Research Article

LMI-Based Stability Criterion for Impulsive Delays Markovian Jumping Time-Delays Reaction-Diffusion BAM Neural Networks via Gronwall-Bellman-Type Impulsive Integral Inequality

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Lyapunov stability theory, variational methods, Gronwall-Bellman-type inequalities theorem, and linear matrices inequality (LMI) technique are synthetically employed to obtain the LMI-based global stochastic exponential stability criterion for a class of time-delays Laplace diffusion stochastic equations with large impulsive range under Dirichlet boundary value, whose backgrounds of physics and engineering are the impulsive Markovian jumping time-delays reaction-diffusion BAM neural networks. As far as the authors know, it is the first time to derive the LMI-based criterion by way of Gronwall-Bellman-type inequalities, which can be easily and efficiently computed by computer Matlab LMI toolbox. And the obtained criterion improves the allowable upper bounds of impulse against those of some previous related literature. Moreover, a numerical example is presented to illustrate the effectiveness of the proposed methods.

1. Introduction

In this paper, we consider the stability of a class of time-delays Laplace diffusion stochastic equations with large impulsive range under Dirichlet boundary value, whose backgrounds of physics and engineering are the impulsive Markovian jumping time-delays reaction-diffusion bidirectional associative memory (BAM) neural networks. In 1987, Kosko [1] introduced originally the BAM neural networks model. Owing to its generalization of the single-layer autoassociative Hebbian correlation to two-layer pattern-matched heteroassociative circuits, the BAM neural networks have been proved to have widespread applications in many areas, such as pattern recognition, automatic control, signal and image processing, artificial intelligence, and parallel computation and optimization problems. Generally, an important precondition of the applications mentioned above is that the equilibrium of the BAM neural networks should be stable to some extent. So the stability analysis for neural networks has been attracting wide publicity ([2–6] and their references therein).

In the real world, the neural networks are often disturbed by environmental noise. The noise may influence the stability of the equilibrium and vary some structure parameters, which usually satisfies the Markov process [7–14]. In addition, diffusion effect exists really in the neural networks when electrons are moving in asymmetric electromagnetic fields. Thereby, reaction-diffusion factor should be considered in any neural networks model [2, 4–7, 15–17]. Besides, time-delays also occur unavoidably owing to the finite switching speed of neurons and amplifiers. On the other hand, impulsive effect inevitably exists in the practical neural networks, affects dynamical behaviors of the systems, and even influences the stability. And hence, the stability of impulsive neural networks was widely investigated [5, 6, 10, 15, 16, 18]. However, there exist some harsh conditions on the impulsive range. For example, in [10, Theorem 3.1], [9, Theorem 1], [15, Theorem 1], [3, Theorem 4.1], [4, Theorem 3.2], and [19, Theorem 3.2], their impulsive condition is similar to

$$C_{kr}^T P_i C_{kr} < P_l. \quad (1)$$

Recently, Gronwall inequalities, Gronwall-Bellman-type inequalities, and their applications have attracted abundant interests [16, 20–23]. In this paper, we will synthetically employ Lyapunov stability theory, variational methods, Gronwall-Bellman-type inequalities theorem, and linear matrices inequality (LMI) technique to derive the LMI-based global stochastic exponential stability criterion for Markovian jumping time-delays reaction-diffusion BAM neural networks with large impulsive range allowable. The main purpose of this paper is to improve the allowable upper bounds of impulse. In our new stability criterion, the harsh condition (1) is unnecessary.

This paper is organized as follows. In Section 2, the new BAM neural network model is formulated, and some necessary preparation knowledge is provided. In Section 3, we firstly employ variational methods to obtain an inequalities lemma and then use the Lyapunov functional method and Schur Complement technique to deduce a LMI-based exponential stability criterion. In Section 4, an example is provided to illustrate the effectiveness of the proposed methods. In the end, Section 5 contains some conclusions of this paper.

Remark 1. In [24, 25], a class of delay differential inequalities ([24, Lemma 2.2] and [25, Lemma 3]) were employed to obtain the stability criteria for deterministic systems. However, a stability criterion of Markovian jumping stochastic system can be obtained via Gronwall-Bellman-type impulsive integral inequality in this paper. To some extent, the restrictive conditions of Gronwall-Bellman-type impulsive integral inequality lemma are simpler than those of the delay differential inequality lemmas ([24, Lemma 2.2] and [25, Lemma 3]), for there are some advantages of the utilization of Gronwall-Bellman-type inequalities (e.g., the allowable upper bounds of time-delays), which will be illustrated in Numerical Example.

2. Model Description and Preliminaries

In 1987, the bidirectional associative memory (BAM) neural networks were introduced by Kosko (see [1, 26]). He set up the following mathematical model [1]:

$$\begin{aligned}\dot{x}_i &= -a_i x_i(t) + \sum_{j=1}^p w_{ji} g_j(y_j(t)) + I_i, \quad i = 1, 2, \dots, n, \\ \dot{y}_j &= -b_j y_j(t) + \sum_{i=1}^n v_{ij} g_i(x_i(t)) + J_j, \quad j = 1, 2, \dots, n.\end{aligned}\quad (2)$$

We can know from the above model that it has generalized the single-layer autoassociative Hebbian correlator to two-layer pattern-matched heteroassociative circuits. Neurons are placed in the two layers, and neurons of the same layer are not connected while the neurons of the different layers are connected. There exists the bidirectional information transfer between the two layers of neurons. Such class of networks has wide applications in many fields such as pattern recognition, associative memory, and artificial intelligence. But the important precondition of these applications is that the system should be stable. So Kosko proved the stability of the above

model in [1]. Since then, the stability analysis of the BAM neural network becomes the most active area of research ([2–6] and their references therein). For a power system of signal transmission, there inevitably is a time lag problem. And the existence of time-delay often results in unstable phenomenon of a network system. Besides, in the real world, impulsive phenomena exist in the process of changing dynamic behaviors. In order to give an exact description of these process, adoption of delay impulsive differential equations for BAM neural networks is a more effective method (see [3, 4, 27–29] and their references therein). In addition, the BAM neural networks are often disturbed by environmental noise. The noise may influence the stability of the equilibrium and vary some structure parameters, which usually satisfies the Markov process ([30, 31] and their references therein). And diffusion effect exists really in the BAM neural networks when electrons are moving in asymmetric electromagnetic fields ([2–6] and their references therein).

Often we assume that there exists an equilibrium point for BAM neural networks. By using the translation transform regarding the equilibrium point, we can actually consider the following BAM neural networks which owns the null solution as its equilibrium point:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \nabla \cdot (D(t, x, u) \circ \nabla u) - A(r(t)) u(t, x) \\ &\quad + W_1(r(t)) f_1(v(t, x)) \\ &\quad + W_2(r(t)) f_2(v(t - \tau(t), x)), \\ &\quad x \in \Omega, \quad t \neq t_k, \\ \frac{\partial v}{\partial t} &= \nabla \cdot (\mathcal{D}(t, x, v) \circ \nabla v) - B(r(t)) v(t, x) \\ &\quad + \Gamma_1(r(t)) g_1(u(t, x)) \\ &\quad + \Gamma_2(r(t)) g_2(u(t - \tau(t), x)), \\ &\quad x \in \Omega, \quad t \neq t_k, \\ \Delta u(t_k, x) &= (C_k(r(t)) - I) u(t_k, x), \Delta v(t_k, x) \\ &= (\mathcal{C}_k(r(t)) - I) v(t_k, x), \\ &\quad x \in \Omega, \quad k = 1, 2, \dots\end{aligned}\quad (3)$$

equipped with the initial condition

$$\begin{aligned}u(s, x; t_0, \varphi) &= \varphi(s, x), \\ v(s, x; t_0, \psi) &= \psi(s, x), \\ t_0 - \tau &\leq s \leq t_0, \quad x \in \Omega,\end{aligned}\quad (4)$$

and the Dirichlet boundary value condition

$$\begin{aligned}u(s, x; t_0, \varphi) &= 0, \\ v(s, x; t_0, \psi) &= 0, \\ x &\in \partial\Omega, \quad t \geq t_0,\end{aligned}\quad (5)$$

where $x \in \Omega$, Ω is a bounded domain in R^m with a smooth boundary $\partial\Omega$ of class $\mathcal{C}^2$ by Ω (see, e.g., [32]), $u = u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$, $v = v(t, x) = (v_1(t, x), v_2(t, x), \dots, v_n(t, x))^T \in R^n$, and $u_i(t, x)$ and $v_j(t, x)$ are state variables of the i th neuron and the j th neuron at time t and in space variable x . $f_i(v) = f_i(v(t, x)) = (f_1^{(i)}(v_1(t, x)), \dots, f_n^{(i)}(v_n(t, x)))^T$, $g_i(u) = g_i(u(t, x)) = (g_1^{(i)}(u_1(t, x)), \dots, g_n^{(i)}(u_n(t, x)))^T$, $i = 1, 2$, and $f_j^{(i)}(u_j(t, x))$ and $g_j^{(i)}(u_j(t, x))$ are neuron activation functions of the j th unit at time t and in space variable x . $\Delta u(t_k, x) = u(t_k^+, x) - u(t_k, x)$ with $u(t_k^-, x) = u(t_k, x)$ and $\Delta v(t_k, x) = v(t_k^+, x) - v(t_k, x)$ with $v(t_k^-, x) = v(t_k, x)$. $(D(t, x, u) \circ \nabla u)$ is Hadamard product of matrices $(D(t, x, u))_{n \times m}$ and ∇u . The definition of Hadamard product may be seen in [7]. $(\widehat{\Omega}, \Upsilon, \mathbb{P})$ is the given probability space where $\widehat{\Omega}$ is sample space, Υ is σ -algebra of subset of the sample space, and $\mathbb{P}$ is the probability measure defined on Υ . Let $S = \{1, 2, \dots, N\}$ and the random form process $\{r(t), t \in [0, +\infty)\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories with generator $\Pi = (\pi_{ij})_{N \times N}$ and transition probability from mode i at time t to mode j at time $t + \Delta t$, $i, j \in S$,

$$\begin{aligned} \mathbb{P}(r(t + \delta) = j \mid r(t) = i) \\ = \begin{cases} \pi_{ij}\delta + o(\delta), & j \neq i \\ 1 + \pi_{ii}\delta + o(\delta), & j = i, \end{cases} \end{aligned} \quad (6)$$

where $\pi_{ij} \geq 0$ is transition probability rate from i to j ($j \neq i$) and $\pi_{ii} = -\sum_{j=1, j \neq i}^S \pi_{ij}$, $\delta > 0$, and $\lim_{\delta \rightarrow 0} o(\delta)/\delta = 0$. For any given $r(t) = r \in S$, we denote $A_r = A(r(t))$, $B_r = B(r(t))$, $W_{ir} = W_i(r(t))$, and $\Gamma_{ir} = \Gamma_i(r(t))$ for $i = 1, 2$. Define the diagonal matrices $C_{kr} = C_k(r(t))$ and $\mathcal{C}_{kr} = \mathcal{C}_k(r(t))$. That is, $C_{kr} = \text{diag}(c_{kr}^{(1)}, c_{kr}^{(2)}, \dots, c_{kr}^{(n)})$ and $\mathcal{C}_{kr} = \text{diag}(\bar{c}_{kr}^{(1)}, \bar{c}_{kr}^{(2)}, \dots, \bar{c}_{kr}^{(n)})$. Also, A_r and B_r both are diagonal matrices. Time-delays $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) < 1 - 1/h$ with $h > 0$.

There exist positive definite diagonal matrices H_i and $\mathcal{H}_i$ such that

$$\begin{aligned} |f_i(u) - f_i(v)| &\leq H_i |u - v|; \\ |g_i(u) - g_i(v)| &\leq \mathcal{H}_i |u - v|, \end{aligned} \quad (7)$$

$$\text{for } u, v \in R^n, i = 1, 2,$$

where we denote $|u| = (|u_1|, |u_2|, \dots, |u_n|)^T$, $|f_i(u)| = (|f_1^{(i)}(u_1)|, |f_2^{(i)}(u_2)|, \dots, |f_n^{(i)}(u_n)|)^T$, and $|g_i(u)| = (|g_1^{(i)}(u_1)|, |g_2^{(i)}(u_2)|, \dots, |g_n^{(i)}(u_n)|)^T$.

Throughout this paper, we assume that $f_i(0) = g_i(0) = 0$ for $i = 1, 2$. Then $u = 0$ and $v = 0$ is the null solution for problem (3)–(5).

Definition 2. For symmetric matrices A and B , one denotes $A < B$ or $B > A$ if matrix $B - A$ is a positive definite matrix. Particularly, $A > 0$ if symmetric matrix A is a positive definite matrix.

Let $u(t, x; \varphi, \psi, i_0)$ and $v(t, x; \varphi, \psi, i_0)$ denote the state trajectory from the initial condition $r(t_0) = i_0$,

$u(t_0 + \theta, x; \varphi, \psi) = \varphi(\theta, x)$, and $v(t_0 + \theta, x; \varphi, \psi) = \psi(\theta, x)$ on $t_0 - \tau \leq \theta \leq t_0$ in $L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n)$. Here, $L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n)$ denotes the family of all $\mathcal{F}_0$ -measurable $\mathcal{C}([-\tau, 0] \times \Omega; R^n)$ -value random variable $\xi = \{\xi(\theta, x) : t_0 - \tau \leq \theta \leq t_0, x \in \Omega\}$ such that $\sup_{t_0 - \tau \leq \theta \leq t_0} \mathbb{E}\|\xi(\theta)\|^2 < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$.

Definition 3. The equilibrium point $u = 0$ and $v = 0$ of problem (3)–(5) is said to be globally stochastically exponential stability if, for every initial condition $\varphi, \psi \in L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n)$, $r(t_0) = i_0$, there exist scalars $c_1 > 0$, $c_2 > 0$, $\beta > 0$, and $\gamma > 0$ such that, for any solution $u(t, x; \varphi, \psi, i_0)$, $v(t, x; \varphi, \psi, i_0)$,

$$\begin{aligned} \mathbb{E}(\|u(t, x; \varphi, \psi, i_0)\|^2) + \mathbb{E}(\|v(t, x; \varphi, \psi, i_0)\|^2) \\ \leq \gamma e^{-\beta(t-t_0)} [c_1 \mathbb{E}(\|\varphi\|_\tau^2) + c_2 \mathbb{E}(\|\psi\|_\tau^2)], \quad t \geq t_0, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \|u(t, x; \varphi, \psi, i_0)\|^2 &= \int_{\Omega} u^T(t, x; \varphi, \psi, i_0) u(t, x; \varphi, \psi, i_0) dx, \\ \|v(t, x; \varphi, \psi, i_0)\|^2 &= \int_{\Omega} v^T(t, x; \varphi, \psi, i_0) v(t, x; \varphi, \psi, i_0) dx, \\ \|\varphi\|_\tau^2 &= \sup_{t_0 - \tau \leq s \leq t_0} \int_{\Omega} \varphi^T(s, x) \varphi(s, x) dx, \\ \|\psi\|_\tau^2 &= \sup_{t_0 - \tau \leq s \leq t_0} \int_{\Omega} \psi^T(s, x) \psi(s, x) dx. \end{aligned} \quad (9)$$

Throughout this paper, we define $\lambda_1 = \min\{\int_{\Omega} |\nabla u|^2 dx : u \in W_0^{1,2}(\Omega), \int_{\Omega} |u|^2 dx = 1\}$, the first eigenvalue of $-\Delta$ in Sobolev space $W_0^{1,2}(\Omega)$ [33].

Lemma 4 ((see [8]) (Gronwall-Bellman-type impulsive integral inequality)). Assume that

- (1) the sequence $\{t_k\}$ satisfies $0 \leq t_0 < t_1 < t_2 < \dots$ with $\lim_{k \rightarrow \infty} t_k = \infty$,
- (2) $q \in PC^1[R^+, R]$ and $q(t)$ is left-continuous at t_k , $k = 1, 2, \dots$,
- (3) $p \in C[R^+, R^+]$ and for $k = 1, 2, \dots$,

$$q(t) \leq c + \int_{t_0}^t p(s) q(s) ds + \sum_{t_0 < t_k < t} \eta_k q(t_k), \quad t \geq t_0, \quad (10)$$

where $\eta_k \geq 0$ and $c = \text{const}$.

Then

$$q(t) \leq c \prod_{t_0 < t_k < t} (1 + \eta_k) e^{\int_{t_0}^t p(s) ds}, \quad t \geq t_0. \quad (11)$$

Lemma 5 (see [34]). Let $x \in R^n$, $y \in R^n$, and $\varepsilon > 0$. Then one has

$$x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y. \quad (12)$$

Lemma 6 (Schur Complement [35]). Given matrices $\mathcal{Q}(t)$, $\mathcal{S}(t)$, and $\mathcal{R}(t)$ with appropriate dimensions, where $\mathcal{Q}(t) = \mathcal{Q}(t)^T$, $\mathcal{R}(t) = \mathcal{R}(t)^T$, then

$$\begin{pmatrix} \mathcal{Q}(t) & \mathcal{S}(t) \\ \mathcal{S}^T(t) & \mathcal{R}(t) \end{pmatrix} > 0, \quad (13)$$

if and only if

$$\mathcal{R}(t) > 0, \quad \mathcal{Q}(t) - \mathcal{S}(t) \mathcal{R}^{-1}(t) \mathcal{S}^T(t) > 0 \quad (14)$$

or

$$\mathcal{Q}(t) > 0, \quad \mathcal{R}(t) - \mathcal{S}^T(t) \mathcal{Q}^{-1}(t) \mathcal{S}(t) > 0, \quad (15)$$

where $\mathcal{Q}(t)$, $\mathcal{S}(t)$, and $\mathcal{R}(t)$ are dependent on t .

3. Main Results

Throughout this paper, we define $\lambda_1 = \min\{\int_{\Omega} |\nabla u|^2 dx : u \in H_0^1(\Omega), \int_{\Omega} |u|^2 dx = 1\}$, the first eigenvalue of $-\Delta$ in Sobolev space $H_0^1(\Omega)$ [18, 33].

Lemma 7. Let $P_r = \text{diag}(p_{r1}, p_{r2}, \dots, p_{rn})$ be a positive definite matrix, $\underline{\alpha} > 0$ with $\underline{\alpha} I \leq P_r$, and let u and v be a solution of problem (3)–(5). Then one has

$$\begin{aligned} \int_{\Omega} u^T P_r \nabla \cdot (D \circ \nabla u) dx &\leq -\lambda_1 \underline{\alpha} d \|u\|^2, \\ \int_{\Omega} v^T P_r \nabla \cdot (\mathcal{D} \circ \nabla v) dx &\leq -\lambda_1 \underline{\alpha} \tilde{d} \|v\|^2, \end{aligned} \quad (16)$$

where $d = \min_{i,j} \{\inf_{[t_0, +\infty) \times \Omega \times R} D_{ij}(t, x, u)\}$ and $\tilde{d} = \min_{i,j} \{\inf_{[t_0, +\infty) \times \Omega \times R} \mathcal{D}_{ij}(t, x, v)\}$.

Proof. Since u and v is a solution of problem (3)–(5), it follows by Gauss formula and the Dirichlet boundary condition that

$$\begin{aligned} &\int_{\Omega} u^T P_r (\nabla \cdot (D \circ \nabla u)) dx \\ &= \int_{\Omega} u^T P_r \left(\sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{1k} \frac{\partial u_1}{\partial x_k} \right), \dots, \right. \\ &\quad \left. \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{nk} \frac{\partial u_n}{\partial x_k} \right) \right)^T dx \\ &= - \sum_{k=1}^m \sum_{j=1}^n \int_{\Omega} p_{rj} D_{jk} \left(\frac{\partial u_j}{\partial x_k} \right)^2 dx \leq -\lambda_1 \underline{\alpha} d \|u\|^2. \end{aligned} \quad (17)$$

Similarly, we can prove that $\int_{\Omega} v^T P_r \nabla \cdot (\mathcal{D} \circ \nabla v) dx \leq -\lambda_1 \underline{\alpha} \tilde{d} \|v\|^2$. Thus, the proof is completed. $\square$

Theorem 8. Assume that there are positive scalars μ and α , satisfying $\inf_{k=1,2,\dots} (t_k - t_{k-1}) \geq \mu$ and

$$\begin{aligned} -\sqrt{1+\alpha} I &\leq C_{kr} \leq \sqrt{1+\alpha} I, \\ -\sqrt{1+\alpha} I &\leq \mathcal{C}_{kr} \leq \sqrt{1+\alpha} I, \end{aligned} \quad (18)$$

$$r \in S, \quad k = 1, 2, \dots$$

Assume, in addition, that there exist positive definite matrices $P_r = \text{diag}(p_{r1}, p_{r2}, \dots, p_{rn})$, $r \in S$, and positive scalars $\underline{\alpha}$, $\bar{\alpha}$, β_1 , and β_2 such that the following LMIs conditions are satisfied:

$$\begin{pmatrix} (-2\lambda_1 d \underline{\alpha} + \beta_1) I - 2P_r A_r & P_r W_{1r} & P_r W_{2r} & \mathcal{H}_1 \\ * & -I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \quad \forall r \in S, \quad (19)$$

$$\begin{pmatrix} (-2\lambda_1 \tilde{d} \underline{\alpha} + \beta_1) I - 2P_r B_r & P_r \Gamma_{1r} & P_r \Gamma_{2r} & H_1 \\ * & -I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \quad \forall r \in S, \quad (20)$$

$$\underline{\alpha} I \leq P_r, \quad \forall r \in S, \quad (21)$$

$$P_r \leq \bar{\alpha} I, \quad \forall r \in S, \quad (22)$$

$$\begin{pmatrix} -\beta_2 I & H_2 \\ * & -I \end{pmatrix} < 0, \quad (23)$$

$$\begin{pmatrix} -\beta_2 I & \mathcal{H}_2 \\ * & -I \end{pmatrix} < 0, \quad (24)$$

where I is the identity matrix, $d = \min_{i,j} \{\inf_{[t_0, +\infty) \times \Omega \times R} D_{ij}(t, x, u)\}$, $\tilde{d} = \min_{i,j} \{\inf_{[t_0, +\infty) \times \Omega \times R} \mathcal{D}_{ij}(t, x, v)\}$, and

$$\frac{\beta_1}{\bar{\alpha}} - \frac{\ln(1+\alpha)}{\mu} - \sum_{j \in S} \frac{\pi_{rj} \bar{\alpha}}{\underline{\alpha}} - \frac{\beta_2 h e^{y^T}}{\underline{\alpha}} > 0, \quad \forall r \in S. \quad (25)$$

Then the equilibrium point $u = 0$ and $v = 0$ of problem (3)–(5) is globally stochastically exponential stability.

Proof. Consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t, r) &= e^{y(t-t_0)} \int_{\Omega} \left(u^T(t, x) P(r(t)) u(t, x) \right. \\ &\quad \left. + v^T(t, x) P(r(t)) v(t, x) \right) dx, \quad \forall r(t) = r \in S. \end{aligned} \quad (26)$$

Let L be the weak infinitesimal operator such that

$$\begin{aligned}
 LV(t, r) = & e^{\gamma(t-t_0)} \int_{\Omega} \left[u^T \left(\gamma P_r + \sum_{j \in S} \pi_{rj} P_j \right) u \right. \\
 & + v^T \left(\gamma P_r + \sum_{j \in S} \pi_{rj} P_j \right) v \Big] dx \\
 & + e^{\gamma(t-t_0)} \int_{\Omega} \left[-2 \sum_{k=1}^m \sum_{j=1}^n p_{rj} D_{jk} \left(\frac{\partial u_j}{\partial x_k} \right)^2 \right. \\
 & - 2u^T P_r A_r u + f_1^T(v) W_{1r}^T P_r u + u^T P_r W_{1r} f_1(v) \\
 & + f_2^T(v(t-\tau(t), x)) W_{2r}^T P_r u \\
 & + u^T P_r W_{2r} f_2(v(t-\tau(t), x)) \\
 & - 2 \sum_{k=1}^m \sum_{j=1}^n p_{rj} \mathcal{D}_{jk} \left(\frac{\partial v_j}{\partial x_k} \right)^2 - 2v^T P_r B_r v \\
 & + g_1^T(u) \Gamma_{1r}^T P_r v + v^T P_r \Gamma_{1r} g_1(u) \\
 & + g_2^T(u(t-\tau(t), x)) \Gamma_{2r}^T P_r v \\
 & \left. + v^T P_r \Gamma_{2r} g_2(u(t-\tau(t), x)) \right] dx. \quad (27)
 \end{aligned}$$

In view of

$$\int_{\Omega} u^T P_j u dx \leq \bar{\alpha} \int_{\Omega} u^T I u dx \leq \bar{\alpha} \int_{\Omega} u^T \frac{1}{\underline{\alpha}} P_r u dx, \quad (28)$$

we have

$$\begin{aligned}
 LV(t, r) \leq & \left(\gamma + \sum_{j \in S} \pi_{rj} \frac{\bar{\alpha}}{\underline{\alpha}} \right) V(t, r) \\
 & + e^{\gamma(t-t_0)} \int_{\Omega} \left[-2 \sum_{k=1}^m \sum_{j=1}^n p_{rj} D_{jk} \left(\frac{\partial u_j}{\partial x_k} \right)^2 \right. \\
 & - 2u^T P_r A_r u + f_1^T(v) W_{1r}^T P_r u + u^T P_r W_{1r} f_1(v) \\
 & + f_2^T(v(t-\tau(t), x)) W_{2r}^T P_r u \\
 & + u^T P_r W_{2r} f_2(v(t-\tau(t), x)) \\
 & - 2 \sum_{k=1}^m \sum_{j=1}^n p_{rj} \mathcal{D}_{jk} \left(\frac{\partial v_j}{\partial x_k} \right)^2 - 2v^T P_r B_r v \\
 & + g_1^T(u) \Gamma_{1r}^T P_r v + v^T P_r \Gamma_{1r} g_1(u) \\
 & + g_2^T(u(t-\tau(t), x)) \Gamma_{2r}^T P_r v \\
 & \left. + v^T P_r \Gamma_{2r} g_2(u(t-\tau(t), x)) \right] dx. \quad (29)
 \end{aligned}$$

It follows by the Lipschitz assumption (7) and Lemma 5 that

$$\begin{aligned}
 & \int_{\Omega} \left[f_2^T(v(t-\tau(t), x)) W_{2r}^T P_r u \right. \\
 & \left. + u^T P_r W_{2r} f_2(v(t-\tau(t), x)) \right] dx \\
 & \leq \int_{\Omega} \left[f_2^T(v(t-\tau(t), x)) f_2(v(t-\tau(t), x)) \right. \\
 & \left. + (W_{2r}^T P_r u)^T (W_{2r}^T P_r u) \right] dx \\
 & \leq \int_{\Omega} \left[v^T(t-\tau(t), x) H_2^2 v(t-\tau(t), x) \right. \\
 & \left. + u^T P_r (W_{2r} W_{2r}^T) P_r u \right] dx. \quad (30)
 \end{aligned}$$

Similarly, we have

$$\begin{aligned}
 & \int_{\Omega} \left[f_1^T(v(t, x)) W_{1r}^T P_r u + u^T P_r W_{1r} f_1(v(t, x)) \right] dx \\
 & \leq \int_{\Omega} \left[v^T H_1^2 v + u^T (P_r W_{1r} W_{1r}^T P_r) u \right] dx, \\
 & \int_{\Omega} \left[g_1^T(u(t, x)) \Gamma_{1r}^T P_r v + v^T P_r \Gamma_{1r} g_1(u(t, x)) \right] dx \\
 & \leq \int_{\Omega} \left[u^T \mathcal{H}_1^2 u + v^T (P_r \Gamma_{1r} \Gamma_{1r}^T P_r) v \right] dx, \quad (31) \\
 & \int_{\Omega} \left[g_2^T(u(t-\tau(t), x)) \Gamma_{2r}^T v \right. \\
 & \left. + v^T \Gamma_{2r} g_2(u(t-\tau(t), x)) \right] dx \\
 & \leq \int_{\Omega} \left[u^T(t-\tau(t), x) \mathcal{H}_2^2 u(t-\tau(t), x) \right. \\
 & \left. + v^T (P_r \Gamma_{2r} \Gamma_{2r}^T P_r) v \right] dx.
 \end{aligned}$$

From the above analysis, Lemma 7, (19), (20), (23), (24), and Schur Complement theorem, we can deduce

$$\begin{aligned}
 LV(t, r) \leq & \left(\gamma + \sum_{j \in S} \pi_{rj} \frac{\bar{\alpha}}{\underline{\alpha}} \right) V(t, r) \\
 & - e^{\gamma(t-t_0)} \int_{\Omega} \left[u^T (2\lambda_1 d \underline{\alpha} I + 2P_r A_r \right. \\
 & - P_r (W_{1r} W_{1r}^T + W_{2r} W_{2r}^T) P_r - \mathcal{H}_1^2) u \\
 & + v^T (2\lambda_1 \tilde{d} \underline{\alpha} I + 2P_r B_r - P_r (\Gamma_{1r} \Gamma_{1r}^T + \Gamma_{2r} \Gamma_{2r}^T) P_r \\
 & - H_1^2) v \Big] dx + e^{\gamma(t-t_0)} \int_{\Omega} \left[u^T(t-\tau(t), x) \right. \\
 & \cdot \mathcal{H}_2^2 u(t-\tau(t), x) + v^T(t-\tau(t), x) H_2^2 v(t \\
 & \left. - \tau(t), x) \right] dx \leq \left(\gamma + \sum_{j \in S} \pi_{rj} \frac{\bar{\alpha}}{\underline{\alpha}} \right) V(t, r)
 \end{aligned}$$

$$\begin{aligned}
& -e^{\gamma(t-t_0)} \int_{\Omega} [\beta_1 (u^T u + v^T v)] dx \\
& + e^{\gamma(t-t_0)} \int_{\Omega} [\beta_2 (u^T(t-\tau(t), x) u(t-\tau(t), x) \\
& + v^T(t-\tau(t), x) v(t-\tau(t), x))] dx \leq \left(\gamma \right. \\
& \left. + \sum_{j \in S} \pi_{rj} \frac{\bar{\alpha}}{\underline{\alpha}} \right) V(t, r) - \frac{\beta_1}{\bar{\alpha}} V(t, r) + \beta_2 \bar{V}(t \\
& - \tau(t), r),
\end{aligned} \tag{32}$$

where $\bar{V}(t-\tau(t), r) = e^{\gamma(t-t_0)} \int_{\Omega} [(u^T(t-\tau(t), x) u(t-\tau(t), x) + v^T(t-\tau(t), x) v(t-\tau(t), x))] dx$.

Then we can derive that, for $t \in (t_k, t_{k+1})$,

$$\begin{aligned}
LV(t, r) & \leq \left(\gamma + \sum_{j \in S} \pi_{rj} \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\beta_1}{\bar{\alpha}} \right) V(t, r) \\
& + \beta_2 \bar{V}(t - \tau(t), r) \\
& \leq \xi V(t, r) + \beta_2 \bar{V}(t - \tau(t), r),
\end{aligned} \tag{33}$$

where $\xi = \gamma - \beta_1/\bar{\alpha} + \max_{r \in S} (\sum_{j \in S} \pi_{rj} \bar{\alpha}/\underline{\alpha})$.

Let $\varepsilon > 0$ small enough, and we can integrate (33) from $t_k + \varepsilon$ to t as follows:

$$\begin{aligned}
\mathbb{E}V(t, r) & \leq \mathbb{E}V(t_k + \varepsilon, r) + \xi \int_{t_k + \varepsilon}^t \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k + \varepsilon}^t \mathbb{E}\bar{V}(s - \tau(s), r) ds.
\end{aligned} \tag{34}$$

Let $\varepsilon \rightarrow 0$, and then we have

$$\begin{aligned}
\mathbb{E}V(t, r) & \leq \mathbb{E}V(t_k^+, r) + \xi \int_{t_k}^t \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k}^t \mathbb{E}\bar{V}(s - \tau(s), r) ds, \\
& t \in (t_k, t_{k+1}), \quad k = 0, 1, 2, \dots
\end{aligned} \tag{35}$$

Let $t = t_{k+1} - \varepsilon$ with $\varepsilon > 0$ small enough, and we can get by (35)

$$\begin{aligned}
\mathbb{E}V(t_{k+1} - \varepsilon, r) & \leq \mathbb{E}V(t_k^+, r) + \xi \int_{t_k}^{t_{k+1} - \varepsilon} \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k}^{t_{k+1} - \varepsilon} \mathbb{E}\bar{V}(s - \tau(s), r) ds.
\end{aligned} \tag{36}$$

Moreover, let $\varepsilon \rightarrow 0$, and then we can get by $\mathbb{E}V(t_{k+1}^-) = \mathbb{E}V(t_{k+1})$

$$\begin{aligned}
\mathbb{E}V(t_{k+1}, r) & \leq \mathbb{E}V(t_k^+, r) + \xi \int_{t_k}^t \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k}^t \mathbb{E}\bar{V}(s - \tau(s), r) ds.
\end{aligned} \tag{37}$$

Combining (35) and (37) results in

$$\begin{aligned}
\mathbb{E}V(t, r) & \leq \mathbb{E}V(t_k^+, r) + \xi \int_{t_k}^t \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k}^t \mathbb{E}\bar{V}(s - \tau(s), r) ds, \\
& t \in (t_k, t_{k+1}], \quad k = 0, 1, 2, \dots
\end{aligned} \tag{38}$$

On the other hand, since both C_{kr} and $\mathcal{C}_{kr}$ are diagonal matrices, we can get by (18)

$$\begin{aligned}
u^T(t_k^+, x) P_r u(t_k^+, x) & = u^T(t_k, x) C_{kr} P_r C_{kr} u(t_k, x) \\
& = u^T(t_k, x) P_r C_{kr}^2 u(t_k, x) \\
& \leq (1 + \alpha) u^T(t_k, x) P_r u(t_k, x), \\
v^T(t_k^+, x) P_r v(t_k^+, x) & = v^T(t_k, x) \mathcal{C}_{kr} P_r \mathcal{C}_{kr} v(t_k, x) \\
& \leq (1 + \alpha) v^T(t_k, x) P_r v(t_k, x).
\end{aligned} \tag{39}$$

Hence,

$$\mathbb{E}V(t_k^+, r) \leq (1 + \alpha) \mathbb{E}V(t_k, r). \tag{40}$$

Synthesizing (38) and (40) results in

$$\begin{aligned}
\mathbb{E}V(t, r) & \leq \mathbb{E}V(t_k, r) + \alpha \mathbb{E}V(t_k, r) \\
& + \xi \int_{t_k}^t \mathbb{E}V(s, r) ds \\
& + \beta_2 \int_{t_k}^t \mathbb{E}\bar{V}(s - \tau(s), r) ds, \\
& t \in (t_k, t_{k+1}], \quad k = 0, 1, 2, \dots
\end{aligned} \tag{41}$$

By induction argument, we have

$$\begin{aligned}
\mathbb{E}V(t_k, r) & \leq \mathbb{E}V(t_{k-1}, r) + \alpha \mathbb{E}V(t_{k-1}, r) \\
& + \xi \int_{t_{k-1}}^{t_k} \mathbb{E}V(s, r) ds + \beta_2 \int_{t_{k-1}}^{t_k} \mathbb{E}\bar{V}(s - \tau(s), r) ds \\
& \vdots \\
\mathbb{E}V(t_1, r) & \leq \mathbb{E}V(t_0, r) + \alpha \mathbb{E}V(t_0, r) \\
& + \xi \int_{t_0}^{t_1} \mathbb{E}V(s, r) ds + \beta_2 \int_{t_0}^{t_1} \mathbb{E}\bar{V}(s - \tau(s), r) ds.
\end{aligned} \tag{42}$$

Hence, for $k = 0, 1, 2, \dots$,

$$\mathbb{E}V(t, r) \leq \mathbb{E}V(t_0, r) + \alpha \mathbb{E}V(t_0, r) \quad t \in (t_k, t_{k+1}]. \quad (43)$$

$$+ \alpha \sum_{t_0 < t_k < t} \mathbb{E}V(t_k, r) + \xi \int_{t_0}^t \mathbb{E}V(s, r) ds$$

Since time-delays $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) < 1 - 1/h$ with $h > 0$, we have

$$\begin{aligned} \int_{t_0}^t \mathbb{E} \widetilde{V}(s - \tau(s), r) ds &= \mathbb{E} \int_{t_0}^t e^{\gamma(s-t_0)} \left[\int_{\Omega} \left(u^T(s - \tau(s), x) u(s - \tau(s), x) + v^T(s - \tau(s), x) v(s - \tau(s), x) \right) dx \right] ds \\ &= \mathbb{E} \int_{t_0 - \tau(t_0)}^{t - \tau(t)} e^{\gamma(\theta + \tau(s) - t_0)} \left[\int_{\Omega} \left(u^T(\theta, x) u(\theta, x) + v^T(\theta, x) v(\theta, x) \right) dx \right] \frac{d\theta}{1 - \dot{\tau}(s)} \\ &\leq h e^{\gamma\tau} \int_{t_0 - \tau(t_0)}^t e^{\gamma(\theta - t_0)} \left[\int_{\Omega} \left(u^T(\theta, x) u(\theta, x) + v^T(\theta, x) v(\theta, x) \right) dx \right] d\theta \\ &\leq h e^{\gamma\tau} \mathbb{E} \left\{ \int_{t_0 - \tau(t_0)}^{t_0} e^{\gamma(\theta - t_0)} \left[\int_{\Omega} \left(u^T(\theta, x) u(\theta, x) + v^T(\theta, x) v(\theta, x) \right) dx \right] d\theta + \frac{1}{\underline{\alpha}} \right. \\ &\quad \cdot \mathbb{E} \int_{t_0}^t e^{\gamma(s-t_0)} \left[\int_{\Omega} \left(u^T(s, x) P_r u(s, x) + v^T(s, x) P_r v(s, x) \right) dx \right] ds \Big\} \\ &\leq h e^{\gamma\tau} \mathbb{E} \int_{t_0 - \tau}^{t_0} e^{\gamma(\theta - t_0)} \left[\int_{\Omega} \left(u^T(\theta, x) u(\theta, x) + v^T(\theta, x) v(\theta, x) \right) dx \right] d\theta + \frac{h e^{\gamma\tau}}{\underline{\alpha}} \int_{t_0}^t \mathbb{E}V(s, r) ds \leq \frac{\tau h}{\gamma} \\ &\quad \cdot e^{\gamma\tau} e^{\gamma(t_0 - t_0)} (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2) + \frac{h e^{\gamma\tau}}{\underline{\alpha}} \int_{t_0}^t \mathbb{E}V(s, r) ds. \end{aligned} \quad (44)$$

It follows by (43) and (44) that, for $r \in S$,

$$\begin{aligned} \mathbb{E}V(t, r) &\leq \left((1 + \alpha) \mathbb{E}V(t_0, r) + \frac{\beta_2 \tau h}{\gamma} e^{\gamma\tau} (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2) \right) \\ &\quad + \left(\xi + \frac{\beta_2 h e^{\gamma\tau}}{\underline{\alpha}} \right) \int_{t_0}^t \mathbb{E}V(s, r) ds \\ &\quad + \alpha \sum_{t_0 < t_k < t} \mathbb{E}V(t_k, r), \quad t \geq t_0. \end{aligned} \quad (45)$$

In addition, for any $t \geq t_0$, we may assume that $t \in (t_k, t_{k+1}]$. Since $\inf_{k=1,2,\dots} (t_k - t_{k-1}) \geq \mu$, one can get

$$(1 + \alpha)^k \leq e^{((t_k - t_0)/\mu) \ln(1 + \alpha)} \leq e^{(\ln(1 + \alpha)/\mu)(t - t_0)}. \quad (46)$$

According to Lemma 4, we can conclude that $r \in S$,

$$\begin{aligned} \mathbb{E}V(t, r) &\leq \left((1 + \alpha) \mathbb{E}V(t_0, r) \right. \\ &\quad \left. + \frac{\beta_2 \tau h}{\gamma} e^{\gamma\tau} (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2) \right) (1 + \alpha)^k \end{aligned}$$

$$\cdot e^{(\xi + \beta_2 h e^{\gamma\tau}/\underline{\alpha})(t - t_0)} \leq \left((1 + \alpha) \bar{\alpha} + \frac{\beta_2 \tau h}{\gamma} e^{\gamma\tau} \right)$$

$$\cdot (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2)$$

$$\cdot e^{(\ln(1 + \alpha)/\mu + \xi + \beta_2 h e^{\gamma\tau}/\underline{\alpha})(t - t_0)}, \quad t \geq t_0. \quad (47)$$

Moreover, we have

$$\begin{aligned} \underline{\alpha} (\mathbb{E} \|u\|^2 + \mathbb{E} \|v\|^2) e^{\gamma(t - t_0)} &\leq \mathbb{E}V(t, r) \\ &\leq \left((1 + \alpha) \bar{\alpha} + \frac{\beta_2 \tau h}{\gamma} e^{\gamma\tau} \right) (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2) \\ &\quad \cdot e^{(\ln(1 + \alpha)/\mu + \xi + \beta_2 h e^{\gamma\tau}/\underline{\alpha})(t - t_0)}, \quad t \geq t_0 \end{aligned} \quad (48)$$

which yields

$$\begin{aligned} \mathbb{E} \|u\|^2 + \mathbb{E} \|v\|^2 &\leq \frac{(1 + \alpha) \bar{\alpha} + (\beta_2 \tau h/\gamma) e^{\gamma\tau}}{\underline{\alpha}} (\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2) \\ &\quad \cdot e^{(\ln(1 + \alpha)/\mu + \xi + \beta_2 h e^{\gamma\tau}/\underline{\alpha} - \gamma)(t - t_0)} \end{aligned}$$

TABLE 1: Ratios between allowable upper bounds of time-delay τ (or impulse $\mathcal{C}$) and maximum of parameters $\mathcal{P}$.

	τ	$\mathcal{C}$	$\mathcal{P}$	$\tau/\mathcal{P}$	$\mathcal{C}/\mathcal{P}$
Our Example 1	19	1.3	2.1	9.048	0.619
[24, Example]	0.5	1.35	2.16	0.2315	0.625
[25, Example 1]	4	1	3	1.333	0.333
[16, Example 1]	1	0.8	3.5	0.2857	0.2286

$$= \frac{(1 + \alpha) \bar{\alpha} + (\beta_2 \tau h / \gamma) e^{\gamma \tau}}{\underline{\alpha}} \left(\mathbb{E} \|\varphi\|_{\tau}^2 + \mathbb{E} \|\psi\|_{\tau}^2 \right) \cdot e^{-(\beta_1 / \bar{\alpha} - \ln(1 + \alpha) / \mu - \beta_2 h e^{\gamma \tau} / \underline{\alpha} - \max_{r \in S} (\sum_{j \in S}^{j \neq r} \pi_{rj} \bar{\alpha} / \underline{\alpha})) (t - t_0)},$$

$$t \geq t_0. \quad (49)$$

Moreover, it follows by (25) that

$$\frac{\beta_1}{\bar{\alpha}} - \frac{\ln(1 + \alpha)}{\mu} - \frac{\beta_2 h e^{\gamma \tau}}{\underline{\alpha}} - \max_{r \in S} \left(\sum_{j \in S}^{j \neq r} \pi_{rj} \bar{\alpha} / \underline{\alpha} \right) > 0. \quad (50)$$

According to Definition 3, the equilibrium point $u = 0$ and $v = 0$ of the problem (3)–(5) is globally stochastically exponential stability. $\square$

Remark 9. In [10, Theorem 3.1] and [9, Theorem 1], their impulsive conditions are similar as $C_{kr}^T P_i C_{kr} < P_i$, $\mathcal{C}_{kr}^T P_i \mathcal{C}_{kr} < P_i$, for all $r, i, l \in S$. But in our Theorem 8, the condition range on C_{kr} and $\mathcal{C}_{kr}$ is enlarged owing to (18).

Remark 10. In [16, Theorem 3.1–3.2] and [36, Theorem 3.1–3.2], the conditions of all the stability criteria can only be verified via calculating by hand. However, we set up LMI-based criterion in our Theorem 8 by Schur Complement technique, which can be verified by computer Matlab LMI toolbox. In the following, Example 1 will offer a skillful method in a numerical example to illustrate more effectiveness and less conservativeness of our Theorem 8. That is, the comprehensive application of computer Matlab LMI toolbox and the method of trial and error in Example 1 improves significantly the allowable upper bounds of time-delays and impulse (see Table 1 and Remark 11 for details).

4. Numerical Example

Example 1. Consider the following Markovian jumping BAM neural networks:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \nabla \cdot (D(t, x, u) \circ \nabla u) - A_r u(t, x) \\ &\quad + W_{1r} f_1(v(t, x)) \\ &\quad + W_{2r} f_2(v(t - \tau(t), x)), \\ x &\in \Omega, \quad t \neq t_k, \end{aligned}$$

$$\frac{\partial v}{\partial t} = \nabla \cdot (\mathcal{D}(t, x, v) \circ \nabla v) - B_r v(t, x)$$

$$+ \Gamma_{1r} g_1(u(t, x))$$

$$+ \Gamma_{2r} g_2(u(t - \tau(t), x)),$$

$$x \in \Omega, \quad t \neq t_k,$$

$$\Delta u(t_k, x) = (C_k(r(t)) - I) u(t_k, x),$$

$$\Delta v(t_k, x) = (\mathcal{C}_k(r(t)) - I) v(t_k, x),$$

$$x \in \Omega, \quad k = 1, 2, \dots,$$

$$u(s, x; t_0, \varphi) = \varphi(s, x),$$

$$v(s, x; t_0, \psi) = \psi(s, x),$$

$$t_0 - \tau \leq s \leq t_0, \quad x \in \Omega,$$

$$u(s, x; t_0, \varphi) = 0,$$

$$v(s, x; t_0, \varphi) = 0,$$

$$x \in \partial\Omega, \quad t \geq t_0, \quad (51)$$

where $u = (u_1, u_2)^T$, $v = (v_1, v_2)^T \in R^2$, $x = (x_1, x_2) \in (0, 10) \times (0, 10) \subset R^2$, and then $\lambda_1 = 0.02\pi^2 = 0.1974$ [15, Remark 2]:

$$D = \begin{pmatrix} 0.008 & 0.006 \\ 0.007 & 0.005 \end{pmatrix},$$

$$\mathcal{D} = \begin{pmatrix} 0.009 & 0.005 \\ 0.007 & 0.006 \end{pmatrix},$$

$$H_1 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.4 \end{pmatrix},$$

$$H_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.5 \end{pmatrix},$$

$$\mathcal{H}_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.4 \end{pmatrix},$$

$$\mathcal{H}_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.3 \end{pmatrix}.$$

(52)

From the above data, we know that $d = 0.005$ and $\tilde{d} = 0.005$. Consider two modes for the Markovian jumping impulsive system (51). For mode 1,

$$A_1 = \begin{pmatrix} 0.9 & 0 \\ 0 & 1.2 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1.8 \end{pmatrix},$$

$$\begin{aligned}
W_{11} &= \begin{pmatrix} 0.11 & 0.003 \\ 0.003 & 0.12 \end{pmatrix}, \\
W_{21} &= \begin{pmatrix} 0.13 & 0.002 \\ 0.002 & 0.12 \end{pmatrix}, \\
\Gamma_{11} &= \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.8 \end{pmatrix}, \\
\Gamma_{21} &= \begin{pmatrix} 0.8 & 0.3 \\ 0.5 & 0.6 \end{pmatrix}, \\
C_{k1} &= \begin{pmatrix} 1.9 & 0 \\ 0 & 1.7 \end{pmatrix}, \\
\mathcal{C}_{k1} &= \begin{pmatrix} 1.8 & 0 \\ 0 & 1.6 \end{pmatrix}.
\end{aligned} \tag{53}$$

For mode 2,

$$\begin{aligned}
A_2 &= \begin{pmatrix} 1.2 & 0 \\ 0 & 1.3 \end{pmatrix}, \\
B_2 &= \begin{pmatrix} 2.1 & 0 \\ 0 & 2 \end{pmatrix}, \\
W_{12} &= \begin{pmatrix} 0.12 & 0.003 \\ 0.003 & 0.11 \end{pmatrix}, \\
W_{22} &= \begin{pmatrix} 0.11 & 0.002 \\ 0.002 & 0.12 \end{pmatrix}, \\
\Gamma_{12} &= \begin{pmatrix} 0.13 & 0.003 \\ 0.003 & 0.16 \end{pmatrix}, \\
\Gamma_{22} &= \begin{pmatrix} 0.17 & 0.002 \\ 0.002 & 0.13 \end{pmatrix}, \\
C_{k2} &= \begin{pmatrix} 2.3 & 0 \\ 0 & 1.9 \end{pmatrix}, \\
\mathcal{C}_{k2} &= \begin{pmatrix} 2 & 0 \\ 0 & 1.7 \end{pmatrix}.
\end{aligned} \tag{54}$$

Let $\pi_{11} = -0.6$, $\pi_{12} = 0.6$, $\pi_{21} = 0.2$, and $\pi_{22} = -0.2$. From the above data, we can select $\alpha = 4.3$. Then it is not difficult to verify that condition (18) holds for any mode $r \in S = \{1, 2\}$. In addition, fix $\mu = 3.5$, $\tau = 19$, $h = 1.2$, and $\gamma = 0.001$. Now

we use the Matlab LMI toolbox to solve the linear matrices inequalities (19)–(24) and then the feasibility data follows:

$$\begin{aligned}
P_1 &= \begin{pmatrix} 12.3228 & 0 \\ 0 & 12.1417 \end{pmatrix}, \\
P_2 &= \begin{pmatrix} 12.1357 & 0 \\ 0 & 12.1246 \end{pmatrix}, \\
\underline{\alpha} &= 11.3765, \\
\bar{\alpha} &= 13.1353, \\
\beta_1 &= 16.8064, \\
\beta_2 &= 0.4689.
\end{aligned} \tag{55}$$

Moreover, substituting the above data into (25) results in

$$\begin{aligned}
\frac{\beta_1}{\bar{\alpha}} - \frac{\ln(1+\alpha)}{\mu} - \sum_{j \in S}^{j \neq 1} \frac{\pi_{1j} \bar{\alpha}}{\underline{\alpha}} - \frac{\beta_2 h e^{\gamma \tau}}{\underline{\alpha}} &= 0.0598 > 0, \\
\frac{\beta_1}{\bar{\alpha}} - \frac{\ln(1+\alpha)}{\mu} - \sum_{j \in S}^{j \neq 2} \frac{\pi_{2j} \bar{\alpha}}{\underline{\alpha}} - \frac{\beta_2 h e^{\gamma \tau}}{\underline{\alpha}} &= 0.5217 > 0,
\end{aligned} \tag{56}$$

which implies that all the assumptions of Theorem 8 are satisfied. And then the equilibrium point $u = 0$ and $v = 0$ of system (51) is globally stochastically exponential stability. On the whole, Example 1 illustrates more effectiveness and less conservativeness of our Theorem 8 than other related literatures (see Table 1 for details).

Remark 11. In Example 1, to verify the condition (25), we need to employ comprehensively the computer Matlab LMI toolbox and the method of trial and error. To compare the upper bounds of time-delay and impulse in various related literature, we need to compute and compare the ratios between the maximum of allowable delay (or impulsive) and maximum of parameters in various related literature, because the maximum of allowable delay may rise as parameters of numerical examples become bigger. From Table 1, the ratios of our Example 1 are bigger than those of [24, 25] to some extent. And hence, there are some advantages of the utilization of Gronwall-Bellman-type inequality compared to that of other inequalities in [24, 25]. Moreover, LMI-based criterion in our Theorem 8 is more effective than hand-computation based criteria of [16, 36].

Remark 12. From the above data in Example 1, it is obvious that condition (18) in Theorem 8 holds. But the similar impulsive conditions $C_{kr}^T P_i C_{kr} < P_l$, $\mathcal{C}_{kr}^T P_i \mathcal{C}_{kr} < P_l$ for all $r, i, l \in S$ in [10, Theorem 3.1] and [9, Theorem 1] can not be satisfied.

Indeed,

$$\begin{aligned}
\mathcal{C}_{k2}^T P_1 \mathcal{C}_{k2} &= \begin{pmatrix} 2 & 0 \\ 0 & 1.7 \end{pmatrix} \begin{pmatrix} 12.3228 & 0 \\ 0 & 12.1417 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1.7 \end{pmatrix} \\
&= \begin{pmatrix} 49.2912 & 0 \\ 0 & 35.0895 \end{pmatrix} > P_r, \quad r = 1, 2.
\end{aligned} \tag{57}$$

This illustrates that the stability criterion of our Theorem 8 can judge what [10, Theorem 3.1] and [9, Theorem 1] can not do, which shows clearly that the allowable upper bounds of impulse are improved significantly against those of [10, Theorem 3.1], [9, Theorem 1], [15, Theorem 1], [3, Theorem 4.1], [4, Theorem 3.2], and [19, Theorem 3.2] owing to employing the Gronwall-Bellman-type impulsive integral inequality.

5. Conclusions

In this paper, Schur Complement technique and Gronwall-Bellman-type impulsive integral inequality are synthetically applied to stability analysis of impulsive delayed Markovian jumping reaction-diffusion bidirectional associative memory (BAM) neural networks, and LMI-based exponential stability criterion is obtained. It is the first time to obtain the LMI-based stability criterion for BAM neural networks via Gronwall-Bellman-type impulsive integral inequality. The main result of this paper improves the allowable upper bounds of impulse in [10, Theorem 3.1], [9, Theorem 1], [15, Theorem 1], [3, Theorem 4.1], [4, Theorem 3.2], and [19, Theorem 3.2]. Finally, a numerical example in Section 4 is presented to illustrate the effectiveness of the proposed method (see Remarks 10 and 11).

Conflict of Interests

All the authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' Contribution

Ruofeng Rao wrote the original paper, and Shouming Zhong examined the original paper in detail before submission. Xionggrui Wang is in charge of the correspondence.

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