Research Article

Co-Design of Event Generator and Dynamic Output Feedback Controller for LTI Systems

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This paper presents a co-design method of the event generator and the dynamic output feedback controller for a linear time-invariant (LTI) system. The event-triggered condition on the sensor-to-controller and the controller-to-actuator depends on the plant output and the controller output, respectively. A sufficient condition on the existence of the event generator and the dynamic output feedback controller is proposed and the co-design problem can be converted into the feasibility of linear matrix inequalities (LMIs). The LTI system is asymptotically stable under the proposed event-triggered controller and also reduces the computing resources with respect to the time-triggered one. In the end, a numerical example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

With plenty of control applications on the digital platforms, event-triggered control becomes more and more popular due to its advantages on control systems with limited resources. For networked control systems, most literature focuses on the performance analysis of the system under the network bandwidth limitation; see [1–4]. However, less traffic requirement, better resource utilization, and better steady-state performance are all the significant factors. Compared with time-triggered control, event-triggered control has its advantages. In the traditional digital control techniques, the controller updates periodically. Analysis and synthesis of the system by using the periodic sampling is much easier than using the aperiodic sampling. However, the periodic sampling leads to a waste of computation and communication resources sometimes. From the resource allocation point of view, the event-triggered control was proposed; see the literature [5–7] and references therein. Event-triggered control is a control scheme in which the controller updates as long as the system state (or output) satisfies a well-designed condition.

The condition is called event generator, which can maintain the necessary properties of the system, such as convergence and stability. By using event-triggered control, computation and communication resources will be utilized only when necessary. Therefore, the event-triggered mechanism is a kind of “on-demand” executive strategy and can guarantee the performance of the system as well.

Due to the advantage of reducing computation and communication resources, event-triggered control has been taken more and more into consideration. It is first presented by Dorf in [5]; several different event-triggered schemes are investigated for a variety of systems with control performances. In [6, 7], the authors experimentally demonstrate the saving resources of event-triggered control while simultaneously preserving the performance of the system. Event-triggered control for the first-order linear stochastic system forces the output variance to be considerably smaller with respect to periodic control [6]. A simple event-based PID controller is studied in [7], which contains a time-triggered event detector and an event-triggered PID controller. The PID controller cannot calculate the control signal unless the variable from

the event detector has enough changes. An asynchronous emulation-based event-triggered feedback approach is taken into account in [8], where a directly digital design strategy is included. In [9], Tabuada proposes an event-triggered mechanism for nonlinear system to guarantee the asymptotical stability of the system and relax the traditional periodic execution requirements. The method proposed in [9] is extended to the exponential input-to-state stability (ISS) in [10]. In [11], Lunze and Lehmann propose a method on event-based state-feedback control for linear systems, with bounded disturbances, in which the sensitivity bound of the event generator can be chosen such that the event-triggered control system approximates the continuous state-feedback control loop.

However, not all of the system states can be directly measured in applications; the event-triggered control based on the system output (a part of the system state information) is much more practical; see, for example, [12–16]. Decentralized event-triggered mechanism is applied to the dynamic output feedback control system in [12]. For guaranteeing the L_∞ performance, the event generator is designed and the bound of interevent time is also provided. The optimal control problem under the event-triggered output feedback control is studied for discrete-time systems in [13] and an upper bound on the optimal cost is presented. The input-to-state stability under the self-triggered dynamic output feedback control is studied in [14], where a discrete-time observer is in cascade with a full state-feedback self-triggered controller. The designed event generator renders the ISS of the system with respect to exogenous disturbances. The result is extended to the decentralized event-triggered system in [15]. In [16], the event-triggered conditions are proposed for three kinds of event-triggered dynamic output feedback control architectures, in which a global lower bound on the inter-sample times is also provided to guarantee the asymptotic stability of the closed-loop system. However, most work in the literature focuses on designing the event generator, does not mention the continuous control algorithm. Most of them assume that the controller has been designed previously. But in fact, the event-triggered controller includes both the event generator and the controller. How to co-design the event generator and the controller simultaneously is much more challengeable. Li and Xu [17, 18] give us a co-design method for both discrete-time linear systems and LPV systems under the state feedback controller. While co-designing the event generator and the controller based on the system output is quite significant from the practical application point of view. To the best of the authors' knowledge, co-designing of event generator and the dynamic output feedback controller has not been investigated. Comparing to the static state feedback controller, the dynamic output feedback controller can often provide more adjustable parameters. It is a challenge to co-design the event generator and the dynamic output feedback controller as well.

The remainder of the paper is organized as follows. In Section 2, the necessary notations and preliminaries are provided. The problem statement of event-triggered dynamic output feedback control is presented in Section 3. Section 4 presents a sufficient condition on how to co-design the event

FIGURE 1: Event-triggered dynamic output feedback control loop.

generator and the dynamic output feedback controller, under which the linear system is asymptotically stable. Numerical simulation results are given in Section 5, which verify the effectiveness of the proposed methods. Finally, conclusions are included in Section 6.

2. Preliminaries

The notation $\|\cdot\|$ is used to denote the Euclidean norm. $X > 0$ ($X < 0$) represents that X is a positive (negative) definite matrix. X^T and X^{-1} denote the transpose and the inverse of matrix X . I represents the unit matrix with appropriate dimensions; Z_0^+ is the set of all nonnegative integers. For simplicity, the form $\begin{pmatrix} A & B \\ * & C \end{pmatrix}$ is equivalent to the symmetric matrix of the form $\begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$. In what follows, if not explicitly stated, matrices are assumed to have compatible dimensions.

The following lemma will be used in the proof of the main result.

Lemma 1 (see [19]). *Consider a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where $S_{11} \in R^{r \times r}$, $S_{12} \in R^{r \times (n-r)}$, $S_{21} \in R^{(n-r) \times r}$, $S_{22} \in R^{(n-r) \times (n-r)}$. Then $S < 0$ only if $S_{11} < 0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$ or equivalently, $S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.*

3. Problem Statement

A linear plant controlled by an event-triggered dynamic output feedback controller is shown in Figure 1. The event generator is an implementation of the event-triggered condition. We need to design an output-dependent event-triggered condition which can be used to decide when the measurement and the control law update. If the output signal y_p measured from the sensor satisfies the event-triggered condition 1, it will be sent to the controller through the zero-order-hold 1 (ZOH 1); otherwise it will not be sent. Similarly, if the output of the controller y_c satisfies the event-triggered condition 2, the output of the controller will be sent to the actuator through the ZOH 2; otherwise it does not transmit. Here we denote the triggered instant by $t_i, i \in Z_0^+$.

We consider a linear time-invariant plant given by

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t), \quad x_p(0) = x_{p0}, \\ y_p(t) &= C_p x_p(t), \end{aligned} \quad (1)$$

where $x_p(t) \in R^n$ denotes the plant state; $x_p(0) = x_{p0}$ is the initial state of the plant; $u_p(t) \in R^m$ is the plant input;

$y_p(t) \in R^q$ is the output of the plant. A_p , B_p , and C_p are the known constant matrices with appropriate dimensions.

The plant is controlled by a dynamic output feedback controller given by

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t), \quad x_c(0) = x_{c0}, \\ y_c(t) &= C_c x_c(t), \end{aligned} \quad (2)$$

where $x_c(t) \in R^n$ denotes the controller state; $x_c(0) = x_{c0}$ is the initial state of the controller; $u_c(t) \in R^q$ is the controller input; $y_c(t) \in R^m$ is the output of the controller. A_c , B_c , and C_c are matrices appropriate dimensions, which will be designed in the following part.

Whether $y_p(t)$ and $y_c(t)$ are sent or not is up to the event generator conditions. That is to say, the controller cannot calculate the control signal unless y_p and y_c in the event generator have enough change. Since $t_i, i \in Z_0^+$ is the triggered instant, $y_p(t)$ and $y_c(t)$ are only sent at t_i ; $\hat{y}_p(t)$ and $\hat{y}_c(t)$ update at the triggered instants. By using the zero-order holder, $\hat{y}_p(t)$ and $\hat{y}_c(t)$ maintain their values until the next triggered instant t_{i+1} arrives. Therefore, $u_p(t) = \hat{y}_c(t)$ and $u_c(t) = \hat{y}_p(t)$ can be expressed by

$$\begin{aligned} \hat{y}_c(t) &= y_c(t_i), \quad t \in [t_i, t_{i+1}), \\ \hat{y}_p(t) &= y_p(t_i), \quad t \in [t_i, t_{i+1}). \end{aligned} \quad (3)$$

Define the errors $e_p(t)$ and $e_c(t)$ as follows:

$$\begin{aligned} e_p(t) &= \hat{y}_p(t) - y_p(t), \\ e_c(t) &= \hat{y}_c(t) - y_c(t). \end{aligned} \quad (4)$$

Combining (1), (2), and (4), we have

$$\dot{x} = \begin{pmatrix} \dot{x}_p \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} A_p & B_p C_c \\ B_c C_p & A_c \end{pmatrix} \begin{pmatrix} x_p \\ x_c \end{pmatrix} + \begin{pmatrix} 0 & B_p \\ B_c & 0 \end{pmatrix} \begin{pmatrix} e_p(t) \\ e_c(t) \end{pmatrix}. \quad (5)$$

Thus the closed-loop system can be described by

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}e(t), \quad (6)$$

where

$$\begin{aligned} \bar{A} &= \begin{pmatrix} A_p & B_p C_c \\ B_c C_p & A_c \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} 0 & B_p \\ B_c & 0 \end{pmatrix}, \\ e(t) &= \begin{pmatrix} e_p(t) \\ e_c(t) \end{pmatrix}, \quad x(t) = \begin{pmatrix} x_p(t) \\ x_c(t) \end{pmatrix}, \\ x_0 &= \begin{pmatrix} x_{p0} \\ x_{c0} \end{pmatrix}. \end{aligned} \quad (7)$$

We present the following event-triggered condition:

$$e_p^T e_p + e_c^T e_c \leq \sigma_1 y_p^T y_p + \sigma_2 y_c^T y_c, \quad \sigma_1 > 0, \quad \sigma_2 > 0 \quad (8)$$

which means $\hat{y}_p(t)$ is updated once if

$$\begin{aligned} e_p^T(t) e_p(t) + e_c^T(t_i) e_c(t_i) \\ \leq \sigma_1 y_p^T(t) y_p(t) + \sigma_2 y_c^T(t_i) y_c(t_i) \end{aligned} \quad (9)$$

is violated and $\hat{y}_c(t)$ is updated once if

$$\begin{aligned} e_p^T(t_i) e_p(t_i) + e_c^T(t) e_c(t) \\ \leq \sigma_1 y_p^T(t_i) y_p(t_i) + \sigma_2 y_c^T(t) y_c(t) \end{aligned} \quad (10)$$

is violated for $t \in [t_i, t_{i+1})$.

The objective of this paper is to co-design the dynamic output feedback controller (2) and the event generator satisfying condition (8) under which the closed-loop system (6) is asymptotically stable.

4. Main Results

In this section, we will give a sufficient condition on the existence of the event generator and the dynamic output feedback controller by using LIMs.

Theorem 2. *If there exist symmetric positive definite matrices $X \in R^{n \times n}$ and $Y \in R^{m \times m}$, matrices $V \in R^{r \times n}$ and $W \in R^{n \times m}$, and scalars $\delta_1 > 0, \delta_2 > 0$ satisfying the following LMIs*

$$\begin{bmatrix} Y A_p^T + A_p Y + V^T B_p^T + B_p V & Y C_p^T & V^T & B_p \\ * & -\delta_1 I & 0 & 0 \\ * & * & -\delta_2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (11)$$

$$\begin{bmatrix} X A_p + A_p^T X + W C_p + C_p^T W^T & W & X B_p & C_p^T \\ * & -I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\delta_1 I \end{bmatrix} < 0, \quad (12)$$

then system (6) under the dynamic output feedback controller (2) and the event-triggered condition (8) is asymptotically stable. The parameters of the controller are as follows:

$$\begin{aligned} A_c &= -(Y - X^{-1}) (A_p + Y A_p^T X + V^T B_p^T X + Y C_p^T W^T \\ &\quad + B_p B_p^T X + \delta_1^{-1} Y C_p^T C_p)^T Y^{-1}, \end{aligned} \quad (13)$$

$$B_c = (Y - X^{-1}) W,$$

$$C_c = V Y^{-1},$$

and the parameters of the event-triggered condition (8) are $\sigma_1 = \delta_1^{-1}, \sigma_2 = \delta_2^{-1}$.

Proof. Choosing a candidate Lyapunov function

$$V_x = x^T P x, \quad (14)$$

we have the following time derivative of V_x along with system (6):

$$\begin{aligned} \dot{V}_x &= 2x^T P (\bar{A}x + \bar{B}e) \\ &= x^T (P \bar{A} + \bar{A}^T P) x + 2x^T P \bar{B}e \\ &\leq x^T (P \bar{A} + \bar{A}^T P + P \bar{B} \bar{B}^T P) x + e^T e. \end{aligned} \quad (15)$$

Letting $V = C_c Y$, and from (11), we use Lemma 1 three times and obtain

$$\begin{aligned} & Y A_p^T + A_p Y + Y^T C_c^T B_p^T + \delta_2^{-1} Y^T C_c^T C_c Y \\ & + \delta_1^{-1} Y C_p^T C_p Y + B_p C_c Y + B_p B_p^T < 0. \end{aligned} \quad (16)$$

Similarly, letting $W = (Y^{-1} - X) B_c$ and applying the Schur lemma to LMI (12) three times, we get

$$\begin{aligned} & X A_p + A_p^T X + (Y^{-1} - X) B_c C_p + C_p^T B_c^T (Y^{-1} - X)^T \\ & + X B_p B_p^T X + \delta_1^{-1} C_p^T C_p \\ & + (Y^{-1} - X) B_c B_c^T (Y^{-1} - X)^T < 0. \end{aligned} \quad (17)$$

Considering

$$\begin{aligned} A_c = - (Y - X^{-1}) (A_p + Y A_p^T X + V^T B_p^T X + Y C_p^T W^T \\ + B_p B_p^T X + \delta_1^{-1} Y C_p^T C_p)^T Y^{-1}, \end{aligned} \quad (18)$$

we know that

$$\begin{aligned} & Y A_c^T (Y^{-1} - X)^T + A_p + Y A_p^T X + Y C_c^T B_p^T X \\ & + Y C_p^T B_c^T (Y^{-1} - X)^T + B_p B_p^T X + \delta_1^{-1} Y C_p^T C_p = 0. \end{aligned} \quad (19)$$

Combining (16), (17), and (19), we have

$$\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} < 0, \quad (20)$$

where

$$\begin{aligned} Q_1 &= Y A_p^T + A_p Y + Y^T C_c^T B_p^T + B_p C_c Y + B_p B_p^T \\ &+ \delta_1^{-1} Y C_p^T C_p Y + \delta_2^{-1} Y^T C_c^T C_c Y, \\ Q_2 &= Y A_c^T (Y^{-1} - X)^T + A_p + Y A_p^T X + Y C_c^T B_p^T X \\ &+ Y C_p^T B_c^T (Y^{-1} - X)^T + B_p B_p^T X + \delta_1^{-1} Y C_p^T C_p, \\ Q_3 &= X A_p + A_p^T X + (Y^{-1} - X) B_c C_p + C_p^T B_c^T (Y^{-1} - X)^T \\ &+ X B_p B_p^T X + \delta_1^{-1} C_p^T C_p \\ &+ (Y^{-1} - X) B_c B_c^T (Y^{-1} - X)^T. \end{aligned} \quad (21)$$

Furthermore, we have

$$\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} = M_1 + M_2 + M_3 + M_4 < 0, \quad (22)$$

where

$$\begin{aligned} M_1 &= \begin{bmatrix} A_p Y + B_p C_c Y & A_p \\ X A_p Y + X B_p C_c Y + (Y^{-1} - X) B_c C_p Y + (Y^{-1} - X) A_c Y & X A_p + (Y^{-1} - X) B_c C_p \end{bmatrix}, \\ M_2 &= \begin{bmatrix} Y A_p^T + Y^T C_c^T B_p^T & Y A_p^T X + Y C_c^T B_p^T X + Y C_p^T B_c^T (Y^{-1} - X)^T + Y A_c^T (Y^{-1} - X)^T \\ A_p^T & A_p^T X + C_p^T B_c^T (Y^{-1} - X)^T \end{bmatrix}, \\ M_3 &= \begin{bmatrix} B_p B_p^T & B_p B_p^T X \\ * & X B_p B_p^T X + (Y^{-1} - X) B_c B_c^T (Y^{-1} - X)^T \end{bmatrix}, \\ M_4 &= \begin{bmatrix} \delta_1^{-1} Y C_p^T C_p Y + \delta_2^{-1} Y^T C_c^T C_c Y & \delta_1^{-1} Y C_p^T C_p \\ * & \delta_1^{-1} C_p^T C_p \end{bmatrix}. \end{aligned} \quad (23)$$

Letting

$$H = \begin{bmatrix} 0 & I \\ Y^{-1} & -I \end{bmatrix}, \quad (24)$$

and premultiplying and postmultiplying LMI (22) by H and H^T , we obtain

$$H M_1 H^T + H M_2 H^T + H M_3 H^T + H M_4 H^T < 0. \quad (25)$$

Since

$$\begin{aligned}
HM_1H^T &= H \begin{bmatrix} A_p Y + B_p C_c Y & & \\ X A_p Y + X B_p C_c Y + (Y^{-1} - X) B_c C_p Y + (Y^{-1} - X) A_c Y & X A_p + (Y^{-1} - X) B_c C_p & \\ & & \end{bmatrix} H^T \\
&= H \begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix} \begin{bmatrix} X & Y^{-1} - X \\ * & X - Y^{-1} \end{bmatrix} \begin{bmatrix} A_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix} \begin{bmatrix} Y & I \\ Y & 0 \end{bmatrix} H^T \\
&= \overline{PA}, \\
HM_2H^T &= H \begin{bmatrix} Y A_p^T + Y^T C_c^T B_p^T & Y C_p^T B_c^T (Y^{-1} - X)^T + Y A_c^T (Y^{-1} - X)^T + Y A_p^T X + Y C_c^T B_p^T X \\ A_p^T & A_p^T X + C_p^T B_c^T (Y^{-1} - X)^T \end{bmatrix} H^T \\
&= H \begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix} \begin{bmatrix} A_p^T & C_p^T B_c^T \\ C_c^T B_p^T & A_c^T \end{bmatrix} \begin{bmatrix} X & Y^{-1} - X \\ * & X - Y^{-1} \end{bmatrix} \begin{bmatrix} Y & I \\ Y & 0 \end{bmatrix} H^T \\
&= \overline{A}^T P, \\
HM_3H^T &= H \begin{bmatrix} B_p B_p^T & B_p B_p^T X \\ * & X B_p B_p^T X + (Y^{-1} - X) B_c B_c^T (Y^{-1} - X)^T \end{bmatrix} H^T \\
&= H \begin{bmatrix} I & 0 \\ X & Y^{-1} - X \end{bmatrix} \begin{bmatrix} 0 & B_p \\ B_c & 0 \end{bmatrix} \begin{bmatrix} 0 & B_c^T \\ B_p^T & 0 \end{bmatrix} \begin{bmatrix} I & X \\ 0 & Y^{-1} - X \end{bmatrix} H^T \\
&= \overline{PBB}^T P, \\
HM_4H^T &= H \begin{bmatrix} \delta_1^{-1} Y C_p^T C_p Y + \delta_2^{-1} Y^T C_c^T C_c Y & \delta_1^{-1} Y C_p^T C_p \\ * & \delta_1^{-1} C_p^T C_p \end{bmatrix} H^T \\
&= H \begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix} \begin{bmatrix} \delta_1^{-1} C_p^T C_p & 0 \\ * & \delta_2^{-1} C_c^T C_c \end{bmatrix} \begin{bmatrix} Y & I \\ Y & 0 \end{bmatrix} H^T \\
&= \begin{bmatrix} \delta_1^{-1} C_p^T C_p & 0 \\ * & \delta_2^{-1} C_c^T C_c \end{bmatrix},
\end{aligned} \tag{26}$$

we know that LMI (25) is equivalent to the following inequality

$$P\overline{A} + \overline{A}^T P + P\overline{BB}^T P + \begin{bmatrix} \delta_1^{-1} C_p^T C_p & 0 \\ * & \delta_2^{-1} C_c^T C_c \end{bmatrix} < 0, \tag{27}$$

which can be rewritten by

$$\begin{aligned}
&x^T (P\overline{A} + \overline{A}^T P + P\overline{BB}^T P) x \\
&+ x^T \begin{bmatrix} \delta_1^{-1} C_p^T C_p & 0 \\ * & \delta_2^{-1} C_c^T C_c \end{bmatrix} x < 0.
\end{aligned} \tag{28}$$

Since

$$\begin{aligned}
e^T e &= e_p^T e_p + e_c^T e_c \leq \sigma_1 y_p^T y_p + \sigma_2 y_c^T y_c \\
&= x^T \begin{bmatrix} \sigma_1 C_p^T C_p & 0 \\ 0 & \sigma_2 C_c^T C_c \end{bmatrix} x \\
&= x^T \begin{bmatrix} \delta_1^{-1} C_p^T C_p & 0 \\ * & \delta_2^{-1} C_c^T C_c \end{bmatrix} x,
\end{aligned} \tag{29}$$

then we obtain that

$$x^T (P\overline{A} + \overline{A}^T P + P\overline{BB}^T P) x + e^T e < 0. \tag{30}$$

Combining (15) and (30), we have

$$\dot{V}_x < 0. \tag{31}$$

Hence, if LMIs (11) and (12) are satisfied, system (6) under the dynamic output feedback (2) with the matrices A_c , B_c ,

and C_c and event generator (8) with $\sigma_1 = \delta_1^{-1}$, $\sigma_2 = \delta_2^{-1}$ is asymptotically stable, where

$$A_c = -\left(Y - X^{-1}\right)\left(A_p + YA_p^T X + V^T B_p^T X + YC_p^T W^T + B_p B_p^T X + \delta_1^{-1} YC_p^T C_p\right)^T Y^{-1}, \quad (32)$$

$$B_c = \left(Y - X^{-1}\right)W,$$

$$C_c = VY^{-1}.$$

The proof is completed. $\square$

Remark 3. Theorem 2 gives us a sufficient condition on how to co-design the event generator and the dynamic output feedback controller. Different from the co-design method in [16–18], this paper focuses on the dynamic output feedback. Moreover, the event-triggered condition in this paper also depends on the system output, which is much easier to realize with respect to the case depending on the system state.

Remark 4. For any given LTI model parameters A_p , B_p , C_p , we can calculate the inequalities (11) and (12) in Theorem 2. If the LMIs have feasible solutions X , Y , V , W , σ_1 , σ_2 , then we know system (6) is asymptotically stable. Furthermore, we obtain the parameters of the event generator and the controller. Therefore, the co-design problem is converted into the feasibility of LMIs, which is easy to be checked by using Matlab/LMI toolbox.

5. A Numerical Example

In this section, a numerical example is given to demonstrate the efficiency of the proposed method above. Consider the plant and its candidate dynamic output feedback controller as follows:

$$\begin{aligned} \dot{x}_p &= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x_p + \begin{bmatrix} 2.2 \\ 2.2 \end{bmatrix} u_p, \\ y_p &= [-1 \quad 4] x_p, \\ \dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t), \\ y_c(t) &= C_c x_c(t). \end{aligned} \quad (33)$$

By solving the LMIs feasibility problem in Theorem 2, we have $X = \begin{bmatrix} 4.1064 & -4.4449 \\ -4.4449 & 5.4109 \end{bmatrix}$, $Y = \begin{bmatrix} 19.3972 & 4.3338 \\ 4.3338 & 4.2859 \end{bmatrix}$, $V = \begin{bmatrix} -9.1681 & -4.7274 \end{bmatrix}$, $W = \begin{bmatrix} -1.8795 \\ 0.7450 \end{bmatrix}$, $\delta_1 = 25.2983$, and $\delta_2 = 28.0767$ and then the parameters of the controller can be calculated as

$$\begin{aligned} A_c &= \begin{bmatrix} 4.5499 & -18.8029 \\ 5.7729 & -19.7737 \end{bmatrix}, & B_c &= \begin{bmatrix} 4.2917 \\ 3.5312 \end{bmatrix}, \\ C_c &= [-0.2922 \quad -0.8075]. \end{aligned} \quad (34)$$

From (8), we obtain that the event-triggered condition is

$$e_p^T e_p + e_c^T e_c \leq 0.0395 y_p^T y_p + 0.0356 y_c^T y_c. \quad (35)$$

FIGURE 2: System state trajectories under the event-triggered dynamic output feedback controller.

FIGURE 3: Event-triggered instant and event-triggered interval.

Given the initial state $x_0 = (40, -20, 40, -20)^T$, we have the trajectory of the closed-loop system shown in Figure 2. It can be seen that the system is asymptotically stable.

Setting the sampling step as 0.05 s, the effect of the event-triggered scheme from sensor to controller and from controller to actuator is shown in Figure 3, where the x -axis means the event-triggered instant and y -axis means the interval between the current event-triggered instant and the last event-triggered instant. After computation, the average sampling time from sensor to controller is 0.15 s, which is 3 times that of the sampling time of the system if there is no event-triggered scheme; the average sampling time from controller to actuator is 0.18 s, which is 3.6 times that of the sampling time of the system if there is no event-triggered scheme. Therefore, the simulation shows that the system resource utilization is greatly saved by using the event-triggered scheme and the unnecessary waste of system resources is reduced.

6. Conclusion

In this paper, we studied the asymptotical stabilization of linear systems based on the event-triggered dynamic output feedback control. We proposed an approach to co-design the event generator and the dynamic output feedback controller.

A sufficient condition was presented in terms of LMIs by using a quadratic Lyapunov function.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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