Research Article

A Hybrid Intelligent Multisensor Positioning Methodology for Reliable Vehicle Navigation

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With the rapid development of intelligent transportation systems worldwide, it becomes more important to realize accurate and reliable vehicle positioning in various environments whether GPS is available or not. This paper proposes a hybrid intelligent multisensor positioning methodology fusing the information from low-cost sensors including GPS, MEMS-based strapdown inertial navigation system (SINS) and electronic compass, and velocity constraint, which can achieve a significant performance improvement over the integration scheme only including GPS and MEMS-based SINS. First, the filter model of SINS aided by multiple sensors is presented in detail and then an improved Kalman filter with sequential measurement-update processing is developed to realize the filtering fusion. Further, a least square support vector machine- (LS SVM-) based intelligent module is designed and augmented with the improved KF to constitute the hybrid positioning system. In case of GPS outages, the LS SVM-based intelligent module trained recently is used to predict the position error to achieve more accurate positioning performance. Finally, the proposed hybrid positioning method is evaluated and compared with traditional methods through real field test data. The experimental results validate the feasibility and effectiveness of the proposed method.

1. Introduction

With the rapid development of intelligent transportation systems, global positioning system (GPS) has been widely utilized in many positioning and navigation applications for land vehicles [1, 2]. The major drawback related to GPS is satellite signal blockage in urban areas, high buildings, and other GPS-denied environments, where its positioning accuracy, reliability, and integrity will be deteriorated [2, 3].

In this situation, GPS is usually incorporated with strapdown inertial navigation system (SINS) to achieve accurate and reliable positioning. SINS is a self-contained system that is not affected by external disturbances. However, the standalone SINS solution drifts with time. Through integrating GPS and SINS, one can achieve superior performance over a standalone GPS or SINS [4–6]. SINS consists of a group of inertial measurement units (IMU), that is, three accelerometers and three gyroscopes, which are mounted

on a moving platform to measure linear accelerations and angular velocities. Given the initial values of navigation parameters, the measurements from IMU can be processed to determine current position, velocity, and attitude of the moving platform with respect to a certain frame of reference [7, 8]. Note that the SINS performance depends on its grade. The high-end navigation or tactical grade SINS can maintain proper positioning performance even after a relatively long period of GPS outage. Unfortunately, the high-end SINS is very expensive and thus not suitable for low-cost applications such as those for typical passenger cars. The SINS utilizing microelectromechanical system (MEMS) technology has been developed rapidly in recent years, which provides a viable solution to achieve mass-market applications in land vehicles for its low cost, low power consumption, and small size [2, 7, 9]. However, the MEMS-based SINS sensors may suffer from large noise, bias, and drift errors, which are much more serious than the high-grade sensors [7, 9, 10]. Even

when GPS outage occurs for a short time, the MEMS-based SINS performance will degrade rapidly.

For the SINS/GPS integration, the Kalman filter (KF) has been widely adopted as the standard optimal estimation tool. However, the limitations of the KF-based SINS/GPS integration approach are the necessity of stochastic modeling of sensor errors, which are difficult to determine especially for MEMS-based gyroscopes and accelerometers, and the need for accurate *a priori* information of the covariance matrices of the noises associated with both SINS and GPS [8, 10, 11].

Alternatively, artificial intelligence (AI) methods have obtained the popularity in recent years due to their abilities to solve the problem of nonlinearity and uncertainty. These AI-based approaches include Random Forest Regression (RFS) [12], Multilayer Perceptron Neural Networks (MLPNNs) [13], Adaptive Neurofuzzy Inference System (ANFIS) [10, 14, 15], and Radial Basis Function Neural Networks (RBFNNs) [8, 16–18].

Generally, although the AI-based methods above perform well for the navigation or tactical grade SINS, they have shown a very limited success when applied to a MEMS-based SINS/GPS navigation system [12]. The main reason is due to the high noise level and bias instability of MEMS-based inertial sensors, which increases the nonlinear complexity of the input/output functional relationship being modeled and limits the AI generalization ability that in turn affects its prediction accuracy. To overcome the standalone MEMS-based SINS limitation when GPS outage occurs, an effective and viable solution is to fuse other sensors, especially low-cost ones in land vehicle applications [2, 9]. Through fusing more information from other sensors or sources, one can take advantage of the complementary properties of different sensors and thus enhance the accuracy and reliability of the positioning system. For instance, if the yaw angle information from low-cost electronic compass is fused into the system during GPS outages, the accumulative error of yaw angle caused by the integration of low-performance MEMS gyroscope will be eliminated, and then the horizontal positioning performance will be improved significantly because it is affected largely by the azimuth accuracy.

In this paper, a hybrid intelligent multisensor positioning methodology is proposed, which integrates the information from low-cost sensors including GPS, MEMS-based IMU and electronic compass, and velocity constraint. An improved Kalman filter (KF) with sequential measurement-update processing is first developed to fuse different information. Further, a least square support vector machine- (LS SVM-) based intelligent module is designed and augmented with the improved KF to form the hybrid positioning system. The hybrid positioning system has two modes of operation, that is, the update and prediction modes, which are dependent on the situation of GPS outage. The novel aspects of this paper can be summarized as follows:

- (1) Instead of only integrating SINS and GPS as presented in much AI-based literature, the proposed hybrid positioning methodology has the capability of fusing more information from other low-cost sensors or sources through combining the improved KF with

the LS SVM-based intelligent algorithm. The distinct advantage is that, without adding much cost, the proposed methodology can significantly improve the positioning performance.

- (2) The proposed methodology exhibits strong generalization ability even when the GPS outage duration exceeds the length of the sliding window designed for training the LS SVM, which has been verified by real field tests. To the authors' knowledge, the performance of the prediction stability is seldom evaluated and discussed in such cases in the existing AI-based literature.

This paper is organized as follows. Section 2 gives an overview of the proposed hybrid positioning methodology for reliable land vehicle navigation. The improved Kalman filter is presented in detail in Section 3. Section 4 describes the design of LS SVM-based intelligent module. Field tests and experimental results are provided in Section 5. Section 6 is devoted to the conclusion.

2. Proposed Hybrid Positioning Methodology

To achieve accurate and reliable positioning performance for land vehicles, a hybrid positioning methodology of augmenting improved KF with LS SVM is proposed, which fuses the information from low-cost sensors including GPS, MEMS-based IMU and electronic compass, and lateral velocity constraint. The augmented system comprises two modes of operation, that is, the update mode and the prediction mode.

When GPS is available, the system operates in the update (i.e., training) mode, as shown in Figure 1(a). It is noted that the Kalman filter is improved here to work with sequential measurement-update processing, which is different from the centralized processing as utilized in the common KF. The measurement update 1 is associated with the measurements of electronic compass and “virtual” lateral velocity (i.e., $\tilde{V}_{y_k} = 0$), while the measurement update 2 is associated with GPS measurements. In actual implementation, two KF processing modules are executed. For the first module, simulated successive w -s GPS signal outages are applied (w denotes the duration of simulated GPS outage) and the position error estimated after the measurement update 1 can be attainable. For the second module, the GPS measurements are available at all times and thus the position error estimated after two sequential measurement updates 1 and 2 is always obtained. The difference between two position errors above at each epoch is then transferred to the LS SVM-based intelligent module as its desired output during the training process. Meanwhile, the time elapsed since GPS is unavailable and the IMU output act as the inputs of LS SVM at the same epoch. Besides, considering the balance between desired accuracy and computation efficiency, a 40 s length sliding window of input/output data pattern (i.e., the window size $w = 40$) is chosen for the adaptive optimization of LS SVM training algorithm.

FIGURE 1: Proposed hybrid positioning methodology. (a) The update mode; (b) the prediction mode.

Whenever the GPS signal is lost due to high buildings, tunnels, or other factors, the measurement update 2 cannot be executed but the measurement update 1 can still be carried out. In this case, the system operation automatically switches to the prediction mode shown in Figure 1(b), where the LS SVM-based intelligent module is utilized to predict the difference between the position error estimated after the measurement update 1 and that estimated after sequential measurement updates 1 and 2. Based on the time elapsed from the moment of GPS outage and the IMU output, the LS SVM uses the latest LS SVM parameters updated just before losing the satellite signals to predict the difference between two position errors mentioned above. Then, utilizing the available position error estimated after the measurement update 1, the position error contained in SINS can be acquired. Finally, such enhanced position error can be removed from the SINS position output to obtain more precise and reliable positioning solution for land vehicles.

3. Improved Kalman Filter

In this section, we describe the construction of the improved Kalman filter that is adopted for this work.

3.1. System State Equation. According to SINS mechanization, the dynamic error model of the SINS navigation parameters (i.e., position, velocity, and attitude) can be described as [19, 20]

$$\begin{aligned} \delta \dot{\mathbf{P}} &= \mathbf{D}_1 \delta \mathbf{P} + \mathbf{D}_2 \delta \mathbf{V}^n, \\ \delta \dot{\mathbf{V}}^n &= \mathbf{f}^n \times \boldsymbol{\psi}^n - (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{V}^n - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \\ &\quad \times \delta \mathbf{V}^n + \mathbf{C}_b^n \delta \mathbf{f}^b, \\ \dot{\boldsymbol{\psi}}^n &= \delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n - (\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \boldsymbol{\psi}^n - \mathbf{C}_b^n \delta \boldsymbol{\omega}_{ib}^b, \end{aligned} \quad (1)$$

where $\delta \mathbf{P} = [\delta L \ \delta \lambda \ \delta h]^T$ is the position error vector (latitude, longitude, and height), $\delta \mathbf{V}^n = [\delta V_E \ \delta V_N \ \delta V_U]^T$ is the velocity error vector (east, north, and up), $\boldsymbol{\psi}^n = [\psi_E \ \psi_N \ \psi_U]^T$ is the attitude error vector, $\mathbf{C}_b^n$ is the strap-down matrix, $\boldsymbol{\omega}_{ie}^n$ is the angular rate vector of the rotation of the Earth relative to the inertial frame, $\boldsymbol{\omega}_{en}^n$ is the angular rate vector of the rotation of the navigation frame relative to the Earth, $\mathbf{f}^n$ and $\mathbf{V}^n$ are specific force and velocity vectors in the navigation frame, respectively, $\delta \mathbf{f}^b$ and $\delta \boldsymbol{\omega}_{ib}^b$ are accelerometer bias and gyro drift vectors in the body frame, respectively, and $\mathbf{D}_1$ and $\mathbf{D}_2$ are two 3×3 matrices whose nonzero elements are functions of the vehicle's latitude (L) and height (h).

In addition, considering inertial sensor residuals, the state vector of Kalman filter (KF) is composed of fifteen states,

$$\mathbf{X} = [\delta L \quad \delta \lambda \quad \delta h \quad \delta V_E \quad \delta V_N \quad \delta V_U \quad \psi_E \quad \psi_N \quad \psi_U \quad \nabla_{bx} \quad \nabla_{by} \quad \nabla_{bz} \quad \varepsilon_{bx} \quad \varepsilon_{by} \quad \varepsilon_{bz}]^T, \quad (2)$$

where ∇_{bx}, ∇_{by} , and ∇_{bz} denote three accelerometer constant biases in the body frame, while $\varepsilon_{bx}, \varepsilon_{by}$, and ε_{bz} are three gyro constant drifts in the body frame.

Based on (1) and inertial sensor residual model [19, 20], the continuous-time system state equation of KF can be expressed as

$$\dot{\mathbf{X}}(t) = \mathbf{F}(t) \cdot \mathbf{X}(t) + \mathbf{G}(t) \cdot \mathbf{W}(t), \quad (3)$$

where $\mathbf{G}$ is the system noise input matrix, $\mathbf{W}$ is the system noise vector, and $\mathbf{F}$ is the system state propagation matrix. The definitions of $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{W}$ are given in Appendix.

According to the method described in [20], (3) is discretized. The discrete-time system state equation is

$$\mathbf{X}(k+1) = \Phi(k+1, k) \mathbf{X}(k) + \Gamma(k) \mathbf{W}(k), \quad (4)$$

where k refers to the discrete-time step, $\mathbf{X}(k)$ is the discrete-time state vector, $\Phi(k+1, k)$ is the discrete-time state transition matrix, $\Gamma(k)$ is the discrete system noise input matrix, and $\mathbf{W}(k)$ is the discrete system noise with covariance

which include nine navigation parameter errors above (i.e., position, velocity, and attitude), three accelerometer biases, and three gyro drifts:

matrix $\mathbf{Q}(k)$. How to determine $\Phi(k+1, k)$, $\Gamma(k)$, and $\mathbf{W}(k)$ is also given in Appendix.

3.2. Measurement Model. According to Figure 1, the KF measurements consist of three parts, that is, the measurement of electronic compass, the lateral velocity constraint, and the GPS measurements.

First, the measurement model about electronic compass-aided SINS can be built up according to the coordinate transformation theory [21]. Here, let H_C represent the vehicle's yaw angle measured by electronic compass and it can be described as

$$H_C = H_t - n_{H_C}, \quad (5)$$

where H_t denotes the true yaw angle and n_{H_C} is the compass measurement Gauss white noise. On the other hand, we suppose that H_S represents the yaw angle calculated by SINS and δH_S denotes the subtraction between H_S and H_t :

$$H_S = H_t + \delta H_S. \quad (6)$$

According to the inertial navigation principle, the relationship between the ideal strapdown matrix $\mathbf{C}_b^n$ and the true attitude angles is

$$\mathbf{C}_b^n = \begin{bmatrix} \cos H_t \cos P_t & \cos H_t \sin P_t \sin R_t - \sin H_t \cos R_t & \cos H_t \sin P_t \cos R_t + \sin H_t \sin R_t \\ \sin H_t \cos P_t & \sin H_t \sin P_t \sin R_t + \cos H_t \cos R_t & \sin H_t \sin P_t \cos R_t - \cos H_t \sin R_t \\ -\sin P_t & \cos P_t \sin R_t & \cos P_t \cos R_t \end{bmatrix}, \quad (7)$$

where P_t and R_t are true pitch and roll angles of the vehicle, respectively.

Correspondingly, the calculated strapdown matrix $\mathbf{C}_b^p$ (the symbol p denotes the SINS's calculated platform frame) can be described as

$$\mathbf{C}_b^p = \begin{bmatrix} \cos H_S \cos P_S & \cos H_S \sin P_S \sin R_S - \sin H_S \cos R_S & \cos H_S \sin P_S \cos R_S + \sin H_S \sin R_S \\ \sin H_S \cos P_S & \sin H_S \sin P_S \sin R_S + \cos H_S \cos R_S & \sin H_S \sin P_S \cos R_S - \cos H_S \sin R_S \\ -\sin P_S & \cos P_S \sin R_S & \cos P_S \cos R_S \end{bmatrix}, \quad (8)$$

where H_S , P_S , and R_S are the yaw, pitch, and roll angles of the vehicle calculated by SINS, respectively.

The relationship between $\mathbf{C}_b^p$ and $\mathbf{C}_b^n$ is [19]

$$\mathbf{C}_b^p = (\mathbf{I} - \boldsymbol{\psi}^n \times) \mathbf{C}_b^n, \quad (9)$$

where $\boldsymbol{\psi}^n \times$ is the skew-symmetric form of the vector $\boldsymbol{\psi}^n$. Expanding (9) in terms of each corresponding element, then we obtain two useful equations; that is,

$$\begin{aligned} \mathbf{C}_b^p(1, 1) &= \mathbf{C}_b^n(1, 1) + \mathbf{C}_b^n(2, 1) \psi_U - \mathbf{C}_b^n(3, 1) \psi_N, \\ \mathbf{C}_b^p(2, 1) &= -\mathbf{C}_b^n(1, 1) \psi_U + \mathbf{C}_b^n(2, 1) + \mathbf{C}_b^n(3, 1) \psi_E, \end{aligned} \quad (10)$$

where $\mathbf{C}_b^p(i, j)$ denotes the i -row and j -column element of $\mathbf{C}_b^p$ ($i = 1, 2, 3$ and $j = 1, 2, 3$). Further, based on (8), the following can be gotten:

$$\begin{aligned} H_S &= \arctan \left[\frac{\mathbf{C}_b^p(2, 1)}{\mathbf{C}_b^p(1, 1)} \right] \\ &= \arctan \left[\frac{\mathbf{C}_b^n(2, 1) - \mathbf{C}_b^n(1, 1) \psi_U + \mathbf{C}_b^n(3, 1) \psi_E}{\mathbf{C}_b^n(1, 1) + \mathbf{C}_b^n(2, 1) \psi_U - \mathbf{C}_b^n(3, 1) \psi_N} \right], \end{aligned} \quad (11)$$

where $\mathbf{C}_b^n(i, j)$ denotes the i -row and j -column element of $\mathbf{C}_b^n$ ($i = 1, 2, 3$ and $j = 1, 2, 3$).

Perturbing (11), the yaw-angle error equation can be obtained as

$$\delta H_S = \frac{\partial H_S}{\partial \psi_E} \psi_E + \frac{\partial H_S}{\partial \psi_N} \psi_N + \frac{\partial H_S}{\partial \psi_U} \psi_U, \quad (12)$$

where $\partial H_S / \partial \psi_E = \mathbf{C}_b^n(1, 1) \mathbf{C}_b^n(3, 1) / ([\mathbf{C}_b^n(1, 1)]^2 + [\mathbf{C}_b^n(2, 1)]^2)$, $\partial H_S / \partial \psi_N = \mathbf{C}_b^n(2, 1) \mathbf{C}_b^n(3, 1) / ([\mathbf{C}_b^n(1, 1)]^2 + [\mathbf{C}_b^n(2, 1)]^2)$, and $\partial H_S / \partial \psi_U = -1$.

Subtracting (5) from (6) and utilizing (12), we can get

$$H_S - H_C = \frac{\partial H_S}{\partial \psi_E} \psi_E + \frac{\partial H_S}{\partial \psi_N} \psi_N + \frac{\partial H_S}{\partial \psi_U} \psi_U + n_{H_C}. \quad (13)$$

Then, we will establish the measurement equation about lateral velocity constraint. It is well-known that, in general driving conditions, the vehicle does not slip or does slip very little. Thus, we can make the reasonable assumption that, for land vehicle positioning applications, the virtual measurement of lateral velocity is zero (i.e., lateral velocity constraint) [22]:

$$\tilde{V}_{Y_b} = 0, \quad (14)$$

where $\tilde{V}_{Y_b}$ denotes the virtually measured lateral velocity of the vehicle in the body frame. Besides, according to the coordinate transformation theory, we have

$$\tilde{V}_{Y_b} = [\mathbf{C}_n^b]_{2,1:3} \mathbf{V}^n - n_{V_{Y_b}}, \quad (15)$$

where $[\mathbf{C}_n^b]_{2,1:3}$ denotes a 1×3 new matrix composed of the 3 elements lying in the second row of $\mathbf{C}_n^b$. $\mathbf{V}^n = [V_E \ V_N \ V_U]^T$ is the true velocity vector of the vehicle in the navigation frame. $n_{V_{Y_b}}$ is the virtual measurement noise that represents the error in the assumption above.

Denoting the vehicle's velocity vector calculated by SINS in the navigation frame as $\hat{\mathbf{V}}^n = [\hat{V}_E \ \hat{V}_N \ \hat{V}_U]^T$, the following equation can be yielded:

$$\hat{V}_{Y_b} = [\mathbf{C}_p^b]_{2,1:3} \cdot \hat{\mathbf{V}}^n = [\mathbf{C}_p^b]_{2,1:3} \cdot (\mathbf{V}^n + \delta \mathbf{V}^n), \quad (16)$$

where $\hat{V}_{Y_b}$ represents the lateral velocity calculated by SINS in the body frame and $\delta \mathbf{V}^n = [\delta V_E \ \delta V_N \ \delta V_U]^T$ is the difference between $\hat{\mathbf{V}}^n$ and $\mathbf{V}^n$. $[\mathbf{C}_p^b]_{2,1:3}$ is a 1×3 new matrix composed of the 3 elements lying in the second row of $\mathbf{C}_p^b$.

Considering $\mathbf{C}_p^b = \mathbf{C}_n^b[\mathbf{I} + \boldsymbol{\psi}^n \times]$ and rearranging (16), we can obtain

$$\begin{aligned} \hat{V}_{Y_b} &= [\mathbf{C}_n^b]_{2,1:3} \mathbf{V}^n + [\mathbf{C}_n^b]_{2,1:3} \delta \mathbf{V}^n \\ &\quad + [\mathbf{C}_n^b(\boldsymbol{\psi}^n \times)]_{2,1:3} \mathbf{V}^n + [\mathbf{C}_n^b(\boldsymbol{\psi}^n \times)]_{2,1:3} \delta \mathbf{V}^n. \end{aligned} \quad (17)$$

Note that the last item of (17) is a second-order small quantity and can be neglected. Based on the skew-symmetric matrix features, we subtract (15) from (17) and then obtain

$$\begin{aligned} \hat{V}_{Y_b} - \tilde{V}_{Y_b} &= \hat{V}_{Y_b} - 0 \\ &= [\mathbf{C}_n^b]_{2,1:3} \delta \mathbf{V}^n - [\mathbf{C}_n^b(\mathbf{V}^n \times)]_{2,1:3} \boldsymbol{\psi}^n + n_{V_{Y_b}}, \end{aligned} \quad (18)$$

where $\mathbf{V}^n \times$ denotes the skew-symmetric matrix formed by the vector $\mathbf{V}^n$.

For convenient discussion later, combining (13) and (18), the measurement model about SINS aided by both electronic compass and lateral velocity constraint can be obtained:

$$\mathbf{Z}_1 = \mathbf{H}_1 \cdot \mathbf{X} + \mathbf{V}_1, \quad (19)$$

where $\mathbf{Z}_1 = [H_S - H_C \ \hat{V}_{Y_b} - 0]^T$ is the measurement information. $\mathbf{H}_1$ is the corresponding observation matrix, which can be determined according to (13) and (18). $\mathbf{V}_1$ is the measurement noise vector with the covariance matrix $\mathbf{R}_1$.

Finally, the measurement model about GPS-aided SINS can be easily built by subtracting the GPS measurement information from the SINS's corresponding output information. When GPS is available, it can provide accurate position and velocity information for the vehicle, that is, $\mathbf{P}_G$ and $\mathbf{V}_G$ shown in Figure 1(a). Thus, the measurement model of GPS-aided SINS can be expressed as

$$\mathbf{Z}_2 = \mathbf{H}_2 \cdot \mathbf{X} + \mathbf{V}_2, \quad (20)$$

where $\mathbf{Z}_2$ and $\mathbf{H}_2$ are GPS measurement information and observation matrix, respectively, and they are discussed in detail in [19]. $\mathbf{V}_2$ is the corresponding measurement noise vector with the covariance matrix $\mathbf{R}_2$.

Combining (19) and (20), the overall measurement equation of the KF can be written as

$$\mathbf{Z} = \mathbf{H} \cdot \mathbf{X} + \mathbf{V}, \quad (21)$$

where $\mathbf{Z}(k) = [\mathbf{Z}_1^T \ \mathbf{Z}_2^T]^T$ is the measurement vector of the KF. $\mathbf{H} = [\mathbf{H}_1^T \ \mathbf{H}_2^T]^T$ is the observation matrix. $\mathbf{V}(k) = [\mathbf{V}_1^T \ \mathbf{V}_2^T]^T$ is the observation noise vector with the covariance matrix $\mathbf{R} = \text{diag}(\mathbf{R}_1, \mathbf{R}_2)$.

3.3. Improved Kalman Filtering Algorithm. For the model described by the system state equation (4) and the measurement equation (21), we can execute the filtering fusion based on the standard KF algorithm, which comprises two phases, that is, *time propagation* and *measurement update*.

(1) *Time Propagation.* Utilizing (4), the equation for state prediction is

$$\mathbf{X}(k+1, k) = \Phi(k+1, k) \mathbf{X}(k). \quad (22)$$

The covariance of the state prediction $\mathbf{P}(k+1, k)$ is

$$\mathbf{P}(k+1, k) = \Phi(k+1, k) \mathbf{P}(k) \Phi(k+1, k)^T + \Gamma(k) \mathbf{Q}(k) \Gamma(k)^T. \quad (23)$$

(2) *Measurement Update.* The Kalman gain can be calculated as

$$\begin{aligned} \mathbf{K}(k+1) &= \mathbf{P}(k+1, k) \cdot \mathbf{H}^T(k+1) \\ &\cdot [\mathbf{H}(k+1) \mathbf{P}(k+1, k) \mathbf{H}^T(k+1) + \mathbf{R}(k+1)]^{-1}. \end{aligned} \quad (24)$$

The estimated state vector after measurement update is

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{X}(k+1, k) \\ &+ \mathbf{K}(k+1) [\mathbf{Z}(k+1) - \mathbf{H}(k+1) \cdot \mathbf{X}(k+1, k)] \end{aligned} \quad (25)$$

and the updated covariance is

$$\mathbf{P}(k+1) = [\mathbf{I} - \mathbf{K}(k+1) \mathbf{H}(k+1)] \mathbf{P}(k+1, k). \quad (26)$$

From (24)–(26), it can be seen that traditional measurement update processes all the measurement information in a centralized mode. The measurement-update results associated with different observation vectors (e.g., $\mathbf{Z}_1$ and $\mathbf{Z}$ here) cannot be acquired in such centralized mode, and thus it is difficult to realize the mechanization proposed in Section 2. Besides, the centralized processing mode has another limitation that it is only suitable for the situation when GPS is available. When the GPS signal is missing, the GPS measurements cannot be obtained and its corresponding measurement update should be excluded, whereas the measurement update associated with electronic compass

and lateral velocity constraint should still be executed. To overcome the limitations of traditional measurement-update processing, a sequential measurement-update processing algorithm is developed here.

To realize sequential measurement-update processing for different observation sources, that is, $\mathbf{Z}_1$ and $\mathbf{Z}_2$, we divide the matrices involved in the overall measurement update into two corresponding subblocks before the measurement update is executed, which is actually equivalent to scalar measurement processing discussed in [23, 24]. Namely, let

$$\begin{aligned} \mathbf{P}_0 &= \mathbf{P}(k+1, k), \\ \mathbf{X}_0 &= \mathbf{X}(k+1, k), \\ \mathbf{H}(k) &= \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}, \\ \mathbf{Z}(k) &= \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix}, \\ \mathbf{R} &= \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix}. \end{aligned} \quad (27)$$

Based on the division above, the measurement update is executed in a sequential order. That is, the measurement update about $\mathbf{Z}_1$ is performed first, and then the measurement update related to GPS (i.e., $\mathbf{Z}_2$) is performed. Specifically, although the time propagation is still executed according to (22)–(23), the measurement update is sequentially executed according to the following two steps.

Step 1. First, execute the measurement update about $\mathbf{Z}_1$,

$$\begin{aligned} \mathbf{K}_1(k+1) &= \mathbf{P}_0(k+1, k) \cdot \mathbf{H}_1^T(k+1) \\ &\cdot [\mathbf{H}_1(k+1) \mathbf{P}_0(k+1, k) \mathbf{H}_1^T(k+1) \\ &+ \mathbf{R}_1(k+1)]^{-1}, \end{aligned} \quad (28)$$

$$\begin{aligned} \mathbf{X}_1(k+1) &= \mathbf{X}_0(k+1, k) + \mathbf{K}_1(k+1) [\mathbf{Z}_1(k+1) \\ &- \mathbf{H}_1(k+1) \cdot \mathbf{X}_0(k+1, k)], \end{aligned}$$

$$\mathbf{P}_1(k+1) = [\mathbf{I} - \mathbf{K}_1(k+1) \mathbf{H}_1(k+1)] \mathbf{P}_0(k+1, k).$$

Step 2. Further, execute the measurement update about $\mathbf{Z}_2$,

$$\begin{aligned} \mathbf{K}_2(k+1) &= \mathbf{P}_1(k+1) \cdot \mathbf{H}_2^T(k+1) \\ &\cdot [\mathbf{H}_2(k+1) \mathbf{P}_1(k+1) \mathbf{H}_2^T(k+1) + \mathbf{R}_2(k+1)]^{-1} \end{aligned} \quad (29)$$

$$\begin{aligned} \mathbf{X}_2(k+1) &= \mathbf{X}_1(k+1) \\ &+ \mathbf{K}_2(k+1) [\mathbf{Z}_2(k+1) - \mathbf{H}_2(k) \cdot \mathbf{X}_1(k+1)] \end{aligned}$$

$$\mathbf{P}_2(k+1) = [\mathbf{I} - \mathbf{K}_2(k+1) \mathbf{H}_2(k+1)] \mathbf{P}_1(k+1).$$

In (28)-(29), $\mathbf{X}_1(k+1)$ and $\mathbf{P}_1(k+1)$ are the estimated state vector and covariance matrix after the measurement update about $\mathbf{Z}_1$, respectively, whereas $\mathbf{X}_2(k+1)$ and $\mathbf{P}_2(k+1)$ are the estimated state vector and covariance matrix after the measurement update about $\mathbf{Z}_2$, respectively. Besides, $\mathbf{X}_2(k+1)$ and $\mathbf{P}_2(k+1)$ also act as the initial values of time propagation of the next time $k+1$; that is, $\mathbf{X}(k+1) = \mathbf{X}_2(k+1)$ and $\mathbf{P}(k+1) = \mathbf{P}_2(k+1)$.

Note that the improved measurement-update processing described above is corresponding to the situation when GPS is available. In such situation, the measurement updates about $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are executed sequentially according to Steps 1 and 2 above. Thus, $\mathbf{X}_1$ and $\mathbf{X}_2$ can easily be obtained, which are utilized for the update mode as shown in Figure 1(a). When the GPS signal is lost, only the observation information about $\mathbf{Z}_1$ is executed and so only the measurement-update algorithm described by Step 1 is executed. In this case, $\mathbf{X}_1$ can still be acquired, which is utilized for the prediction mode as illustrated in Figure 1(b). Therefore, the improved measurement-update processing provides an effective and feasible solution for the proposed methodology.

4. Design of LS SVM-Based Intelligent Module

The least square support vector machine (LS SVM) is a statistical learning theory which adopts a least squares linear system as the loss function instead of the quadratic program [25, 26]. LS SVM is based on the principle of structure risk minimization, which minimizes an upper bound on the generalization error, and thus its generalization performance is superior to that of the common AI methods adopting empirical risk minimization [27, 28]. The excellent generalization ability of LS SVM has been verified to be very effective to solve both regression and classification problems, and it has also been successfully applied for solving many engineering problems.

4.1. LS SVM Algorithm. Consider a given training set of N data points $\{\mathbf{x}_k, y_k\}_{k=1}^N$ with input data $\mathbf{x}_k \in \mathbf{R}^n$ and output $y_k \in r$ where $\mathbf{R}^n$ is the n -dimensional vector space and r is the one-dimensional vector space. In the feature space, the LS SVM model takes the form

$$f(\mathbf{x}) = \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}) + b, \quad (30)$$

where $\boldsymbol{\omega} \in \mathbf{R}^{n_h}$ is an adjustable weight vector, $\boldsymbol{\varphi}(\cdot) : \mathbf{R}^n \rightarrow \mathbf{R}^{n_h}$ is the nonlinear function that maps the input data into a higher dimensional feature space, and $b \in r$ is the scalar threshold.

In the LS SVM for function estimation, the following optimization problem is formulated:

$$\begin{aligned} \min \quad & J(\boldsymbol{\omega}, b, e) = \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{\omega} + \frac{1}{2} \gamma \sum_{k=1}^N e_k^2 \\ \text{s.t.} \quad & y_k = \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + e_k, \quad k = 1, \dots, N, \end{aligned} \quad (31)$$

where e_k is the error variable and $\gamma \geq 0$ is a regularization constant. Smaller γ can avoid the overfitting problem in cases where data are noisy.

For (31), the Lagrangian is given by

$$\begin{aligned} L_{\text{LS SVM}} = & \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{\omega} + \frac{1}{2} \gamma \sum_{k=1}^N e_k^2 \\ & - \sum_{k=1}^N \alpha_k \{ \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + e_k - y_k \}, \end{aligned} \quad (32)$$

where $\alpha_k \in r$ ($k = 1, \dots, N$) are the Lagrange multipliers. Then, the conditions for optimality are

$$\begin{aligned} \frac{\partial L_{\text{LS SVM}}}{\partial \boldsymbol{\omega}} = 0 & \longrightarrow \boldsymbol{\omega} = \sum_{k=1}^N \alpha_k \boldsymbol{\varphi}(\mathbf{x}_k) \\ \frac{\partial L_{\text{LS SVM}}}{\partial b} = 0 & \longrightarrow \sum_{k=1}^N \alpha_k = 0 \\ \frac{\partial L_{\text{LS SVM}}}{\partial e_k} = 0 & \longrightarrow \alpha_k = \gamma e_k, \quad k = 1, \dots, N \\ \frac{\partial L_{\text{LS SVM}}}{\partial \alpha_k} = 0 & \longrightarrow \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + e_k - y_k = 0, \\ & k = 1, \dots, N. \end{aligned} \quad (33)$$

Eliminating $\boldsymbol{\omega}$ and e_k will yield a linear system instead of a quadratic programming problem

$$\begin{bmatrix} 0 & \mathbf{1}_N^T \\ \mathbf{1}_N & \boldsymbol{\Omega} + \frac{\mathbf{I}_N}{\gamma} \end{bmatrix} \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{Y} \end{bmatrix}, \quad (34)$$

where $\mathbf{Y} = [y_1, \dots, y_N]^T$, $\mathbf{1}_N = [1, \dots, 1]^T$, and $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_N]^T$. $\mathbf{I}_N$ is $N \times N$ identity matrix. $\boldsymbol{\Omega} \in \mathbf{R}^{N \times N}$ is the kernel matrix defined by

$$\Omega_{ij} = \boldsymbol{\varphi}(\mathbf{x}_i)^T \boldsymbol{\varphi}(\mathbf{x}_j) = K(\mathbf{x}_i, \mathbf{x}_j), \quad i, j = 1, \dots, N. \quad (35)$$

The radial basis function (RBF) has been used here as the kernel function $K(\cdot, \cdot)$, which is given by

$$K(\mathbf{x}, \mathbf{x}_k) = \exp \left\{ -\frac{\|\mathbf{x} - \mathbf{x}_k\|_2^2}{2\sigma^2} \right\}, \quad k = 1, \dots, N, \quad (36)$$

where $\|\mathbf{x} - \mathbf{x}_k\|_2^2$ is the squared Euclidean distance between the two feature vectors and σ is the width of RBF.

Further, the result of the LS SVM model for function estimation becomes

$$f(\mathbf{x}) = \sum_{k=1}^N \alpha_k K(\mathbf{x}, \mathbf{x}_k) + b, \quad (37)$$

where α_k and b are the solutions to (34). The design values of γ and σ can be determined during the training of LS SVM.

4.2. Input/Output Space Design of LS SVM. For the highly nonlinear input/output modeling problem discussed here, the LS SVM algorithm can provide an effective and viable solution. Besides, for land vehicle applications, the horizontal positioning performance is generally the main concern, as presented in a large body of literature [4, 9, 10, 12, 13]. Therefore, to simplify the LS SVM network structure and improve computational efficiency, the latitudinal and longitudinal components of the difference between the position error estimated after the measurement update 1 and that estimated after sequential measurement updates 1 and 2 will be selected as the outputs of LS SVM; that is,

$$\text{Outputs}_{\text{LS SVM}} = [\delta\tilde{L} \quad \delta\tilde{\lambda}], \quad (38)$$

where $\delta\tilde{L} = \mathbf{X}_1[1] - \mathbf{X}_2[1]$ is the latitudinal component and $\delta\tilde{\lambda} = \mathbf{X}_1[2] - \mathbf{X}_2[2]$ is the longitudinal component. $\mathbf{X}_1[1]$ and $\mathbf{X}_1[2]$ denote the first and second elements of the state vector $\mathbf{X}_1$, respectively, while $\mathbf{X}_2[1]$ and $\mathbf{X}_2[2]$ are the first and second elements of the state vector $\mathbf{X}_2$, respectively. As discussed above, $\mathbf{X}_i$ is the estimated state vector after the i th measurement update ($i = 1, 2$).

Also for simplifying the LS SVM structure, only the parameters that are the main factors affecting the outputs of LS SVM will be selected as its inputs. According to the mechanism of the inertial navigation, the fundamental error sources affecting the outputs (38) mainly include the measurement errors of IMU sensors, especially the longitudinal and lateral accelerometers and the yaw gyro and the time since the GPS signal is lost. Therefore, the inputs of LS SVM here are chosen as

$$\text{Inputs}_{\text{LS SVM}} = [f_x \quad f_y \quad \omega_z \quad T], \quad (39)$$

where f_x , f_y , and ω_z are the longitudinal and lateral accelerations and the yaw rate, respectively, which are output by IMU. T denotes the time elapsed since the GPS signal is lost. From (39), it can be found that the adopted inputs have the advantage that they are all easy to determine.

5. Field Tests and Results

To evaluate the performance of the hybrid intelligent multisensor positioning methodology proposed in this paper, field tests were conducted on a Buick Sail SRV vehicle. It was equipped with a low-cost NovAtel Superstar II GPS receiver with 1 Hz update rate, Crossbow MEMS-based IMU VG440CA-200 inertial sensors sampled at 100 Hz, and SDI electronic compass that provides the yaw angle measurement at 1 Hz. The low-cost IMU VG440CA-200 adopted here includes three gyroscopes and three accelerometers, which are all made based on MEMS technology. The sensor accuracies (1σ) are 0.05 m/s and 3 m for the GPS velocity and position when its signal is available and 0.15° for the yaw angle of electronic compass. The field tests were carried out along a representative route in an open environment, as shown in Figure 2. Since GPS signals were available all the time, the SINS/GPS/Electronic Compass/Velocity Constraint integrated solution based on KF can be obtained and utilized

FIGURE 2: Field test trajectory with 8 GPS outages indicated.

as a reference for performance evaluation. During the tests, all sensor data were recorded and then the positioning methods were evaluated using the logged data.

Figure 2 shows the trajectory of this selected test route, which includes typical driving conditions such as straight motion, curved driving, and 90° turns. Throughout the whole test, a sufficient number of satellites were available. To cover different driving conditions and dynamics and assess the positioning performance comprehensively, eight simulated 40 s GPS signal outages are intentionally introduced at eight typical locations along the trajectory, which are also marked in numeric labels and shown in Figure 2. The position error discussed in the following denotes the horizontal Euclidean distance error between the estimated position and the corresponding reference, which is the main concern for land vehicle positioning.

5.1. Performance Comparison between KF and Proposed Method. Table 1 gives a quantitative comparison of the RMS and maximum position errors between the traditional KF method and the proposed method for the simulated eight 40 s GPS outages. From Table 1, it is clear that the proposed hybrid positioning method outperforms the KF and offers much better accuracy for all GPS outages, except for outage 4. For outage 4, the maximum position error of KF is 17.97 m, which is better than that of the proposed method, that is, 21.18 m, but the difference between them is small and only about 3 m. Besides, as far as the RMS value of the position error is concerned, the proposed method is superior to the KF, which achieves 38.1% improvement in accuracy during this GPS outage. For the remaining seven outages, the proposed method achieves the significant improvement in the positioning accuracy over the KF method. The proposed method can maintain the maximum position errors within the range of 5~22 m (the RMS range is 3~11 m), as opposed to the range of 17~59 m (the RMS range is 12~26 m) achieved by the traditional KF. For the remaining seven outages, the average improvement is more than 56% whether for the RMS error or for the maximum error.

To further illustrate the improvements of the proposed method over the traditional KF method, three representative GPS outages, that is, 1, 5, and 8, are chosen to show their

TABLE 1: Statistics of position errors for KF and proposed method.

Outage number	RMS			Maximum		
	KF	Proposed	Improvement (%)	KF	Proposed	Improvement (%)
1	19.56	8.37	57.2	28.54	14.77	48.3
2	17.95	6.87	61.7	35.15	13.99	60.2
3	13.93	8.67	37.8	28.27	19.59	30.7
4	12.58	7.79	38.1	17.97	21.18	—
5	18.84	3.81	79.8	36.96	5.77	84.4
6	17.75	10.46	41.1	31.43	19.87	36.8
7	25.97	8.70	66.5	58.05	15.01	74.1
8	15.01	5.51	63.3	31.22	13.11	58.0

FIGURE 3: Positioning results during GPS outage 1.

FIGURE 4: Positioning results during GPS outage 5.

trajectories, as illustrated in Figures 3–5. Figure 3 shows their positioning results for GPS outage 1, which is introduced in a 90° turn where the driving maneuvers change greatly. During this GPS outage, the maximum value of the position error of traditional KF is 28.54 m, whereas that of the proposed method is reduced to 14.77 m, which means a 48.3% performance improvement. Figure 4 gives the positioning results for GPS outage 5, which is in a typical driving scenario where the vehicle goes through a straight-curved-straight road. The solution of the proposed method follows the true trajectory more closely and provides an approximately 80% better accuracy than the traditional KF for both the RMS and maximum values of the position error. Figure 5 shows the positioning results for GPS outage 8, during which the vehicle goes through a continuously curved road. Similarly, the proposed method provides more accurate positioning outputs than the traditional KF, with about 60% performance improvement.

5.2. Performance Evaluation in Longer GPS Outage Conditions. Note that, in Section 5.1, we only discuss and analyze

FIGURE 5: Positioning results during GPS outage 8.

FIGURE 6: Position errors for three GPS outages (extended to 50 s): (a) during outage 1; (b) during outage 5; (c) during outage 8.

the positioning performance for the 40 s GPS outage duration, which is equal to the designed sliding window size (i.e., 40 s). However, in practice, the period of GPS outage cannot be determined in advance. Therefore, the positioning performance should also be evaluated in conditions where the GPS outage time exceeds the size of the window for training to verify the generalization ability of the positioning system. Such evaluation is seldom discussed in the existing literature.

To assess the performance in longer GPS outage conditions, the size of the sliding window for training is kept unchanged but the GPS outage duration is extended from 40 s to 50 s. Besides, to compare with traditional neural network methods, the radial basis function (RBF) network as utilized in much literature is also designed. Specifically, the RBF method here denotes that the LS SVM module in Figure 1 is replaced by the RBF network with the same inputs/outputs and other modules in Figure 1 are kept unchanged. The model parameters of RBF are determined mainly via trial to achieve high training accuracy and good computational efficiency, which include 0.01 for the MSE (mean squared error) goal, 3

for the spread of radial basis functions, 40 for the maximum number of neurons, and 25 for the number of neurons to add between displays. Three typical GPS outages above, that is, 1, 5, and 8, are still chosen to be analyzed. Figure 6 shows the position errors for three extended GPS outages. Figures 7–9 illustrate the positioning results for these three extended GPS outages (i.e., 1, 5, and 8), respectively. In Figures 8–9, the locally enlarged plots are also shown.

From Figure 6, both the proposed and RBF methods can significantly enhance the positioning accuracy compared with the traditional KF when the GPS outage is less than 40 s. Moreover, they exhibit similar positioning performance within 40 s GPS outage. However, once the GPS outage is longer than 40 s, the prediction stability of RBF deteriorates, while that of the proposed method is still good and reliable, which is especially remarkable for outages 5 and 8, as shown in Figures 6(b)–6(c), 8, and 9. That is, the generalization performance of the proposed method is obviously superior to that of traditional RBF method. The main reason is that the overfitting problem easily occurs for the RBF, which cannot ensure the generalization ability of RBF once the prediction

FIGURE 7: Positioning results during GPS outage 1 (extended to 50 s).

FIGURE 9: Positioning results during GPS outage 8 (extended to 50 s).

FIGURE 8: Positioning results during GPS outage 5 (extended to 50 s).

data is beyond the space covered by the training sample. For example, for the extended GPS outage 5, the maximum value of the position error of RBF reaches 51.42 m as indicated in Figure 6(b), which is very close to that of KF, that is, 55.40 m. In contrast, the maximum value of the position error of the proposed method is only 12.36 m, which achieves an accuracy improvement of 76% and 78% over RBF and KF, respectively. Figure 8 clearly shows that the proposed method can provide more accurate positioning solution, which is much closer to the true trajectory.

6. Conclusions

To achieve reliable positioning performance even in adverse GPS environments, a hybrid intelligent multisensor positioning methodology is proposed for land vehicles. The proposed method can fuse the information from low-cost sensors including GPS, MEMS-based SINS, electronic compass, and lateral velocity constraint. For such information, the improved KF with sequential measurement-update processing is first developed to realize the filtering fusion. Further, the LS SVM-based intelligent module is designed and augmented with the improved KF to form the hybrid positioning system. When GPS works well, the LS SVM module is trained and updated using the recent input/output data. Once GPS outage occurs, it switches to predict the position error to achieve more accurate positioning performance.

The field tests on typical roads have been performed for evaluation of the proposed methodology. The experimental results verify that the proposed method can significantly improve the positioning accuracy over the traditional KF method for most conditions under GPS outages. In addition, it has shown much stronger prediction stability than other AI-based methods such as RBF, which indicates that it can achieve more satisfactory generalization ability.

Appendix

Consider

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{PP} & \mathbf{F}_{PV} & \mathbf{F}_{P\psi} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{F}_{VP} & \mathbf{F}_{VV} & \mathbf{F}_{V\psi} & \mathbf{C}_b^n & \mathbf{0}_{3 \times 3} \\ \mathbf{F}_{\psi P} & \mathbf{F}_{\psi V} & \mathbf{F}_{\psi\psi} & \mathbf{0}_{3 \times 3} & -\mathbf{C}_b^n \\ \mathbf{0}_{6 \times 3} & \mathbf{0}_{6 \times 3} & \mathbf{0}_{6 \times 3} & \mathbf{0}_{6 \times 3} & \mathbf{0}_{6 \times 3} \end{bmatrix}, \quad (\text{A.1})$$

where all the subcomponents in $\mathbf{F}$ are discussed in detail in [19]. For instance,

$$\begin{aligned} \mathbf{F}_{\psi P} &= \begin{bmatrix} 0 & 0 & \frac{V_N}{(R_n + h)^2} \\ -\omega_{ie} \sin L & 0 & \frac{-V_E}{(R_e + h)^2} \\ \omega_{ie} \cos L + \frac{V_E \sec^2 L}{R_e + h} & 0 & \frac{-V_E \tan L}{(R_e + h)^2} \end{bmatrix}, \\ \mathbf{F}_{\psi V} &= \begin{bmatrix} 0 & \frac{-1}{R_n + h} & 0 \\ \frac{1}{R_e + h} & 0 & 0 \\ \frac{\tan L}{R_e + h} & 0 & 0 \end{bmatrix}, \\ \mathbf{F}_{\psi\psi} &= \begin{bmatrix} 0 & \omega_{ie} \sin L + \frac{V_E \tan L}{R_e + h} & -\left(\omega_{ie} \cos L + \frac{V_E}{R_e + h}\right) \\ -\left(\omega_{ie} \sin L + \frac{V_E \tan L}{R_e + h}\right) & 0 & -\frac{V_N}{R_n + h} \\ \omega_{ie} \cos L + \frac{V_E}{R_e + h} & \frac{V_N}{R_n + h} & 0 \end{bmatrix}, \end{aligned} \quad (\text{A.2})$$

where $R_n = a(1 - 2f + 3f \sin^2 L)$, $R_e = a(1 + f \sin^2 L)$. a is the Earth semimajor axis length, and f is the Earth ellipsoid oblateness. ω_{ie} is the Earth rotation rate. L is the latitude of the vehicle location, and h is the altitude of the vehicle location. V_E and V_N are the east and north velocity components, respectively. One has

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{C}_b^n & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & -\mathbf{C}_b^n \\ \mathbf{0}_{6 \times 3} & \mathbf{0}_{6 \times 3} \end{bmatrix} \quad (\text{A.3})$$

$$\mathbf{W} = [w_{ax} \ w_{ay} \ w_{az} \ w_{gx} \ w_{gy} \ w_{gz}],$$

where w_{ax} , w_{ay} , and w_{az} denote three accelerometer white noises, while w_{gx} , w_{gy} , and w_{gz} are three gyro white noises. One has

$$\Phi(k+1, k) \approx \mathbf{I} + \mathbf{F}(t_k)(t_{k+1} - t_k), \quad (\text{A.4})$$

where t_{k+1} and t_k are the discrete time. Consider

$$\begin{aligned} \Gamma(k) &\approx \mathbf{G}(t_k)(t_{k+1} - t_k) \\ \mathbf{W}(k) &= \frac{1}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} \mathbf{W}(t) dt. \end{aligned} \quad (\text{A.5})$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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