Research Article

LSSVM-Based Rock Failure Criterion and Its Application in Numerical Simulation

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A rock failure criterion is very important for the prediction of the failure of rocks or rock masses in rock mechanics and engineering. Least squares support vector machines (LSSVM) are a powerful tool for addressing complex nonlinear problems. This paper describes a LSSVM-based rock failure criterion for analyzing the deformation of a circular tunnel under different *in situ* stresses without assuming a function form. First, LSSVM was used to represent the nonlinear relationship between the mechanical properties of rock and the failure behavior of the rock in order to construct a rock failure criterion based on experimental data. Then, this was used in a hypothetical numerical analysis of a circular tunnel to analyze the mechanical behavior of the rock mass surrounding the tunnel. The Mohr-Coulomb and Hoek-Brown failure criteria were also used to analyze the same case, and the results were compared; these clearly indicate that LSSVM can be used to establish a rock failure criterion and to predict the failure of a rock mass during excavation of a circular tunnel.

1. Introduction

The failure of rock has been one of the most active research areas in the engineering of rock structures since the establishment of the International Society for Rock Mechanics (ISRM) in 1962. Ulusay and Hudson gave a detailed description of the importance of the ability to predict rock failure in practical engineering [1]. Rock failure criteria include the theoretical basis of methods of predicting when and how rock materials fail under the action of external loads and are therefore essential for optimization of engineering design and construction in rock masses. Over the past several decades, a number of failure criteria have been proposed (e.g., Mohr-Coulomb, Hoek-Brown, and Griffith's and Drucker-Prager criteria) and developed to predict the failure of rock [2–11]. Recently, ISRM suggested methods have been published as criteria for the prediction of rock failure [12, 13]. Both the Mohr-Coulomb and Hoek-Brown criteria are generally considered to be reliable predictors of rock failure. In these two models, the major principal stress σ_1 is seen as function

of the uniaxial compressive strength (σ_c), the minor principal stress (σ_3), and some constants, that is, $f(\sigma_1, \sigma_3) = 0$. These constants can often be determined by laboratory experiment, *in situ* tests, and back-analysis.

Empirical failure criteria established under specific conditions such as rock type, stress state, or functional form are not generic and cannot be adopted uncritically for any set of conditions [10] but must be established experimentally for any given set of rock formation and stress conditions.

For this reason, artificial neural network (ANN) computational models are widely used to establish rock failure criteria for geotechnical engineering problems. This is due to the fact that ANN perform function approximation without requiring additional assumptions of the function form. More detailed descriptions of the ANN-based failure criterion method have been reported by, for example, Meulenkamp and Grima, Singh et al., Çanakci and Pala, Tiryaki, Zorlu et al., and Rafiai and Jafari [14–20]. However, a disadvantage of ANN for establishing rock failure criteria is its slow convergence and overfitting. To overcome these limitations,

in the present study a LSSVM-based rock failure criterion was developed based on experimental data. Support vector machines (SVM) have been widely used in geotechnical engineering for nonlinear mapping [21–23].

This paper is organized as follows. Section 2 focuses on the formulation of LSSVM. A LSSVM-based rock failure criterion is described in detail in Section 3. In Section 4, a hypothetical numerical experiment is used to verify the LSSVM-based rock failure criterion by comparing with earlier failure criteria. Finally, conclusions are given in Section 5.

2. Least-Squares Support Vector Machine (LSSVM)

LSSVM models are an alternative formulation of SVM regression proposed by Suykens and Vandewalle [24]. Suppose there is a given training set of N data points $\{x_k, y_k\}$ ($k = 1, 2, \dots, N$) with input data $x_k \in R^n$ and output $y_k \in r$, where R^n is an N -dimensional vector space and r is a one-dimensional vector space; LSSVM models in feature space are given by

$$y(x) = w^T \phi(x) + b, \quad (1)$$

where the nonlinear mapping $\phi(\cdot)$ represents the input data in a higher-dimensional feature space; $w \in R^n$; $b \in r$; w is an adjustable weight vector; and b is the scalar threshold. For function estimation in LSSVM, the following optimization problem is formulated:

$$\begin{aligned} &\text{Minimize} \quad \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^N e_k^2 \\ &\text{Subjected to} \quad y(x) = w^T \phi(x) + b + e_k, \quad k = 1, \dots, N, \end{aligned} \quad (2)$$

where γ is the regularization parameter for determining the trade-off between the fitting error minimization and smoothness, and e_k is an error variable.

The Lagrangian ($L(w, b, e, \alpha)$) for the above optimization problem (2) is

$$\begin{aligned} L(w, b, e; \alpha) = & \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^N e_k^2 \\ & - \sum_{k=1}^N \alpha_k \{y_k [w^T \phi(x_k) + b] - 1 + e_k\}, \end{aligned} \quad (3)$$

where α_k is the Lagrange multiplier. The conditions for optimality are given by

$$\frac{\partial L}{\partial w} = 0 \implies w = \sum_{k=1}^N \alpha_k \phi(x_k),$$

$$\frac{\partial L}{\partial b} = 0 \implies \sum_{k=1}^N \alpha_k = 0,$$

$$\frac{\partial L}{\partial e_k} = 0 \implies \alpha_k = \gamma e_k, \quad k = 1, \dots, N,$$

$$\frac{\partial L}{\partial \alpha_k} = 0 \implies w^T \phi(x_k) + b + e_k - y_k = 0, \quad k = 1, \dots, N \quad (4)$$

without regard to e_k and w ; the solution is given by the following set of linear equations:

$$\Phi \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}, \quad (5)$$

where $y = [y_1, \dots, y_N]$, $\mathbf{1} = [1, \dots, 1]$, $\alpha = [\alpha_1, \dots, \alpha_N]$, and Mercer's theorem is applied within the Ω matrix, $\Omega = \phi(x_k)^T \phi(x_l) = K(x_k, x_l)$, $k, l = 1, \dots, N$, where $K(x_k, x_l)$ is the kernel function, and the matrix Φ is invertible by choosing $\gamma > w$ and can be written as

$$\Phi = \begin{bmatrix} 0 & \mathbf{1}^T \\ \mathbf{1} & \Omega + \gamma^{-1} I \end{bmatrix}. \quad (6)$$

Then the analytical solution of α and b is given by

$$\begin{bmatrix} b \\ \alpha \end{bmatrix} = \Phi^{-1} \begin{bmatrix} 0 \\ y \end{bmatrix}. \quad (7)$$

The LSSVM model can then be expressed as

$$y(x) = \sum_{k=1}^N \alpha_k K(x, x_k) + b. \quad (8)$$

In (8), $K(x, x_k)$ is the kernel function, composed of the following three forms.

(1) Polynomial kernel:

$$K(X, Y) = ((X \cdot Y) + 1)^d, \quad d = 1, 2, \dots, n. \quad (9)$$

(2) Radial kernel:

$$K(X, Y) = \exp \left\{ -\frac{|X - Y|^2}{2\sigma^2} \right\}. \quad (10)$$

(3) Sigmoidal kernel:

$$K(X, Y) = \tanh(\varphi(X \cdot Y) + \theta). \quad (11)$$

The brief procedure LSSVM is the following.

Step 1. Build the training samples (x_k, y_k) based on rock mechanical experiment.

Step 2. Get (6) according to the LSSVM algorithm.

Step 3. Get the analytical solution of α and b in (7).

Step 4. Get the LSSVM model (8).

FIGURE 1: The flowchart of LSSVM-based failure criterion.

3. LSSVM-Based Rock Failure Criteria

In order to better predict the behavior failure of rock masses, LSSVM was used in this work to represent the nonlinear relationship between rock properties and rock failure and thus to establish a LSSVM-based rock failure criterion. Similar to traditional failure criteria in form, LSSVM-based rock failure criteria can be defined as

$$\text{SVM}(\sigma_1, \sigma_3) = 0, \quad (12)$$

where σ_1 and σ_3 are the major and minor principal stresses, respectively. These can be obtained by laboratory experiment and field testing. The LSSVM-based rock failure criterion can be rewritten in the form:

$$\sigma_1 = \sum_{k=1}^N \alpha_k K(\sigma_3, \sigma_{3k}) + b, \quad (13)$$

where σ_{3k} is the series of experimental data and α_k and b are calculated using the algorithm introduced in Section 2. Once a LSSVM-based rock failure criterion is established, it can be used in numerical analysis to predict rock failure. In this work, Excel VBA software was used to execute the code of the LSSVM-based rock failure criterion.

To implement the LSSVM-based failure criteria in a numerical model, it is useful to express it as an equivalent Mohr-Coulomb criterion with variable parameters. The parameters of the equivalent criterion are obtained as

$$\sigma_c^{\text{eq}} = \sigma_1 - q^{\text{eq}} \sigma_3 = \sum_{k=1}^N \alpha_k K(\sigma_3, \sigma_{3k}) + b - q^{\text{eq}} \sigma_3, \quad (14)$$

$$q^{\text{eq}} = \frac{\partial \sigma_1}{\partial \sigma_3} = \frac{\partial \left(\sum_{k=1}^N \alpha_k K(\sigma_3, \sigma_{3k}) \right)}{\partial \sigma_3} = \sum_{k=1}^N \alpha_k \frac{\partial (K(\sigma_3, \sigma_{3k}))}{\partial \sigma_3}. \quad (15)$$

The RBF kernel function (Equation (10)) was adopted:

$$\frac{\partial (K(\sigma_3, \sigma_{3k}))}{\partial \sigma_3} = -\frac{1}{\sigma^2} K(\sigma_3, \sigma_{3k}). \quad (16)$$

Substituting (16) into (15) gives the following form:

$$q^{\text{eq}} = -\frac{1}{\sigma^2} \sum_{k=1}^N \alpha_k K(\sigma_3, \sigma_{3k}), \quad (17)$$

and the equivalent Mohr-Coulomb criterion may be calculated from the following equations:

$$c^{\text{eq}} = \frac{\sigma_c^{\text{eq}}}{2\sqrt{q^{\text{eq}}}}, \quad (18)$$

$$\varphi^{\text{eq}} = 2 \tan^{-1} \left(\sqrt{q^{\text{eq}}} \right) - \frac{\pi}{2}, \quad (19)$$

where c^{eq} and φ^{eq} are the instantaneous values of cohesion and friction angle, respectively.

LSSVM-based rock failure criterion presents the mechanical character of rock through combining L-SVM and rock failure criterion. The brief procedure is showed in Figure 1.

TABLE I: Experimental data and the values of Lagrange multiplier α_k .

Number	σ_3 (Mpa)	σ_1 (Mpa)	α_k	Number	σ_3 (Mpa)	σ_1 (Mpa)	α_k
1	0.0000	0.0000	-138.4852	41	3.8699	17.8247	0.1287
2	0.0853	2.7979	97.6169	42	3.9471	18.0397	0.0845
3	0.2398	4.0913	5.5845	43	4.0244	18.2534	0.0837
4	0.3170	4.6287	2.2133	44	4.1016	18.4657	0.0829
5	0.3943	5.1224	1.6698	45	4.1788	18.6769	0.0822
6	0.4715	5.5829	1.3160	46	4.2561	18.8868	0.1220
7	0.5487	6.0173	1.5167	47	4.4105	19.3033	0.1210
8	0.7032	6.8251	1.2040	48	4.4878	19.5098	0.0800
9	0.7804	7.2046	0.6538	49	4.5650	19.7153	0.1193
10	0.8577	7.5708	0.5713	50	4.7195	20.1231	0.1186
11	0.9349	7.9254	0.7297	51	4.7967	20.3255	0.0787
12	1.0894	8.6045	0.6295	52	4.8740	20.5269	0.0785
13	1.1666	8.9311	0.3662	53	4.9512	20.7273	0.1175
14	1.2439	9.2501	0.4865	54	5.1057	21.1255	0.1173
15	1.3983	9.8678	0.4344	55	5.1829	21.3232	0.0781
16	1.4756	10.1677	0.2606	56	5.2601	21.5200	0.0781
17	1.5528	10.4620	0.3552	57	5.3374	21.7160	0.1171
18	1.7073	11.0360	0.3247	58	5.4919	22.1055	0.1172
19	1.7845	11.3162	0.1989	59	5.5691	22.2990	0.0783
20	1.8617	11.5922	0.1876	60	5.6463	22.4918	0.0784
21	1.9390	11.8644	0.2615	61	5.7236	22.6838	0.1178
22	2.0935	12.3979	0.2442	62	5.8780	23.0655	0.1182
23	2.1707	12.6597	0.1526	63	5.9553	23.2553	0.0790
24	2.2479	12.9183	0.1459	64	6.0325	23.4443	0.1190
25	2.3252	13.1740	0.1398	65	6.1870	23.8203	0.1195
26	2.4024	13.4268	0.1342	66	6.2642	24.0073	0.0800
27	2.4796	13.6769	0.1914	67	6.3415	24.1937	0.1206
28	2.6341	14.1696	0.1826	68	6.4959	24.5644	0.1212
29	2.7113	14.4124	0.1165	69	6.5732	24.7488	0.0813
30	2.7886	14.6529	0.1678	70	6.6504	24.9327	0.0816
31	2.9431	15.1275	0.1616	71	6.7276	25.1159	0.0820
32	3.0203	15.3617	0.1040	72	6.8049	25.2985	0.1237
33	3.0975	15.5940	0.1015	73	6.9593	25.6620	0.1244
34	3.1748	15.8245	0.0992	74	7.0366	25.8429	0.0835
35	3.2520	16.0532	0.1447	75	7.1138	26.0232	0.0839
36	3.4065	16.5054	0.1410	76	7.1911	26.2030	0.1267
37	3.4837	16.7290	0.0918	77	7.3455	26.5610	0.1276
38	3.5609	16.9511	0.0903	78	7.4228	26.7392	0.0856
39	3.6382	17.1717	0.0889	79	7.5000	26.9169	15.4377
40	3.7154	17.3908	0.1309

FIGURE 2: Comparison of major principal stress σ_1 between the experimental data and predicted value for training samples.

FIGURE 3: Comparison of major principal stress σ_1 between the experimental data and predicted value for testing samples.

4. Numerical Experiment

4.1. Matching the Experimental Data. In this work, the RBF kernel function was adopted for pattern analysis or recognition, with the following parameters: $\sigma = 2$. The value of b is 14.5061. The values of α_k are listed in Table 1. As in any other use of LSSVM, the SVM must be trained and tested; 98 data sets from triaxial compression tests, consisting of 79 training samples and 19 testing samples, were used as the experimental data in this study (seen in Table 1). Once the performance

of the SVM model was satisfactory, it was used to represent the nonlinear relationship between rock properties and the mechanical behavior of the rock to establish the LSSVM-based rock failure criterion.

As shown in Figures 2 and 3, the predicted LSSVM values and experimental values of σ_1 were almost identical, indicating that the LSSVM-based failure criterion model effectively simulated the failure behavior. Figure 4 shows a comparison between the failure envelopes determined by the LSSVM, Mohr-Coulomb, and Hoek-Brown models and those

FIGURE 4: Comparison of failure envelopes for different rock failure criteria.

FIGURE 5: A circular tunnel model, stress condition, and rock mass properties.

from the experimental data. It can be seen from Figure 4 that σ_1 predicted by the LSSVM model agreed well both with the experimental data and the Hoek-Brown model at large stresses but differed a little from the Mohr-Coulomb model; however, σ_1 differed a little from the Hoek-Brown model at small stresses. These results indicate that the LSSVM-based rock failure criterion trained on experimental data predicts the failure behavior of rock with reasonable accuracy.

4.2. Numerical Analysis Using the LSSVM-Based Rock Failure Criterion. To verify the feasibility of the LSSVM-based rock failure criterion in numerical analysis, it was combined with FLAC3D modeling code to simulate the failure behavior of a 5 m radius circular rock tunnel as a hypothetical numerical case. An initial *in situ* stress and gravity was presupposed in the numerical model, and the experimental data from

Section 4.1 was used. The properties of the rock mass listed in Figure 5 are based on the Mohr-Coulomb model. The numerical model is built using FLAC3D together with the algorithm in Section 3. The horizontal displacements are almost the same as the value of Mohr-Coulomb by FLAC3D (seen in Figure 6). The stress of surrounding rock is in well agreement with the law of Mohr-Coulomb by FLAC3D (seen in Figure 7). The horizontal displacements and the horizontal, vertical, and shear stresses induced in the surrounding rock by the excavation show that the LSSVM model results were almost identical to those obtained by the Mohr-Coulomb model. The induced horizontal and vertical stresses calculated by the two models are also shown as contour plots in Figures 8 and 9. It shows the mechanical character of rock was presented by LSSVM-based rock failure criteria. Overall, the results show that the LSSVM-based rock failure criteria

FIGURE 6: Comparison of surrounding rock displacement of tunnel obtained by LSSVM and Mohr-Coulomb model.

FIGURE 7: Comparison of horizontal stress of tunnel obtained by LSSVM and Mohr-Coulomb model (a) σ_x , (b) σ_y , (c) σ_{xy} .

FIGURE 8: Comparison of horizontal stress contour in surrounding rock mass obtained by LSSVM and Mohr-Coulomb model.

FIGURE 9: Comparison of vertical stress contour in surrounding rock mass obtained by LSSVM and Mohr-Coulomb model.

FIGURE 10: Comparison of surrounding rock mass displacement obtained by LSSVM and Mohr-Coulomb model in different *in situ* stress level (s_y is the vertical *in situ* stress).

FIGURE 11: Comparison of surrounding rock stress of tunnel between LSSVM and Mohr-Coulomb model in different *in situ* stress level (a) σ_x , (b) σ_y , (c) τ_{xy} (s_y is the vertical *in situ* stress).

present well the mechanical behavior and character and can be used for numerical analysis.

To verify the LSSVM-based rock failure criterion for different initial *in situ* stress states, the circular tunnel was investigated at different vertical stress $\sigma_{yy} = 5, 10$, and 15 MPa and corresponding horizontal stress $\sigma_{xx} = 1.2\sigma_{yy}$ in each case. The maximum error and maximum relative error of horizontal displacements are about 0.08 and 2.8% , respectively. The maximum relative error of vertical stress and shear stress is about 18% at the beginning of excavation (seen Figure 11(b)) and then it will be less than 5% with the excavation. Using the LSSVM and Mohr-Coulomb models, the calculated horizontal displacements in the rock surrounding the tunnel are shown in Figure 10 and the major principal stresses are shown in Figure 11. Those show it is feasible to combine LSSVM rock failure criterion with numerical analysis.

The displacements for both models agree well for the three initial *in situ* stress states (Figure 10), and the principal stresses are also similar for both models (Figure 11), with σ_y and τ_{xy} showing small differences near the wall of tunnel. These results generally reflect the above results (in which $\sigma_{yy} = 1$ MPa).

The LSSVM and Hoek-Brown models were also compared. Figures 12–14 show the results for the same initial stress states as above, and it is seen that displacements in the surrounding rock agreed well with both the Hoek-Brown and Mohr-Coulomb results. Some small differences are evident in σ_{xx} and τ_{xy} compared to the Mohr-Coulomb model, but the results are consistent with the Hoek-Brown model because, as shown in Figure 4, the LSSVM and Hoek-Brown failure envelopes are similar. Thus, the proposed failure criterion can be used in numerical analysis to effectively reflect the mechanical behavior of rock.

FIGURE 12: Displacement and stresses induced by excavation at $\sigma_y = 5$ MPa.

5. Conclusions

The results of comparisons between the proposed LSSVM-based rock failure criterion and the Mohr-Coulomb and Hoek-Brown criteria and experimental data demonstrated clearly that LSSVM provided an effective rock failure criterion for the purpose of numerical analysis. Comparisons of the displacements in the rock surrounding a circular tunnel from the LSSVM, Mohr-Coulomb and Hoek-Brown models, and experimental data showed that the LSSVM model mapped the nonlinear relationship between the mechanical properties of the rock and its failure behavior.

List of Symbols

N :	Number of samples
R^N :	N -dimensional vector space
r :	One-dimensional vector space
x_k, y_k :	Input and output of training samples
$\varphi(\cdot)$:	Nonlinear mapping in a higher-dimensional feature space
w :	Adjustable weight vector
b :	Scalar threshold
γ :	Regularization parameter
e_k :	Error variable

FIGURE 13: Displacement and stresses induced by excavation at $\sigma_y = 10$ MPa.

α_k : Lagrange multiplier
 $k(\cdot)$: Kernel function
 Φ : Matrix of kernel function
 l : $1 \times N$ matrix
 σ_1 : Major principal stress
 σ_3 : Minor principal stress
 σ_{yy} : Vertical *in situ* stress
 σ_{xx} : Horizontal *in situ* stress
 σ_x : Horizontal stress of surrounding rock mass
 σ_y : Vertical stress of surrounding rock mass
 τ_{xy} : Shear stress
 σ_{3k} : Experimental data
 c_{eq} : Instantaneous values of cohesion

φ_{eq} : Instantaneous values of friction angle
 $SVM(\cdot)$: LSSVM-based rock failure criteria
 σ : Parameter of RBF kernel function.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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FIGURE 14: Displacement and stresses induced by excavation at $\sigma_y = 15$ MPa.

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