} h_p \Delta P}{\mu_o \ln(r_e/r_w)}, \quad (8)$$

where h_p is the thickness of opened oil-bearing zone, r_e is the drainage radius, r_w is the well radius, and ΔP is the pressure drop.

Under the perforated interval, oil recovery can be calculated from the spherical radial flow as [9]

$$Q_2 = \frac{2\pi KK_{owi} r_w \Delta P}{\mu_o}. \quad (9)$$

If (8) is divided by (9), a new coefficient can be defined as follows:

$$C = \frac{Q_1}{Q_2} = \frac{h_p}{\alpha r_w \ln(r_e/r_w)}. \quad (10)$$

Total oil output can be calculated as

$$Q = Q_1 + Q_2. \quad (11)$$

Oil recovery from spherical radial flow is

$$Q_2 = \frac{Q}{1 + C}. \quad (12)$$

Oil seepage velocity can be derived from spherical radial flow:

$$v_o = \frac{Q_2}{2\pi R^2}. \quad (13)$$

So the vertical seepage velocity of oil is

$$\begin{aligned} v_o &= \frac{Q_2}{2\pi R^2} \sin \theta, \\ \sin \theta &= \frac{h-z}{R}, \\ R &= \sqrt{r^2 + (h-z)^2}, \end{aligned} \quad (14)$$

where h is the height from OWC to well bottom, R is the radius of the flow sphere, and r is the radius of the circle formed from tangent OWC plane of the flow sphere.

Equations (7), (12), and (14) result in the following equation:

$$v_w = \frac{QM_{ow}(h-z)}{2\pi\phi(1-S_{or}-S_{wi})(1+C)(r^2+(h-z)^2)^{3/2}}, \quad (15)$$

$$M_{ow} = \frac{\mu_o K_{wor}}{\mu_w K_{owi}}, \quad (16)$$

where M_{ow} is the oil-water mobility ratio.

Oil-water contact rises along with the production rate of oil [10, 11]; the corresponding uplift amplitude is

$$\Delta h = \frac{Qt}{\pi r_e^2 \phi (1-S_{or}-S_{wi})}. \quad (17)$$

Distance covered by water of OWC is

$$dz = (v_w + \beta) dt, \quad (18)$$

$$\beta = \frac{Q}{\pi r_e^2 \phi (1-S_{or}-S_{wi})}. \quad (19)$$

Equations (15), (17), and (18) yield

$$\begin{aligned} dz &= \left[\frac{QM_{ow}(h-z)}{2\pi\phi(1-S_{or}-S_{wi})(1+C)(r^2+(h-z)^2)^{3/2}} \right. \\ &\quad \left. + \beta \right] dt. \end{aligned} \quad (20)$$

Equation (20) is the geometrical equation of OWC water which takes well axis as the origin of radius r . Given certain value for r , coning height of the OWC water at different times can be concluded.

When radius $r = 0$, that is, at the well bottom axis, coning height is exactly h , so (20) can be simplified as

$$dz = \left[\frac{QM_{ow}}{2\pi\phi(1-S_{or}-S_{wi})(1+C)(h-z)^2} + \beta \right] dt. \quad (21)$$

TABLE 1: Breakthrough time calculated from different prediction method.

Prediction method	Breakthrough time, d
Zhu [1]	214.77
Tang [2]	53.70
Xiong [4]	57.75
Li [3]	3.81
This paper	3.54

Let $H = QM_{ow}/2\pi\phi(1-S_{or}-S_{wi})(1+C)$; taking (19) into (21) results in

$$dt = \left(\frac{(h-z)^2}{H + \beta(h-z)^2} \right) dz. \quad (22)$$

Expressing (22) and integrating in h from 0 to h yield the expression for bottom water breakthrough time:

$$t_{bt} = \int_0^h \left(\frac{(h-z)^2}{H + \beta(h-z)^2} \right) dz. \quad (23)$$

Applying Matlab, (23) results in its analytical expression as follows:

$$t_{bt} = \frac{h}{\beta} - \frac{\sqrt{H} \arctan(h\sqrt{\beta/H})}{\beta^{3/2}}. \quad (24)$$

Equation (24) is right expression for breakthrough time.

3. Case Study

The case is a well from a certain bottom water reservoir in Tarim oilfield; related parameters are as follows: h_o is 22 m, h_p is 5 m, r_e is 500 m, r_w is 0.1 m, K is 0.46 D, B_o is 1.7, Φ is 0.19, μ_w is 0.96 mPa·s, μ_o is 6.3 mPa·s, surface production rate Q_u is 250 m³/d, S_{wi} is 0.22, S_{or} is 0.19, K_{owi} is 0.807, K_{owr} is 0.19, and t_{bt} is 3.5 d. Table 1 compares the calculated results from different prediction methods for breakthrough time.

Bottom water coning profiles for well X determined by our method are described in Figures 2, 3, and 4.

Comparing the predicted breakthrough time with the real one, it is clear that Zhu's equations yield the largest deviation; Tang's equations report more precise result than Zhu's but cannot avoid substantial deviation; Xiong et al. present even more accurate outcome; however deviation still exists obviously because of the ignorance of OWC movement, especially when oil output is large. This paper calculates the most accurate result which puts forward smaller deviation than Li's method.

As is shown in Figures 3 and 4, for the bottom water coning profile versus distance to well axis and the corresponding three-dimensional profile for 3.48 days, coning rate becomes higher in the position near the well axis. In addition, there is a sharp rising period of bottom water before its breakthrough. So for the sake of postponing production period without water time, controlling measures should be taken during the period from 80 to 85 percent of the entire breakthrough time.

FIGURE 2: Height of bottom water coning versus production time.

FIGURE 3: Bottom water coning profile versus distance to well axis.

4. Discussion

Shengju postulates that bottom water moves along the straight generatrix of the cone body, which enlarges the swept volume of the coning bottom water, thus obtaining a longer breakthrough time; Renxuan ignores the impact of the mobility ratio of oil and water, irreducible water saturation, and the residual oil saturation after water flooding, so he also presents a longer breakthrough time; without consideration of the movement of OWC, Li and Xiong et al. report longer breakthrough time too. This paper proposed a more precise method from which the calculated prediction time is much closer to the actual time.

FIGURE 4: Three-dimensional profile bottom water coning for 3.48 days.

FIGURE 5: Damaged sections and undamaged sections of the oil-bearing zone.

Usually sections near the well could be damaged as a result of well drilling and completion, thus leading to the change of permeability. Therefore, (10) is modified as

$$C = \frac{h_p}{\alpha r_w (\ln(r_e/r_w) + S)}, \quad (25)$$

where S is the well skin factor, which can be calculated by [10]

$$S = \left(\frac{K}{K_s} - 1 \right) \ln \frac{r_s}{r_w}, \quad (26)$$

where K_s is the permeability of the damaged inner sections and r_s is the radius of the damaged inner sections. As is shown in Figure 5, the section marked in blue represents the damaged sections, and the damaged and undamaged sections are assumed to be homogeneous, respectively.

FIGURE 6: The impact of surface recovery on the predicted production period without water.

FIGURE 8: The impact of porosity on the predicted production period without water.

FIGURE 7: The impact of crude oil volume on the predicted production period without water.

FIGURE 9: The impact of oil/water viscosity ratio on production period without water.

When oil well is a perfect well, that is, the skin factor $S = 0$, (25) expresses the same conditions like (10); when oil well is not perfect, in other words, $S > 0$, recovery rate of not being perfect will be lower than the perfect one, and the predicted breakthrough time of being not perfect will be shorter than the perfect one. Similarly, if oil well is super-perfect well, that is, $S < 0$, produced oil of the super-perfect well will be more than the perfect one, with longer predicted breakthrough time. So if the influence caused by drilling and completion is considered, the prediction method will yield more accurate results.

Figures 6, 7, 8, and 9 illustrate the impact of surface recovery, crude oil volume factor, formation porosity, and oil/water viscosity ratio in terms of the predicted production period without water.

From Figures 6 and 7, on the one hand, we can see that the predicted production period without water will be shorter

with the increase of surface recovery rate and crude oil volume factor. On the other hand, the predicted production period without water falls dramatically at the early time, followed by a slight decrease, while crude oil volume factor causes a relatively stable drop.

From Figure 8, it can be concluded that the predicted production period without water increases evenly; to rephrase it, bottom water rise up at a uniform speed with the growing of formation porosity.

According to Figure 9, the predicted water-free production time falls with the increase of oil/water viscosity ratio. It is attractive that when oil/water viscosity is relatively small, the predicted water-free production time experiences rapid changes. Consequently, oil/water viscosity ratio will influence bottom water coning to some extent, which is consistent with Yu et al.'s proposal [12].

5. Conclusion

Considering oil and water viscosity, initial irreducible water saturation, residual oil saturation, and OWC contact movement, this paper proposed a prediction method of bottom water coning profile and bottom water breakthrough time in bottom water reservoir without barrier. Based on theory of fluids flow in porous media, this paper assumes that the flow models are plane radial flow for opened reservoir sections and spherical radial flow for unopened reservoir sections. Compared with other prediction methods, this method can best match the real bottom water breakthrough time, and the relative deviation is only 1.14 percent. Therefore, this method has considerable potential for predicting water breakthrough time in bottom water oil reservoir without barrier.

Nomenclature

h_o :	The thickness of the oil-bearing zone, m
h_p :	The opened thickness of oil-bearing zone, m
ϕ :	Porosity
K :	Permeability, D
r_e :	Drainage radius, m
r_w :	Well radius, m
Q :	Underground production, m ³ /d
B_o :	Underground oil volume factor
μ_o :	Oil viscosity, mPa·s
μ_w :	Water viscosity, mPa·s
S_{or} :	Residual oil saturation
S_{wi} :	Initial irreducible water saturation
K_{owi} :	Oil relative permeability of initial irreducible water saturation
K_{wor} :	Water relative permeability of residual oil saturation
t_{bt} :	The bottom water breakthrough time, d
M_{ow} :	Oil-water mobility ratio, dimensionless
S :	The well skin factor.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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