Research Article

A Novel Probabilistic Approach for Vehicle Position Prediction in Free, Partial, and Full GPS Outages

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In this paper, a novel framework is developed with the intention of continuously predicting vehicle position even in the challenging environments such as partial and full GPS outages. To achieve this, the Bayesian-Sparse Random Gaussian Prediction (B-SRGP) approach is proposed where the sparse random Gaussian matrix which obeys the restricted isometry property with high probability is adopted to handle the measurement model. During the full GPS outages, the proposed method fuses all available INS measurements to improve the vehicle position prediction whereas in free outages only the GPS data are processed. Besides, the Bayesian inference is used to specifically deal with the vehicle position prediction in partial GPS outages where data from both GPS and INS are taken as inputs. In all cases, measurement noises are assumed to be non-Gaussian distributed and follow the generalized error distribution. The performance of B-SRGP is evaluated with respect to real-world data collected using Smartphone-based vehicular sensing model. The proposed method is tested when measurement noises are both Gaussian and non-Gaussian distributed and also compared with the existing prediction methods. Experimental results confirm that B-SRGP presents higher accuracy prediction and lower mean-squared prediction error for vehicle position when measurement noises are non-Gaussian distributed.

1. Introduction

Nowadays, vehicle trajectory prediction is one of the major problems in intelligent transportation systems (ITS) where many researchers are widely interested in reliable safety applications. For instance, in order to avoid fatal accidents or collisions between vehicles, vehicle information should be accurately predicted and therefore drivers and pedestrians can be warned by in-vehicle systems at the earliest possible moment. Meanwhile, predicting the position of a moving vehicle is still a challenge case in the fields of engineers and scientists. Actually, the global positioning system (GPS) is used as a source of accurate position information. In contrast, GPS is not for all time a perfect vehicle positioning system in urban areas since its signals are blocked due to high buildings, tunnels, thick tree cover, and so forth. This is the so-called “GPS outage” in the literature. Based on the number of available GPS satellites in space, one can identify three kinds of outages: *full* GPS outage which occurs when signals

of all available satellites reflect on different objects such as ground block of buildings, before arriving to the GPS receiver. In this case, GPS receiver is unable to adequately produce any location estimates. Moreover, GPS outage is said to be *partial* when the number of GPS satellites in view is less than four [1]. The authors in [1] mentioned that, during the partial GPS outages, the GPS location cannot be computed. Nevertheless, we believe that when the number of satellites is less than four, the GPS information is provided but with lower accuracy as also supported by the authors in [2, 3]. Finally, the GPS measurement is reliable if the signals are coming from at least four GPS satellites [4, 5]. This is the so-called *free* GPS outage in this study. Even though the inertial sensor system (INS) accuracy deteriorates with time due to the possible inherent sensor errors, it is usually used for navigation as a stand-alone system to bridge the full GPS outages until GPS information is available again [6, 7]. For example, the odometer of the vehicle has been used in [8] as dead reckoning sensors to complement the GPS during the full outages.

Although the on-board vehicle sensors such as GPS receiver and inertial sensors (odometer, the steering angle sensor) are relatively accurate for predicting the vehicle information, they demand additional hardware to the vehicle (e.g., the steering angle sensor and odometer) and do not work under low visibility of the satellites (e.g., GPS receiver) [9].

Alternatively, the authors in [9] introduced the use of the *Smartphone* on board a vehicle in order to determine the vehicle position. In addition, Mok et al. [5] proposed a combination of different sensors such as accelerometer and digital compass that are embedded into the Smartphone to assist GPS positioning to increase the accuracy level for the vehicle tracking. This new advanced technology based on mobile computing system has gained a large popularity nowadays in various applications such as in target localization [10] and tracking systems [11]. Other benefits of choosing the sensors embedded in Smartphone over the ones on board the vehicle for vehicle positioning can also be found in [9].

1.1. Literature Review. Over the past few years, different prediction methods such as Kalman Filter (KF), Gaussian process for regression (GPR), radial basis function (RBF), and Dempster-Shafer (DS) theory augmented by Support Vector Machines (SVM), known as DS-SVM [12], have been developed for bridging GPS outages. Meanwhile, due to the limited knowledge of the system's dynamic model and measurement noise, FK method suffers from the divergence and provides inaccurate prediction information. In fact, there are several considerable drawbacks to the use of KF in vehicular navigation application which can be found in [13]. Besides, the main aim of GPR is to generate a nonlinear regression model that predicts all possible inputs based on all observed variables. Although this method is flexible and simple to implement, it is more computer-intensive demanding [14]. To forecast the measurement update of KF method, the authors in [7, 15] have introduced the prediction method based on radial basis function (RBF) neural network coupled with time series analysis. In [7], for instance, when GPS outages occur, the outputs of RBF neural network are used straightly to correct the INS results. In [15], RBF neural network is used to predict measurement of GPS/SINS KF in order to guarantee the regular operation of the filter in the system during the GPS outages. Furthermore, with the intention of improving the positioning accuracy during outages [12], the trained SVM model is used to correct the INS error and the DS contributes when the GPS signals are available. All these studies revealed that the proposed prediction methods present the higher position accuracy during the GPS outages compared to some predefined methods in the literature. However, the authors did not deal with the partial GPS outages which occur when less than four satellites are available or when the four satellites in view present a higher Geometric Dilution of Precision (GDOP) [3]. Recall that the GDOP values decrease as the angles between the satellites increase and, consequently, the accuracy increases [16]. What is more, Boucher and Noyer [1] introduced the hybrid method based on the combination of KF and particle filter (PF) to handle the partial GPS outages. In their study, they mentioned that when GPS fails, the

filter fuses all available pseudorange measures to improve the vehicle positioning. The used GPS receiver was not, however, reliable since it could not get any signal from less than four satellites. Based on all the aforementioned prediction methods, one can remark that the sensor measurements are assumed to be white noise and follow the Gaussian distribution. On the contrary, there is evidence that, in several engineering systems and physical environments, noises in principle follow a non-Gaussian distribution [17, 18].

1.2. Objectives and Contributions. The main objective of this study is to develop a prediction approach-based low-cost GPS/INS model integrated in mobile computing device taking into account the challenging environments and thus providing a better vehicle positioning accuracy even during the full and partial GPS outages. To achieve this, the Bayesian-Sparse Random Gaussian Prediction (B-SRGP) algorithm is introduced. This new algorithm takes advantages of the Sparse Random Gaussian (SRG) matrix which is considered as the measurement model and obeys the restricted isometry property (RIP) [19] with high probability. This property is helpful in the sense that it is used to evaluate the quality of the measurement matrix. Since our approach deals with the free, full, and GPS outages, the parameter adjusting the GPS propagation weight is assumed to be monitored by the Gaussian model [12]. Moreover, this parameter is defined based on logical inclusive OR in order to randomly determine the free, partial, and full GPS outages of the system. During the free and full GPS outages, the Sparse Random Gaussian Prediction (SRGP) method is applied to predict the vehicle position. Additionally, when the GPS measurements are available but with low weight, that is, when the number of satellites in view is less than four, the partial GPS outage is detected. In this case, the Bayesian inference is applied to SRGP method to model the inputs from both the INS and GPS measurements in order to provide the reliable prediction accuracy. The size of sliding window is introduced to control the flow data generated by both INS and GPS which are required for the prediction accuracy. Bayesian inference is preferred since not only does it consider the data to be fixed and the parameters to be random but also it uses prior distributions which allow the usage of more information. The measurement noises are assumed to be non-Gaussian distributed and follow the generalized error distribution (GED) with nonzero shape parameter. This assumption has experimentally a positive effect on the position prediction accuracy.

The performance of the proposed B-SRGP is evaluated with respect to real-world data collected using Smartphone-based vehicular sensing model [9] instead of using the on-board vehicle sensors [2, 3]. Indeed, with the intention of effectively predicting the vehicle position in free, partial, and full GPS outages based on the proposed method, the test vehicle was driven along different roads in Hunan University area with different velocities for about 25 minutes. The proposed method is tested when the measurement noises are both Gaussian and non-Gaussian distributed and is denoted by "B-SRGP (gn)" and "B-SRGP (non-gn)," respectively, where "gn" stands for "Gaussian noise" and "non-gn" represents

“non-Gaussian noise.” In fact, based on maximum likelihood estimation (MLE) method, zeroing the shape parameter increases the estimated noise variance (or covariance matrix) which in turn increases sensibly the likelihood function and therefore the position prediction accuracy. B-SRGP is also compared with the existing prediction methods such as KF and Multivariate Adaptive Regression Splines (MARS). Experimental results reveal that B-SRGP presents the good performance in terms of prediction accuracy of the vehicle position and mean-squared prediction error (MSPE) when the measurement noises are non-Gaussian distributed.

In the next section, we formally express the problem statement where the Sparse Random Gaussian (SRG) matrix for prediction model and the fusion of the GPS and INS data based on predictor matrix are introduced. Section 3 presents in detail the Bayesian-Sparse Random Gaussian Prediction (B-SRGP) for vehicle position during the free, partial, and full GPS outages. Section 4 is devoted to the experiment and evaluation results where the determination of the partial and full GPS outages as well as the comparison results based on the adopted prediction methods is highlighted. Conclusion and future work are presented in Section 5.

2. Problem Statement

2.1. Sparse Random Gaussian (SRG) Matrix for Prediction Model. In the statistical signal processing framework, one can predict the values of a continuous dependent variable X from a set of independent variables Y using the following nonlinear model:

$$Y_i = f(X_i) + \xi_i, \quad i = 1, \dots, n, \quad (1)$$

where Y_i represents the $n \times 1$ response or measurement vector. The variable $X_i \in \mathbb{R}^p$ indicates the $p \times 1$ input to be predicted. Here, ξ_i stands for the $n \times 1$ error vector and is supposed to be non-Gaussian distributed whereas $f(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is an unknown prediction function. This function can be transformed as

$$Y_i = FX_i + \xi_i, \quad i = 1, \dots, n, \quad (2)$$

where F stands for the $n \times p$ measurement matrix or predictor matrix, with $n < p$. The crucial task here is how to design the predictor matrix F to guarantee that it preserves the information X_i . To achieve this, we assume that F is a sparsified random Gaussian matrix whose entries F_{ij} are given by [20]

$$F_{ij} = \begin{cases} N\left(0, \frac{1}{\lambda}\right) & \text{w.p. } \lambda \\ 0 & \text{w.p. } 1 - \lambda, \end{cases} \quad (3)$$

where $\lambda \in [0, 1]$ is the measurement sparsification parameter. Moreover, since there are more unknowns than the number of equations ($n < p$), the measurement matrix F becomes singular and leads (2) to an ill-conditioned system. Therefore, for better prediction, F should satisfy the restricted isometry property (RIP) [19]. Indeed, the matrix F satisfies the (s, δ_s) -RIP with restricted isometry constant (RIC) $\varepsilon_s \in [0, 1]$ if

$(1 - \varepsilon_s)\|X_i\|_2^2 \leq \|F_s X_i\|_2^2 \leq (1 + \varepsilon_s)\|X_i\|_2^2$, where s ($n \leq s \leq n \times p$) represents the nonvanishing entries of the sparse matrix F . This property is also used to evaluate the quality of the measurement matrix.

Moreover, according to [21], the sparsified random Gaussian matrix F satisfies the RIP with high probability if $p \left((1 - \varepsilon_s)\|X_i\|_2^2 \leq \|F_s X_i\|_2^2 \leq (1 + \varepsilon_s)\|X_i\|_2^2 \right) \geq r$, where the constants $r = 1 - 2(12/\varepsilon_s)^k \exp(-q(\varepsilon_s/2)n)$ and $q(\varepsilon_s/2) = \varepsilon_s^2(1 - \varepsilon_s/48)$. The variable k represents the dimension of the subspace. After the proof, it is shown that the matrix F obeys the RIP with the probability $p \left((1 - \varepsilon_s)\|X_i\|_2^2 \leq \|F_s X_i\|_2^2 \leq (1 + \varepsilon_s)\|X_i\|_2^2 \right) \geq 1 - \varepsilon_s$. The choice of the sparsified random Gaussian matrix over the deterministic matrix is due to the fact that it is very hard to get the latter matrix with $n \times p$ size that satisfies the RIP because it requires the larger size of n and smaller sparsity level [22, 23]. More benefits of choosing the random matrix as the measurement matrix can be found in [24].

2.2. GPS and INS Data Fusion-Based Predictor Matrix. In this paper, we assume that the measurement device comprises both the global positioning system (GPS) and the Inertial Navigation System (INS). Based on (2), the measurements of GPS and INS are given by Y_t^{GPS} and Y_t^{INS} where $Y_t^{\text{GPS}} = FX_t^{\text{GPS}} + \xi_t^{\text{GPS}}$ and $Y_t^{\text{INS}} = F_t X_t^{\text{INS}} + \xi_t^{\text{INS}}$, respectively, where F_t is the s -sparse predictor matrix constructed based on (3). The variable X_t^{GPS} indicates the position in terms of latitude and longitude over the sample time t , for example, whereas X_t^{INS} represents data provided by INS such as the acceleration, direction angle, and angular velocity. Moreover, variables ξ_t^{GPS} and ξ_t^{INS} are the GPS and INS measurement noises which show the dependence of Y_t^{GPS} and Y_t^{INS} on the variables other than predictor variables X_t^{GPS} and X_t^{INS} . These noises are considered as non-Gaussian distributed and follow the generalized error distribution (GED) [25]. The probability density function (PDF) related to ξ_t^{GPS} and ξ_t^{INS} is therefore defined as

$$p_{\text{GPS/INS}}(\xi_t | b) = M(b) \sigma_{\text{GPS/INS}}^{-1} \exp \left[-N(b) \left| \frac{\xi_t - \mu_{\text{GPS/INS}}}{\sigma_{\text{GPS/INS}}} \right|^{2/(1+b)} \right], \quad (4)$$

where $M(b) = \{\Gamma[3(1+b)/2]\}^{1/2}/(1+b)\{\Gamma[(1+b)/2]\}^{3/2}$, $N(b) = \{\Gamma[3(1+b)/2]/\Gamma[(1+b)/2]\}^{1/(1+b)}$, and $\Gamma(\cdot)$ is the gamma function.

The quantities $\mu_{\text{GPS/INS}}$ and $\sigma_{\text{GPS/INS}}$ are location and scale parameters, respectively, and $\sigma_{\text{GPS/INS}}$ represents specifically the standard deviation of the sample. The variable b is a shape parameter.

Proposition 1. *The generalized error distribution (GED) becomes the normal or Gaussian distribution, if the shape parameter b is zero and is as follows:*

$$p_{\text{GPS/INS}}(\xi_t | b=0) = \frac{1}{\sqrt{2\pi}} \sigma_{\text{GPS/INS}}^{-1} \exp \left[-\frac{1}{2} \left| \frac{\xi_t - \mu_{\text{GPS/INS}}}{\sigma_{\text{GPS/INS}}} \right|^2 \right]. \quad (5)$$

FIGURE 1: The generalized error distribution with different values of the shape b . The curve where $(b = 0)$ indicates the Gaussian distribution.

Proof. Assume that the shape vanishes; that is, $b = 0$; then, (4) becomes

$$p(\xi_t, b=0, \sigma, \mu) = M(0) \sigma^{-1} \exp \left[-N(0) \left| \frac{\xi - \mu}{\sigma} \right|^2 \right], \quad (6)$$

where $M(0) = \{\Gamma[3/2]\}^{1/2} / \{\Gamma[1/2]\}^{3/2} = ((1/2)\sqrt{\pi})^{1/2} / (\sqrt{\pi})^{3/2} = 1/\sqrt{2\pi}$ and $N(0) = \{\Gamma[3/2]/\Gamma[1/2]\} = (1/2)\sqrt{\pi}/\sqrt{\pi} = 1/2$.

Then, plugging these results into (6) leads to the following Gaussian distribution:

$$p(\xi_t, b=0, \sigma, \mu) = \frac{1}{\sqrt{2\pi}} \sigma^{-1} \exp \left[-\frac{1}{2} \left| \frac{\xi_t - \mu}{\sigma} \right|^2 \right]. \quad (7)$$

□

For more precision, Figure 1 plots the generalized error distribution (GED) with $\mu = 0$, $\sigma = 1$, and different values of the shape b with the intention of proving graphically Proposition 1 where the curve $(b = 0)$ represents the Gaussian distribution.

In this study, we consider also that the response variable Y which depends on both Y_t^{GPS} and Y_t^{INS} is defined as $Y = A_{\text{GPS}} Y_t^{\text{GPS}} + A_{\text{INS}} Y_t^{\text{INS}}$, where A_{GPS} and A_{INS} stand for the weight related to GPS and INS, respectively. Hereafter, we will suppose that A_{INS} also depends on the GPS weight in order to regulate the contribution of GPS for the accurate navigation solution in the system. The GPS weight denoted as α_{GPS} is assumed to be Gaussian distributed and is computed based on the Gaussian PDF [12] as follows:

$$\alpha = \left[(2\pi)^{1/2} |D|_{\text{GPS}} \right]^{-1} \cdot \exp \left[-\frac{1}{2} (\text{GPS} - \mu_{\text{GPS}})^T D_{\text{GPS}}^{-1} (\text{GPS} - \mu_{\text{GPS}}) \right], \quad (8)$$

where D_{GPS} and $|D|_{\text{GPS}}$ indicate the covariance matrix of GPS and its determinant, respectively, whereas μ_{GPS} represents the mean value of GPS measurement. Moreover, we assume that the estimated response $\hat{Y}$ is given by

$$\hat{Y} = A_{\text{GPS}} \hat{Y}_t^{\text{GPS}} + A_{\text{INS}} \hat{Y}_t^{\text{INS}} \quad \text{subject to } (A) \equiv \begin{cases} A_{\text{GPS}} = \alpha_{\text{GPS}} \\ A_{\text{INS}} = 1 - \alpha_{\text{GPS}}, \end{cases} \quad (9)$$

where $\hat{Y}_t^{\text{GPS}} = F_s X_t^{\text{GPS}}$ and $\hat{Y}_t^{\text{INS}} = F_s X_t^{\text{INS}}$ are the estimated measurements provided by GPS and INS, respectively. Since the GPS weight is defined using Gaussian PDF, its values vary from 0 to 1 [26] (i.e., $\alpha_{\text{GPS}} \in [0, 1]$). Indeed, the GPS weight α_{GPS} is calculated by integrating the PDF α where the random variable varies from negative infinity to a certain limit d , for example, that is, $]-\infty, d[$. In this case, $A_{(\cdot)}$ is equal to $\vee \alpha_{\text{GPS}}$ where “ $\vee$ ” indicates the logical inclusive OR. This is due to the fact that the information can come, at the same time, from both the GPS and INS but is monitored by the GPS weight α_{GPS} .

3. Bayesian-Sparse Random Gaussian Prediction (B-SRGP) Method during the Free, Partial, and Full GPS Outages

This section describes the formulation of free, partial, and full GPS outages based on the weight of GPS, α_{GPS} , and the adopted technique to handle the evoked cases. Then, based on α_{GPS} , the three cases can be derived as follows.

3.1. Availability of GPS Signals (Free Outage). During the process, if $\alpha_{\text{GPS}} = 1$, that is, when the number of available satellites is at least four [4], it is clear that the system ignores the INS contribution and GPS will be solely taken into account at 100% (see (9) and Figure 2). Then, no GPS outage occurs. In this case, the system uses only the GPS measurements to predict the state of $\hat{Y}$ ($\hat{Y}_t = \hat{Y}_t^{\text{GPS}}$). Therefore, a predictive distribution for unknown data can then be obtained by conditioning on the known data as $p(\hat{Y}_t | Y) = p(\hat{Y}_t^{\text{GPS}} | Y)$ and the likelihood related to the GPS measurement noise is proportional to

$$L_{\xi^{\text{GPS}}}(\hat{Y}_t | Y) \sim (\hat{Y}_t^{\text{GPS}} - Y^{\text{GPS}})^T (\hat{R}_t^{\text{GPS}})^{-1} (\hat{Y}_t^{\text{GPS}} - Y^{\text{GPS}}), \quad (10)$$

where $\hat{R}_t^{\text{GPS}}$ is the estimated covariance matrix. In this paper, this matrix is estimated via the Maximum Likelihood Estimation (MLE) [27] and is given by $\hat{R}_t^{\text{GPS}} = \text{diag}(\hat{\sigma}_{\text{GPS}}^2/n, 2\hat{\sigma}_{\text{GPS}}^4/n)$. In addition, n represents the sample size and $\hat{\sigma}_{\text{GPS}}^2$ is the estimated variance defined in terms of shape parameter b as follows:

$$\hat{\sigma}_{\text{GPS}}^2 = \frac{2N(b) \sum_{i=1}^n |\xi_i^{\text{GPS}} - \hat{\mu}_{\text{GPS}}|^{2/(1+b)}}{n(1+b)}, \quad \text{for } b \neq -1. \quad (11)$$

FIGURE 2: Block diagram during the free, partial, and full GPS outages.

3.2. Full GPS Outage. Similarly, if $\alpha_{\text{GPS}} = 0$, that is, when GPS signals are absent (full GPS outage), the system assigns the 100% confidence to the INS (see (9) and Figure 2) and is used to estimate the state of $\hat{Y}$ ($\hat{Y} = \hat{Y}^{\text{INS}}$). Therefore, $p(\hat{Y}_t | Y) = p(\hat{Y}_t^{\text{INS}} | Y)$ and the likelihood related to the INS measurement noise is proportional to

$$L_{\xi^{\text{INS}}}(\hat{Y}_t | Y) \sim (\hat{Y}_t^{\text{INS}} - Y^{\text{INS}})^T (\hat{R}_t^{\text{INS}})^{-1} (\hat{Y}_t^{\text{INS}} - Y^{\text{INS}}), \quad (12)$$

where $\hat{R}_t^{\text{INS}}$ stands for the estimated covariance matrix. Its variance in terms of shape parameter b is given by

$$\hat{\sigma}_{\text{INS}}^2 = \frac{2N(b) \sum_{i=1}^n |\xi_i^{\text{INS}} - \hat{\mu}_{\text{INS}}|^{2/(1+b)}}{n(1+b)}, \quad \text{for } b \neq -1. \quad (13)$$

In (11) and (13), $\hat{\mu}_{(\cdot)}$ represents the estimated mean (via MLE) related to the GPS and INS measurement noises, respectively, and is defined as

$$\hat{\mu}_{(\cdot)} = \sum_{i=1}^n |\xi_i^{(\cdot)} - \mu_{(\cdot)}|^{(1-b)/(1+b)}, \quad \text{for } b \neq \{-1, 1\}. \quad (14)$$

In the following, we provide the proof of (11) (or (13)) and (14).

In general, recall that for each sample G , for example, consider $\theta(g)$ a parameter value at which $L(\theta | g)$ attains its maximum as function of θ . A maximum likelihood estimator of the parameter θ based on G is $\hat{\theta}(g)$. Then, if the likelihood function is C^2 , the MLEs of θ_i are the θ_m 's which maximize the likelihood function and the necessary conditions for an optimum are given by

$$\frac{\partial}{\partial \theta_i} L(\theta_1, \theta_2, \dots, \theta_m | g_1, g_2, \dots, g_n) = 0, \quad (15)$$

where $g = [g_1, g_2, \dots, g_n]^T$ and $\theta = [\theta_1, \theta_2, \dots, \theta_m]^T$ and such that

$$L(\theta_1, \theta_2, \dots, \theta_l | g_1, g_2, \dots, g_n) = \prod_{i=1}^n f(g_i | \theta_1, \theta_2, \dots, \theta_m) \quad (16)$$

is the joint PDF (or likelihood) of $G_1, \dots, G_n$. Moreover, MLE is equivalent to the maximum log-likelihood estimation which is provided by

$$l(\theta_1, \theta_2, \dots, \theta_l | g_1, g_2, \dots, g_n) = \sum_{i=1}^n \log(f(g_i | \theta_1, \theta_2, \dots, \theta_m)). \quad (17)$$

Proof of (11) (or (13)). The covariance matrix of the MLEs is the Hessian matrix which is the inverse of the matrix of second partial derivatives (in terms of the log-likelihood function, L). Then, the estimated covariance matrix $\hat{R}_\xi$ related to the measurement noise ξ is given by

$$\begin{aligned}\hat{R}_\xi &= \begin{pmatrix} \text{var}(\hat{\mu}_\xi) & \text{cov}(\hat{\mu}_\xi, \hat{\sigma}_\xi^2) \\ \text{cov}(\hat{\mu}_\xi, \hat{\sigma}_\xi^2) & \text{var}(\hat{\sigma}_\xi^2) \end{pmatrix} \\ &= \begin{pmatrix} \frac{\partial^2 \log(L(\xi_t))}{\partial \hat{\mu}_\xi \partial \hat{\mu}_\xi} & \frac{\partial^2 \log(L(\xi_t))}{\partial \hat{\mu}_\xi \partial \hat{\sigma}_\xi^2} \\ \frac{\partial^2 \log(L(\xi_t))}{\partial \hat{\mu}_\xi \partial \hat{\sigma}_\xi^2} & \frac{\partial^2 \log(L(\xi_t))}{\partial \hat{\sigma}_\xi^2 \partial \hat{\sigma}_\xi^2} \end{pmatrix}^{-1} \\ &= \begin{pmatrix} \frac{n}{\hat{\sigma}_\xi^2} & 0 \\ 0 & \frac{n \hat{\sigma}_\xi^2}{\hat{\sigma}_\xi^6} - \frac{n}{2 \hat{\sigma}_\xi^4} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{n}{\hat{\sigma}_\xi^2} & 0 \\ 0 & \frac{n}{2 \hat{\sigma}_\xi^4} \end{pmatrix}^{-1} \\ &= \begin{pmatrix} \frac{\hat{\sigma}_\xi^2}{n} & 0 \\ 0 & \frac{2 \hat{\sigma}_\xi^4}{n} \end{pmatrix}.\end{aligned}\quad (18)$$

Let us now determine the estimated variance $\hat{\sigma}_\xi^2$ based on MLE. Indeed, the likelihood function is computed as follows:

$$\begin{aligned}L &= \prod_{i=1}^n p(\xi^i, b, \sigma, \mu) = p(\xi^1, \xi^2, \dots, \xi^n) \\ &= (M(b) \sigma_\xi^{-1})^n \exp \left[-N(b) \left| \frac{\xi^1 - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right] \\ &\quad \cdot \exp \left[-N(b) \left| \frac{\xi^2 - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right] \\ &\quad \cdots \exp \left[-N(b) \left| \frac{\xi^n - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right] = (M(b) \sigma_\xi^{-1})^n \\ &\quad \cdot \exp \left[\sum_{i=1}^n \left(-N(b) \left| \frac{\xi^i - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right) \right].\end{aligned}\quad (19)$$

The maximum log-likelihood estimation is defined as $\log L = l = n \log(M(b) \sigma_\xi^{-1}) - \sum_{i=1}^n (N(b) |(\xi^i - \mu_\xi)/\sigma_\xi|^{2/(1+b)})$.

Using (15), the ML estimation of the variance σ_ξ^2 related to the measurement noise ξ is as follows:

$$\begin{aligned}\frac{\partial}{\partial \sigma_\xi^2} (\log L) &= \frac{\partial}{\partial \sigma_\xi^2} \left[n \log(M(b) (\sigma_\xi^2)^{-1/2}) \right. \\ &\quad \left. - \sum_{i=1}^n \left(\frac{N(b)}{(\sigma_\xi^2)^{1/(1+b)}} |\xi^i - \mu_\xi|^{2/(1+b)} \right) \right] = 0 \\ &\iff \frac{\partial}{\partial \sigma_\xi^2} \left[n \log M(b) - \frac{n}{2} \log \sigma_\xi^2 \right. \\ &\quad \left. - \frac{N(b)}{(\sigma_\xi^2)^{1/(1+b)}} \sum_{i=1}^n (|\xi^i - \mu_\xi|^{2/(1+b)}) \right] = 0 \\ &\iff -\frac{n}{2\sigma_\xi^2} + \frac{N(b)}{1+b} (\sigma_\xi^2)^{-(2+b)/(1+b)} \\ &\quad \cdot \sum_{i=1}^n (|\xi^i - \mu_\xi|^{2/(1+b)}) = 0 \\ &\iff -\frac{n}{2} + \frac{N(b)}{(1+b)\sigma_\xi^2} \sum_{i=1}^n (|\xi^i - \mu_\xi|^{2/(1+b)}) = 0 \\ &\iff \hat{\sigma}_\xi^2 = \frac{2N(b)}{n(1+b)} \sum_{i=1}^n (|\xi^i - \hat{\mu}_\xi|^{2/(1+b)}).\end{aligned}\quad (20)$$

Then, the estimated variance $\hat{\sigma}_\xi^2$ of σ_ξ^2 is provided by

$$\hat{\sigma}_\xi^2 = \frac{2N(b) \sum_{i=1}^n |\xi^{(i)} - \hat{\mu}_\xi|^{2/(1+b)}}{n(1+b)}, \quad \text{for } b \neq -1. \quad (21)$$

□

Proof of (14). Based on (15), the ML estimation of the mean μ_ξ related to the measurement noise ξ is obtained as follows:

$$\begin{aligned}\frac{\partial}{\partial \mu_\xi} (\log L) &= \frac{\partial}{\partial \mu_\xi} \left[n \log(M(b) \sigma_\xi^{-1}) \right. \\ &\quad \left. - \sum_{i=1}^n \left(N(b) \left| \frac{\xi^i - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right) \right] = 0 \\ &\iff \frac{\partial}{\partial \mu_\xi} \left[\sum_{i=1}^n \left(\left| \frac{\xi^i - \mu_\xi}{\sigma_\xi} \right|^{2/(1+b)} \right) \right] \\ &= \sum_{i=1}^n \left(\left| \xi^i - \mu_\xi \right|^{(1-b)/(1+b)} \right) = 0.\end{aligned}\quad (22)$$

Then, the estimated (minimum) mean $\hat{\mu}_\alpha$ of μ_α is provided by

$$\hat{\mu}_\xi = \sum_{i=1}^n |\xi^i - \mu_\xi|^{(1-b)/(1+b)}, \quad \text{for } b \neq \{-1, 1\}. \quad (23)$$

□

Consequently, based on (21) and (23), the estimated mean and variance related to the Gaussian distribution (see (7)) are provided by

$$\begin{aligned} \sum_{i=1}^n |\xi^{(i)} - \hat{\mu}_\xi| &= 0 \iff \hat{\mu}_\xi = \bar{\xi} = \frac{1}{n} \sum_{i=1}^n \xi^{(i)}, \\ \hat{\sigma}_\xi^2 &= \frac{\sum_{i=1}^n |\xi^{(i)} - \bar{\xi}|^2}{n}. \end{aligned} \quad (24)$$

3.3. Partial GPS Outage. This particular situation happens if $\alpha_{\text{GPS}} \in]0, 1[$, that is, if the number of available satellites is less than four or if the geometry of the four selected satellites is not adequate (a greater angle between the satellites provides a better measurement and therefore the good position accuracy [16]). In this case, there is interaction between the dependent variables X_t^{GPS} and X_t^{INS} . The selection of these local variables is made based on (8) which affects different values to $\alpha_{\text{GPS}} \in]0, 1[$.

Position Prediction Based Bayesian Inference during the Partial GPS Outages. In the case of partial GPS outages, the basic idea is to use the available information from both X_t^{GPS} and X_t^{INS} . To achieve this, the Bayesian inference is applied in order to optimally detect which information is more accurate for good position prediction. Indeed, assuming that the prior probabilities $p(X_t^{\text{GPS}})$ and $p(X_t^{\text{INS}})$ for X_t^{GPS} and X_t^{INS} , respectively, are known and based on (9), the Bayes theorem provides the following conditional probabilities:

$$\begin{aligned} p(X_t^{\text{GPS}} | \hat{Y}_t, \alpha_{\text{GPS}}) &= \frac{p(\hat{Y}_t | X_t^{\text{GPS}}) p(X_t^{\text{GPS}} | \alpha_{\text{GPS}})}{p(\hat{Y}_t | \alpha_{\text{GPS}})}, \\ p(X_t^{\text{INS}} | \hat{Y}_t, \alpha_{\text{GPS}}) &= \frac{p(\hat{Y}_t | X_t^{\text{INS}}) p(X_t^{\text{INS}} | \alpha_{\text{GPS}})}{p(\hat{Y}_t | \alpha_{\text{GPS}})}, \end{aligned} \quad (25)$$

where $p(X_t^{\text{GPS}} | \hat{Y}_t)$ and $p(X_t^{\text{INS}} | \hat{Y}_t)$ are the posterior probabilities related to the prior probabilities $p(X_t^{\text{GPS}})$ and $p(X_t^{\text{INS}})$ whereas $p(\hat{Y}_t | X_t^{\text{GPS}})$ and $p(\hat{Y}_t | X_t^{\text{INS}})$ are the probability densities from which the training data were drawn. Recall that the denominator in (25) can be expressed in terms of the numerator as $p(\hat{Y}_t | \alpha_{\text{GPS}}) \approx p(\hat{Y}_t | X_t^{\text{GPS}}) p(X_t^{\text{GPS}} | \alpha_{\text{GPS}}) + p(\hat{Y}_t | X_t^{\text{INS}}) p(X_t^{\text{INS}} | \alpha_{\text{GPS}})$ which acts as a normalization factor. Now the main problem is how to decide which information is coming from either the GPS or the INS to assign to the new output $\hat{Y}_t$. Based on [26], one can consider that $\hat{Y}_t$ is assigned to the data with larger posterior

probability; that is, the response $\hat{Y}_t$ is, for instance, assigned to GPS data if $p(X_t^{\text{GPS}} | \hat{Y}_t) > p(X_t^{\text{INS}} | \hat{Y}_t)$. This condition is verified if

$$p(\hat{Y}_t | X_t^{\text{GPS}}) > \frac{p(\hat{Y}_t | X_t^{\text{INS}}) p(X_t^{\text{INS}} | \alpha_{\text{GPS}})}{p(X_t^{\text{GPS}})}. \quad (26)$$

Otherwise, the response is assigned to INS data. This method can be practical for detecting new data coming from the GPS or INS that have low probability during the process and for which the predictions may be of low accuracy. However, under the condition with which $\alpha_{\text{GPS}} \in]0, 1[$, this cannot provide the accurate position prediction since the GPS or INS weight is still insufficient.

To overcome this weakness, we adopt the combination of data from both INS and GPS by summing (25) to get the following predictive distribution:

$$\begin{aligned} p(\hat{Y}_t^{\text{GPS} \wedge \text{INS}}) &= \sum_{k=1}^K p(X_t^k | \hat{Y}_t) \text{ such that } \|\hat{Y}_t^{\text{GPS} \wedge \text{INS}}\|_1 \geq \delta, \end{aligned} \quad (27)$$

where $K = 2$ stands for GPS and INS, respectively, and $\|\cdot\|_1$ represents the 1-norm function. The $\text{GPS} \wedge \text{INS}$ notation, here, indicates that information is from both GPS and INS where “ $\wedge$ ” is the AND logical operator. In this case, as the measurements from GPS and INS are received at the fusion center, the K -scan sliding window [28] approach is adopted to update the tracks and control the flow data generated by both INS and GPS which are required for the prediction accuracy. The value $\delta = (\theta - \tau)K$ is a regulation parameter where θ is the predefined support threshold and τ is the bound parameter error. The block diagram during the free, partial, and full GPS outages is presented in Figure 2.

3.4. Bayesian-Sparse Random Gaussian Prediction (B-SRGP) Algorithm for Vehicle Position. Algorithm 1 aims to predict the vehicle position using the Bayesian-Sparse Random Gaussian Prediction (B-SRGP) method during the free, partial, and full GPS outages. In order to define the predictor matrix F based on RIP, the optimization problem $\hat{X}_i = \min \|F_i X_i - Y_i\|_2^2$ is resolved using the conjugate gradient algorithm [29]. In addition, the Sparse Random Gaussian matrix is determined based on M iterations. The vehicle information provided by the GPS and the INS (Gyroscope and Accelerometer) integrated in Smartphone comprises the vehicle position (latitude L_t and longitude l_t), the angular velocity v_t , and acceleration a_t , respectively, at epoch t . During the free and full GPS outages, the vehicle information is provided by GPS and INS, respectively, according to (8), (9), (10), and (12). The Bayesian inference is used in order to handle the problem related to the partial GPS outages as described by (27).

4. Experiment and Evaluation Results

With the intention of effectively predicting the vehicle position in free, partial, and full GPS outages based on the

TABLE 1: Specifications of the Smartphone sensors.

Sensor name	Sensor type	Voltage (V)	Typical current (mA)	Range	Noise (RMS)
GPS receiver	BCM 4752	2.46–2.54	0.7	About 3 meters	1 (m)
Accelerometer	MPU 6500	1.8	0.45	$\pm 2\text{ g}/4\text{ g}/8\text{ g}$ ($\text{g} = \text{ms}^{-2}$)	6.1 (mg)
Gyroscope	MPU 6500	1.8	3.2	$\pm 250/500/1000\text{ dps}$	0.1 (dps)

Input: $\{X^{\text{GPS}}, X^{\text{INS}}\}$: predictor vectors; $\{Y^{\text{GPS}}, Y^{\text{INS}}\}$: measurement vectors; D^{GPS} : covariance matrix of GPS; M : number of iterations to obtain the Sparse Random Gaussian matrix.

Output: the predicted measurement of the vehicle position $\hat{Y}(\tilde{X})$.

- (1) Initialization: At epoch $t = 0$:
- (i) Set $X_0^{\text{GPS}} = \{L_0, I_0\}$ the initial vehicle position provided by GPS and $X_0^{\text{INS}} = \{a_0, v_0\}$, the initial angular velocity and acceleration provided by INS;
- (ii) Set initial values $\{n, p, s, T, b, \alpha_0, \delta, K\}$
- (2) **for** $t = 1$ to T **do**
- (3) **for** $m = 1$ to M **do**
- (4) Compute the random Gaussian matrix F_m according to (3);
- (5) $F_t \leftarrow F_m$
- (6) **end for**
- (7) Explore the measurement matrix F_t based on RIP such that the optimization problem is resolved via [29];
- (8) Generate $\hat{R}_t^{\text{INS}}, \hat{R}_t^{\text{GPS}}, \hat{\mu}_t^{\text{GPS}}$ and $\hat{\mu}_t^{\text{INS}}$ using MLE (see (11), (13) and (14));
- (9) Calculate the likelihoods $L_{\xi^{\text{GPS}}}(\hat{Y}_t | Y)$ and $L_{\xi^{\text{INS}}}(\hat{Y}_t | Y)$ according to (10) and (12);
- (10) Generate and evaluate the GPS weight α_t^{GPS} according to (8);
- (11) **if** $\alpha_t^{\text{GPS}} = 1$, (Free GPS outage) **then**
- (12) Predict vehicle position such that $p(\hat{Y}_t | Y) = p(\hat{Y}_t^{\text{GPS}} | Y)$ based on (9) and (10);
- (13) **end if**
- (14) **if** $\alpha_t^{\text{GPS}} = 0$ (Full GPS outage), **then**
- (15) Predict vehicle position such that $p(\hat{Y}_t | Y) = p(\hat{Y}_t^{\text{INS}} | Y)$ based on (9) and (12);
- (16) **Otherwise** ($0 < \alpha_t^{\text{GPS}} < 1$, Partial GPS outage)
- (17) Compute $p(\hat{Y}_t^{\text{GPS/INS}}) = \sum_{k=1}^K p(X_t^k | \hat{Y}_t)$ according to (27);
- (18) Predict the vehicle position such that $\|\hat{Y}_t^{\text{GPS/INS}}\|_1 \geq \delta$
- (19) **end if**
- (20) **end for**
- (21) Return the predicted measurement of the vehicle position $\hat{Y}(\tilde{X})$.

ALGORITHM 1: Bayesian-Sparse Random Gaussian Prediction (B-SRGP) algorithm.

proposed method, the test vehicle was driven along different roads in Hunan University area with different velocities for about 25 minutes. The Smartphone, Samsung Galaxy S4, on board the vehicle was with Android OS v4.2.2 as Operating System. This device not only comprises an Assisted-GPS (A-GPS) support and GLONASS but also includes the usual accelerometer, gyroscope, proximity, compass, and ambient light sensors for improving urban navigation performance. The technical specifications of the Smartphone sensors are highlighted in Table 1.

Although the device comprises two navigation systems, data from different satellites in view were collected using only A-GPS during the road experiments.

Moreover, for the purpose of computing the vehicle position (latitude and longitude), the Earth-Centered Earth Fixed (ECEF) system was utilized. This system was then changed into the coordinate system based on the Universal Transverse Mercator (UTM) projection system which is the most commonly used method in China.

(a) *Determination of the Partial and Full GPS Outages.* In order to show the location of the full and partial GPS outages, the number of satellites in-view system was used. The Geometric Dilution of Precision (GDOP) technique could be also adopted since, even though several satellites are available, the smaller angle between the four selected satellites leads to inaccuracy position. This case is also regarded as the partial GPS outage [3]. However, due to the technical limitations, the strategy related to the recorded satellites in view is preferred instead of using GDOP in order to detect different types of outages. The number of satellites in view which determines the free, partial, and full GPS outages during the road experiments is shown in Figure 3.

As can be seen from Figure 3, nine full GPS outages have been intentionally introduced in different periods on both the latitude and longitude from 73 to 109, from 268 to 307, from 532 to 562, from 602 to 643, from 699 to 748, from 780 to 827, from 991 to 1041, from 1111 to 1163, and from 1290 to 1345 seconds, respectively, with the lengths of 36, 39, 30, 41,

FIGURE 3: Number of satellites in view during the road test. The plot shows the number of satellites available at each time (second) where four (4) satellites are taken as minimum threshold for vehicle position accuracy.

51, 47, 50, 52, and 55 seconds. Even though the imbedded GPS receiver can detect signals from several satellites, its design is limited to the demodulation of signals from at most six satellites. Beside the nine artificial full GPS outages, the used system shows five natural partial outages distributed on latitude and longitude. It should be noted that two natural partial GPS outages were detected on latitude from 123 to 140 and from 351 to 378 seconds, respectively, with the lengths of 17 and 27 seconds and three were detected on longitude from 135 to 159, from 893 to 907, and from 1081 to 1101 seconds with lengths of 24, 14, and 30 seconds, respectively.

(b) Comparison and Analysis Results. In this experiment, at each time $t = 1, \dots, T$ ($T = 1410$ s), different values related to the vehicle position (the latitude and longitude) were provided by GPS integrated in Smartphone. The test was started with $X_0 = [490659.0, 2219445.0]$ as initial values which represent, respectively, the latitude [m] and longitude [m]. To achieve the better accuracy using the proposed method, the measurement matrix F_t defined as Sparse Random Gaussian (SRG) matrix of size $n = 4$ and $p = 6$ with only $s = 14$ nonzero coefficients was randomly generated after $M = 18$ iterations. For satisfying the RIP, the fixed RIC $\delta_s = 0.41$ [30] is considered which permits the canonical convex optimization problem for sparse approximation as well as guaranteeing the high probability. When the GPS signals are considerably available (the weight parameter $\alpha_{\text{GPS}} = 1$), the GPS error covariance matrix and the mean are estimated based on (11) and (14), respectively, where the scale parameter is set to $b = 2$ (for non-Gaussian measurement noise). In this case, the vehicle position was predicted using all available GPS information.

FIGURE 4: Vehicle position prediction in terms of longitude and latitude errors using the adopted methods KF, MARS, B-SRGP (non-gn), and B-SRGP (gn) in free, partial, and full outages. The black circle symbols show the detected partial outages on the latitude and longitude, respectively.

During the full GPS outage (the weight parameter $\alpha_{\text{GPS}} = 0$), the system assigns the 100% confidence to INS which means that all available INS data (from gyroscope and accelerometer sensors) are used for bridging the vehicle position prediction. Similarly, the INS error covariance matrix and the mean are estimated based on (13) and (14), respectively. Finally, during the partial GPS outage (the weight parameter $\alpha_{\text{GPS}} \in]0, 1[$), the available GPS information is completed by the INS data coming from the gyroscope and accelerometer sensors in order to successfully provide the reliable vehicle position prediction.

In this study, we have compared the performance of our proposed B-SRGP algorithm by taking KF [7] and Multivariate Adaptive Regression Splines (MARS) [31] as references which were the most prediction algorithms extensively used in the literature. Moreover, the effect of measurement noise on our proposed method B-SRGP is tested by considering the measurement noise to be non-Gaussian distributed (following GED) on one hand and Gaussian distributed on the other hand. This latter distribution was determined by zeroing the scale parameter $b = 0$ (see Proposition 1 and its proof). In this case, for simplicity, the proposed method is denoted “B-SRGP (gn)” and “B-SRGP (non-gn),” respectively, where “gn” stands for “Gaussian noise” and “non-gn” represents “non-Gaussian noise” as shown in Figure 4.

This assessment was conducted based not only on different simulated free outages but also on nine artificial simulated full outages as well as five natural partial outages of different durations as shown in Figures 3 and 4. A comparison to KF, MARS, B-SRGP (non-gn), and B-SRGP (gn) was performed for both the longitude and latitude which constitute the vehicle position.

The vehicle position error based latitude and longitude coordinates are computed by comparing the predicted vehicle position observed from the measurement with the corresponding *true* vehicle position. Figure 4 indicates the longitude position and the latitude position error curves of the four methods mentioned above. In fact, let us analyze the results taking into account the three kinds of outages.

(i) *Free and Full GPS Outages.* As mentioned in Section 2.2, the proposed method uses the GPS and the INS data to predict the vehicle position. The required measurements which are based only on the sparsity of the optimized singular matrix are sensibly reduced since, in our case, approximately 41.67 percent of matrix elements are zeros. The second factor that proves the appropriateness of B-SRGP (non-gn) for INS/GPS applications is based on the consideration of measurement noise as non-Gaussian distributed. Indeed, setting the shape parameter as greater than zero, that is, $b = 2$, reduces the prediction error because the estimated noise variance and therefore the covariance matrix which constitutes the likelihood functions become small [32]. These two main factors contribute to increasing the position prediction accuracy. In addition, obviously, B-SRGP (gn) is as accurate as B-SRGP (non-gn) in free GPS outages. However, the main difference resides in the full GPS outages. The disparity is also due to the non-Gaussianity related to the measurement noise. In fact, zeroing the shape parameter not only leads to the Gaussian distribution (5) but also increases the estimated noise variance which in turn increases the likelihood function expressed as a function of the estimated covariance matrix. This augments consequently the prediction errors [32] for the vehicle position.

Figure 4 shows that the vehicle position prediction errors related to MARS are very close to the errors related to B-SRGP (non-gn) and B-SRGP (gn) in both free and full GPS outages because MARS performs well in the noisy environment and has fairly low variance due to the constrained form of its spline basis functions. Moreover, MARS is flexible enough to model nonlinearity and variable interactions as B-SRGP (non-gn) or B-SRGP (gn). However, B-SRGP (non-gn) or B-SRGP (gn) is slightly better than MARS because this latter fails to predict well when the sample size is small [33, 34] (in our case, the sample size is set to $s = 14$, non-zero entries of the random Gaussian matrix). From Figure 4, one can also remark that KF is the worst predictor of other adopted predictors. Indeed, due to the limited knowledge of the system's dynamic model and measurement noise, KF method suffers from the divergence and provides inaccurate prediction information. Moreover, the weakness in discernibility of some of the error states may lead to unstable estimates of other error states [13].

(ii) *Partial GPS Outages.* To achieve the reliable results, the regulation parameter is set such that the support threshold $\theta = 0.5$ and the bound parameter error $\tau = 0.05$ [35]. Moreover, 2-scan sliding window is used as $K = 2$ systems (INS and GPS) (see (27)). Through the zoom (see Figure 5) of the four out of five partial GPS outages plotted in Figure 4, one can see that the proposed method also provides the

good performance in terms of vehicle position prediction. The aforementioned comparison analysis about the adopted methods is relatively identical for the partial GPS outages. Generally, based on B-SRGP (non-gn), the prediction accuracy during the partial GPS outages is better than the prediction accuracy during the full GPS outages. Indeed, during the partial GPS outages, the INS and GPS data are complemented and the Bayesian inference uses prior distributions which allow the usage of more information. Besides, the selection of the sliding window also contributes to increasing the vehicle position prediction. For all the simulated free, partial, and full GPS outages considered in this study, the proposed B-SRGP algorithm is more efficient than other adopted methods for the vehicle position prediction when the measurement noises are non-Gaussian distributed.

The performance of our proposed method is also evaluated statistically using a standard equation (28) to calculate mean-squared prediction error (MSPE):

$$\text{MSPE} = \frac{1}{n} \sum_{X_i \in \{X\}_{\text{GPSVINS}}} \left(Y_{\text{true}}(X_i) - \hat{Y}_{\text{pred}}(X_i) \right)^2, \quad (28)$$

$$i = 1, \dots, n,$$

where $Y_{\text{true}}(X_i)$ is the true measurement at the position $X_i \in \{X\}_{\text{GPSVINS}}$ and $\hat{Y}_{\text{pred}}(X_i)$ represents the predicted measurement at the position $X_i \in \{X\}_{\text{GPSVINS}}$ using different adopted methods. Here, $\{X\}_{\text{GPSVINS}}$ indicates the set of measured positions using GPS and/or INS.

Figure 6 plots the MSPE for all adopted prediction methods based on the components of the vehicle position (latitude and longitude) during the full GPS outages taking into account the length of each outage. Recall that low MSPE implies the high confidence for the prediction method. Specifically, the difference between B-SRGP (gn) and B-SRGP (non-gn) in terms of MSPE is related to the large measurement noise variance of B-SRGP (gn) which produces the large value of MSPE. The same observations are applied on the partial outages plotted in Figure 7.

In addition, the MSP errors increase as the lengths of the outages increase. Indeed, let us consider, for instance, the three last full outages on longitude (see Figure 6) with 50, 52, and 55 seconds of lengths, respectively. The MSP longitude errors are 0.55, 1.14, and 1.49 m, respectively, for B-SRGP (non-gn); 27.69, 33.90, and 29.8 m, respectively, for B-SRGP (gn); 5.70, 7.72, and 9.61 m, respectively, for MARS; and 23.91, 28.14, and 30.55 m, respectively, for KF. These results are consistent because the full outage lengths where only data from INS are used to replace GPS information are not extremely large. This observation is also supported by the authors in [7] who demonstrated that, in general, bridging by inertial sensors can be applied for only very short time.

In terms of averages, as it is stated in Table 2, it can be seen that the B-SRGP (non-gn) has the lower average of MSPE than other adopted prediction methods based on the nine full outages and five partial outages located on both latitude and longitude. Although B-SRGP (gn) outperforms MARS in free and partial GPS outages (see Figure 4), we can notice from Table 2 that MARS is rather better than B-SRGP (gn) in terms

FIGURE 5: Highlighting the position prediction errors in different partial outages distributed on the latitude and longitude under the adopted methods: red for B-SRGP (gn), magenta for B-SRGP (non-gn), blue for KF, and green for MARS.

TABLE 2: MSP position errors average during full GPS outage and partial GPS outage [m].

	B-SRGP (non-gn)	MARS	KF	B-SRGP (gn)
Full GPS outage				
Latitude	0.26	0.96	15.42	13.25
Longitude	0.68	4.78	21.64	21.90
Partial GPS outage				
Latitude	0.07	3.35	5.0	2.05
Longitude	0.13	3.89	4.39	2.37

of MSPE in full GPS outages. The main reasons are due to the good characteristics of MARS already previously explained while B-SRGP (gn) suffers from the drawbacks related to the Gaussian measurement noise, especially when measurement data are from only INS.

In sum, the above experimental results reveal that B-SRGP (non-gn) presents higher accuracy prediction and lower mean-squared prediction error (MSPE) for vehicle position when compared to the other adopted methods.

Although our proposed method is reliable for predicting the vehicle position even in challenging environments, it faces some limitations. Indeed, if, for instance, the size of the measurement matrix tends to infinity, that is, $p \rightarrow \infty$ (or $n \rightarrow \infty$), this leads to the higher computational complexity of our proposed method which was not treated in this study. In other words, while MARS is suitable for handling fairly large datasets [35], the proposed B-SRGP is still not applicable to very large-scale problems due to its computational complexity. This implies that MARS would become better than our proposed method when large amounts of data were taken into account.

5. Conclusion and Future Work

In this paper, we propose an approach based on B-SRGP for vehicle position prediction. The originality lies in the use of a Sparse Random Gaussian matrix as a matrix measurement for vehicle position prediction as well as the sensors integrated in Smartphone to collect the dataset in urban city where free and natural partial GPS outages were detected and full GPS

FIGURE 6: Comparison of the adopted methods in terms of MSPE related to the vehicle position prediction during the nine full GPS outages detected on latitude and longitude, respectively.

FIGURE 7: Comparison of the adopted methods in terms of MSPE related to the vehicle position prediction during the two partial GPS outages and the three partial GPS outages detected on latitude and longitude, respectively.

outages were intentionally introduced. In addition, the Bayesian inference is introduced to deal with the partial GPS outages where the sliding window size is used to regulate the flow data generated by both INS and GPS which are required for the prediction accuracy. The benefits of our prediction method are shown on simulation results through both the

recorder dataset and the statistical evaluation using mean-squared prediction error (MSPE). Moreover, it is demonstrated that the prediction ability of the proposed algorithm B-SRGP is superior to the conventional KF and MARS. Besides, when the proposed algorithm is tested under the assumption that the measurement noises are Gaussian and non-Gaussian distributed, the experiment results revealed that it presents higher accuracy prediction and lower mean-squared prediction error (MSPE) when the measurement noises are non-Gaussian distributed. Although the B-SRGP is able to predict the vehicle position during the free, partial, and full outages, it may suffer from the higher computational complexity when the size of the measurement matrix becomes larger (tends to infinity). What is more, even though the experiment results showed that MSP errors increase with the lengths of the outages, this study did not evaluate the effectiveness of the B-SRGP when the period of full or partial GPS outage is extra long (e.g., more than 100 seconds). It is therefore our duty to deal with the aforementioned challenging cases in the future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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