Research Article

Stabilization of a Class of Switched Positive Nonlinear Systems

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The problem of switching stabilization for a class of switched positive nonlinear systems (switched positive homogeneous cooperative system (SPHCS) in the continuous-time context and switched positive homogeneous order-preserving system (SPHOS) in the discrete-time context) is studied by using average dwell time (ADT) approach, where the positive subsystems are possibly all unstable. To tackle this problem, a new class of ADT switching is first defined, which is different from the previous defined ADT switching in the literature. Then, the proposed ADT is designed via analyzing the weighted l_∞ norm of the considered system's state. A sufficient condition of stabilization for SPHCSs with unstable positive subsystems is derived in continuous-time context. Furthermore, a sufficient condition for SPHOSs under the assumption that all modes are possibly unstable is also obtained. Finally, a numerical example is given to demonstrate the advantages and effectiveness of our developed results.

1. Introduction

Switched system is a class of hybrid systems that involves a coupling between continuous dynamics and discrete events, which has wide application areas such as traffic control, process control, network control systems, automotive industry, and mechanical systems. Due to their numerous applications, switched systems have been a hot research topic in the past decades [1–5]. Most efforts in researches of switched systems are mainly devoted to the dynamic behavior analysis and property characterization, for example, controllability, observability, realizability, optimized performance, and especially the stability. The authors in [6] applied a class of Lyapunov-like function to study the problem of stability for switched systems comprising unstable subsystems.

On the other hand, many physical systems encountered in practice involve state variables that are always confined to be nonnegative. For instance, density of a object, absolute temperatures, and concentration of substances in chemical processes are always positive. Such systems are generally termed as positive systems whose states and output are positive whenever the initial conditions and input are nonnegative [7]. Numerous models having positive behavior

can be found in many areas such as biology, sociology, and communication networks. Given their practical importance, recently, positive system has been paid much attention by a large number of researchers [8–14]. As it is well known, positivity of the system state can yield some interesting properties and bring about some efficiently tackling techniques to positive linear systems [7, 15]. For example, the so-called copositive linear Lyapunov function with positive vector parameters can be particularly chosen as a system energy function for switched positive linear systems. In the most recent few years, positive nonlinear systems have also been noticed due to their important applications [16–18]. A valuable issue naturally arises: whether those properties, results, and methods for positive linear systems can be extended to positive nonlinear systems. In [19], the authors proved that, same to positive linear systems with constant delay, the stability of homogeneous cooperative system is also independent of delays. Afterwards, the authors of [20] extended this fact to continuous-time homogeneous cooperative systems and discrete-time homogeneous monotone systems with bounded time-varying delays.

Furthermore, switched positive system that possesses the essences of both switched systems and positive systems is

recently studied due to its wide applications in practice. As a special class of switched systems and positive systems, some relevant methods applicable to positive systems or switched systems are still suitable for switched positive systems [21, 22]. But anyway, it should be pointed out that those methods may not be as efficient as they are for general switched systems or positive systems. Therefore, many researchers have recently focused on exploring efficient approaches and results for switched positive systems. The authors of [23] introduced a multiple linear copositive Lyapunov function (MLCLF) to investigate the problem of stability for switched positive systems. Then, the authors of [24] presented a generalization of copositive types of Lyapunov function for stability analysis of switched positive systems. Note that those works all consider switched positive linear systems. So far there are few results reported for switched positive nonlinear systems which are however of both theoretical and practical importances.

As far as the stabilization of switched positive systems is concerned, how to find an appropriate switching signals to guarantee system stability is an important issue. Switching signals that may be either autonomous or controlled generally include arbitrary switching signals, stochastic switching signals, and constraint switching signals. Time constrained switching signal that is viewed as an important class of constraint switching signals naturally has been extensively investigated during the past several decades because of its strong applicability. Time constrained switching signal can be also called slow switching, and it is classified into three types: dwell time (DT), average dwell time (ADT), and mode-dependent average dwell time switching (MDADT).

In the recent few years, time constrained switching has been successfully used for switching stabilization of both general switched systems and switched positive systems. To list a few, when all the subsystems of a switched linear system are stable, the author of [25] had shown that the system is exponentially stable if the DT is sufficiently large. It was proved in [26–28] that ADT turns out to be very useful for analysis and synthesis of switched systems. The ADT was extended to MDADT in [29] for stabilization of general switched systems. In [23], the ADT switching was utilized for switched positive linear systems. It is noted that, most recently, the time constrained switching stabilization for switched (positive) linear systems with partly/all unstable subsystems have been paid some attentions [6, 30, 31]. However, such an attempt is just a start, and the slow switching stabilization for switched (positive) systems with possibly all unstable subsystems has not been fully solved, especially for switched positive nonlinear systems.

In summary, the existing time constrained switching stabilization results for switched positive systems are mainly focused on the linear case, and those promising ideas therein are not applicable for switched positive nonlinear systems. Such a problem becomes more complicated once all the nonlinear subsystems become unstable. To the best of the authors' knowledge, up to now, there is no literature reporting the switching stabilization problem for switched positive nonlinear systems with possibly all unstable subsystems, even in the linear case.

All the above observations motivate us to carry out the present work. Compared with some existing results, our results can solve the switching stabilization problem of switched positive nonlinear systems though all the subsystems are unstable. The most important of the paper lies in that a new class of ADT is proposed and the weighted l_∞ norm of the considered system's state is introduced to tackle the considered problem. The layout of the paper is organized as follows. Section 2 reviews some necessary definitions of switched positive nonlinear systems and defines a new concept of ADT which characterizes a different set of switching signals from traditional ADT. In Section 3, stabilization criteria for switched positive nonlinear systems with possibly all unstable subsystems via proposed ADT switching are derived in continuous-time and discrete-time case. Section 4 provides a numerical example to demonstrate the feasibility and effectiveness of the proposed techniques, and Section 5 concludes the paper.

Notations. In this paper, the notations used are standard. $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}_+^n$ denote the field of real numbers, n -dimensional Euclidean space, and the nonnegative orthant of $\mathbb{R}^n$, respectively, and $\mathbb{R}_0^n$ stands for $\mathbb{R}_+^n \setminus \{0\}$, and Metzler is a matrix whose off diagonal entries are nonnegative. The notation $\|\cdot\|$ refers to the Euclidean norm. For $\mathbf{x} \in \mathbb{R}^n$, x_i denotes the i th component of $\mathbf{x}$. In addition, $\mathbf{x} \geq \mathbf{y}$ (or $\mathbf{x} > \mathbf{y}$) mean that all entries of vector $\mathbf{x} \geq \mathbf{y}$ (or $\mathbf{x} > \mathbf{y}$).

2. Problem Formulation and Preliminaries

This section presents some definitions and preliminary results which will be used throughout the paper. We first recall some preliminaries about positive nonlinear systems for the introduction of switched positive nonlinear systems hereafter. Consider the following nonlinear system:

$$\partial \mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t)), \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector; $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth functions vector, and is homogeneous cooperative in the continuous-time case (homogeneous order-preserving in the discrete-time case); the symbol ∂ denotes the derivative operator in the continuous-time context $\partial \mathbf{x}(t) = (d/dt)\mathbf{x}(t)$ and the shift forward operator in the discrete-time case $\partial \mathbf{x}(t) = \mathbf{x}(t+1)$.

Definition 1. System (1) is said to be positive if and only if, for every nonnegative initial state, its state is nonnegative.

Definition 2. A continuous vector field $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is said to be homogeneous if, for all $\mathbf{x}(t) \in \mathbb{R}^n$, and all real $\lambda > 0$, $\mathbf{f}(\lambda \mathbf{x}) = \lambda \mathbf{f}(\mathbf{x})$.

Definition 3. A continuous vector field $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$, which is $\mathcal{C}^1$ on $\mathbb{R}^n \setminus \{0\}$, is said to be cooperative if the Jacobian $(\partial \mathbf{f} / \partial \mathbf{x})(\mathbf{a})$ is Metzler for all $\mathbf{a} \in \mathbb{R}_0^n$.

Definition 4. A continuous vector field $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is said to be order-preserving on $\mathbb{R}_+^n$, if $\mathbf{f}(\mathbf{x}) > \mathbf{f}(\mathbf{y})$ for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^n$ such that $\mathbf{x} \geq \mathbf{y}$.

Lemma 5. If $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is cooperative, then for any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}_0^n$ satisfying $\mathbf{x} \geq \mathbf{y}$ and $x_i = y_i$, $f_i(\mathbf{x}) \geq f_i(\mathbf{y})$.

Remark 6. It should be pointed out that, if $\mathbf{f}$ is defined by homogeneous and cooperative vector field in the continuous-time context (or $\mathbf{f}$ is defined by homogeneous and order-preserving vector field in the discrete-time case), the nonlinear system (1) is positive, which means that for every nonnegative initial condition $\mathbf{x}(0) \in \mathbb{R}_+^n$, the corresponding state trajectory $\mathbf{x}(t) \in \mathbb{R}_+^n$ for all $t \geq 0$.

In this paper, we consider the following SPHCS (SPHOS in the discrete-time case) consisting of a family of subsystems (1):

$$\dot{\mathbf{x}}(t) = \mathbf{f}_{\sigma(t)}(\mathbf{x}(t)), \quad (2)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector; $\sigma(t)$ is a switching signal which is a piecewise constant function from the right of time and takes its values in the finite set $S = \{1, \dots, m\}$, where $m > 1$ is the number of subsystems. $\mathbf{f}_p : \mathbb{R}^n \rightarrow \mathbb{R}^n$ are smooth functions for any $\sigma(t) = p \in S$. Moreover, all the subsystems in system (1) may be unstable. Also, for a switching sequence, $0 < t_1 < \dots < t_p < t_{p+1} < \dots$, $\sigma(t)$ may be either autonomous or controlled. When $t \in [t_p, t_{p+1})$, we say $\sigma(t_p)$ th mode is active. With respect to switching law $\sigma(t)$, the following exponential stability definition of system (2) is given, and we denote time by k in the discrete-time case.

Definition 7. Switched system (2) with switching $\sigma(t)$ is said to be globally uniformly exponentially stable (GUES), if there exist constants $\alpha > 0$, $\beta > 0$ (resp., $0 < \lambda < 1$), such that the solution of the system satisfies $\|\mathbf{x}(t)\| \leq \alpha e^{-\beta(t-t_0)} \|\mathbf{x}(t_0)\|$, $\forall t \geq t_0$ (resp., $\|\mathbf{x}(k)\| \leq \alpha \lambda^{-(k-k_0)} \|\mathbf{x}(t_0)\|$, $\forall k \geq k_0$, with any initial conditions $\mathbf{x}(t_0)$ (or $\mathbf{x}(k_0)$)).

Due to the fact that all the positive nonlinear subsystems of system (2) may be unstable, we need to design time constrained switching sequences, such that the switched nonlinear system (2) is GUES. In order to achieve this goal, let us first define the following ADT switching.

Definition 8. For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_2, t_1)$ denote the number of discontinuities of $\sigma(t)$ in the interval (t_1, t_2) . One says that $\sigma(t)$ has an average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that

$$N_\sigma(t_2, t_1) \geq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad \forall t_2 \geq t_1 \geq 0. \quad (3)$$

3. Main Results

In this section, we consider the problem of stabilization for switched positive nonlinear system (2) with our proposed ADT switching.

First, we are in a position to provide the continuous-time version of stabilization condition in the case that the $\mathbf{f}_p$ is defined in homogeneous and cooperative vector field.

Theorem 9 (continuous-time version). Consider SPHCS (2) possibly composed of all unstable modes. If there exist a set of vectors $\mathbf{v}_p > 0$, $p \in S$ and two positive numbers $\eta > 0$ and $0 < \mu < 1$ such that $\forall i = 1, 2, \dots, n$, $\forall p \in S$

$$\mathbf{f}_p(\mathbf{v}_p) > 0, \quad (4)$$

$$\frac{f_{pi}(\mathbf{v}_{pi})}{v_{pi}} - \eta < 0, \quad (5)$$

$$\|\mathbf{x}(t_p^+)\|_{\mathbf{v}_p}^\infty - \mu \|\mathbf{x}(t_p^-)\|_{\mathbf{v}_q}^\infty \leq 0, \quad (6)$$

where the $\|\mathbf{x}(t_p)\|_{\mathbf{v}_p}^\infty$ is the weight l_∞ norm defined as

$$\|\mathbf{x}(t_p)\|_{\mathbf{v}_p}^\infty = \max_{1 \leq i \leq n} \frac{|x(t_p)_i|}{v_i}, \quad (7)$$

then SPHCS (2) is globally uniformly exponentially stable for every switching signal $\sigma(t)$ with average dwell time:

$$\tau_a \leq \tau_a^* = \frac{-\ln \mu}{\eta}. \quad (8)$$

Proof. For any $T > 0$, $t_0 = 0$, we denote $t_1, t_2, \dots, t_p, t_{p+1}, \dots, t_{N_{\sigma(T,0)}}$ as the switching times on time interval $[0, T]$. Then, we consider the function

$$W(t) = e^{-\eta(t-t_{\sigma(t)})} \|\mathbf{x}(t)\|_{\mathbf{v}_{\sigma(t)}}^\infty. \quad (9)$$

It is immediately clear that $W(t)$ is a continuously differentiable function. When $t \in [t_p, t_{p+1})$, we get from (9) that

$$W(t) = e^{-\eta(t-t_p)} \|\mathbf{x}(t)\|_{\mathbf{v}_p}^\infty. \quad (10)$$

It will show that $W(t)$ is nonincreasing when $t \in [t_p, t_{p+1})$. Next, we suppose that there exists a point-in-time $\hat{t} \in [t_p, t_{p+1}]$, such that

$$W(\hat{t}) = \|\mathbf{x}(t_p)\|_{\mathbf{v}_p}^\infty. \quad (11)$$

Then, one can obtain from (7) and (11) that

$$W(\hat{t}) = e^{-\eta(t-t_{\sigma(t)})} \max_{1 \leq i \leq n} \frac{|x_i(\hat{t})|}{v_{pi}} = e^{-\eta(t-t_p)} \frac{x_k(\hat{t})}{v_{pk}}. \quad (12)$$

This together with (11) yields

$$\begin{aligned} x_k(\hat{t}) &= \|\mathbf{x}(\hat{t})\|_{\mathbf{v}_p}^\infty v_{pk} \\ &= e^{\eta(\hat{t}-t_p)} \|\mathbf{x}(t_p)\|_{\mathbf{v}_p}^\infty v_{pk}, \\ \mathbf{x}(\hat{t}) &\leq \|\mathbf{x}(\hat{t})\|_{\mathbf{v}_p}^\infty \mathbf{v}_p \\ &= e^{\eta(\hat{t}-t_p)} \|\mathbf{x}(t_p)\|_{\mathbf{v}_p}^\infty \mathbf{v}_p. \end{aligned} \quad (13)$$

Moreover, since $\mathbf{f}_p : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $p \in S$ is in homogeneous and cooperative vector field, based on Lemma 5, it can be derived from (5), (12), and (13) that

$$\begin{aligned} \dot{W}(t) &= e^{-\eta(\hat{t}-t_p)} \frac{f_{pk}(\mathbf{x}(\hat{t}))}{\nu_{pk}} - \eta e^{-\eta(\hat{t}-t_p)} \frac{x_k(\hat{t})}{\nu_{pk}} \\ &\leq e^{-\eta(\hat{t}-t_p)} \frac{f_{pk}(e^{\eta(\hat{t}-t_p)} \|\mathbf{x}(t_p)\|_{\nu_p}^\infty \nu_p)}{\nu_{pk}} \\ &\quad - \eta e^{-\eta(\hat{t}-t_p)} \frac{e^{\eta(\hat{t}-t_p)} \|\mathbf{x}(t_p)\|_{\nu_p}^\infty \nu_{pk}}{\nu_{pk}} \\ &= e^{-\eta(\hat{t}-t_p)} \frac{e^{\eta(\hat{t}-t_p)} \|\mathbf{x}(t_p)\|_{\nu_p}^\infty f_{pk}(\nu_p)}{\nu_{pk}} - \eta \|\mathbf{x}(\hat{t})\|_{\nu_p}^\infty \\ &= \|\mathbf{x}(t_p)\|_{\nu_p}^\infty \left(\frac{f_{pk}(\nu_p)}{\nu_{pk}} - \eta \right) < 0. \end{aligned} \quad (14)$$

Note that if (11) holds, we can get $\dot{W}(\hat{t}) < 0$, which implies that the upper-right Dini derivative $D^+(W(\hat{t})) < 0$. Thus, it can be seen that $W(t)$ is nonincreasing when $t \in [t_p, t_{p+1})$. This together with (10) gives that for $\forall t \in [t_p, t_{p+1})$

$$\begin{aligned} W(t) &= e^{-\eta(t-t_p)} \|\mathbf{x}(t)\|_{\nu_p}^\infty \\ &\leq W(t_p) \\ &= \|\mathbf{x}(t_p)\|_{\nu_p}^\infty. \end{aligned} \quad (15)$$

According to (6) and (15), one can obtain that

$$\begin{aligned} \|\mathbf{x}(t_{p+1}^+)\|_{\nu_p}^\infty &\leq \mu \|\mathbf{x}(t_{p+1}^-)\|_{\nu_p}^\infty \\ &\leq \mu e^{\eta(t_{p+1}-t_p)} \|\mathbf{x}(t_p)\|_{\nu_p}^\infty. \end{aligned} \quad (16)$$

By integrating this for $t \in [t_p, t_{p+1})$, one gets that

$$\begin{aligned} \|\mathbf{x}(T^-)\|_{\nu_{N_\sigma}}^\infty &\leq e^{\eta(T-t_{N_\sigma})} \|\mathbf{x}(t_{N_\sigma})\|_{\nu_{N_\sigma}}^\infty \\ &= e^{\eta(T-t_{N_\sigma})} \|\mathbf{x}(t_{N_\sigma}^+)\|_{\nu_{N_\sigma}}^\infty \\ &\leq \mu e^{\eta(T-t_{N_\sigma-1})} \|\mathbf{x}(t_{N_\sigma-1})\|_{\nu_{N_\sigma-1}}^\infty \\ &\vdots \\ &\leq \mu^{N_\sigma} e^{\eta T} \|\mathbf{x}(0)\|_{\nu_0}^\infty. \end{aligned} \quad (17)$$

Moreover, it can be derived from (3) and (17) that

$$\begin{aligned} \|\mathbf{x}(T)\|_{\nu_{N_\sigma}}^\infty &\leq \mu^{N_\sigma} e^{\eta T} \|\mathbf{x}(0)\|_{\nu_0}^\infty \\ &\leq e^{\eta T} e^{(N_\sigma+T/\tau_a) \ln \mu} \|\mathbf{x}(0)\|_{\nu_0}^\infty \\ &= e^{N_\sigma \ln \mu} e^{(\eta + \ln \mu / \tau_a) T} \|\mathbf{x}(0)\|_{\nu_0}^\infty. \end{aligned} \quad (18)$$

Therefore, by denoting $\alpha = e^{N_\sigma \ln \mu}$, $\beta = -(\eta + \mu/\tau_a)$, one can get from (18) that

$$\|\mathbf{x}(T)\| \leq \alpha e^{-\beta T} \|\mathbf{x}(0)\|, \quad \forall T \geq 0. \quad (19)$$

By Definition 7, we conclude that SPHCS (2) is GUES by our proposed ADT switching signals (3) satisfying (8) if the conditions (4)–(6) hold. This completes the proof. $\square$

Remark 10. It has been shown in [19] that if there exists a vector $\nu_p > 0$ satisfying $\mathbf{f}(\nu) > 0$, the corresponding homogeneous cooperative system is not globally asymptotically stable. Thus, all the homogeneous cooperative subsystems of SPHCS (2) may be unstable, if (4) holds. Under the ADT designed in Theorem 9, the stabilization can be achieved even if all the subsystems are unstable. On the other hand, it should be pointed out that, different from the existing results where the lower bound of the ADT should be specified a priori, we need to set the upper bound in advance in our results.

Next, the following theorem is provided for the discrete-time version of stabilization condition for SPHOS (2) where the $\mathbf{f}_p$ is defined in homogeneous and order-preserving vector field.

Theorem 11 (discrete-time version). *Consider SPHOS (2) possibly composed of all unstable modes. If there exist a set of vectors $\nu_p > 0$, $p \in S$ and two positive numbers $0 < \eta < 1$ and $0 < \mu < 1$ such that $\forall i = 1, 2, \dots, n, \forall p \in S$*

$$\mathbf{f}_p(\nu_p) > \nu_p, \quad (20)$$

$$\frac{f_{pi}(\nu_{pi})}{\nu_{pi}} - \frac{1}{\eta} \leq 0, \quad (21)$$

$$\|\mathbf{x}(k_p^+)\|_{\nu_p}^\infty - \mu \|\mathbf{x}(k_p^-)\|_{\nu_q}^\infty \leq 0, \quad (22)$$

where the $\|\mathbf{x}(t_p)\|_{\nu_p}^\infty$ is the weight l_∞ norm defined as

$$\|\mathbf{x}(t_p)\|_{\nu_p}^\infty = \max_{1 \leq i \leq n} \frac{|x(t_p)_i|}{\nu_i}, \quad (23)$$

then SPHOS (2) is globally uniformly exponentially stable for every switching signal σ with average dwell time:

$$\tau_a \leq \tau_a^* = \frac{\ln \mu}{\ln \eta}. \quad (24)$$

Proof. For any $K > 0$, $k_0 = 0$, we denote $k_1, k_2, \dots, k_p, k_{p+1}, \dots, k_{N_{\sigma(K,0)}}$ as the switching times on time interval $[0, T]$. Then, we consider the function

$$W(t) = \eta^{k-k_{\sigma(k)}} \|\mathbf{x}(k)\|_{\nu_{\sigma(k)}}^\infty. \quad (25)$$

When $k \in [k_p, k_{p+1})$, we get from (25) that

$$W(k) = \eta^{k-k_p} \|\mathbf{x}(k)\|_{\nu_p}^\infty. \quad (26)$$

Next, it will be shown that $W(k) \leq \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty$ when $k \in [k_p, k_{p+1})$. First, when $k = k_p$, the above inequality obviously holds. Then, we suppose that $k > k_p$; for a given $k \in [k_p, k_{p+1})$ the above inequality is satisfied and it proves that it is also correct for $k + 1 \in (k_p, k_{p+1})$. Due to the fact that for $k \in [k_p, k_{p+1})$, $W(k) \leq \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty$ is true, one can obtain that

$$W(k) = \eta^{k-k_p} \max_{1 \leq i \leq n} \frac{|x_i(k)|}{\nu_{pi}} \leq \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty, \quad (27)$$

which implies that

$$\mathbf{x}(k) \leq \eta^{k-k_p} \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty \mathbf{v}_p. \quad (28)$$

Moreover, since $\mathbf{f}_p : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $p \in S$ is defined in homogeneous and order-preserving vector field, it can be seen from (21), (26), and (28) that

$$\begin{aligned} W(k+1) &= \eta^{k+1-k_p} \|\mathbf{x}((k+1)_p)\|_{\mathbf{v}_p}^\infty \\ &= \eta^{k+1-k_p} \|\mathbf{f}_p(\mathbf{x}(k))\|_{\mathbf{v}_p}^\infty \\ &\leq \eta^{k+1-k_p} \left\| \mathbf{f}_p \left(\eta^{k-k_p} \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty \mathbf{v}_p \right) \right\|_{\mathbf{v}_p}^\infty \\ &= \eta^{k+1-k_p} \eta^{k_p-k} \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty \|\mathbf{f}_p(\mathbf{v}_p)\|_{\mathbf{v}_p}^\infty \\ &= \eta \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty \|\mathbf{f}_p(\mathbf{v}_p)\|_{\mathbf{v}_p}^\infty \\ &= \eta \max_{1 \leq i \leq n} \frac{|f_{pi}(\mathbf{v}_p)|}{\nu_{pi}} \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty \\ &\leq \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty. \end{aligned} \quad (29)$$

Thus, we can obtain that (27) is right for $\forall k \in [t_p, t_{p+1})$. Combining this together with (22), one can get

$$\begin{aligned} \|\mathbf{x}(k_{p+1}^+)\|_{\mathbf{v}_p}^\infty &\leq \mu \|\mathbf{x}(k_{p+1}^-)\|_{\mathbf{v}_p}^\infty \\ &\leq \mu \eta^{k_{p+1}-k_p} \|\mathbf{x}(k_p)\|_{\mathbf{v}_p}^\infty. \end{aligned} \quad (30)$$

Similar to (17) and (18), we have for $\forall K > 0$

$$\begin{aligned} \|\mathbf{x}(K^-)\|_{\mathbf{v}_{N_0}}^\infty &\leq \mu^{N_0} \eta^{-K} \|\mathbf{x}(0)\|_{\mathbf{v}_0}^\infty \\ &\leq \exp \left\{ \left(\frac{K}{\tau_a} + N_0 \right) \ln \mu \right\} \exp \{ -K \ln \eta \} \|\mathbf{x}(0)\|_{\mathbf{v}_0}^\infty \\ &= \exp \{ N_0 \ln \mu \} \exp \left\{ \left(\frac{\ln \mu}{\tau_a} - \ln \eta \right) K \right\} \|\mathbf{x}(0)\|_{\mathbf{v}_0}^\infty. \end{aligned} \quad (31)$$

Therefore, by denoting $\alpha = \exp \{ N_0 \ln \mu \}$, $\lambda = \exp \{ (\ln \mu / \tau_a - \ln \eta) \}$; one can get from (31) that

$$\|\mathbf{x}(K)\| \leq \alpha \lambda^K \|\mathbf{x}(0)\|, \quad \forall K \geq 0. \quad (32)$$

We conclude that SPHOS (2) is GUES by our proposed ADT switching signals (3) satisfying (24) if the conditions (20)–(22) hold. This completes the proof. $\square$

FIGURE 1: State response of the subsystem Σ_1 .

Remark 12. In Theorems 9 and 11, based on a novel proposed switching signals, sufficient conditions of stabilization for a class of switched positive nonlinear systems are obtained, where the positive subsystems are possibly all unstable.

4. A Numerical Example

We provide the following numerical example in this section to verify our main results developed in this paper.

Example 1. Consider a switched nonlinear system consisting of two homogeneous cooperative subsystems described by

$$\begin{aligned} \Sigma_1 &= \begin{cases} \dot{x}_1(t) = f_{11}(x_1(t), x_2(t)) \\ \quad = -2x_1(t) + 4x_2(t) - 3\sqrt{x_1^2(t) + x_2^2(t)}, \\ \dot{x}_2(t) = f_{12}(x_1(t), x_2(t)) \\ \quad = -0.2x_1(t) + 0.1x_2(t) \\ \quad \quad + 0.01\sqrt{x_1^2(t) + x_2^2(t)}, \end{cases} \\ \Sigma_2 &= \begin{cases} \dot{x}_1(t) = f_{21}(x_1(t), x_2(t)) \\ \quad = -2x_1(t) + 3x_2(t) + 1.2\sqrt{x_1^2(t) + x_2^2(t)}, \\ \dot{x}_2(t) = f_{22}(x_1(t), x_2(t)) \\ \quad = 1.2x_1(t) - x_2(t) + 0.6\sqrt{x_1^2(t) + x_2^2(t)}. \end{cases} \end{aligned} \quad (33)$$

It is seen in (33) that $\mathbf{f}_1, \mathbf{f}_2$ are defined in homogeneous cooperative vector field, which means that the switched nonlinear system is a switched positive system. There exist a vector $\mathbf{v}_1 = [3, 7.8] > 0$ with $\mathbf{f}(\mathbf{v}_1) = [0.1289, 0.2636] > 0$ and a vector $\mathbf{v}_2 = [3, 1.2] > 0$ with $\mathbf{f}(\mathbf{v}_2) = [1.8754, 0.4613] > 0$. Thus, both of them are unstable, and the state trajectories are shown in Figures 1 and 2.

Next, we are interested in designing a kind of switching signal $\sigma(t)$ with property (3) to asymptotically stabilize the system. Furthermore, we generate two possible switching sequences by $\tau_a = 0.5$ and $\tau_a = 2$, respectively; the corresponding state responses of the system under initial state

FIGURE 2: State response of the subsystem Σ_2 .

FIGURE 3: State responses of switched nonlinear system (33) under switching signal $\sigma(t)$ with $\tau_a = 0.5$.

condition $x(0) = [10 \ 15]^T$ are shown in Figures 3 and 4, from which we can see that the switched nonlinear system is stable under $\tau_a = 0.5$ but unstable under $\tau_a = 2$.

5. Conclusions

The stabilization problem for switched positive nonlinear systems composed of possibly all unstable subsystems are studied in both continuous-time and discrete-time domains by using ADT switching. As a first attempt, a new class of ADT switching signal is proposed, and then it is designed for the system via the analysis of the weight l_∞ norm of the system. Two sufficient stabilization conditions for the underlying systems are derived. The highlight of the paper lies in that it is the first time the stabilization is solved for our considered switched positive nonlinear system where the system is possibly composed of all unstable subsystems. A numerical example is provided to show the effectiveness of our proposed approach. In our future work, we aim at

FIGURE 4: State responses of switched nonlinear system (33) under switching signal $\sigma(t)$ with $\tau_a = 2$.

developing numerically easily checked conditions for the considered systems, and the problem of stabilization for switched positive nonlinear system with delays also needs to be studied.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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