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*Research Article*

# Adaptive Output Feedback Sliding Mode Control for Complex Interconnected Time-Delay Systems

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We extend the decentralized output feedback sliding mode control (SMC) scheme to stabilize a class of complex interconnected time-delay systems. First, sufficient conditions in terms of linear matrix inequalities are derived such that the equivalent reduced-order system in the sliding mode is asymptotically stable. Second, based on a new lemma, a decentralized adaptive sliding mode controller is designed to guarantee the finite time reachability of the system states by using output feedback only. The advantage of the proposed method is that two major assumptions, which are required in most existing SMC approaches, are both released. These assumptions are (1) disturbances are bounded by a known function of outputs and (2) the sliding matrix satisfies a matrix equation that guarantees the sliding mode. Finally, a numerical example is used to demonstrate the efficacy of the method.

## 1. Introduction

Advancement in the field of engineering has led to increasingly complex large-scale systems [1]. In addition, time-delay systems often feature in real-world problems, for example, chemical processes, biological systems, economic systems, and hydraulic/pneumatic systems. Time delay commonly leads to a degradation and/or instability in system performance (e.g., [2, 3]). The stability of interconnected time-delay systems has therefore been the focus of much research, which has achieved useful results [4–8]. However, the solutions proposed by previous studies necessarily require that all state variables are available for measurements.

In many practical systems, the state variables are not accessible for direct measurement or the number of measuring devices is limited. Recently, various control approaches have been employed to overcome the above obstacles. In [9–11], based on the assumption that each isolated subsystem is of triangular form and includes internal dynamics, a class of decentralized stabilizing dynamic output feedback controller was proposed for interconnected time-delay systems. In [12], based on two adaptive neural networks, a class of decentralized stabilizing output feedback controllers was proposed for a class of uncertain nonlinear interconnected time-delay systems with immeasurable states and triangular structures. In [13], based on adaptive fuzzy control theory, a decentralized robust output feedback controller was proposed for a class of strict-feedback nonlinear interconnected time-delay systems. In [14], a new adaptive robust state observer was designed for a class of uncertain interconnected systems with multiple time-varying delays. By including fuzzy logic systems and fuzzy state observer, the authors of [15] presented an adaptive decentralized fuzzy output feedback control for interconnected systems when system states cannot be measured. The work in [16] investigated the issue of robust and reliable decentralized $H_\infty$ tracking control for fuzzy interconnected time-delay systems. In [1], based on Lyapunov stability theory and the corresponding linear matrix inequalities (LMI), the design of a dynamic output feedback controller was proposed for uncertain interconnected systems of neutral type. The authors of [17] proposed two new stability criteria of the synchronization state for interconnected time-delay systems. The above work obtained important results related to decentralized control using only output variables. However, it should be noted that most of the existing results for interconnected time-delay

systems can only be obtained when the systems conform to a special structure [9–13]. The approaches proposed by [14–17] cannot be applied for interconnected time-delay systems with mismatched parameter uncertainties in the state matrix of each isolated subsystem. Therefore, it is important to develop a decentralized adaptive output feedback sliding mode control (SMC) law to stabilize interconnected time-delay systems with a more general structure.

Sliding mode control is a robust fast-response control strategy that has been successfully applied to a wide variety of practical engineering systems [2, 3, 18]. Generally speaking, SMC is attained by applying a discontinuous control law to drive state trajectories onto a sliding surface and force them to remain on it thereafter (this process is called reaching phase), and then to keep the state trajectories moving along the surface towards the origin with the desired performance (such motion is called sliding mode) [2, 3, 18]. Earlier work on decentralized adaptive SMC mainly focused on interconnected systems or nonlinear systems that satisfy the matching condition [19–22]. If the matching condition is not satisfied, then the mismatched uncertainty will affect the dynamics of the system in sliding mode. Thus, system behavior in sliding mode is not invariant to mismatched uncertainty. Many techniques, such as [23–25], have been applied to deal with mismatched uncertainties in sliding mode. The authors of [23] proposed a decentralized SMC law for a class of mismatched uncertain interconnected systems by using two sets of switching surfaces. In [24], a decentralized dynamic output feedback based on a linear controller was proposed for the same systems. In [25], by using a multiple-sliding surface, a new control scheme was presented for a class of decentralized multi-input perturbed systems. However, time delays are not included in the above approaches [23–25]. The existence of delay usually leads to a degradation and/or instability in system performance [2, 3]. In the limited available literature, results on applying sliding mode techniques to interconnected time-delay systems are very few [2, 3, 18]. A decentralized model reference adaptive control scheme was proposed for interconnected time-delay systems in [18]. An interconnected time-delayed system with dead-zone input via SMC in which all system state variables are available for feedback was considered in [2]. The authors of [3] investigated the global decentralized stabilization of a class of interconnected time-delay systems with known and uncertain interconnections. Their proposed approach uses only output variables. Based on Lyapunov stability theory, they designed a composite sliding surface and analyzed the stability of the associated sliding motion. As a result, the stability of interconnected time-delay systems is assured under certain conditions, the most important of which are that the disturbances must be bounded by a known function of outputs and that the sliding matrix must satisfy a matrix equation in order to guarantee sliding mode. However, in practical cases, these assumptions are difficult to achieve. Therefore, it would be worthwhile to design a decentralized adaptive output feedback SMC scheme for complex interconnected time-delay systems with a more general structure in which two of the above limitations are eliminated. To the best of our knowledge, no decentralized adaptive output feedback SMC scheme has so far been proposed for interconnected time-delay systems with unknown disturbance, mismatched parameter uncertainties in the state matrix, and mismatched interconnections and without the measurements of the states.

In this technical note, we extend the concept of decentralized output feedback sliding mode controller, introduced by Yan et al. in [3], for the aim of stabilizing complex interconnected time-delay systems. The main contributions of this paper are as follows.

(i) The interconnected time-delay systems investigated in this study include mismatched parameter uncertainties in the state matrix, mismatched interconnections, and unknown disturbance. Therefore, we consider a more general structure than the one considered in [2, 3, 18–25].

(ii) This approach uses the output information completely in the sliding surface and controller design. Therefore, conservatism is reduced and robustness is enhanced.

(iii) The two major limitations in [3] are both eliminated (disturbances must be bounded by a known function of outputs and the sliding matrix must satisfy a matrix equation in order to guarantee sliding mode). Hence, the proposed method can be applied to a wider class of interconnected time-delay systems.

*Notation.* The notation used throughout this paper is fairly standard. $X^T$ denotes the transpose of matrix $X$. $I_n$ and $0_{n\times m}$ are used to denote the $n\times n$ identity matrix and the $n\times m$ zero matrix, respectively. The subscripts $n$ and $n\times m$ are omitted where the dimension is irrelevant or can be determined from the context. $\|x\|$ stands for the Euclidean norm of vector $x$ and $\|A\|$ stands for the matrix induced norm of the matrix $A$. The expression $A > 0$ means that $A$ is a symmetric positive definite. $R^n$ denotes the $n$-dimensional Euclidean space. For the sake of simplicity, sometimes function $x_i(t)$ is denoted by $x_i$.

## 2. Problem Formulations and Preliminaries

We consider a class of interconnected time-delay systems that is decomposed into $L$ subsystems. The state space representation of each subsystem is described as follows:

$$\begin{aligned}\dot{x}_i &= \left(A_i + \Delta A_i\right) x_i + B_i\left(u_i + G_i\left(t, x_i, x_{id_i}\right)\right)\\ &\quad + \sum_{\substack{j=1\\ j\neq i}}^{L}\left[H_{ij} + \Delta H_{ij}\left(t, x_j, x_{jd_j}\right)\right] x_{jd_j},\\ y_i &= C_i x_i,\end{aligned} \tag{1}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, and $y_i \in R^{p_i}$ with $m_i < p_i < n_i$ are the state variables, inputs, and outputs of the $i$th subsystem, respectively. The triplet $(A_i, B_i, C_i)$ and $H_{ij}$ represent known constant matrices of appropriate dimensions. The notations $x_{id_i} := x_i(t-d_i)$ and $y_{id_i} := y_i(t-d_i)$ represent delayed states and delayed outputs, respectively. The symbol $d_i := d_i(t)$ is

the time-varying delay, which is assumed to be known and is bounded by $\overline{d}_i$ for all $d_i$ where $\overline{d}_i > 0$ is constant. The initial conditions are given by $x_i(t) = \chi_i(t)$ $(t \in [-\overline{d}_i, 0])$, where $\chi_i(t)$ are continuous in $[-\overline{d}_i, 0]$ for $i = 1, 2, 3, \ldots, L$. The matrices $\Delta A_i x_i$ and $\Delta H_{ij}(t, x_j, x_{jd_j})$ represent mismatched parameter uncertainties in the state matrix and mismatched uncertain interconnections with $\operatorname{rank}[B_i : \Delta A_i : \Delta H_{ij}] > \operatorname{rank}(B_i) = m_i$. The matrix $B_i G_i(t, x_i, x_{id_i})$ is the disturbance input. In this paper, only output variables $y_i$ are assumed to be available for measurements.

For system (1), the following basic assumptions are made for each subsystem in this paper.

*Assumption 1.* All the pairs $(A_i, B_i)$ are completely controllable.

*Assumption 2.* The matrices $B_i$ and $C_i$ are full rank and $\operatorname{rank}(C_i B_i) = m_i$.

*Assumption 3.* The exogenous disturbance $G_i(t, x_i, x_{id_i})$ is assumed to be bounded and to satisfy the following condition:

$$\left\| G_i\left(t, x_i, x_{id_i}\right) \right\| \le c_i + b_i \left\| x_i \right\|, \tag{2}$$

where $b_i$ and $c_i$ are unknown bounds which are not easily obtained due to the complicated structure of the uncertainties in practical control systems.

*Assumption 4.* The mismatched parameter uncertainties in the state matrix of each isolated subsystem are satisfied as $\Delta A_i = D_i \Delta F_i(x_i(t), t) E_i$, where $\Delta F_i(x_i(t), t)$ is unknown but bounded as $\|\Delta F_i(x_i(t), t)\| \le 1$ and $D_i$, $E_i$ are known matrices of appropriate dimensions.

*Assumption 5.* The mismatched uncertain interconnections are given as $\Delta H_{ij} = D_{ij} \Delta F_{ij}(t, x_j, x_{jd_j}) E_{ij}$, where $\Delta F_{ij}(t, x_j, x_{jd_j})$ is unknown but bounded as $\|\Delta F_{ij}(t, x_j, x_{jd_j})\| \le 1$ and $D_{ij}$, $E_{ij}$ are any nonzero matrices of appropriate dimensions.

*Remark 1.* Assumption $\operatorname{rank}(C_i B_i) = m_i$ is a limitation on the triplet $(A_i, B_i, C_i)$ and has been utilized in most existing output feedback SMCs, for example [3, 26, 27]. This assumption guarantees the existence of the output sliding surface. Assumptions 4 and 5 were used in [6, 27].

*Remark 2.* There are two major assumptions in [3].

(i) The exogenous disturbances are bounded by a known function of outputs $y_i$. That is $\|G_i(t, x_i, x_{id_i})\| \le g_i(t, y_i, y_{id_i})$, where $g_i(t, y_i, y_{id_i})$ is known. This condition is quite restrictive.

(ii) The sliding matrix $F_i$ satisfies $\Gamma_i C_i = F_i C_i A_i$ to guarantee sliding condition $S_i(x_i) = F_i y_i = 0$. This limitation is really quite strong.

In this paper, a decentralized adaptive output feedback SMC scheme is proposed for complex interconnected time-delay systems where the two above limitations are eliminated.

For later use, we will need the following lemma.

**Lemma 3** (see [3, 26]). *Consider the following interconnected system:*

$$\begin{aligned} \dot{x}_i &= A_{ii} x_i + B_i u_i + \sum_{\substack{j=1 \\ j \ne i}}^{L} A_{ij} x_j, \\ y_i &= C_i x_i, \end{aligned} \tag{3}$$

*where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, and $y_i \in R^{p_i}$ are the state variables, inputs, and outputs of the ith subsystem, respectively. Under assumption $\operatorname{rank}(C_i B_i) = m_i$, it follows from [3, 26] that there exists a coordinate transformation $x_i \to z_i = T_i x_i$ such that the interconnected system (3) has the following regular form:*

$$\begin{aligned} \dot{z}_i &= \begin{bmatrix} A_{ii1} & A_{ii2} \\ A_{ii3} & A_{ii4} \end{bmatrix} z_i + \sum_{\substack{j=1 \\ j \ne i}}^{L} \begin{bmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{bmatrix} z_j + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} u_i, \\ y_i &= \begin{bmatrix} 0 & C_{i2} \end{bmatrix} z_i, \end{aligned} \tag{4}$$

*where $T_i A_{ii} T_i^{-1} = \left[\begin{smallmatrix} A_{ii1} & A_{ii2} \\ A_{ii3} & A_{ii4} \end{smallmatrix}\right]$, $T_i A_{ij} T_j^{-1} = \left[\begin{smallmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{smallmatrix}\right]$, and $T_i B_i = \left[\begin{smallmatrix} 0 \\ B_{i2} \end{smallmatrix}\right]$, $C_i T_i^{-1} = \begin{bmatrix} 0 & C_{i2} \end{bmatrix}$. The matrices $B_{i2} \in R^{m_i \times m_i}$ and $C_{i2} \in R^{p_i \times p_i}$ are nonsingular and $A_{ii1}$ is stable.*

## 3. Sliding Mode Control Design for Complex Interconnected Time-Delay Systems

In this section, we design a new decentralized adaptive output feedback SMC scheme for the system (1). There are three steps involved in the design of our decentralized adaptive output feedback SMC scheme. In the first step, a proper sliding function is constructed such that the sliding surface is designed to be dependent on output variables only. In the second step, we derive sufficient conditions in terms of LMI for the existence of a sliding surface guaranteeing asymptotic stability of the sliding mode dynamic. In the final step, based on a new Lemma, we design a decentralized adaptive output feedback sliding mode controller, which assures that the system states reach the sliding surface in finite time and stay on it thereafter.

*3.1. Sliding Surface Design.* Let us first design a sliding surface, which depends on only output variables. Since $\operatorname{rank}(C_i B_i = m_i)$, it follows from Lemma 3 that there exists a coordinate transformation $z_i = T_i x_i$ such that the system (1) has the following regular form:

$$\begin{aligned} \dot{z}_i = &\left( \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} + \begin{bmatrix} D_{i1} \\ D_{i2} \end{bmatrix} \Delta F_i \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix} \right) z_i \\ &+ \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \left[ u_i + G_i\left(t, T_i^{-1} z_i, T_i^{-1} z_{id_i}\right) \right] \end{aligned}$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{L}\left(\begin{bmatrix} H_{ij1} & H_{ij2}\\ H_{ij3} & H_{ij4}\end{bmatrix}+\begin{bmatrix} D_{ij1}\\ D_{ij2}\end{bmatrix}\Delta F_{ij}\begin{bmatrix} E_{ij1} & E_{ij2}\end{bmatrix}\right)z_{jd_j},$$

$$y_i=\begin{bmatrix}0 & C_{i2}\end{bmatrix}z_i, \tag{5}$$

where $T_i=\begin{bmatrix}T_{i1}\\ T_{i2}\end{bmatrix}$, $T_i^{-1}=\begin{bmatrix}W_{i1} & W_{i2}\end{bmatrix}$, $T_iA_iT_i^{-1}=\begin{bmatrix}A_{i1} & A_{i2}\\ A_{i3} & A_{i4}\end{bmatrix}$, $T_iH_{ij}T_j^{-1}=\begin{bmatrix}H_{ij1} & H_{ij2}\\ H_{ij3} & H_{ij4}\end{bmatrix}$, $T_iD_i\Delta F_iE_iT_i^{-1}=\begin{bmatrix}D_{i1}\\ D_{i2}\end{bmatrix}\Delta F_i\begin{bmatrix}E_{i1} & E_{i2}\end{bmatrix}$, $T_iD_{ij}\Delta F_{ij}E_{ij}T_j^{-1}=\begin{bmatrix}D_{ij1}\\ D_{ij2}\end{bmatrix}\Delta F_{ij}\begin{bmatrix}E_{ij1} & E_{ij2}\end{bmatrix}$, and $T_iB_i=\begin{bmatrix}0\\ B_{i2}\end{bmatrix}$, $C_iT_i^{-1}=\begin{bmatrix}0 & C_{i2}\end{bmatrix}$. The matrices $B_{i2}\in R^{m_i\times m_i}$ and $C_{i2}\in R^{p_i\times p_i}$ are non-singular and $A_{i1}$ is stable.

Letting $z_i=\begin{bmatrix}z_{i1}\\ z_{i2}\end{bmatrix}$, where $z_{i1}\in R^{n_i-m_i}$ and $z_{i2}\in R^{m_i}$, the first equation of (5) can be rewritten as

$$\begin{aligned}\dot z_{i1}&=\left(A_{i1}+D_{i1}\Delta F_iE_{i1}\right)z_{i1}+\left(A_{i2}+D_{i1}\Delta F_iE_{i2}\right)z_{i2}\\&\quad+\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(H_{ij1}+D_{ij1}\Delta F_{ij}E_{ij1}\right)z_{j1d_j}\\&\qquad+\left(H_{ij2}+D_{ij1}\Delta F_{ij}E_{ij2}\right)z_{j2d_j}\Big],\end{aligned} \tag{6}$$

$$\begin{aligned}\dot z_{i2}&=\left(A_{i3}+D_{i2}\Delta F_iE_{i1}\right)z_{i1}+\left(A_{i4}+D_{i2}\Delta F_iE_{i2}\right)z_{i2}\\&\quad+B_{i2}\left[u_i+G_i\left(t,T_i^{-1}z_i,T_i^{-1}z_{id_i}\right)\right]\\&\quad+\sum_{\substack{j=1\\ j\neq i}}^{L}\Big[\left(H_{ij3}+D_{ij2}\Delta F_{ij}E_{ij1}\right)z_{j1d_j}\\&\qquad+\left(H_{ij4}+D_{ij2}\Delta F_{ij}E_{ij2}\right)z_{j2d_j}\Big].\end{aligned} \tag{7}$$

Obviously, the system (6) represents the sliding-motion dynamic of the system (5), and, hence, the corresponding sliding surface can be chosen as follows:

$$\sigma_i\left(x_i\right)=K_iC_{i2}^{-1}y_i=0, \tag{8}$$

where $K_i=\begin{bmatrix}F_{i1} & F_{i2}\end{bmatrix}=\begin{bmatrix}0_{m_i\times(p_i-m_i)} & F_{i2}\end{bmatrix}$, $F_{i2}=\Xi_iP_i\Xi_i^T$, the matrix $P_i\in R^{(n_i-m_i)\times(n_i-m_i)}$ is defined later, and the matrix $\Xi_i\in R^{m_i\times(n_i-m_i)}$ is selected such that $F_{i2}$ is nonsingular. Then, by using the second equation of (5), we have

$$\begin{aligned}\sigma_i\left(x_i\right)&=K_iC_{i2}^{-1}y_i=K_iC_{i2}^{-1}\begin{bmatrix}0 & C_{i2}\end{bmatrix}z_i\\&=K_i\begin{bmatrix}N_i & 0_{(p_i-m_i)\times m_i}\\ 0_{m_i\times(n_i-m_i)} & I_{m_i}\end{bmatrix}z=\begin{bmatrix}F_{i1}N_i & F_{i2}\end{bmatrix}z_i\\&=F_{i2}z_{i2}=0,\end{aligned} \tag{9}$$

where $N_i=\begin{bmatrix}0_{(p_i-m_i)\times(n_i-p_i)} & I_{(p_i-m_i)}\end{bmatrix}$. In addition, the Newton-Leibniz formula is defined as

$$z_{i2d_i}=z_{i2}\left(t-d_i\right)=z_{i2}\left(t\right)-\int_{t-d_i}^{t}\dot z_{i2}\left(s\right)ds. \tag{10}$$

Therefore, in sliding modes $\sigma_i(x_i)=0$ and $\dot\sigma_i(x_i)=0$, we have $z_{i2}=0$ and $z_{j2d_j}=0$. Then, from the structure of systems (6)-(7), the sliding mode dynamics of the system (1) associated with the sliding surface (8) is described by

$$\dot z_{i1}=\left(A_{i1}+D_{i1}\Delta F_iE_{i1}\right)z_{i1}+\sum_{\substack{j=1\\ j\neq i}}^{L}\left(H_{ij1}+D_{ij1}\Delta F_{ij}E_{ij1}\right)z_{j1d_j}. \tag{11}$$

*3.2. Asymptotically Stable Conditions by LMI Theory.* Now we are in position to derive sufficient conditions in terms of linear matrix inequalities (LMI) such that the dynamics of the system (11) in the sliding surface (8) is asymptotically stable. Let us begin with considering the following LMI:

$$\begin{bmatrix}\Psi_i & P_iD_{i1} & E_{i1}^T\\ D_{i1}^TP_i & -\varphi_iI_{m_i} & 0\\ E_{i1} & 0 & -\varphi_i^{-1}I_{m_i}\end{bmatrix}<0,\quad i=1,2,\ldots,L, \tag{12}$$

where $\Psi_i=A_{i1}^TP_i+P_iA_{i1}+((L-1)/\varepsilon_i)P_i+\sum_{j=1,j\neq i}^{L}(q\varepsilon_jH_{ji1}^TP_jH_{ji1}+\overline{\varphi}_i^{-1}P_iD_{ij1}D_{ij1}^TP_i+\widehat{q}\overline{\varphi}_jE_{ji1}^TE_{ji1})$, $P_i\in R^{(n_i-m_i)\times(n_i-m_i)}$ is any positive matrix, and $L$ is the number of subsystems and the scalars $q>1$, $\widehat{q}>1$, $\varphi_i>0$, $\varepsilon_i>0$, $\overline{\varphi}_i>0$, $i=1,2,\ldots,L$. Then, we can establish the following theorem.

**Theorem 4.** *Suppose that LMI (12) has solution $P_i>0$ and the scalars $q>1$, $\widehat{q}>1$, $\varphi_i>0$, $\varepsilon_i>0$, $\overline{\varphi}_i>0$, $i=1,2,\ldots,L$. Suppose also that the SMC law is*

$$\begin{aligned}u_i\left(t\right)=-\left(F_{i2}B_{i2}\right)^{-1}\Big(&\kappa_i\eta_i\left(t\right)+\overline{\kappa}_i\left\|y_i\right\|+\widehat{\kappa}_i\left\|y_{id_i}\right\|\\&+\zeta_i\left(t\right)+\alpha_i\Big)\frac{\sigma_i}{\left\|\sigma_i\right\|},\quad i=1,2,\ldots,L,\end{aligned} \tag{13}$$

*where $\kappa_i=\|F_{i2}\|(\|A_{i3}\|+\|D_{i2}\|\|E_{i1}\|)+\sum_{j=1,j\neq i}^{L}\beta_i\|F_{j2}\|(\|H_{ji3}\|+\|D_{ji2}\|\|E_{ji1}\|)$, $\overline{\kappa}_i=\|F_{i2}\|(\|A_{i4}\|+\|D_{i2}\|\|E_{i2}\|)\|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|$, $\widehat{\kappa}_i=\sum_{j=1,j\neq i}^{L}\|F_{j2}\|(\|H_{ji4}\|+\|D_{ji2}\|\|E_{ji2}\|)\|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|$, and the scalars $\alpha_i>0$, $\beta_i>1$, and the time functions $\zeta_i(t)$ and $\eta_i(t)$ will be designed later. The sliding surface is given by (8). Then, the dynamics of system (11) restricted to the sliding surface $\sigma_i(x_i)=0$ is asymptotically stable.*

Before proofing Theorem 4, we recall the following lemmas.

**Lemma 5** (see [27]). *Let $X$, $Y$, and $F$ be real matrices of suitable dimension with $F^TF\le I$; then, for any scalar $\varphi>0$, the following matrix inequality holds:*

$$XFY+Y^TF^TX^T\le\varphi^{-1}XX^T+\varphi Y^TY. \tag{14}$$

**Lemma 6** (see [28]). *The linear matrix inequality:*

$$\begin{bmatrix}Q\left(x\right) & \Pi\left(x\right)\\ \Pi\left(x\right)^T & R\left(x\right)\end{bmatrix}>0, \tag{15}$$

*where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $\Pi(x)$ depend affinely on $x$, is equivalent to $R(x) > 0$, $Q(x) - \Pi(x)R(x)^{-1}\Pi(x)^T > 0$.*

**Lemma 7.** *Assume that $x \in R^n$, $y \in R^n$, $N \in R^{n\times n}$, and $N$ is a positive definite matrix. Then, the inequality*

$$x^T N y + y^T N x \le \frac{1}{\varepsilon} x^T N x + \varepsilon y^T N y \tag{16}$$

*holds for all $\varepsilon > 0$.*

*Proof of Lemma 7.* For any $n \times n$ matrix $N > 0$, $N^{1/2}$ is well defined and $N^{1/2} > 0$. Let vector

$$\vartheta = \sqrt{\frac{1}{\varepsilon}} N^{1/2} x - \sqrt{\varepsilon} N^{1/2} y. \tag{17}$$

Then, we have

$$\begin{aligned}\vartheta^T \vartheta &= \left(\sqrt{\frac{1}{\varepsilon}} N^{1/2} x - \sqrt{\varepsilon} N^{1/2} y\right)^T \left(\sqrt{\frac{1}{\varepsilon}} N^{1/2} x - \sqrt{\varepsilon} N^{1/2} y\right) \\ &= \frac{1}{\varepsilon} x^T N x - x^T N y - y^T N x + \varepsilon y^T N y.\end{aligned} \tag{18}$$

Since $\vartheta^T \vartheta \ge 0$, it is obvious that

$$x^T N y + y^T N x \le \frac{1}{\varepsilon} x^T N x + \varepsilon y^T N y. \tag{19}$$

The proof is completed. □

*Proof of Theorem 4.* Now we are going to prove that the system (11) is asymptotically stable. Let us first consider the following positive definition function:

$$V = \sum_{i=1}^{L} z_{i1}^T P_i z_{i1}, \tag{20}$$

where the matrix $P_i \in R^{(n_i - m_i)\times(n_i - m_i)}$ is defined in (12). Then, the time derivative of $V$ along the state trajectories of system (11) is given by

$$\begin{aligned}\dot{V} = &\sum_{i=1}^{L} z_{i1}^T \left(A_{i1}^T P_i + P_i A_{i1} + P_i D_{i1} \Delta F_i E_{i1} \right. \\ &\left. + E_{i1}^T \Delta F_i^T D_{i1}^T P_i\right) z_{i1} \\ &+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \left(z_{j1d_j}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1} z_{j1d_j} \right. \\ &+ z_{i1}^T P_i D_{ij1} \Delta F_{ij} E_{ij1} z_{j1d_j} \\ &\left. + z_{j1d_j}^T E_{ij1}^T \Delta F_{ij}^T D_{ij1}^T P_i z_{i1}\right).\end{aligned} \tag{21}$$

Applying Lemma 5 to (21) yields

$$\begin{aligned}\dot{V} \le &\sum_{i=1}^{L} z_{i1}^T \left(A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i \right. \\ &\left. + \varphi_i E_{i1}^T E_{i1}\right) z_{i1} \\ &+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \left(z_{j1d_j}^T H_{ij1}^T P_i z_{i1} + z_{i1}^T P_i H_{ij1} z_{j1d_j} \right. \\ &+ \overline{\varphi}_i^{-1} z_{i1}^T P_i D_{ij1} D_{ij1}^T P_i z_{i1} \\ &\left. + \overline{\varphi}_i z_{j1d_j}^T E_{ij1}^T E_{ij1} z_{j1d_j}\right),\end{aligned} \tag{22}$$

where the scalars $\varphi_i > 0$ and $\overline{\varphi}_i > 0$. By Lemma 7, it follows that for any $\varepsilon_i > 0$

$$\begin{aligned}&\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \left(z_{i1}^T P_i H_{ij1} z_{j1d_j} + z_{j1d_j}^T H_{ij1}^T P_i z_{i1}\right) \\ &\quad \le \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \left(\frac{1}{\varepsilon_i} z_{i1}^T P_i z_{i1} + \varepsilon_i z_{j1d_j}^T H_{ij1}^T P_i H_{ij1} z_{j1d_j}\right).\end{aligned} \tag{23}$$

From (22) and (23), it is obvious that

$$\begin{aligned}\dot{V} \le &\sum_{i=1}^{L} z_{i1}^T \left(A_{i1}^T P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^T P_i \right. \\ &\left. + \varphi_i E_{i1}^T E_{i1}\right) z_{i1} \\ &+ \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \left(\varepsilon_i z_{j1d_j}^T H_{ij1}^T P_i H_{ij1} z_{j1d_j} + \frac{1}{\varepsilon_i} z_{i1}^T P_i z_{i1} \right. \\ &+ \overline{\varphi}_i z_{j1d_j}^T E_{ij1}^T E_{ij1} z_{j1d_j} \\ &\left. + \overline{\varphi}_i^{-1} z_{i1}^T P_i D_{ij1} D_{ij1}^T P_i z_{i1}\right).\end{aligned} \tag{24}$$

Then, by using (24) and properties

$$\begin{aligned}&\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \varepsilon_i z_{j1d_j}^T H_{ij1}^T P_i H_{ij1} z_{j1d_j} \\ &\qquad = \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \varepsilon_j z_{i1d_i}^T H_{ji1}^T P_j H_{ji1} z_{i1d_i}, \\ &\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \overline{\varphi}_i z_{j1d_j}^T E_{ij1}^T E_{ij1} z_{j1d_j} \\ &\qquad = \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\ne i}}^{L} \overline{\varphi}_j z_{i1d_i}^T E_{ji1}^T E_{ji1} z_{i1d_i},\end{aligned} \tag{25}$$

it generates

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{L} z_{i1}^{T} \Big( & A_{i1}^{T} P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^{T} P_i \\ & + \varphi_i E_{i1}^{T} E_{i1} \Big) z_{i1} \\ + \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \Big( & \varepsilon_j z_{i1d_i}^{T} H_{ji1}^{T} P_j H_{ji1} z_{i1d_i} \\ & + \frac{1}{\varepsilon_i} z_{i1}^{T} P_i z_{i1} + \overline{\varphi}_j z_{i1d_i}^{T} E_{ji1}^{T} E_{ji1} z_{i1d_i} \\ & + \overline{\varphi}_i^{-1} z_{i1}^{T} P_i D_{ij1} D_{ij1}^{T} P_i z_{i1} \Big).\end{aligned} \tag{26}$$

According to Assumption 5, $E_{ij}$ is a free-choice matrix. Therefore, we can easily select matrix $E_{ij}$ such that the matrix $E_{ji1}^{T} E_{ji1}$ is semipositive definite. Since the $z_{i1}$ for $i = 1, 2, \ldots, L$ are independent of each other, then, from equation (31) of paper [3], the following is true:

$$V\left(z_{11d_1}, z_{21d_2}, z_{31d_3}, \ldots, z_{n1d_n}\right) \le qV\left(z_{11}, z_{21}, z_{31}, \ldots, z_{n1}\right) \tag{27}$$

for $q > 1$, and is equivalent to

$$\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \varepsilon_j z_{i1d_i}^{T} H_{ji1}^{T} P_j H_{ji1} z_{i1d_i} \le q \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \varepsilon_j z_{i1}^{T} H_{ji1}^{T} P_j H_{ji1} z_{i1} \tag{28}$$

which implies that

$$\sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \overline{\varphi}_j z_{i1d_i}^{T} E_{ji1}^{T} E_{ji1} z_{i1d_i} \le \widehat{q} \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \overline{\varphi}_j z_{i1}^{T} E_{ji1}^{T} E_{ji1} z_{i1}, \tag{29}$$

where the scalar $\widehat{q} > 1$. Thus, from (26), (28), and (29), we achieve

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{L} z_{i1}^{T} \Bigg[ & A_{i1}^{T} P_i + P_i A_{i1} + \varphi_i^{-1} P_i D_{i1} D_{i1}^{T} P_i \\ & + \varphi_i E_{i1}^{T} E_{i1} + \frac{L-1}{\varepsilon_i} P_i \\ & + \sum_{\substack{j=1 \\ j\neq i}}^{L} \Big( q\varepsilon_j H_{ji1}^{T} P_j H_{ji1} + \overline{\varphi}_i^{-1} P_i D_{ij1} D_{ij1}^{T} P_i \\ & + \widehat{q} \overline{\varphi}_j E_{ji1}^{T} E_{ji1} \Big) \Bigg] z_{i1}.\end{aligned} \tag{30}$$

In addition, by applying Lemma 6, LMI (12) is equivalent to the following inequality:

$$\begin{aligned} & A_{i1}^{T} P_i + P_i A_{i1} + \varphi_i E_{i1}^{T} E_{i1} \\ & \quad + \frac{L-1}{\varepsilon_i} P_i + \varphi_i^{-1} P_i D_{i1} D_{i1}^{T} P_i \\ & \quad + \sum_{\substack{j=1 \\ j\neq i}}^{L} \Big( q\varepsilon_j H_{ji1}^{T} P_j H_{ji1} \\ & \qquad + \overline{\varphi}_i^{-1} P_i D_{ij1} D_{ij1}^{T} P_i + \widehat{q} \overline{\varphi}_j E_{ji1}^{T} E_{ji1} \Big) < 0. \end{aligned} \tag{31}$$

According to (30) and (31), it is easy to get

$$\dot{V} < 0. \tag{32}$$

The inequality (32) shows that LMI (12) holds, which further implies that the sliding motion (11) is asymptotically stable. □

*Remark 8.* Theorem 4 provides a new existence condition of the sliding surface in terms of strict LMI, which can be easily worked out using the LMI toolbox in Matlab.

*Remark 9.* Compared to recent LMI methods [1, 5–7], the proposed method offers less number of matrix variables in LMI equations making it easier to find a feasible solution.

In order to design a new decentralized adaptive output feedback sliding mode control scheme for complex interconnected time-delay system (1), we establish the following lemma.

**Lemma 10.** *Consider a class of interconnected time-delay systems that is decomposed into L subsystems*

$$\dot{v}_i = \left(A_{ii} + \Delta A_{ii}\right) v_i + \sum_{\substack{j=1 \\ j\neq i}}^{L} A_{ij} v_{jd_j}, \tag{33}$$

*where* $v_i = \left[\begin{smallmatrix} v_{i1} \\ v_{i2} \end{smallmatrix}\right]$ *are the state variables of the ith subsystem with* $v_{i1} \in R^{n_i - m_i}$ *and* $v_{i2} \in R^{m_i}$. *The matrix* $A_{ii} = \left[\begin{smallmatrix} A_{ii1} & A_{ii2} \\ A_{ii3} & A_{ii4} \end{smallmatrix}\right]$ *is known matrices of appropriate dimensions. The matrices* $\Delta A_{ii} = \left[\begin{smallmatrix} \Delta A_{ii1} & \Delta A_{ii2} \\ \Delta A_{ii3} & \Delta A_{ii4} \end{smallmatrix}\right]$ *and* $A_{ij} = \left[\begin{smallmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{smallmatrix}\right]$ *are unknown matrices of appropriate dimensions. The notation* $v_{id_i} := v_i(t - d_i)$ *represents delayed states. The symbol* $d_i := d_i(t)$ *is the time-varying delay, which is assumed to be known and is bounded by* $\overline{d}_i$ *for all* $d_i$. *The initial conditions are given by* $v_i(t) = \chi_i(t)$ $(t \in [-\overline{d}_i, 0])$, *where* $\chi_i(t)$ *are continuous in* $[-\overline{d}_i, 0]$ *for* $i = 1, 2, 3, \ldots, L$. *If the matrix* $A_{ii1}$ *is stable then* $\sum_{i=1}^{L} \|v_{i1}(t)\|$

*is bounded by* $\sum_{i=1}^{L}\phi_i(t)$ *for all time, where* $\phi_i(t)$ *is the solution of*

$$\dot{\phi}_i(t) = \widehat{k}_i\phi_i(t) + k_i\left[\left(\|A_{ii2}\| + \|\Delta A_{ii2}\|\right)\|v_{i2}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|A_{ji2}\|\,\|v_{i2d_i}\|\right], \quad i = 1,2,\ldots,L \tag{34}$$

*in which* $\widehat{k}_i = k_i(\|\Delta A_{ii1}\| + \sum_{j=1,j\neq i}^{L}\beta_i\|A_{ji1}\|) + \lambda_i < 0$, $k_i > 0$. $\lambda_i$ *is the maximum eigenvalue of the matrix* $A_{ii1}$ *and the scalar* $\beta_i > 1$.

*Proof of Lemma 10.* We are now in the position to prove Lemma 10. From (33), it is obvious that

$$\dot{v}_{i1}(t) = \left(A_{ii1} + \Delta A_{ii1}\right)v_{i1} + \left(A_{ii2} + \Delta A_{ii2}\right)v_{i2} + \sum_{\substack{j=1\\ j\neq i}}^{L}\left(A_{ij1}v_{j1d_j} + A_{ij2}v_{j2d_j}\right). \tag{35}$$

From system (35), we have

$$v_{i1}(t) = \exp\left(A_{ii1}\right)v_{i1}(0) + \int_0^t \exp\left(A_{ii1}(t-\tau)\right) \times \left[\Delta A_{ii1}v_{i1} + \left(A_{ii2} + \Delta A_{ii2}\right)v_{i2} + \sum_{\substack{j=1\\ j\neq i}}^{L}\left(A_{ij1}v_{j1d_j} + A_{ij2}v_{j2d_j}\right)\right]d\tau. \tag{36}$$

According to (36), we obtain

$$\begin{aligned}\|v_{i1}(t)\| \le{}& \|\exp\left(A_{ii1}t\right)\|\,\|v_{i1}(0)\| + \int_0^t \|\exp\left(A_{ii1}(t-\tau)\right)\|\left(\|A_{ii2}\| + \|\Delta A_{ii2}\|\right) \times \|v_{i2}\|\,d\tau \\ &+ \int_0^t \|\exp\left(A_{ii1}(t-\tau)\right)\|\left[\|\Delta A_{ii1}\|\,\|v_{i1}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|A_{ij1}\|\,\|v_{j1d_j}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|A_{ij2}\|\,\|v_{j2d_j}\|\right]d\tau.\end{aligned} \tag{37}$$

The stable matrix $A_{ii1}$ implies that $\|\exp(A_{ii1}t)\| \le k_i\exp(\lambda_i t)$ for some scalars $k_i > 0, i = 1,2,\ldots,L$. Therefore, the above inequality can be rewritten as

$$\begin{aligned}\|v_{i1}(t)\|\exp\left(-\lambda_i t\right) \le{}& k_i\|v_{i1}(0)\| + \int_0^t k_i\exp\left(-\lambda_i\tau\right)\left(\|A_{ii2}\| + \|\Delta A_{ii2}\|\right) \times \|v_{i2}\|\,d\tau \\ &+ \int_0^t k_i\exp\left(-\lambda_i\tau\right) \times \left[\|\Delta A_{ii1}\|\,\|v_{i1}(t)\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|A_{ij1}\|\,\|v_{j1d_j}\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\|A_{ij2}\|\,\|v_{j2d_j}\|\right]d\tau.\end{aligned} \tag{38}$$

Let $s_i(t)$ be the right side term of the inequality (38)

$$s_i(t) = k_i\|v_{i1}(0)\| + \int_0^t k_i\exp\left(-\lambda_i\tau\right)\left(\|A_{ii2}\| + \|\Delta A_{ii2}\|\right)\|v_{i2}\|\,d\tau + \int_0^t k_i\exp\left(-\lambda_i\tau\right)\left[\|\Delta A_{ii1}\|\,\|v_{i1}\|\right.$$

$$+\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij1}\right\|\left\|v_{j1d_j}\right\| + \sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij2}\right\|\left\|v_{j2d_j}\right\|\Bigg] d\tau. \tag{39}$$

Then, by taking the time derivative of $s_i(t)$, we can get that

$$\begin{aligned}\frac{d}{dt}s_i(t) &= k_i\exp(-\lambda_i t)\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| \\ &+ k_i\exp(-\lambda_i t)\Bigg[\left\|\Delta A_{ii1}\right\|\left\|v_{i1}\right\| \\ &+\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij1}\right\|\left\|v_{j1d_j}\right\| \\ &+\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij2}\right\|\left\|v_{j2d_j}\right\|\Bigg].\end{aligned} \tag{40}$$

For the above equation, we multiply the term $(1/k_i)\exp(\lambda_i t)$ on both sides; then

$$\begin{aligned}\frac{1}{k_i}\exp(\lambda_i t)\frac{d}{dt}s_i(t) &= \left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| \\ &+\left\|\Delta A_{ii1}\right\|\left\|v_{i1}\right\| + \sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij1}\right\|\left\|v_{j1d_j}\right\| \\ &+\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ij2}\right\|\left\|v_{j2d_j}\right\|.\end{aligned} \tag{41}$$

Then, by taking the summation of both sides of the above equation, we have

$$\begin{aligned}&\sum_{i=1}^{L}\frac{1}{k_i}\exp(\lambda_i t)\frac{d}{dt}s_i(t) \\ &\quad= \sum_{i=1}^{L}\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| + \sum_{i=1}^{L}\left\|\Delta A_{ii1}\right\|\left\|v_{i1}\right\| \\ &\quad+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\| + \sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ji1}\right\|\left\|v_{i1d_i}\right\|.\end{aligned} \tag{42}$$

Since the $v_{i1}$ for $i = 1, 2, \ldots, L$ are independent of each other, then, from equation (32) of paper [3], it is clear that

$$\left\|v_{i1d_i}\right\| \le \beta_i\left\|v_{i1}\right\|, \quad i = 1, 2, \ldots, L \tag{43}$$

for some scalars $\beta_i > 1, i = 1, 2, \ldots, L$. Then, by substituting (43) into (42), we achieve

$$\begin{aligned}&\sum_{i=1}^{L}\frac{1}{k_i}\exp(\lambda_i t)\frac{d}{dt}s_i(t) \\ &\quad= \sum_{i=1}^{L}\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| + \sum_{i=1}^{L}\left\|\Delta A_{ii1}\right\|\left\|v_{i1}\right\| \\ &\quad+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\| + \sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\beta_i\left\|A_{ji1}\right\|\left\|v_{i1}\right\|.\end{aligned} \tag{44}$$

For the above equation, we multiply the term $k_i\exp(-\lambda_i t)$ to both sides. Since $\|v_{i1}\|\exp(-\lambda_i t) \le s_i(t)$, one can get that

$$\begin{aligned}\sum_{i=1}^{L}\frac{d}{dt}s_i(t) &\le \sum_{i=1}^{L}k_i\exp(-\lambda_i t) \\ &\times\left[\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| + \sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\|\right] \\ &+\sum_{i=1}^{L}\overline{k}_i s_i(t),\end{aligned} \tag{45}$$

where $\overline{k}_i = k_i\left(\left\|\Delta A_{ii1}\right\| + \sum_{j=1,j\neq i}^{L}\beta_i\left\|A_{ji1}\right\|\right)$. For the above inequality, we multiply the term $\exp(-\overline{k}_i t)$ to both sides, then

$$\begin{aligned}&\sum_{i=1}^{L}\frac{d}{dt}\left[s_i(t)\exp\left(-\overline{k}_i t\right)\right] \\ &\quad\le \sum_{i=1}^{L}k_i\exp(-\lambda_i t) \\ &\quad\times\left[\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\| + \sum_{\substack{j=1\\j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\|\right] \\ &\quad\times\exp\left(-\overline{k}_i t\right).\end{aligned} \tag{46}$$

Since $\|v_{i1}\|\exp(-\lambda_i t) \le s_i(t)$, integrating the above inequality on both sides, we obtain

$$\begin{aligned}&\sum_{i=1}^{L}\left\|v_{i1}\right\| \\ &\quad\le \sum_{i=1}^{L}k_i\left\|v_{i1}(0)\right\|\exp\left(\left(\overline{k}_i+\lambda_i\right)t\right)\end{aligned}$$

$$+\sum_{i=1}^{L}\left\{\int_0^t k_i\exp\left(-\lambda_i\tau\right)\left[\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\|+\sum_{\substack{j=1\\ j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\|\right]\times\exp\left(-\bar{k}_i\tau\right)d\tau\right\}\exp\left(\bar{k}_it\right)\exp\left(\lambda_it\right)$$

$$=\sum_{i=1}^{L}\left\{\phi_i\left(0\right)\exp\left(\left(\bar{k}_i+\lambda_i\right)t\right)+\int_0^t k_i\exp\left[\left(\bar{k}_i+\lambda_i\right)\left(t-\tau\right)\right]\times\left[\left(\left\|A_{ii2}\right\|+\left\|\Delta A_{ii2}\right\|\right)\left\|v_{i2}\right\|+\sum_{\substack{j=1\\ j\neq i}}^{L}\left\|A_{ji2}\right\|\left\|v_{i2d_i}\right\|\right]d\tau\right\}$$

$$=\sum_{i=1}^{L}\phi_i\left(t\right)\quad\text{if }\phi_i\left(0\right)\geq k_i\left\|v_{i1}\left(0\right)\right\|, \tag{47}$$

where the time function $\phi_i(t)$ satisfies (34). Hence, we can see that $\sum_{i=1}^{L}\phi_i(t)\geq\sum_{i=1}^{L}\|v_{i1}\|$ for all time, if $\phi_i(0)$ is sufficiently large. □

*Remark 11.* It is obvious that the time function $\phi_i(t)$ is dependent on only state variable $v_{i2}$. Therefore, we can replace state variable $v_{i1}$ by a function of state variable $v_{i2}$ in controller design. This feature is very useful in controller design using only output variables.

*3.3. Decentralized Adaptive Output Feedback Sliding Mode Controller Design.* Now, we are in the position to prove that the state trajectories of system (1) reach sliding surface (8) in finite time and stay on it thereafter. In order to satisfy the above aims, the modified decentralized adaptive output feedback sliding mode controller is selected to be

$$u_i\left(t\right)=-\left(F_{i2}B_{i2}\right)^{-1}\left(\kappa_i\eta_i\left(t\right)+\bar{\kappa}_i\left\|y_i\right\|+\hat{\kappa}_i\left\|y_{id_i}\right\|+\zeta_i\left(t\right)+\alpha_i\right)\frac{\sigma_i}{\left\|\sigma_i\right\|},\quad i=1,2,\ldots,L, \tag{48}$$

where $\kappa_i=\|F_{i2}\|(\|A_{i3}\|+\|D_{i2}\|\|E_{i1}\|)+\sum_{j=1,j\neq i}^{L}\beta_i\|F_{j2}\|(\|H_{ji3}\|+\|D_{ji2}\|\|E_{ji1}\|)$, $\bar{\kappa}_i=\|F_{i2}\|(\|A_{i4}\|+\|D_{i2}\|\|E_{i2}\|)\|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|$, $\hat{\kappa}_i=\sum_{j=1,j\neq i}^{L}\|F_{j2}\|(\|H_{ji4}\|+\|D_{ji2}\|\|E_{ji2}\|)\|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|$, and the scalars $\alpha_i>0$ and $\beta_i>1$. The adaptive law is defined as

$$\zeta_i\left(t\right)\geq\hat{b}_i\left\|F_{i2}\right\|\left\|B_{i2}\right\|\times\left(\left\|W_{i1}\right\|\eta_i\left(t\right)+\left\|W_{i2}\right\|\left\|F_{i2}^{-1}\right\|\left\|K_iC_{i2}^{-1}\right\|\left\|y_i\right\|\right)+\left\|F_{i2}\right\|\left\|B_{i2}\right\|\hat{c}_i+\breve{q}_i\frac{c_i^2}{4}, \tag{49}$$

where $\hat{b}_i$ and $\hat{c}_i$ are the solution of the following equations:

$$\dot{\hat{b}}_i=\bar{q}_i\left\|F_{i2}\right\|\left\|B_{i2}\right\|\times\left(\left\|W_{i1}\right\|\eta_i\left(t\right)+\left\|W_{i2}\right\|\left\|F_{i2}^{-1}\right\|\left\|K_iC_{i2}^{-1}\right\|\left\|y_i\right\|\right),\qquad\dot{\hat{c}}_i=\tilde{q}_i\left(-\breve{q}_i\hat{c}_i+\left\|F_{i2}\right\|\left\|B_{i2}\right\|\right) \tag{50}$$

in which $\left[W_{i1}\quad W_{i2}\right]=T_i^{-1}$ and the scalars $\bar{q}_i>0$, $\tilde{q}_i>0$, and $\breve{q}_i>0$. The time function $\eta_i(t)$ will be designed later. It should be pointed out that controller (48) uses only output variables.

Now let us discuss the reaching conditions in the following theorem.

**Theorem 12.** *Suppose that LMI (12) has solution $P_i>0$ and the scalars $q>1$, $\hat{q}>1$, $\varphi_i>0$, $\varepsilon_i>0$, $\bar{\varphi}_i>0$, $i=1,2,\ldots,L$. Consider the closed loop of system (1) with the above decentralized adaptive output feedback sliding mode controller (48) where the sliding surface is given by (8). Then, the state trajectories of system (1) reach the sliding surface in finite time and stay on it thereafter.*

*Proof of Theorem 12.* We consider the following positive definite function:

$$V=\sum_{i=1}^{L}\left(\left\|\sigma_i\right\|+\frac{0.5}{\bar{q}_i}\tilde{b}_i^2+\frac{0.5}{\tilde{q}_i}\tilde{c}_i^2\right), \tag{51}$$

where $\tilde{b}_i(t)=b_i-\hat{b}_i(t)$ and $\tilde{c}_i(t)=c_i-\hat{c}_i(t)$. Then, the time derivative of $V$ along the trajectories of (9) is given by

$$\dot{V}=\sum_{i=1}^{L}\left(\frac{\sigma_i^T}{\left\|\sigma_i\right\|}F_{i2}\dot{z}_{i2}-\frac{1}{\bar{q}_i}\tilde{b}_i\dot{\hat{b}}_i-\frac{1}{\tilde{q}_i}\tilde{c}_i\dot{\hat{c}}_i\right). \tag{52}$$

Substituting (7) into (52), we have

$$\dot{V}=\sum_{i=1}^{L}\frac{\sigma_i^T}{\left\|\sigma_i\right\|}F_{i2}\left[\left(A_{i3}+D_{i2}\Delta F_iE_{i1}\right)z_{i1}+\left(A_{i4}+D_{i2}\Delta F_iE_{i2}\right)z_{i2}\right]$$

$$+\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}\left(u_i+G_i\left(t,T_i^{-1}z_i,T_i^{-1}z_{id_i}\right)\right)$$
$$-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{b}_i\dot{\hat{b}}_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{c}_i\dot{\hat{c}}_i$$
$$+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}\left[\left(H_{ij3}+D_{ij2}\Delta F_{ij}E_{ij1}\right)z_{j1d_j}\right.$$
$$\left.+\left(H_{ij4}+D_{ij2}\Delta F_{ij}E_{ij2}\right)z_{j2d_j}\right]. \tag{53}$$

From (53), properties $\|AB\| \leq \|A\|\|B\|$ and $\|\Delta F_i\| \leq 1$, $\|\Delta F_{ij}\| \leq 1$ generate

$$\begin{aligned}\dot{V} \leq{}& \sum_{i=1}^{L}\|F_{i2}\|\left[\left(\|A_{i3}\|+\|D_{i2}\|\|E_{i1}\|\right)\|z_{i1}\|\right.\\&\left.+\left(\|A_{i4}\|+\|D_{i2}\|\|E_{i2}\|\right)\|z_{i2}\|\right]\\&+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\|F_{i2}\|\left[\left(\|H_{ij3}\|+\|D_{ij2}\|\|E_{ij1}\|\right)\|z_{j1d_j}\|\right.\\&\left.+\left(\|H_{ij4}\|+\|D_{ij2}\|\|E_{ij2}\|\right)\|z_{j2d_j}\|\right]\\&+\sum_{i=1}^{L}\|F_{i2}\|\|B_{i2}\|\|G_i\|+\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}u_i\\&-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{b}_i\dot{\hat{b}}_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{c}_i\dot{\hat{c}}_i.\end{aligned} \tag{54}$$

Since $\|G_i\| \leq c_i + b_i\|x_i\|$ and $x_i = W_{i1}z_{i1} + W_{i2}z_{i2}$, where $\left[W_{i1}\ \ W_{i2}\right] = T_i^{-1}$, we obtain

$$\begin{aligned}\dot{V} \leq{}& \sum_{i=1}^{L}\|F_{i2}\|\left[\left(\|A_{i3}\|+\|D_{i2}\|\|E_{i1}\|\right)\|z_{i1}\|\right.\\&\left.+\left(\|A_{i4}\|+\|D_{i2}\|\|E_{i2}\|\right)\|z_{i2}\|\right]\\&+\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}u_i\\&+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\|F_{i2}\|\left[\left(\|H_{ij3}\|+\|D_{ij2}\|\|E_{ij1}\|\right)\|z_{j1d_j}\|\right.\\&\left.+\left(\|H_{ij4}\|+\|D_{ij2}\|\|E_{ij2}\|\right)\|z_{j2d_j}\|\right]\\&+\sum_{i=1}^{L}b_i\|F_{i2}\|\|B_{i2}\|\left(\|W_{i1}\|\|z_{i1}\|+\|W_{i2}\|\|z_{i2}\|\right)\\&+\sum_{i=1}^{L}\|F_{i2}\|\|B_{i2}\|c_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{b}_i\dot{\hat{b}}_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{c}_i\dot{\hat{c}}_i.\end{aligned} \tag{55}$$

The facts $\sum_{i=1}^{L}\sum_{j=1,j\neq i}^{L}\|F_{i2}\|(\|H_{ij3}\|+\|D_{ij2}\|\|E_{ij1}\|)\|z_{j1d_j}\| = \sum_{i=1}^{L}\sum_{j=1,j\neq i}^{L}\|F_{j2}\|(\|H_{ji3}\|+\|D_{ji2}\|\|E_{ji1}\|)\|z_{i1d_i}\|$ and $\sum_{i=1}^{L}\sum_{j=1,j\neq i}^{L}\|F_{i2}\|(\|H_{ij4}\|+\|D_{ij2}\|\|E_{ij2}\|)\|z_{j2d_j}\| = \sum_{i=1}^{L}\sum_{j=1,j\neq i}^{L}\|F_{j2}\|(\|H_{ji4}\|+\|D_{ji2}\|\|E_{ji2}\|)\|z_{i2d_i}\|$ imply that

$$\begin{aligned}\dot{V} \leq{}& \sum_{i=1}^{L}\|F_{i2}\|\left[\left(\|A_{i3}\|+\|D_{i2}\|\|E_{i1}\|\right)\|z_{i1}\|\right.\\&\left.+\left(\|A_{i4}\|+\|D_{i2}\|\|E_{i2}\|\right)\|z_{i2}\|\right]\\&+\sum_{i=1}^{L}\frac{\sigma_i^T}{\|\sigma_i\|}F_{i2}B_{i2}u_i\\&+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\|F_{j2}\|\left[\left(\|H_{ji3}\|+\|D_{ji2}\|\|E_{ji1}\|\right)\|z_{i1d_i}\|\right.\\&\left.+\left(\|H_{ji4}\|+\|D_{ji2}\|\|E_{ji2}\|\right)\|z_{i2d_i}\|\right]\\&+\sum_{i=1}^{L}b_i\|F_{i2}\|\|B_{i2}\|\left(\|W_{i1}\|\|z_{i1}\|+\|W_{i2}\|\|z_{i2}\|\right)\\&+\sum_{i=1}^{L}\|F_{i2}\|\|B_{i2}\|c_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{b}_i\dot{\hat{b}}_i-\sum_{i=1}^{L}\frac{1}{\overline{q}_i}\tilde{c}_i\dot{\hat{c}}_i.\end{aligned} \tag{56}$$

Equation (9) implies that

$$\begin{aligned}\|z_{i2}\| &= \|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|\|y_i\|,\\ \|z_{i2d_i}\| &= \|F_{i2}^{-1}\|\|K_iC_{i2}^{-1}\|\|y_{id_i}\|.\end{aligned} \tag{57}$$

In addition, let $v_{i1} = z_{i1}$, $v_{i2} = z_{i2}$, $v_{j1d_j} = z_{j1d_j}$, $v_{j2d_j} = z_{j2d_j}$, $A_{ii1} = A_{i1}$, $\Delta A_{ii1} = D_{i1}\Delta F_iE_{i1}$, $A_{ii2} = A_{i2}$, $\Delta A_{ii2} = D_{i1}\Delta F_iE_{i2}$, $A_{ij1} = (H_{ij1} + D_{ij1}\Delta F_{ij}E_{ij1})$, $A_{ij2} = (H_{ij2} + D_{ij1}\Delta F_{ij}E_{ij2})$, and $\phi_i(t) = \eta_i(t)$. Then, by applying Lemma 10 to the system (6), we obtain

$$\sum_{i=1}^{L}\|z_{i1}\| \leq \sum_{i=1}^{L}\eta_i(t), \tag{58}$$

where $\eta_i(t)$ is the solution of

$$\dot{\eta}_i(t) = \hat{k}_i\eta_i(t) + k_i\left[\left(\|A_{i2}\|+\|D_{i1}\Delta F_iE_{i2}\|\right)\|z_{i2}\|+\sum_{\substack{j=1\\j\neq i}}^{L}\|H_{ji2}+D_{ji1}\Delta F_{ji}E_{ji2}\|\|z_{i2d_i}\|\right] \tag{59}$$

in which $\hat{k}_i = (\overline{k}_i + \lambda_i) < 0$ and $\overline{k}_i = k_i(\|D_{i1}\Delta F_iE_{i1}\| + \sum_{j=1,j\neq i}^{L}\beta_i\|H_{ji1} + D_{ji1}\Delta F_{ji}E_{ji1}\|)$. $\lambda_i$ is the maximum eigenvalue of the matrix $A_{i1}$ and the scalars $k_i > 0$, $\beta_i > 1$.

From (57) and $\|\Delta F_i\| \leq 1$, $\|\Delta F_{ji}\| \leq 1$, (59) can be rewritten as

$$\begin{aligned}\dot{\eta}_i(t) &= \hat{k}_i \eta_i(t) \\ &\quad + k_i \Bigg[ \left(\|A_{i2}\| + \|D_{i1}\| \|E_{i2}\|\right) \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_i\| \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{L} \left(\|H_{ji2}\| + \|D_{ji1}\| \|E_{ji2}\|\right) \|F_{i2}^{-1}\| \\ &\quad \times \|K_i C_{i2}^{-1}\| \|y_{id_i}\| \Bigg],\end{aligned} \tag{60}$$

where $\hat{k}_i = (\bar{k}_i + \lambda_i) < 0$ and $\bar{k}_i = k_i(\|D_{i1}\|\|E_{i1}\| + \sum_{j=1,j\neq i}^{L} \beta_i(\|H_{ji1}\| + \|D_{ji1}\|\|E_{ji1}\|))$. By (56), (57), and (58), we have

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^{L} \|F_{i2}\| \Big[ \left(\|A_{i3}\| + \|D_{i2}\| \|E_{i1}\|\right) \eta_i \\ &\quad + \left(\|A_{i4}\| + \|D_{i2}\| \|E_{i2}\|\right) \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_i\| \Big] \\ &\quad + \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j\neq i}}^{L} \|F_{j2}\| \Big[ \beta_i \left(\|H_{ji3}\| + \|D_{ji2}\| \|E_{ji1}\|\right) \eta_i \\ &\quad + \left(\|H_{ji4}\| + \|D_{ji2}\| \|E_{ji2}\|\right) \\ &\quad \times \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_{id_i}\| \Big] \\ &\quad + \sum_{i=1}^{L} b_i \|F_{i2}\| \|B_{i2}\| \left(\|W_{i1}\| \eta_i + \|W_{i2}\| \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_i\|\right) \\ &\quad + \sum_{i=1}^{L} \|F_{i2}\| \|B_{i2}\| c_i + \sum_{i=1}^{L} \frac{\sigma_i^T}{\|\sigma_i\|} F_{i2} B_{i2} u_i \\ &\quad - \sum_{i=1}^{L} \frac{1}{\bar{q}_i} \tilde{b}_i \dot{\hat{b}}_i - \sum_{i=1}^{L} \frac{1}{\tilde{q}_i} \tilde{c}_i \dot{\hat{c}}_i.\end{aligned} \tag{61}$$

By substituting the controller (48) into (61), it is clear that

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^{L} b_i \|F_{i2}\| \|B_{i2}\| \left(\|W_{i1}\| \eta_i + \|W_{i2}\| \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_i\|\right) \\ &\quad - \sum_{i=1}^{L} \zeta_i - \sum_{i=1}^{L} \alpha_i + \sum_{i=1}^{L} \|F_{i2}\| \|B_{i2}\| c_i \\ &\quad - \sum_{i=1}^{L} \frac{1}{\bar{q}_i} \tilde{b}_i \dot{\hat{b}}_i - \sum_{i=1}^{L} \frac{1}{\tilde{q}_i} \tilde{c}_i \dot{\hat{c}}_i.\end{aligned} \tag{62}$$

Considering (50) and (62), the above inequality can be rewritten as

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^{L} \hat{b}_i \|F_{i2}\| \|B_{i2}\| \left(\|W_{i1}\| \eta_i + \|W_{i2}\| \|F_{i2}^{-1}\| \|K_i C_{i2}^{-1}\| \|y_i\|\right) \\ &\quad - \sum_{i=1}^{L} \zeta_i - \sum_{i=1}^{L} \alpha_i + \sum_{i=1}^{L} \|F_{i2}\| \|B_{i2}\| \hat{c}_i \\ &\quad + \sum_{i=1}^{L} \breve{q}_i \left[ -\left(\hat{c}_i - \frac{c_i}{2}\right)^2 + \frac{c_i^2}{4} \right].\end{aligned} \tag{63}$$

By applying (49) to (63), we achieve

$$\dot{V} \leq -\sum_{i=1}^{L} \alpha_i - \sum_{i=1}^{L} \breve{q}_i \left(\hat{c}_i - \frac{c_i}{2}\right)^2 < 0. \tag{64}$$

The above inequality implies that the state trajectories of system (1) reach the sliding surface $\sigma_i(x_i) = 0$ in finite time and stay on it thereafter. □

*Remark 13.* From sliding mode control theory, Theorems 4 and 12 together show that the sliding surface (8) with the decentralised adaptive output feedback SMC law (48) guarantee that (1) at any initial value the state trajectories will reach the sliding surface in finite time and stay on it thereafter; and (2) the system (1) in sliding mode is asymptotically stable.

*Remark 14.* The SMC scheme is often discontinuous which causes "chattering" in the sliding mode. This chattering is highly undesirable because it may excite high-frequency unmodelled plant dynamics. The most common approach to reduce the chattering is to replace the discontinuous function $\sigma_i/\|\sigma_i\|$ by a continuous approximation such as $\sigma_i/(\|\sigma_i\| + \mu_i)$, where $\mu_i$ is a positive constant [29]. This approach guarantees not asymptotic stability but ultimate boundedness of system trajectories within a neighborhood of the origin depending on $\mu_i$.

*Remark 15.* The proposed controller and sliding surface use only output variables, while the bounds of disturbances are unknown. Therefore, this approach is very useful and more realistic, since it can be implemented in many practical systems.

## 4. Numerical Example

To verify the effectiveness of the proposed decentralized adaptive output feedback SMC law, our method has been applied to interconnected time-delay systems composed of two third-order subsystems, which is modified from [3].

The first subsystem's dynamics is given as

$$\begin{aligned}\dot{x}_1 &= (A_1 + \Delta A_1) x_1 + B_1 \left(u_1 + G_1\left(x_1, x_{1d_1}, t\right)\right) \\ &\quad + (H_{12} + \Delta H_{12}) x_{2d_2}, \\ y_1 &= C_1 x_1,\end{aligned} \tag{65}$$

where $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \in R^3$, $u_1 \in R^1$, $y_1 = \begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} \in R^2$, $A_1 = \begin{bmatrix} -8 & 0 & 1 \\ 0 & -8 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $H_{12} = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.02 & 0 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}$. The mismatched parameter uncertainties in the state matrix of the first subsystem are $\Delta A_1 = D_1 \Delta F_1 E_1$ with $D_1 = \begin{bmatrix} -0.02 & 0.1 & 0.02 \end{bmatrix}^T$, $E_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \end{bmatrix}$, and $\Delta F_1 = 0.4\sin^2(x_{11}^2 + t \times x_{12} + x_{13} + t \times x_{11}x_{12})$. The mismatched uncertain interconnections with the second subsystem are $\Delta H_{12} = D_{12}\Delta F_{12}E_{12}$ with $D_{12} = \begin{bmatrix} 0.1 & 0.02 & 0.1 \end{bmatrix}^T$, $\Delta F_{12} = 0.2\sin^2(x_{23d_2} + x_{23}x_{22d_2} + t \times x_{21}x_{22})$, and $E_{12} = \begin{bmatrix} 0.1 & 0.02 & 0.1 \end{bmatrix}$. The exogenous disturbance in the first subsystem is $\|G_1(x_1, x_{1d_1}, t)\| \le c_1 + b_1\|x_1\|$, where $b_1$ and $c_1$ can be selected by any positive value.

The second subsystem's dynamics is given as

$$\begin{aligned} \dot{x}_2 &= \left(A_2 + \Delta A_2\right) x_2 + B_2 \left(u_2 + G_2\left(x_2, x_{2d_2}, t\right)\right) \\ &\quad + \left(H_{21} + \Delta H_{21}\right) x_{1d_1}, \\ y_2 &= C_2 x_2, \end{aligned} \tag{66}$$

where $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix} \in R^3$, $u_2 \in R^1$, $y_2 = \begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} \in R^2$, $A_2 = \begin{bmatrix} -6 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $H_{21} = \begin{bmatrix} 0.1 & 0.02 & 0.1 \\ 0 & 0.1 & 0.1 \\ 0.02 & 0.1 & 0.02 \end{bmatrix}$. The mismatched parameter uncertainties in the state matrix of the second subsystem are $\Delta A_2 = D_2 \Delta F_2 E_2$ with $D_2 = \begin{bmatrix} 0.1 & 0.02 & -0.1 \end{bmatrix}^T$, $E_2 = \begin{bmatrix} 0.1 & 0.1 & 0.03 \end{bmatrix}$, and $\Delta F_2 = 0.45\sin(x_{21} + x_{23}^2 + t \times x_{22} + x_{21}x_{22})$. The mismatched uncertain interconnection with the first subsystem is $\Delta H_{21} = D_{21}\Delta F_{21}E_{21}$ with $D_{21} = \begin{bmatrix} 0.1 & 0.1 & -0.1 \end{bmatrix}^T$, $E_{21} = \begin{bmatrix} 0.02 & 0.02 & 0.1 \end{bmatrix}$, and $\Delta F_{21} = 0.5\sin^3(x_{11d_1} + t \times x_{12d_1} + x_{11}x_{12}x_{13})$. The exogenous disturbance in the second subsystem is $\|G_2(x_2, x_{2d_2}, t)\| \le c_2 + b_2\|x_2\|$ where $b_2$ and $c_2$ can be selected by any positive value.

For this work, the following parameters are given as follows: $\varphi_1 = 0.6$, $\varphi_2 = 3.9$, $\overline{\varphi}_1 = 0.08$, $\overline{\varphi}_2 = 0.05$, $\varepsilon_1 = 2$, $\varepsilon_2 = 3$, $q = 100$, $\widehat{q} = 100$, $\breve{q}_1 = 4$, $\breve{q}_2 = 3$, $\overline{q}_1 = \overline{q}_2 = \widetilde{q}_1 = \widetilde{q}_2 = 1$, $\beta_1 = 80$, $\beta_2 = 150$, $k_1 = 1.002$, $k_2 = 1.01$, $b_1 = 0.8$, $b_2 = 0.1$, $c_1 = 0.3$, $c_2 = 0.5$, $\alpha_1 = 0.05$, $\alpha_2 = 0.06$. According to the algorithm given in [3], the coordinate transformation matrices for the first subsystem and the second subsystem are $T_1 = T_2 = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. By solving LMI (12), it is easy to verify that conditions in Theorem 4 are satisfied with positive matrices $P_1 = \begin{bmatrix} 0.2104 & -0.0017 \\ -0.0017 & 0.2305 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0.2669 & -0.0266 \\ -0.0266 & 0.2517 \end{bmatrix}$. The matrices $\Xi_1$ and $\Xi_2$ are selected to be $\Xi_1 = \begin{bmatrix} 0.2236 & 0.6708 \end{bmatrix}$ and $\Xi_2 = \begin{bmatrix} 0.8 & -0.4 \end{bmatrix}$. From (8), the sliding surface for the first subsystem and the second subsystem are $\sigma_1 = \begin{bmatrix} 0 & 0 & -0.1137 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \end{bmatrix}^T = 0$ and $\sigma_2 = \begin{bmatrix} 0 & 0 & -0.2281 \end{bmatrix} \begin{bmatrix} x_{21} & x_{22} & x_{23} \end{bmatrix}^T = 0$. Theorem 4 showed that the sliding motion associated with the sliding surfaces $\sigma_1$ and $\sigma_2$ is globally asymptotically stable. The time functions $\eta_1(t)$ and $\eta_2(t)$ are the solution of $\dot{\eta}_1(t) = -7.954\eta_1(t) + 2.015\|y_1\| + 0.22\|y_{1d_1}\|$ and $\dot{\eta}_2(t) = -5.796\eta_2(t) + 2.024\|y_2\| + 0.215\|y_{2d_2}\|$, respectively. From Theorem 12, the decentralized

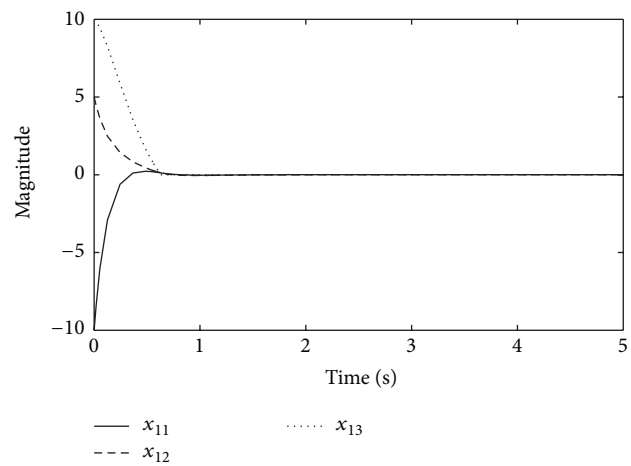


Figure 1: Time responses of states $x_{11}$ (solid), $x_{12}$ (dashed), and $x_{13}$ (dotted).

adaptive output feedback sliding mode controller for the first subsystem and the second subsystem are

$$\begin{aligned} u_1(t) &= \Big(\zeta_1 + 0.05 + 1.0862\eta_1(t) + 0.00028\|y_1\| \\ &\quad + 0.0068\|y_{1d_1}\|\Big)\frac{\sigma_1}{\|\sigma_1\|}, \end{aligned} \tag{67}$$

$$\begin{aligned} u_2(t) &= \Big(\zeta_2 + 0.06 + 1.1048\eta_2(t) + 0.000684\|y_2\| \\ &\quad + 0.0125\|y_{2d_2}\|\Big)\frac{\sigma_2}{\|\sigma_2\|}, \end{aligned} \tag{68}$$

where $\zeta_1 \ge 0.113\widehat{b}_1(\eta_1(t) + \|y_1\|) + 0.1137\widehat{c}_1 + 0.022$, $\dot{\widehat{b}}_1 = 0.113(\eta_1(t) + \|y_1\|)$, $\dot{\widehat{c}}_1 = -4\widehat{c}_1 + 0.113$, $\zeta_2 \ge 0.228\widehat{b}_2(\eta_2(t) + \|y_2\|) + 0.2281\widehat{c}_2 + 0.0625$, $\dot{\widehat{b}}_2 = 0.228(\eta_2(t) + \|y_2\|)$, and $\dot{\widehat{c}}_2 = -4\widehat{c}_2 + 0.2281$. Figures 5 and 6 imply that the chattering occurs in control input. In order to eliminate chattering phenomenon, the discontinuous controllers (67) and (68) are replaced by the following continuous approximations:

$$\begin{aligned} u_1(t) &= \Big(\zeta_1 + 0.05 + 1.0862\eta_1(t) + 0.00028\|y_1\| \\ &\quad + 0.0068\|y_{1d_1}\|\Big)\frac{\sigma_1}{\|\sigma_1\| + 0.0001}, \end{aligned} \tag{69}$$

$$\begin{aligned} u_2(t) &= \Big(\zeta_2 + 0.06 + 1.1048\eta_2(t) + 0.000684\|y_2\| \\ &\quad + 0.0125\|y_{2d_2}\|\Big)\frac{\sigma_2}{\|\sigma_2\| + 0.0001}. \end{aligned} \tag{70}$$

From Figures 7 and 8, we can see that the chattering is eliminated.

The time-delays chosen for the first subsystem and the second subsystem are $d_1(t) = 2 - \sin(t)$ and $d_2(t) = 1 - 0.5\cos(t)$. The initial conditions for two subsystems are selected to be $\chi_1(t) = \begin{bmatrix} -10 & 5 & 10 \end{bmatrix}^T$ and $\chi_2(t) = \begin{bmatrix} 10 & -8 & -10 \end{bmatrix}^T$, respectively. By Figures 1, 2, 3, 4, 5, 6, 7, and 8 it is clearly seen that the proposed controller is effective in

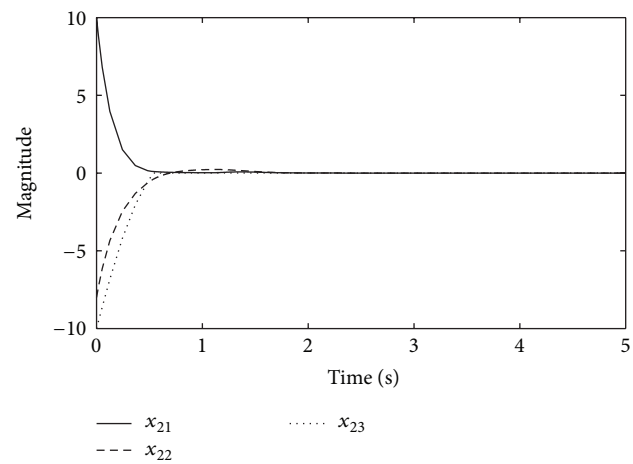


FIGURE 2: Time responses of states $x_{21}$ (solid), $x_{22}$ (dashed), and $x_{23}$ (dotted).

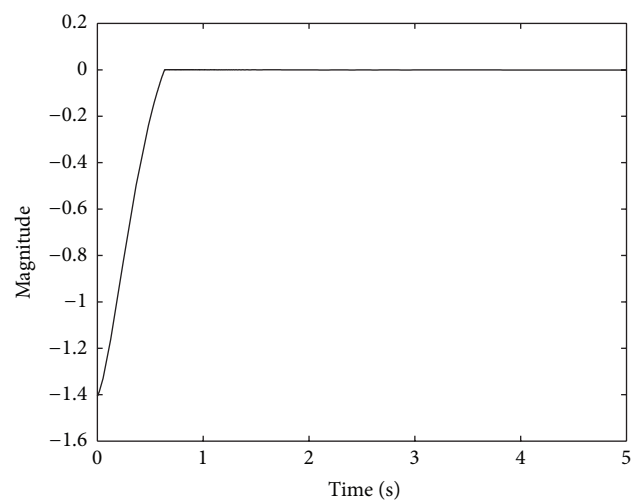


FIGURE 3: Time responses of sliding function $\sigma_1$.

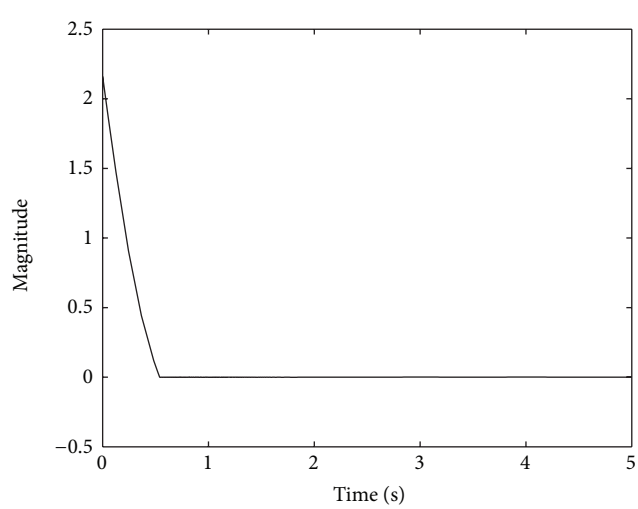


FIGURE 4: Time responses of sliding function $\sigma_2$.

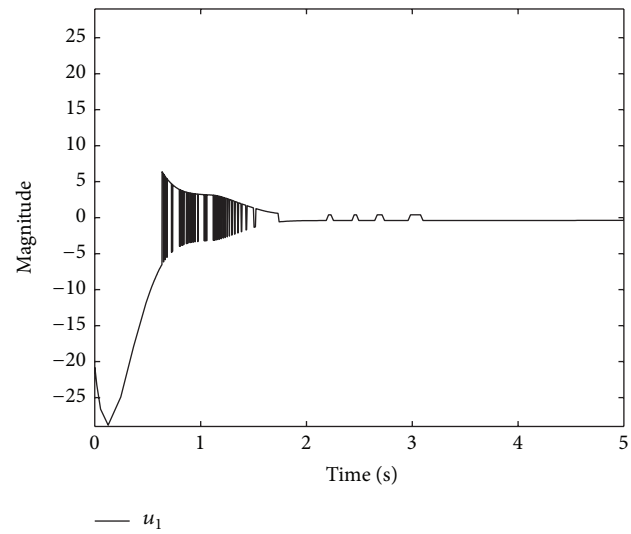


FIGURE 5: Time responses of discontinuous control input $u_1$ (67).

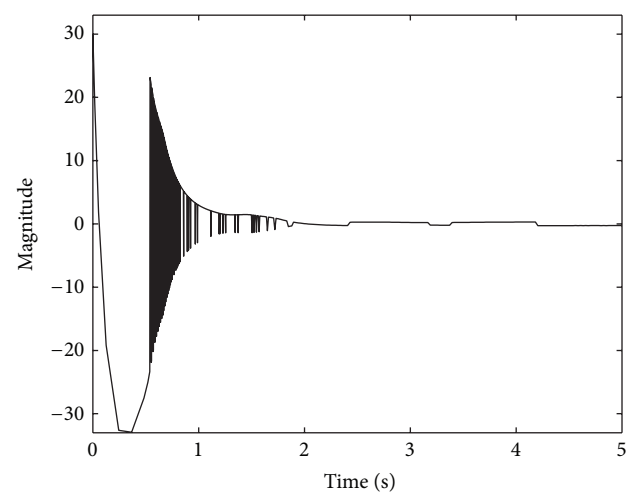


FIGURE 6: Time responses of discontinuous control input $u_2$ (68).

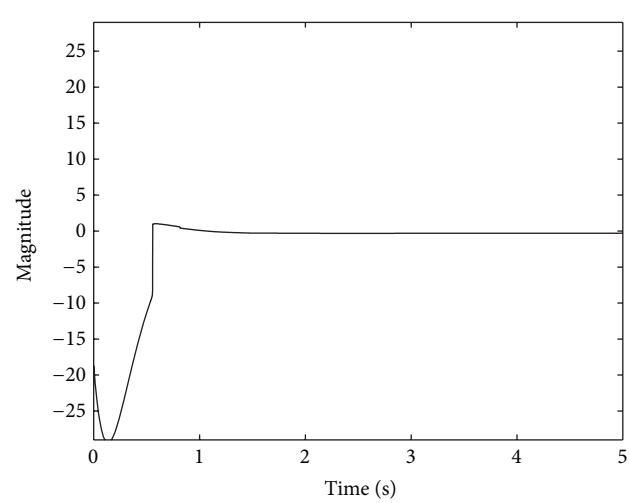


FIGURE 7: Time responses of continuous control input $u_1$ (69).

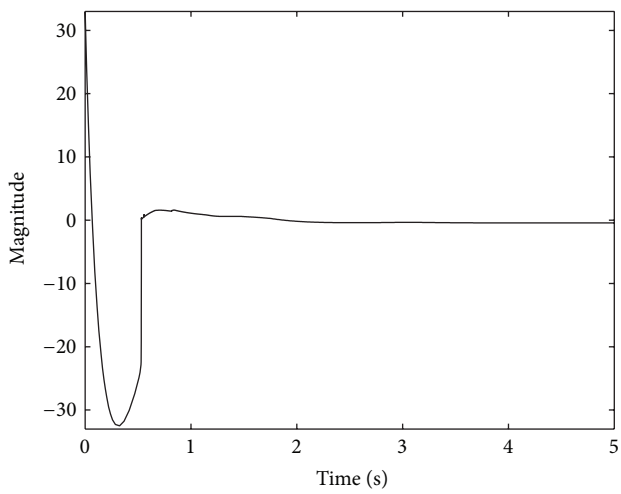


Figure 8: Time responses of continuous control input $u_2$ (70).

dealing with matched and mismatched uncertainties and the system has a good performance.

## 5. Conclusion

In this paper, a decentralized adaptive SMC law is proposed to stabilize complex interconnected time-delay systems with unknown disturbance, mismatched parameter uncertainties in the state matrix, and mismatched interconnections. Furthermore, in these systems, the system states are unavailable and no estimated states are required. This is a new problem in the application of SMC to interconnected time-delay systems. By establishing a new lemma, the two major limitations of SMC approaches for interconnected time-delay systems in [3] have been removed. We have shown that the new sliding mode controller guarantees the reachability of the system states in a finite time period, and moreover the dynamics of the reduced-order complex interconnected time-delay system in sliding mode is asymptotically stable under certain conditions.

## Conflict of Interests

The authors declare that they have no conflict of interests regarding to the publication of this paper.

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