Research Article

Application of AR Model in the Analysis of Preearthquake Ionospheric Anomalies

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Earthquake ionosphere coupling phenomenon is one of the hot research topics using the Global Positioning System (GPS). Taking Lushan earthquake in April 2013 as an example, this paper firstly establishes ionosphere TEC models and determines the optimal model based on Autoregression model by analyzing the TEC values of the epicenter area detected by GPS. Then it makes predictions about the ionosphere data, obtains the background value, and conducts anomaly analysis by using the optimal model. Finally the correlation between ionosphere anomalies and earthquake is analyzed quantitatively by presenting data diagrams explicitly; then a new method to do short-term and imminent earthquake prediction is proposed in the end.

1. Introduction

An earthquake, as a result of a sudden release of energy in the Earth's crust that creates seismic waves, is one of the major natural disasters which may cause huge property damage and casualties. Sichuan is one of earthquake-prone areas of China, where Wenchuan 8.0 earthquake took place. Until 12:00 on September 18, 2008, the earthquake resulted in 69,227 deaths, 374,643 being injured, and 17,923 being missing, and the latest strong earthquake Lushan 7.0 earthquake resulted in 1.52 million victims as well as a damaged area of 12,500 square kilometers. According to the website of China Seismological Bureau, until 14:30 on April 24, 2013, the earthquake caused a total of 196 deaths, 21 people missing, and 11,470 people injured. Earthquake prediction and forecasting, especially short-Pro forecasting, are still in the exploratory stage [1]. In order to advance earthquake prediction research, many scientists continue to explore new methods of seismic monitoring, and the relationship between the earthquake gestation period ionospheric disturbances and seismic coupling has become a hot topic [2]. Leonard and Barnes firstly discovered the coupling relationship between earthquakes and ionospheric when they observed the ionospheric anomaly in the fore-shock of Alaska earthquake [3]. Liu et al. in Taiwan's National

Central University did a survey about ionospheric anomalies in earthquakes of magnitude > 6 during the period 1994 to 1999, which showed that 1–6 days prior to the earthquake ionosphere f₀F₂ decreased significantly in value at local time 12:00–17:00 compared with the value 15 days before the earthquake and they regard this phenomenon as an earthquake precursory to predict earthquake [4].

Traditional methods usually study the ionosphere by using ionospheric altimeter. Currently there are about 200 ionosphere altimeters distributed around the world, but only a few are running [4]. With the development of GNSS technology, especially the in-depth study of GPS technology, the use of dual-frequency GPS observation techniques solver ionospheric delay to obtain ionospheric total electron density (Total Electron Content, TEC) technology matures. Thousands of ground-based GPS receiving stations that are distributed all over the world improve the TEC spatial and temporal resolution, which will be more conducive to the sustained simulation research of the distribution and exception analysis of the ionosphere.

Currently, the main methods of ionospheric anomalies coupling studies include quartile method and sliding window method. Xinzhou recently proposed a preseismic ionospheric anomaly detection using time series method. The above

methods all detect by selecting a normal period of time or a period of TEC value as the background without explaining the reasons for selecting the time window. The scale and change of the ionosphere, affected by crustal movements and solar activity, are periodic and certain. Therefore, adopting time series method (ARMA model) can better take into account the uncertainty components and be able to calculate changes in the ionosphere TEC uncertainty [5].

2. Principle of AR Model

Autoregression model (AR model) is a method of processing statistical time series and making predictions about values of variables currently and afterwards based on the value of the same variable at different stages. The formula is

$$X_t = \sum_{i=1}^{t-p} \varphi_i X_i + a_t, \quad (1)$$

where p is the autoregressive order and a_t is the constant term. To establish Autoregression model, we should reasonably determine its order p firstly which can be normally set within a range of the order, then make a parameter estimation within the scope of the various model, and test the significance of the parameters. Finally determine the order by using fixed-order criteria. We use the linear hypothesis method for model order. The principle is as follows: observational data set $(x_1, x_2, \dots, x_n)$, first set of order p , establish Autoregression model,

$$x_t = \sum_{i=1}^p b_i x_{t-i}. \quad (2)$$

Then consider $p - 1$ model, $b_p = 0$, obtain $p - 1$ order model, and obtain the model parameter estimation and residual sum of squares, denoted S_p, S_{p-1} . Assuming a linear method.

The formula is

$$S_p = S_{p-1} + R. \quad (3)$$

Suppose $H_0 : b_p = 0$ is true; F distribution can be used for statistics:

$$F = \frac{R/1}{S_p/(n-p)} = \frac{S_{p-1} - S_p}{S_p/(n-p)}. \quad (4)$$

Select significant level α , molecular degrees of freedom for 1, and denominator degrees of freedom $n - p$; look up table F_α ; if $F > F_\alpha$, then H_0 is not established, $b_p \neq 0$. Since there are significant differences between P order and $p - 1$ order model, p order should be used. Otherwise $F < F_\alpha$; then accept H_0 . It means P order and $p - 1$ order model have no significant difference; then $p - 1$ order should be used [6].

3. The Establishment of AR Model

Autoregression model in the study of ionospheric anomalies needed no more information, but it must comply with the following two conditions.

TABLE 1: TEC correlation analysis table.

	2011.3 TECu	2012.3 TECu	2013.3 TECu
2011.3 TECu	1		
2012.3 TECu	0.912770639	1	
2013.3 TECu	0.899358422	0.924532001	1

- (1) It must be autocorrelated; if autocorrelation coefficient (R) is less than 0.5, then it should not be used to predict the results which will be highly inaccurate.
- (2) Autoregression model only applies to the forecast and its early-related phenomenon, which is influenced by its own historical factors [6].

For these reasons, the paper must first verify Lushan ionospheric electron content relevance, that is, different trends within the same month of the year whether converge. In probability theory and statistics, correlation (correlation, also known as the correlation coefficient or correlation coefficient) shows the strength and direction of the linear relationship between two random variables. Therefore, the correlation efficient may be used to determine whether they converge. Specific experiments are as follows: by using the TEC data in March 2011, March 2012, and March 2013, the TEC data prior to Lushan earthquake can be obtained with the interpolation method. The correlation coefficient can be calculated with the TEC data in March 2011 data and March 2012 and March 2013, respectively. The formula is as follows:

$$\rho(\text{TEC}_{12}, \text{TEC}_{13}) = \frac{E((\text{TEC}_{12} - \mu_{\text{TEC}_{12}})(\text{TEC}_{13} - \mu_{\text{TEC}_{13}}))}{\sigma_{\text{TEC}_{12}} \sigma_{\text{TEC}_{13}}}. \quad (5)$$

When $|\rho| < 0.4$ for low linear correlation, when 0.4e correlation coefficient is calculated using the interpolated data shown in Table 1 with March 2011 data.

As can be seen from Table 1, the same month in different years TEC has a very high correlation, Lushan local ionosphere can be determined with different years, and TEC in March has a great similarity. The correlation between TEC in March 2012 and that in March 2013 is up to 0.925. Figure 1 shows the time series of TEC Lushan three years of observations, Figure 2 Lushan March 2011 and March 2013 TEC time series of observations, and Figure 3 Lushan March 2012 and March 2013 TEC time series of observations.

The Autoregression model data is eventually based on the TEC observation data in March 2012 according to the correlation analysis. By doing the regression analysis of the TEC data in March 2012, we get the following results.

AR(2) Autoregression model is as follows:

$$\begin{aligned} \text{TEC}_t &= 1.5337\text{TEC}_{t-1} - 0.8427\text{TEC}_{t-2} + 8.5887; \\ &\vdots \end{aligned} \quad (6)$$

FIGURE 1: Lushan in March 2011, March 2012, and March 2013 TEC time series.

AR(8) Autoregression model is as follows:

$$\begin{aligned} \text{TEC}_t = & 1.1328\text{TEC}_{t-1} - 0.6135\text{TEC}_{t-2} \\ & - 0.0597\text{TEC}_{t-3} - 0.2058\text{TEC}_{t-4} + 0.1808\text{TEC}_{t-5} \\ & - 0.1270\text{TEC}_{t-6} - 0.2659\text{TEC}_{t-7} + 0.0034\text{TEC}_{t-8} \\ & + 26.5854; \end{aligned} \quad (7)$$

AR(16) Autoregression model is as follows:

$$\begin{aligned} \text{TEC}_t = & 1.0879\text{TEC}_{t-1} - 0.49\text{TEC}_{t-2}^{9000} \\ & - 0.0767\text{TEC}_{t-3} - 0.0296\text{TEC}_{t-4} \\ & + 0.1781\text{TEC}_{t-5} - 0.1704\text{TEC}_{t-6} \\ & - 0.0633\text{TEC}_{t-7} - 0.0825\text{TEC}_{t-8} \\ & + 0.1676\text{TEC}_{t-9} - 0.1811\text{TEC}_{t-10} \\ & + 0.0768\text{TEC}_{t-11} + 0.4795\text{TEC}_{t-12} \\ & - 0.4374\text{TEC}_{t-13} + 0.0629\text{TEC}_{t-14} \\ & + 0.1222\text{TEC}_{t-15} - 0.1222\text{TEC}_{t-16} \\ & + 13.5605. \end{aligned} \quad (8)$$

The predicted value of TEC in March 2013 is obtained by using the 16-step model equations and then we compare it with the actual value. S is calculated by the sum of error square formula:

$$S = \sum (\text{TEC}_t - \widehat{\text{TEC}}_t)^2. \quad (9)$$

Inside the formula $\widehat{\text{TEC}}_t$ is the predicted value, TEC is actual value, and S is the sum of error square. The results

FIGURE 2: Lushan in March 2011 and March 2013 TEC time series.

FIGURE 3: Lushan in March 2012 and March 2013 TEC time series.

are shown in Table 2. Figure 4 shows the sum of error square trends, and Figure 5 presents the F trends.

Figure 4 shows that the sum of error square decreases with the increase of the order, but the significant difference exists only when $p = 8$ by comparing the data in Figures 4 and 5, according to hypothesis testing formulas and molecular dof 1, denominator dof ∞ , and degree of confidence $\alpha = 0.05$, checking F distribution table $F_\alpha = 3.84$. So we determine the Autoregression model of order 8 as the optimal model:

$$\begin{aligned} \text{TEC}_t = & 1.1328\text{TEC}_{t-1} - 0.6135\text{TEC}_{t-2} \\ & - 0.0597\text{TEC}_{t-3} - 0.2058\text{TEC}_{t-4} \\ & + 0.1808\text{TEC}_{t-5} - 0.127\text{TEC}_{t-6} \\ & - 0.2659\text{TEC}_{t-7} + 0.0034\text{TEC}_{t-8} + 26.5854. \end{aligned} \quad (10)$$

TABLE 2: Comparison of different Autoregression model's predictions.

Order	Second-order	Third-order	Fourth-order	Fifth-order	Sixth-order
S_{residual}	8330.168247	8226.200168	7827.606801	7301.419726	5755.198635
Observations	370	369	368	367	366
F		4.663662494	18.7391067	26.44837084	98.3314313
Order	Seventh-order	Eighth-order	Ninth-order	Tenth-order	Eleventh-order
S_{residual}	5365.003372	5343.676889	5063.582368	4944.146682	4368.569797
Observations	365	364	363	362	361
F	26.5463526	1.452715028	20.07952149	8.74482927	47.56322207
Order	Twelfth-order	Thirteenth-order	Fourteenth-order	Fifteenth-order	Sixteenth-order
S_{residual}	4297.146322	3890.81946	3841.13286	3817.8232	3754.323933
Observations	360	359	358	357	356
F	5.983610742	37.49116218	4.630874185	2.179657889	6.021254327

FIGURE 4: Sum of error square trends.

FIGURE 5: F trends.

4. Analysis of Ionospheric Anomalies of Lushan Earthquake

The predicted value $\widehat{\text{TEC}}$ is calculated based on formula (10) and 2013 TEC observations 3 months ago. And standard deviation is obtained as follows:

$$s = \sqrt{\frac{\sum (\widehat{\text{TEC}}_i - \overline{\widehat{\text{TEC}}})^2}{N - 1}} \quad (11)$$

so $s = 4.174$.

TEC values in April are predicted based on AR(8). The results are shown in Figure 6.

Take the upper and lower boundaries:

$$\begin{aligned} \text{up} &= 2s, \\ \text{low} &= -2s. \end{aligned} \quad (12)$$

By comparing the predicted value in April and the actual value in April 2013, we obtain the results in Figure 7.

Based on the above analysis of the ionosphere anomaly formulas, we can further analyze the difference between TEC predicted values and actual values in Lushan regions.

5. Conclusions

Using autoregressive model to make predictions and do anomaly analysis of Lushan earthquakes ionosphere, we can draw the following conclusions.

- (1) Since April 6th, 2013, the ionospheric TEC over the epicenter displays obvious anomalies with little disturbance by space weather. Therefore it can be inferred that they are caused by the coming earthquake and there is duration of 9 days. This is consistent with the Wu Yun's results which showed 10 days before the 3 big earthquakes in Asia; ionospheric TEC over the epicenter and its vicinities displays obvious abnormal disturbances. Here the first TEC abnormal changes appeared about 14 days prior to the seismic event.
- (2) On April 13th 2013, the abnormal disturbances (TEC) of ionosphere appear and abnormal value reached 18

FIGURE 6: April 2013 Lushan TEC predictive value.

FIGURE 7: Lushan TEC difference between March 15, 2013, and March 31, 2013.

TECu with longitude span of 25° and latitude span of 5° in the spatial distribution of anomalies. Abnormal peak is located near the epicenter near the equator. The ionospheric abnormalities 7 days before the main shock are indeed related to the preparation process of Lushan earthquake.

Based on the above analysis, we can draw the conclusion that AR model can be applied effectively to establish model with the optimal background data to analyze preearthquake ionospheric anomalies.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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