Research Article

Asynchronous $\mathcal{H}_\infty$ Estimation for Two-Dimensional Nonhomogeneous Markovian Jump Systems with Randomly Occurring Nonlocal Sensor Nonlinearities

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This paper is devoted to the problem of asynchronous $\mathcal{H}_\infty$ estimation for a class of two-dimensional (2D) nonhomogeneous Markovian jump systems with nonlocal sensor nonlinearity, where the nonlocal measurement nonlinearity is governed by a stochastic variable satisfying the Bernoulli distribution. The asynchronous estimation means that the switching of candidate filters may have a lag to the switching of system modes, and the varying character of transition probabilities is considered to reside in a convex polytope. The jumping process of the error system is modeled as a two-component Markov chain with extended varying transition probabilities. A stochastic parameter-dependent approach is provided for the design of $\mathcal{H}_\infty$ filter such that, for randomly occurring nonlocal sensor nonlinearity, the corresponding error system is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance index. Finally, a numerical example is used to illustrate the effectiveness of the developed estimation method.

1. Introduction

Two-dimensional (2D) systems have recently received considerable attention from scientific communities for their potential applications in various areas, such as image data processing and transmission, multidimensional digital filtering, and process control [1]. The analysis and synthesis of 2D systems are much more complicated than those of one-dimensional systems due primarily to their structural complexity. As a result, a great amount of effort has been invested in such systems, and many interesting and important results have been derived so far [2–6]. In the context of state estimation, filtering problems for 2D systems have also been deeply studied. To mention a few, the minimum mean-square state estimation has been addressed in [7, 8], the $\mathcal{H}_\infty$ filtering problem has been tackled in [9–19], and the $l_2 - l_\infty$ filtering problem has been considered in [20].

As is well known, Markovian jump linear systems, which were first introduced in [21], have been widely used to model a large variety of physical systems that experience abrupt

changes in their structure and parameters. The transition probabilities play a crucial role in determining the behavior and performance of Markovian jump systems. In the case of time-varying transition probabilities, the Markov chain is viewed as nonhomogeneous. Most recently, a number of outstanding analysis and design results have been obtained for Markovian jump systems with nonhomogeneous transition probabilities [22, 23].

In particular, the filter design for Markovian jump systems has also been extensively investigated [23–25]. A popular solution is to find less conservative mode-dependent filters such that the resulting filtering error system is stable and satisfies certain performance. Most of the mode-dependent methods available rely on the ideal assumption that the switches of filters are strictly synchronized with those of the system modes. However, perfect synchronization is not always possible in practical situations owing to operations related to identifying the system mode and specifying the matched filter [26]. Thus, it seems more practicable and significant to design asynchronous filters for Markovian jump

systems, especially for nonhomogeneous Markovian jump systems.

Moreover, sensor nonlinearities arise frequently under harsh filtering environments including both uncontrollable elements and aggressive conditions. It is worth pointing out that filtering techniques concerning the sensor nonlinearities usually provide a relatively reliable solution. State estimation related to Markov jump systems with sensor nonlinearities has been developed in terms of many sorts of methods [27]. However, to the best of the authors' knowledge, the problem of asynchronous $\mathcal{H}_\infty$ estimation for 2D nonhomogeneous Markovian jump systems with nonlocal sensor nonlinearity has not been fully resolved, despite its deep practical implications.

Therefore, in this paper, we will handle the problem of asynchronous $\mathcal{H}_\infty$ estimation for 2D nonhomogeneous Markovian jump systems with randomly occurring nonlocal sensor nonlinearity. A stochastic variable satisfying the Bernoulli binary distribution is employed to characterize the nonlocal nonlinear measurement behavior. The varying character of transition probabilities is described by means of a specified polytope, and the jumping process of the error system is represented by a two-component Markov chain with extended varying transition probabilities. The $\mathcal{H}_\infty$ analysis result is derived by a stochastic parameter-dependent approach. The existence condition of the desired filter is then obtained such that, for randomly occurring nonlocal sensor nonlinearity, the corresponding error system is mean-square asymptotically stable and has a guaranteed $\mathcal{H}_\infty$ performance level. A numerical example is provided to show the effectiveness of the proposed design method.

Notations. The notation used throughout the paper is fairly standard. The superscript T stands for matrix transposition, $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, and $P > 0$ means that P is real symmetrical and positive definite. $\mathbb{E}\{\cdot\}$ stands for the expectation operation. $l_2\{[0, \infty), [0, \infty)\}$ is the space of square summable sequences on $\{[0, \infty), [0, \infty)\}$. $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix, for a vector x the diagonal matrix defined by the entries of x is denoted by $\text{diag}(x)$. For any symmetric matrix, $(*)$ represents a symmetric term. Moreover, for each integer q , $\mathbf{1}_q$ denotes the vector in $\mathbb{R}^q$ defined by $\mathbf{1}_q = [1 \cdots 1]^T$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

Consider the 2D Markovian jump system with sensor nonlinearity in the Roesser model:

$$\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = A(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + B(r_{i,j}) w_{i,j},$$

$$y_{i,j} = (1 - \delta_k) C(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix}$$

$$+ \delta_k \varphi \left(C(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) + D(r_{i,j}) w_{i,j},$$

$$z_{i,j} = H(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + L(r_{i,j}) w_{i,j}, \quad (1)$$

where $x_{i,j}^h \in \mathbb{R}^{n_1}$ and $x_{i,j}^v \in \mathbb{R}^{n_2}$ represent the horizontal and vertical states, respectively, $y_{i,j} \in \mathbb{R}^r$ is the measured output, $z_{i,j} \in \mathbb{R}^p$ is the objective signal to be estimated, and $w_{i,j} \in \mathbb{R}^q$ is the noise signal which belongs to $l_2\{[0, \infty), [0, \infty)\}$. The system matrices are functions of $r_{i,j}$, which is a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{J} = \{1, 2, \dots, s\}$ with mode transition probabilities:

$$\begin{aligned} \pi_{mn}^{ij} &= \Pr \{r_{i+1,j} = n \mid r_{i,j} = m\} \\ &= \Pr \{r_{i,j+1} = n \mid r_{i,j} = m\}, \end{aligned} \quad (2)$$

where $\pi_{mn}^{ij} \geq 0$ and $\sum_{n=1}^s \pi_{mn}^{ij} = 1$. To simplify the notation, the system matrices are denoted by $\mathcal{S}_m = (A_m, B_m, C_m, D_m, H_m, L_m)$, when $r_{i,j} = m \in \mathcal{J}$. Here, π_{mn}^{ij} are the entries of the transition matrix Π^{ij} , which is assumed to be of the form

$$\Pi^{ij} = \sum_{g=1}^N \alpha_g^{ij} \Pi^g, \quad (3)$$

where $\alpha_g^{ij} \geq 0$, $\sum_{g=1}^N \alpha_g^{ij} = 1$ and Π^g denote the vertices of the polytope.

Remark 1. It is worth stressing that, different from the homogeneous Markov chain concerned in the recently developed techniques for asynchronous filter design [18, 19, 27], the polytopic time-varying model of Markov chain under consideration is more practicable in the Markovian jump systems field. The homogeneous Markov chain has a critical assumption that the transition probabilities need to be known exactly. This ideal requirement sometimes inevitably limits the application of the derived results, since it can be hard or costly to obtain the precise information about the transition probabilities in practice. In view of that, it seems more reasonable to consider that the transition probabilities are affected by varying parameters.

The function φ represents the nonlocal sensor nonlinearity satisfying the following nonlocal sector condition [28]:

$$\begin{aligned} (\varphi(\theta) - K_1\theta - F_1)^T (\varphi(\theta) - K_2\theta - F_2) &\leq 0, \\ \theta &\in \mathbb{R}^n, \|\theta\| \geq v_c, \end{aligned} \quad (4)$$

where K_1 and K_2 are matrices, F_1 and F_2 are vectors, and v_c is a positive real number. Moreover, the nonlinear function $\varphi(\theta)$ can be decomposed into a linear and a nonlinear part as

$$\varphi(\theta) = \varphi_p(\theta) + K_1\theta + F_1, \quad (5)$$

where the nonlinearity $\varphi_p(\theta)$ satisfies

$$\varphi_p^T(\theta) (\varphi_p(\theta) - K\theta - F) \leq 0 \quad (6)$$

with $K = K_2 - K_1$ and $F = F_2 - F_1$. The stochastic variable δ_k , which is introduced to account for the phenomena of randomly occurring nonlocal sensor nonlinearity, is Bernoulli sequence taking the values of 1 and 0 with

$$\begin{aligned} \Pr\{\delta_k = 1\} &= \mathbb{E}\{\delta_k\} = \mu, \\ \Pr\{\delta_k = 0\} &= 1 - \mathbb{E}\{\delta_k\} = 1 - \mu. \end{aligned} \quad (7)$$

Remark 2. It can be found that when $F_1 = 0$, $F_2 = 0$, and $v_c = 0$, the nonlocal sector nonlinearity in (4) reduces to the conventional sector nonlinearity [27], which implies that the nonlocal sector nonlinearity in (4) covers the conventional sector nonlinearity as a special case. Hence, the case considered in the sequel is more general.

For the asynchronous phenomenon considered, we are interested in estimating the objective signal $z_{i,j}$ by a filter as follows:

$$\begin{aligned} \begin{bmatrix} \hat{x}_{i+1,j}^h \\ \hat{x}_{i,j+1}^v \end{bmatrix} &= \left[(1 - \sigma_k) A_{fm} + \sigma_k A_{fz} \right] \begin{bmatrix} \hat{x}_{i,j}^h \\ \hat{x}_{i,j}^v \end{bmatrix} \\ &\quad + \left[(1 - \sigma_k) B_{fm} + \sigma_k B_{fz} \right] y_{i,j}, \\ \hat{z}_{i,j} &= \left[(1 - \sigma_k) C_{fm} + \sigma_k C_{fz} \right] \begin{bmatrix} \hat{x}_{i,j}^h \\ \hat{x}_{i,j}^v \end{bmatrix} \\ &\quad + \left[(1 - \sigma_k) D_{fm} + \sigma_k D_{fz} \right] y_{i,j}, \end{aligned} \quad (8)$$

where $\hat{x}_{i,j}^h \in \mathbb{R}^{n_1}$ and $\hat{x}_{i,j}^v \in \mathbb{R}^{n_2}$ are the filter states, $\hat{z}_{i,j} \in \mathbb{R}^p$ is the estimation of $z_{i,j}$, $\mathcal{F}_m = (A_{fm}, B_{fm}, C_{fm}, D_{fm})$ and $\mathcal{F}_z = (A_{fz}, B_{fz}, C_{fz}, D_{fz})$ are the filter gains corresponding to the current and previous stages, respectively, $\forall m, z \in \mathcal{J}$, and σ_k is a Bernoulli distributed white sequence specified by

$$\begin{aligned} \Pr\{\sigma_k = 1\} &= \mathbb{E}\{\sigma_k\} = \eta, \\ \Pr\{\sigma_k = 0\} &= 1 - \mathbb{E}\{\sigma_k\} = 1 - \eta. \end{aligned} \quad (9)$$

In addition, σ_k , δ_k , and $r_{i,j}$ are mutually independent.

In view of (1) and (8), the estimation error $\bar{z}_{i,j} = z_{i,j} - \hat{z}_{i,j}$ can be described by the following model:

$$\begin{aligned} \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix} &= \bar{A}_{m,l} \begin{bmatrix} \bar{x}_{i,j}^h \\ \bar{x}_{i,j}^v \end{bmatrix} + \bar{M}_l \varphi_p \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) + \bar{M}_l F_1 \\ &\quad + \bar{B}_{m,l} w_{i,j} + (\mu - \delta_k) \\ &\quad \cdot \left(\hat{A}_{m,l} \begin{bmatrix} \hat{x}_{i,j}^h \\ \hat{x}_{i,j}^v \end{bmatrix} + \hat{M}_l \varphi_p \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) + \hat{M}_l F_1 \right), \\ \bar{z}_{i,j} &= \bar{C}_{m,l} \begin{bmatrix} \bar{x}_{i,j}^h \\ \bar{x}_{i,j}^v \end{bmatrix} + \bar{N}_l \varphi_p \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) + \bar{N}_l F_1 \\ &\quad + \bar{D}_{m,l} w_{i,j} + (\mu - \delta_k) \\ &\quad \cdot \left(\hat{C}_{m,l} \begin{bmatrix} \hat{x}_{i,j}^h \\ \hat{x}_{i,j}^v \end{bmatrix} + \hat{N}_l \varphi_p \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) + \hat{N}_l F_1 \right), \end{aligned} \quad (10)$$

where $\bar{x}_{i,j}^h = [x_{i,j}^{hT} \hat{x}_{i,j}^{hT}]^T$, $\bar{x}_{i,j}^v = [x_{i,j}^{vT} \hat{x}_{i,j}^{vT}]^T$ and

$$\begin{aligned} \bar{A}_{m,l} &= \Lambda^T \begin{bmatrix} A_m & 0 \\ (1 - \mu) B_{fl} C_m + \mu B_{fl} K_1 C_m & A_{fl} \end{bmatrix} \Lambda, \\ \bar{M}_l &= \Lambda^T \begin{bmatrix} 0 \\ \mu B_{fl} \end{bmatrix}, \\ \hat{A}_{m,l} &= \Lambda^T \begin{bmatrix} 0 & 0 \\ B_{fl} C_m - B_{fl} K_1 C_m & 0 \end{bmatrix} \Lambda, \\ \hat{M}_l &= \Lambda^T \begin{bmatrix} 0 \\ -B_{fl} \end{bmatrix}, \\ \bar{B}_{m,l} &= \Lambda^T \begin{bmatrix} B_m \\ B_{fl} D_m \end{bmatrix}, \\ \bar{C}_{m,l} &= [H_m - (1 - \mu) D_{fl} C_m - \mu D_{fl} K_1 C_m \quad -C_{fl}] \Lambda, \\ \bar{N}_l &= -\mu D_{fl}, \\ \hat{C}_{m,l} &= [D_{fl} K_1 C_m - D_{fl} C_m \quad 0] \Lambda, \\ \hat{N}_l &= D_{fl}, \\ \bar{D}_{m,l} &= L_m - D_{fl} D_m, \end{aligned} \quad (11)$$

$$\Lambda = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

$$\forall m, l \in \mathcal{J}.$$

Interestingly, the jumping process $\{\mathcal{S}_m, \mathcal{F}_l\}$ of the error system in (10) forms a two-component Markov chain $\bar{r}_{i,j}$ on $\mathcal{J} \times \mathcal{J}$ with the extended varying transition probabilities $\bar{\pi}_{(m,l)(n,u)}^{ij} = \Pr\{\mathcal{S}_n, \mathcal{F}_u \mid \mathcal{S}_m, \mathcal{F}_l\}$ given by

$$\bar{\pi}_{(m,l)(n,u)}^{ij} = \begin{cases} \sum_{g=1}^N \alpha_g^{ij} \pi_{mn}^g, & u = n, u = l, \\ \sum_{g=1}^N \alpha_g^{ij} \eta \pi_{mn}^g, & u \neq n, u = l, \\ \sum_{g=1}^N \alpha_g^{ij} (1 - \eta) \pi_{mn}^g, & u = n, u \neq l, \\ 0, & u \neq n, u \neq l. \end{cases} \quad (12)$$

The following definitions for the error system in (10) are necessary to formulate the considered problem. For more details, refer to [15] and the references therein.

Definition 3. System (10) is said to be mean-square asymptotically stable if for $w_{i,j} = 0$ and bounded boundary conditions, the following holds:

$$\lim_{i+j \rightarrow \infty} \mathbb{E} \left\{ \|\bar{x}_{i,j}\|^2 \right\} = 0, \quad (13)$$

where $\bar{x}_{i,j} = [\bar{x}_{i,j}^{hT} \bar{x}_{i,j}^{vT}]^T$.

Definition 4. Given a scalar $\gamma > 0$, system (10) is said to be mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level γ , if it is mean-square asymptotically stable, and under zero initial conditions satisfies

$$\|\bar{z}_{i,j}\|_E < \gamma \|w_{i,j}\|_2, \quad (14)$$

for all nonzero $w_{i,j}$, where

$$\begin{aligned} \|\bar{z}_{i,j}\|_E &= \sqrt{\mathbb{E} \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \|\bar{z}_{i,j}\|^2 \right\}}, \\ \|w_{i,j}\|_2 &= \sqrt{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \|w_{i,j}\|^2}. \end{aligned} \quad (15)$$

Then, the estimation problem of interest is stated as follows: given $\gamma > 0$, design a filter of the form in (8) such that the error system in (10) with extended varying transition probabilities (12) is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance level γ .

3. $\mathcal{H}_\infty$ Filter Design

This section provides a procedure for designing the asynchronous $\mathcal{H}_\infty$ filter, which guarantees that the error system with extended varying transition probabilities is mean-square asymptotically stable and has a prescribed disturbance attenuation level in the $\mathcal{H}_\infty$ sense. First, we derive an $\mathcal{H}_\infty$ analysis criterion to check if the $\mathcal{H}_\infty$ norm of the error system in (10) is bounded by the asynchronous filter. The corresponding analysis result is summarized in the following theorem.

Theorem 5. Consider system (10) with extended varying transition probabilities (12) and let $\gamma > 0$ be a given constant. If there exist matrices $P_{m,l}^g = \text{diag}\{P_{m,l}^{hg}, P_{m,l}^{vg}\} > 0$ and scalars $\tau_{m,l}^g > 0$, $\rho_{m,l}^g > 0$, $g = 1, \dots, N$, $\forall (m, l) \in \mathcal{F} \times \mathcal{F}$, such that

$$\begin{bmatrix} -\mathcal{P}_{n,u}^g & 0 & 0 & 0 & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} \\ * & -\mathcal{P}_{n,u}^g & 0 & 0 & 0 & \Omega_{26} & \Omega_{27} & \Omega_{28} & 0 \\ * & * & -I & 0 & 0 & \bar{C}_{m,l} & \bar{N}_l & \Omega_{38} & \bar{D}_{m,l} \\ * & * & * & -I & 0 & \Omega_{46} & \Omega_{47} & \Omega_{48} & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & 0 & 0 & 0 \\ * & * & * & * & * & -\sum_{g=1}^N \alpha_g^{ij} P_{m,l}^g & \Omega_{67} & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned} \mathcal{P}_{n,u}^g &= \sum_{(n,u) \in \mathcal{F} \times \mathcal{F}} \sum_{g=1}^N \sum_{q=1}^N \alpha_g^{ij} \beta_q^{ij} \bar{\pi}_{(m,l)(n,u)}^g P_{n,u}^q, \\ \alpha_g^{ij} &\geq 0, \quad \sum_{g=1}^N \alpha_g^{ij} = 1, \quad \beta_q^{ij} \geq 0, \quad \sum_{q=1}^N \beta_q^{ij} = 1, \\ \Omega_{16} &= \mathcal{P}_{n,u}^g \bar{A}_{m,l}, \\ \Omega_{17} &= \mathcal{P}_{n,u}^g \bar{M}_l, \\ \Omega_{18} &= \mathcal{P}_{n,u}^g \bar{M}_l \text{diag}(F_1), \\ \Omega_{19} &= \mathcal{P}_{n,u}^g \bar{B}_{m,l}, \\ \Omega_{26} &= \sqrt{\lambda} \mathcal{P}_{n,u}^g \hat{A}_{m,l}, \\ \Omega_{27} &= \sqrt{\lambda} \mathcal{P}_{n,u}^g \hat{M}_l, \\ \Omega_{28} &= \sqrt{\lambda} \mathcal{P}_{n,u}^g \hat{M}_l \text{diag}(F_1), \\ \Omega_{38} &= \bar{N}_l \text{diag}(F_1), \\ \Omega_{46} &= \sqrt{\lambda} \hat{C}_{m,l}, \\ \Omega_{47} &= \sqrt{\lambda} \hat{N}_l, \\ \Omega_{48} &= \sqrt{\lambda} \hat{N}_l \text{diag}(F_1), \\ \Omega_{55} &= -\mu \sum_{g=1}^N \alpha_g^{ij} \tau_{m,l}^g I, \\ \Omega_{56} &= \mu \sum_{g=1}^N \alpha_g^{ij} \tau_{m,l}^g C_m \Lambda_1, \\ \Omega_{67} &= \mu \sum_{g=1}^N \alpha_g^{ij} \rho_{m,l}^g \Lambda_1^T C_m^T K^T, \\ \Omega_{77} &= -2\mu \sum_{g=1}^N \alpha_g^{ij} \rho_{m,l}^g I, \\ \Omega_{78} &= \mu \sum_{g=1}^N \alpha_g^{ij} \rho_{m,l}^g \text{diag}(F), \\ \Omega_{88} &= -\mu \frac{\gamma^2}{r} \sum_{g=1}^N \alpha_g^{ij} \tau_{m,l}^g I, \end{aligned} \quad (17)$$

and $\lambda = \mu(1 - \mu)$, then the system in (10) is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance index γ .

Proof. First, we handle the stochastic stability of system (10) with $w_{i,j} \equiv 0$. Construct the following index:

$\mathcal{L}_{i,j}$

$$= \mathbb{E} \left\{ \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix}^T \begin{bmatrix} P^h(\bar{r}_{i+1,j}) & 0 \\ 0 & P^v(\bar{r}_{i,j+1}) \end{bmatrix} \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix} \right. \\ \left. - \bar{x}_{i,j}^T P(\bar{r}_{i,j}) \bar{x}_{i,j} \mid \bar{x}_{i,j}, \bar{r}_{i,j} = (m, l) \right\}, \quad (18)$$

where $P(\bar{r}_{i,j}) = \text{diag}\{P^h(\bar{r}_{i,j}), P^v(\bar{r}_{i,j})\}$, which is represented as $P_{m,l}$ when $\bar{r}_{i,j} = (m, l)$. It follows from expression (18) that

$$\mathcal{L}_{i,j} = \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix}^T \sum_{(n,u) \in \mathcal{J} \times \mathcal{J}} \left(\sum_{g=1}^N \alpha_g^{ij} \pi_{(m,l)(n,u)}^g \right) \\ \cdot \left(\sum_{q=1}^N \beta_q^{ij} P_{n,u}^q \right) \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix} - \bar{x}_{i,j}^T \sum_{g=1}^N \alpha_g^{ij} P_{m,l}^g \bar{x}_{i,j}. \quad (19)$$

Collectively considering expression (19) and system (10) with $w_{i,j} = 0$, we have

$$\mathcal{L}_{i,j} = \begin{bmatrix} \bar{A}_{m,l} \bar{x}_{i,j} + \bar{M}_l \varphi_p + \bar{M}_l F_1 \end{bmatrix}^T \\ \cdot \mathcal{P}_{n,u}^q \begin{bmatrix} \bar{A}_{m,l} \bar{x}_{i,j} + \bar{M}_l \varphi_p + \bar{M}_l F_1 \end{bmatrix} \\ + \lambda \begin{bmatrix} \hat{A}_{m,l} \bar{x}_{i,j} + \hat{M}_l \varphi_p + \hat{M}_l F_1 \end{bmatrix}^T \\ \cdot \mathcal{P}_{n,u}^q \begin{bmatrix} \hat{A}_{m,l} \bar{x}_{i,j} + \hat{M}_l \varphi_p + \hat{M}_l F_1 \end{bmatrix} \\ - \bar{x}_{i,j}^T \sum_{g=1}^N \alpha_g^{ij} P_{m,l}^g \bar{x}_{i,j}. \quad (20)$$

It is inferred from the sensor nonlinearity constraint in (6) that

$$\xi_1 = 2\mu \sum_{g=1}^N \alpha_g^{ij} \rho_{m,l}^g \varphi_p^T \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) \\ \cdot \left(\varphi_p \left(C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \right) - K C_m \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} - F \right) \leq 0, \quad (21)$$

$$\xi_2 = 2\mu \sum_{g=1}^N \alpha_g^{ij} \tau_{m,l}^g (v_c^2 - \gamma_{i,j}^T \gamma_{i,j}) \leq 0.$$

By the Schur complement property, the inequality in (16) yields $\mathcal{L}_{i,j} - \xi_1 - \xi_2 < 0$, which means that

$$\lim_{i+j \rightarrow \infty} \mathbb{E} \left\{ \|\bar{x}_{i,j}\|^2 \right\} = 0. \quad (22)$$

Therefore, the system is mean-square asymptotically stable.

Next, to establish the $\mathcal{H}_\infty$ performance for the system, we introduce the following index:

$$\mathcal{J}_{i,j} = \mathbb{E} \left\{ \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix}^T \right. \\ \cdot \begin{bmatrix} P^h(\bar{r}_{i+1,j}) & 0 \\ 0 & P^v(\bar{r}_{i,j+1}) \end{bmatrix} \begin{bmatrix} \bar{x}_{i+1,j}^h \\ \bar{x}_{i,j+1}^v \end{bmatrix} - \bar{x}_{i,j}^T P(\bar{r}_{i,j}) \\ \cdot \bar{x}_{i,j} + \bar{z}_{i,j}^T \bar{z}_{i,j} - \gamma^2 w_{i,j}^T w_{i,j} \mid \bar{x}_{i,j}, \bar{r}_{i,j} = (m, l) \left. \right\}. \quad (23)$$

It is shown from (23) that

$$\mathcal{J}_{i,j} = \zeta_{i,j}^T \Phi_{m,l} \zeta_{i,j}, \quad (24)$$

where $\zeta_{i,j} = [\bar{x}_{i,j}^T \ \varphi_p^T \ \mathbf{1}_r \ w_{i,j}^T]^T$ and

$$\Phi_{m,l} = \Sigma_1^T \mathcal{P}_{n,u}^q \Sigma_1 + \Sigma_2^T \mathcal{P}_{n,u}^q \Sigma_2 + \Sigma_3^T \Sigma_3 + \Sigma_4^T \Sigma_4 - \Sigma_5, \quad (25)$$

with

$$\Sigma_1 = \begin{bmatrix} \bar{A}_{m,l} & \bar{M}_l & \bar{M}_l \text{diag}(F_1) & \bar{B}_{m,l} \end{bmatrix}, \\ \Sigma_2 = \begin{bmatrix} \sqrt{\lambda} \hat{A}_{m,l} & \sqrt{\lambda} \hat{M}_l & \sqrt{\lambda} \hat{M}_l \text{diag}(F_1) & 0 \end{bmatrix}, \\ \Sigma_3 = \begin{bmatrix} \bar{C}_{m,l} & \bar{N}_l & \bar{N}_l \text{diag}(F_1) & \bar{D}_{m,l} \end{bmatrix}, \\ \Sigma_4 = \begin{bmatrix} \sqrt{\lambda} \hat{C}_{m,l} & \sqrt{\lambda} \hat{N}_l & \sqrt{\lambda} \hat{N}_l \text{diag}(F_1) & 0 \end{bmatrix}, \quad (26) \\ \Sigma_5 = \begin{bmatrix} \sum_{g=1}^N \alpha_g^{ij} P_{m,l}^g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma^2 I \end{bmatrix}.$$

By applying Schur's complement again, the inequality in (16) guarantees $\mathcal{J}_{i,j} - \xi_1 - \xi_2 < 0$, which implies that $\|\bar{z}_{i,j}\|_E < \gamma \|w_{i,j}\|_2$. Thus, the system is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance level. The proof is completed. $\square$

Remark 6. In Theorem 5, an $\mathcal{H}_\infty$ analysis criterion for the underlying system is established by considering a nonhomogeneous Markovian process under asynchronous switching. By a closer inspection, it is found that the obtained condition reveals the relationship between the asynchronous switching and the $\mathcal{H}_\infty$ performance level.

The developments above lead to the asynchronous filtering result in the next theorem.

Theorem 7. The system in (10) with extended varying transition probabilities (12) is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance index $\gamma > 0$, if there exist matrices $P_{1m,l}^g = \text{diag}\{P_{1m,l}^{hg}, P_{1m,l}^{vg}\} > 0$, $P_{3m,l}^g = \text{diag}\{P_{3m,l}^{hg}, P_{3m,l}^{vg}\} > 0$, $P_{2m,l}^g = \text{diag}\{P_{2m,l}^{hg}, P_{2m,l}^{vg}\}$, $X_l, S_l, U_l, \bar{A}_{fl}, \bar{B}_{fl}, \bar{C}_{fl}$,

$\bar{D}_{fl}$, and scalars $\tau_{m,l}^g > 0$, $\rho_{m,l}^g > 0$, $g = 1, \dots, N$, $\forall (m, l) \in \mathcal{I} \times \mathcal{I}$, such that

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & 0 & 0 & 0 & 0 & \Psi_{18} & \mu \bar{B}_{fl} & \mu \bar{B}_{fl} \text{diag}(F_1) & \Psi_{19} \\ * & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & \Psi_{28} & \mu \bar{B}_{fl} & \mu \bar{B}_{fl} \text{diag}(F_1) & \Psi_{29} \\ * & * & \Psi_{11} & \Psi_{12} & 0 & 0 & 0 & \Psi_{38} & -\sqrt{\lambda} \bar{B}_{fl} & -\sqrt{\lambda} \bar{B}_{fl} \text{diag}(F_1) & 0 \\ * & * & * & \Psi_{22} & 0 & 0 & 0 & \Psi_{38} & -\sqrt{\lambda} \bar{B}_{fl} & -\sqrt{\lambda} \bar{B}_{fl} \text{diag}(F_1) & 0 \\ * & * & * & * & -I & 0 & 0 & \Psi_{58} & -\mu \bar{D}_{fl} & -\mu \bar{D}_{fl} \text{diag}(F_1) & \Psi_{59} \\ * & * & * & * & * & -I & 0 & \Psi_{68} & \sqrt{\lambda} \bar{D}_{fl} & \sqrt{\lambda} \bar{D}_{fl} \text{diag}(F_1) & 0 \\ * & * & * & * & * & * & -\mu \tau_{m,l}^g I & \Psi_{78} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & 0 & 0 \\ * & * & * & * & * & * & * & * & -2\mu \rho_{m,l}^g I & \mu \rho_{m,l}^g \text{diag}(F) & 0 \\ * & * & * & * & * & * & * & * & * & -\mu \frac{v_c^2}{r} \tau_{m,l}^g I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (27)$$

where

$$\begin{aligned} \Psi_{11} &= \sum_{(n,u) \in \mathcal{I} \times \mathcal{I}} \bar{\pi}_{(m,l)(n,u)}^g P_{1n,u}^q - X_l^T - X_l, \\ \Psi_{12} &= \sum_{(n,u) \in \mathcal{I} \times \mathcal{I}} \bar{\pi}_{(m,l)(n,u)}^g P_{2n,u}^q - U_l^T - S_l, \\ \Psi_{22} &= \sum_{(n,u) \in \mathcal{I} \times \mathcal{I}} \bar{\pi}_{(m,l)(n,u)}^g P_{3n,u}^q - U_l^T - U_l, \\ \Psi_{18} &= [X_l^T A_m + (1 - \mu) \bar{B}_{fl} C_m + \mu \bar{B}_{fl} K_1 C_m \quad \bar{A}_{fl}], \\ \Psi_{19} &= X_l^T B_m + \bar{B}_{fl} D_m, \\ \Psi_{28} &= [S_l^T A_m + (1 - \mu) \bar{B}_{fl} C_m + \mu \bar{B}_{fl} K_1 C_m \quad \bar{A}_{fl}], \\ \Psi_{29} &= S_l^T B_m + \bar{B}_{fl} D_m, \\ \Psi_{38} &= [\sqrt{\lambda} \bar{B}_{fl} C_m - \sqrt{\lambda} \bar{B}_{fl} K_1 C_m \quad 0], \\ \Psi_{58} &= [H_m - (1 - \mu) \bar{D}_{fl} C_m - \mu \bar{D}_{fl} K_1 C_m \quad -\bar{C}_{fl}], \\ \Psi_{59} &= L_m - \bar{D}_{fl} D_m, \\ \Psi_{68} &= [\sqrt{\lambda} \bar{D}_{fl} K_1 C_m - \sqrt{\lambda} \bar{D}_{fl} C_m \quad 0], \\ \Psi_{78} &= [\mu \tau_{m,l}^g C_m \quad 0], \\ \Psi_{88} &= \begin{bmatrix} -P_{1m,l}^g & -P_{2m,l}^g \\ * & -P_{3m,l}^g \end{bmatrix}, \\ \Psi_{89} &= \begin{bmatrix} \mu \rho_{m,l}^g C_m^T K^T \\ 0 \end{bmatrix}. \end{aligned} \quad (28)$$

In this case, the admissible filter gains are given by

$$\begin{aligned} A_{fl} &= U_l^{-T} \bar{A}_{fl}, \\ B_{fl} &= U_l^{-T} \bar{B}_{fl}, \\ C_{fl} &= \bar{C}_{fl}, \\ D_{fl} &= \bar{D}_{fl}. \end{aligned} \quad (29)$$

Proof. First, consider the system in (10) and define the matrices

$$\begin{aligned} P_{m,l}^{hg} &= \begin{bmatrix} P_{1m,l}^{hg} & P_{2m,l}^{hg} \\ * & P_{3m,l}^{hg} \end{bmatrix}, \\ P_{m,l}^{vg} &= \begin{bmatrix} P_{1m,l}^{vg} & P_{2m,l}^{vg} \\ * & P_{3m,l}^{vg} \end{bmatrix}, \end{aligned} \quad (30)$$

$$G_l = \begin{bmatrix} G_{1l} & G_{2l} \\ G_{4l} & G_{3l} \end{bmatrix},$$

$$G_{il} = \begin{bmatrix} X_{il} & S_{il} \\ U_{il} & U_{il} \end{bmatrix}, \quad i = 1, \dots, 4.$$

Notice that

$$\Lambda P_{m,l}^g \Lambda^T = \begin{bmatrix} P_{1m,l}^g & P_{2m,l}^g \\ * & P_{3m,l}^g \end{bmatrix},$$

$$\Lambda G_l \Lambda^T = \begin{bmatrix} X_l & S_l \\ U_l & U_l \end{bmatrix},$$

$$\begin{aligned}
& \Lambda G_l^T \bar{A}_{m,l} \Lambda^T \\
&= \begin{bmatrix} X_l^T A_m + (1 - \mu) \bar{B}_{fl} C_m + \mu \bar{B}_{fl} K_1 C_m & \bar{A}_{fl} \\ S_l^T A_m + (1 - \mu) \bar{B}_{fl} C_m + \mu \bar{B}_{fl} K_1 C_m & \bar{A}_{fl} \end{bmatrix}, \\
& \Lambda \sqrt{\lambda} G_l^T \hat{A}_{m,l} \Lambda^T = \begin{bmatrix} \sqrt{\lambda} \bar{B}_{fl} C_m - \sqrt{\lambda} \bar{B}_{fl} K_1 C_m & 0 \\ \sqrt{\lambda} \bar{B}_{fl} C_m - \sqrt{\lambda} \bar{B}_{fl} K_1 C_m & 0 \end{bmatrix}, \\
& \Lambda G_l^T \bar{M}_l = \begin{bmatrix} \mu \bar{B}_{fl} \\ \mu \bar{B}_{fl} \end{bmatrix}, \\
& \Lambda G_l^T \bar{M}_l \text{diag}(F_1) = \begin{bmatrix} \mu \bar{B}_{fl} \text{diag}(F_1) \\ \mu \bar{B}_{fl} \text{diag}(F_1) \end{bmatrix}, \\
& \Lambda \sqrt{\lambda} G_l^T \hat{M}_l = \begin{bmatrix} -\sqrt{\lambda} \bar{B}_{fl} \\ -\sqrt{\lambda} \bar{B}_{fl} \end{bmatrix}, \\
& \Lambda \sqrt{\lambda} G_l^T \hat{M}_l \text{diag}(F_1) = \begin{bmatrix} -\sqrt{\lambda} \bar{B}_{fl} \text{diag}(F_1) \\ -\sqrt{\lambda} \bar{B}_{fl} \text{diag}(F_1) \end{bmatrix}, \\
& \Lambda G_l^T \bar{B}_{m,l} = \begin{bmatrix} X_l^T B_m + \bar{B}_{fl} D_m \\ S_l^T B_m + \bar{B}_{fl} D_m \end{bmatrix}, \\
& \bar{C}_{m,l} \Lambda^T \\
&= [H_m - (1 - \mu) \bar{D}_{fl} C_m - \mu \bar{D}_{fl} K_1 C_m \quad -\bar{C}_{fl}], \\
& \sqrt{\lambda} \hat{C}_{m,l} \Lambda^T = [\sqrt{\lambda} \bar{D}_{fl} K_1 C_m - \sqrt{\lambda} \bar{D}_{fl} C_m \quad 0], \\
& \bar{N}_l = -\mu \bar{D}_{fl}, \\
& \bar{N}_l \text{diag}(F_1) = -\mu \bar{D}_{fl} \text{diag}(F_1), \\
& \sqrt{\lambda} \hat{N}_l = \sqrt{\lambda} \bar{D}_{fl}, \\
& \sqrt{\lambda} \hat{N}_l \text{diag}(F_1) = \sqrt{\lambda} \bar{D}_{fl} \text{diag}(F_1), \\
& \mu \tau_{m,l}^g C_m \Lambda_1 \Lambda^T = [\mu \tau_{m,l}^g C_m \quad 0], \\
& \mu \rho_{m,l}^g K C_m \Lambda_1 \Lambda^T = [\mu \rho_{m,l}^g K C_m \quad 0], \\
& \bar{D}_{m,l} = L_m - \bar{D}_{fl} D_m.
\end{aligned} \tag{31}$$

Further let the following matrices of the filter

$$\begin{aligned}
& \bar{A}_{fl} = U_l^T A_{fl}, \\
& \bar{B}_{fl} = U_l^T B_{fl}, \\
& \bar{C}_{fl} = C_{fl}, \\
& \bar{D}_{fl} = D_{fl}.
\end{aligned} \tag{32}$$

Then, in terms of the condition in (27), we get

$$\begin{bmatrix} \bar{\Omega}_{11} & 0 & 0 & 0 & 0 & \bar{\Omega}_{16} & \bar{\Omega}_{17} & \bar{\Omega}_{18} & \bar{\Omega}_{19} \\ * & \bar{\Omega}_{11} & 0 & 0 & 0 & \bar{\Omega}_{26} & \bar{\Omega}_{27} & \bar{\Omega}_{28} & 0 \\ * & * & -I & 0 & 0 & \bar{C}_{m,l} & \bar{N}_l & \Omega_{38} & \bar{D}_{m,l} \\ * & * & * & -I & 0 & \Omega_{46} & \Omega_{47} & \Omega_{48} & 0 \\ * & * & * & * & \bar{\Omega}_{55} & \bar{\Omega}_{56} & 0 & 0 & 0 \\ * & * & * & * & * & -P_{m,l}^g & \bar{\Omega}_{67} & 0 & 0 \\ * & * & * & * & * & * & \bar{\Omega}_{77} & \bar{\Omega}_{78} & 0 \\ * & * & * & * & * & * & * & \bar{\Omega}_{88} & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{33}$$

where

$$\begin{aligned}
& \bar{\Omega}_{11} = \bar{\mathcal{P}}_{n,u}^q - G_l^T - G_l, \\
& \bar{\Omega}_{16} = G_l^T \bar{A}_{m,l}, \\
& \bar{\Omega}_{17} = G_l^T \bar{M}_l, \\
& \bar{\Omega}_{18} = G_l^T \bar{M}_l \text{diag}(F_1), \\
& \bar{\Omega}_{19} = G_l^T \bar{B}_{m,l}, \\
& \bar{\Omega}_{26} = \sqrt{\lambda} G_l^T \hat{A}_{m,l}, \\
& \bar{\Omega}_{27} = \sqrt{\lambda} G_l^T \hat{M}_l, \\
& \bar{\Omega}_{28} = \sqrt{\lambda} G_l^T \hat{M}_l \text{diag}(F_1), \\
& \bar{\Omega}_{55} = -\mu \tau_{m,l}^g I, \\
& \bar{\Omega}_{56} = \mu \tau_{m,l}^g C_m \Lambda_1, \\
& \bar{\Omega}_{67} = \mu \rho_{m,l}^g \Lambda_1^T C_m^T K^T, \\
& \bar{\Omega}_{77} = -2\mu \rho_{m,l}^g I, \\
& \bar{\Omega}_{78} = \mu \rho_{m,l}^g \text{diag}(F), \\
& \bar{\Omega}_{88} = -\mu \frac{v_c^2}{r} \tau_{m,l}^g I, \\
& \bar{\mathcal{P}}_{n,u}^q = \sum_{(n,u) \in \mathcal{F} \times \mathcal{F}} \bar{\pi}_{(m,l)(n,u)}^g P_{n,u}^q.
\end{aligned} \tag{34}$$

Multiplying inequality (33) by the adequate coefficients and adding the resulting inequalities, together with the consideration of the fact that $\bar{\mathcal{P}}_{n,u}^q - G_l^T - G_l \geq -G_l^T (\bar{\mathcal{P}}_{n,u}^q)^{-1} G_l$, yield inequality (16) in Theorem 5. Therefore, it follows from Theorem 5 that system (10) with extended varying transition probabilities (12) is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance level. Meanwhile, the filter gains in (29) follow immediately from (32). This completes the proof. $\square$

Remark 8. According to Theorem 7, the asynchronous $\mathcal{H}_\infty$ filter can be designed for the addressed Markovian jump system with varying transition probabilities in the presence of randomly occurring nonlocal sensor nonlinearity. By solving the convex problem contained in Theorem 7, the $\mathcal{H}_\infty$ performance γ can be optimized in terms of the feasibility of the corresponding condition. The result in Theorem 7 indicates that the different η in (12) sparks off the different optimal γ achieved for the system in (10). Thus, the effect of the asynchronous behavior can be readily comprehended by comparing the $\mathcal{H}_\infty$ performance indexes.

4. Numerical Example

In this section, an example is presented to demonstrate the merits of the proposed approach. Consider a 2D Markovian jump system with two operation modes and the parameters as follows:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 0.12 & 0 \\ 1 & 0.29 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} 0.83 & 0 \\ 1 & 0.92 \end{bmatrix}, \\
 C_1 &= [0.12 \ 1], \\
 C_2 &= [0.83 \ 1], \\
 B_1 &= B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\
 D_1 &= D_2 = [0 \ 1], \\
 H_1 &= H_2 = [0 \ 1], \\
 K_1 &= 0, \\
 K_2 &= 5, \\
 F_1 &= 0, \\
 F_2 &= -1, \\
 v_c &= 0.25.
 \end{aligned} \tag{35}$$

The varying transition probability matrix is assumed to be included in a polytope defined by its vertices:

$$\begin{aligned}
 \Pi^1 &= \begin{bmatrix} 0.27 & 0.73 \\ 0.35 & 0.65 \end{bmatrix}, \\
 \Pi^2 &= \begin{bmatrix} 0.59 & 0.41 \\ 0.85 & 0.15 \end{bmatrix}.
 \end{aligned} \tag{36}$$

Moreover, the probabilities μ and η are set as 0.2 and 0.35, respectively.

FIGURE 1: Paths of $r_{i,j}$, σ_k , and δ_k .

By applying the filter design method in Theorem 7, the minimum $\mathcal{H}_\infty$ cost is obtained $\gamma^* = 10.7273$ as well as the resulting filter gain matrices:

$$\begin{aligned}
 A_{f1} &= \begin{bmatrix} 0.0602 & 0.0163 \\ 0.2327 & 0.3538 \end{bmatrix}, \\
 B_{f1} &= \begin{bmatrix} -0.0069 \\ -0.0681 \end{bmatrix}, \\
 C_{f1} &= [-0.7647 \ -0.8920], \\
 A_{f2} &= \begin{bmatrix} 0.1724 & 0.0834 \\ 1.0948 & 0.4952 \end{bmatrix}, \\
 B_{f2} &= \begin{bmatrix} -0.0238 \\ -0.0982 \end{bmatrix}, \\
 C_{f2} &= [-1.0738 \ -0.7933].
 \end{aligned} \tag{37}$$

Given paths of $r_{i,j}$, σ_k , and δ_k demonstrated in Figure 1, the system initial condition $x_{i,0}^v = 1$, $0 \leq i \leq 29$, the disturbance signal

$$\begin{aligned}
 w_{i,j} &= \begin{cases} 0.3e^{-0.3j} \sin(0.5\pi j), & 1 \leq i \leq 15, 1 \leq j \leq 15, \\ 0, & \text{otherwise,} \end{cases} \tag{38}
 \end{aligned}$$

and the sensor nonlinearity $\varphi(\theta) = K_2\theta + F_2$, by using the achieved filter, it is seen that $\bar{z}_{i,j}$ of the error system converges

FIGURE 2: Filtering error response $\bar{z}_{i,j}$.

to zero, shown in Figure 2, which means that the obtained filter is effective in spite of the asynchronous switching and randomly occurring nonlocal sensor nonlinearity.

5. Conclusion

This paper has addressed the problem of asynchronous $\mathcal{H}_\infty$ estimation for a class of 2D Markovian jump systems with varying transition probabilities in the presence of randomly occurring nonlocal sensor nonlinearity. The jumping process of the estimation error system is modeled by the Markov chain with extended varying transition probabilities. The existence condition of asynchronous $\mathcal{H}_\infty$ filters has been derived to ensure the mean-square asymptotic stability and $\mathcal{H}_\infty$ performance level of the error system. A numerical example has been provided that highlights the effectiveness of the developed estimation approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

- [1] C. Du, L. Xie, and C. Zhang, *H_∞ Control and Filtering of Two-Dimensional Systems*, Springer, Berlin, Germany, 2002.
- [2] L. Wu and H. Gao, "Sliding mode control of two-dimensional systems in Roesser model," *IET Control Theory and Applications*, vol. 2, no. 4, pp. 352–364, 2008.
- [3] L. Ntogramatzidis and M. Cantoni, "LQ optimal control for 2D Roesser models of finite extent," *Systems & Control Letters*, vol. 58, no. 7, pp. 482–490, 2009.
- [4] T. Kaczorek, "Practical stability and asymptotic stability of positive fractional 2D linear systems," *Asian Journal of Control*, vol. 12, no. 2, pp. 200–207, 2010.
- [5] Z. Feng, L. Xu, M. Wu, and Y. He, "Delay-dependent robust stability and stabilisation of uncertain two-dimensional discrete systems with time-varying delays," *IET Control Theory and Applications*, vol. 4, no. 10, pp. 1959–1971, 2010.
- [6] Q. Zhu and G. Hu, "Stability and absolute stability of a general 2-D non-linear FM second model," *IET Control Theory and Applications*, vol. 5, no. 1, pp. 239–246, 2011.
- [7] M. Sebek, "Polynomial solution of 2-D Kalman-Bucy filtering problem," *IEEE Transactions on Automatic Control*, vol. 37, no. 10, pp. 1530–1533, 1992.
- [8] J. Woods and C. Radewan, "Kalman filtering in two dimensions," *IEEE Transactions on Information Theory*, vol. 23, no. 4, pp. 473–482, 1977.
- [9] H. Gao, J. Lam, C. Wang, and S. Xu, "Robust H_∞ filtering for 2D stochastic systems," *Circuits, Systems and Signal Processing*, vol. 23, no. 6, pp. 479–505, 2004.
- [10] H. Gao, X. Meng, and T. Chen, "New design of robust H_∞ filters for 2-D systems," *IEEE Signal Processing Letters*, vol. 15, pp. 217–220, 2008.
- [11] S. Chen and I. Fong, "Robust filtering for 2-D state-delayed systems with NFT uncertainties," *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 274–285, 2006.
- [12] D. Peng and X. Guan, " H_∞ filtering of 2-D discrete state-delayed systems," *Multidimensional Systems and Signal Processing*, vol. 20, no. 3, pp. 265–284, 2009.
- [13] N. Hoang, H. Tuan, T. Nguyen, and S. Hosoe, "Robust mixed generalized H_2/H_∞ filtering of 2-D nonlinear fractional transformation systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 12, pp. 4697–4706, 2005.
- [14] S. Xu, J. Lam, Y. Zou, Z. Lin, and W. Paszke, "Robust H_∞ filtering for uncertain 2-D continuous systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 5, pp. 1731–1738, 2005.
- [15] L. Wu, P. Shi, H. Gao, and C. Wang, " H_∞ filtering for 2D Markovian jump systems," *Automatica*, vol. 44, no. 7, pp. 1849–1858, 2008.
- [16] C. Souza, L. Xie, and D. Coutinho, "Robust filtering for 2-D discrete-time linear systems with convex-bounded parameter uncertainty," *Automatica*, vol. 46, no. 4, pp. 673–681, 2010.
- [17] H. Xu and Y. Zou, "Robust H_∞ filtering for uncertain two-dimensional discrete systems with state-varying delays," *International Journal of Control, Automation and Systems*, vol. 8, no. 4, pp. 720–726, 2010.
- [18] R. Zhang, Y. Zhang, C. Hu, M. Meng, and Q. He, "Asynchronous H_∞ filtering for a class of two-dimensional Markov jump systems," *IET Control Theory and Applications*, vol. 6, no. 7, pp. 979–984, 2012.
- [19] R. Zhang, Y. Zhang, Y. Zhao, J. Liao, and B. Li, "Extended $\mathcal{H}_\infty$ estimation for two-dimensional Markov jump systems under asynchronous switching," *Mathematical Problems in Engineering*, vol. 2013, Article ID 734271, 7 pages, 2013.
- [20] L. Wu, Z. Wang, H. Gao, and C. Wang, " H_∞ and l_2-l_∞ filtering for two-dimensional linear parameter-varying systems," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 12, pp. 1129–1154, 2007.

- [21] N. Krasovskii and È. Lidskiĭ, "Analytical design of controllers in systems with random attributes. I. Statement of the problem, method of solving," *Automation and Remote Control*, vol. 22, pp. 1021–1294, 1961.
- [22] S. Aberkane, "Stochastic stabilization of a class of nonhomogeneous Markovian jump linear systems," *Systems and Control Letters*, vol. 60, no. 3, pp. 156–160, 2011.
- [23] L. Zhang, " H_∞ estimation for discrete-time piecewise homogeneous Markov jump linear systems," *Automatica*, vol. 45, no. 11, pp. 2570–2576, 2009.
- [24] C. Souza, A. Trofino, and K. Barbosa, "Mode-independent H_∞ filters for Markovian jump linear systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 11, pp. 1837–1841, 2006.
- [25] L. Zhang and E. Boukas, "Mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 6, pp. 1462–1467, 2009.
- [26] L. Zhang, N. Cui, M. Liu, and Y. Zhao, "Asynchronous filtering of discrete-time switched linear systems with average dwell time," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 5, pp. 1109–1118, 2011.
- [27] Z. Wu, P. Shi, H. Su, and J. Chu, "Asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180–186, 2014.
- [28] V. Andrieu, C. Prieur, S. Tarbouriech, and D. Arzelier, "Global asymptotic stabilization of systems satisfying two different sector conditions," *Systems & Control Letters*, vol. 60, no. 8, pp. 570–578, 2011.