Research Article

Terminal Sliding Mode Control with Unidirectional Auxiliary Surfaces for Hypersonic Vehicles Based on Adaptive Disturbance Observer

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A novel flight control scheme is proposed using the terminal sliding mode technique, unidirectional auxiliary surfaces and the disturbance observer model. These proposed dynamic attitude control systems can improve control performance of hypersonic vehicles despite uncertainties and external disturbances. The terminal attractor is employed to improve the convergence rate associated with the critical damping characteristics problem noted in short-period motions of hypersonic vehicles. The proposed robust attitude control scheme uses a dynamic terminal sliding mode with unidirectional auxiliary surfaces. The nonlinear disturbance observer is designed to estimate system uncertainties and external disturbances. The output of the disturbance observer aids the robust adaptive control scheme and improves robust attitude control performance. Finally, simulation results are presented to illustrate the effectiveness of the proposed terminal sliding mode with unidirectional auxiliary surfaces.

1. Introduction

It is well known that the near space hypersonic vehicle (NHV) as a novel aerospace aircraft plays an important role in military and civil applications [1]. Compared with traditional flight vehicles, the NHV has significant difference in flight airspace, working states, and mission patterns. In addition, it possesses many advantages over traditional spacecraft, such as lower launch cost, maintainability, speed of redeployment, and reusability. Consequently, the NHV concept has garnered considerable interest over the last few years. On the other hand, the NHV dynamics is severely nonlinear, time-varying, highly uncertain, and strongly coupled. It also suffers different external disturbances with changes in the flight environment, propulsion system perturbation, atmospheric disturbances, and other factors. Therefore, robust flight control of the NHV is a challenging problem. Many advanced control schemes have been presented in the literatures for control of NHVs. For example, Chen and Jiang [2] developed robust attitude control for NHVs with time-varying

disturbances. Zhao et al. [3, 4] proposed a fault-tolerant control scheme based on a second-order dynamic terminal sliding mode. Their model calls for a near space vehicle (NSV) attitude dynamic system in the presence of model parameter uncertainties and external disturbances. He et al. [5] designed an adaptive output feedback fault-tolerant controller for the longitudinal dynamics of a generic hypersonic flight vehicle. The robust sliding mode control strategy was applied to the NSV attitude dynamics. In [6], an adaptive nonsingular terminal sliding mode approach was presented for the attitude control of NHV in the presence of parameter uncertainties and external disturbances. In [7], a second-order dynamic terminal sliding mode control was proposed for the NSV with high speed, violent changes of aerodynamic parameters, and large external disturbance. The adaptive attitude control was studied for the NSV with dynamic uncertainties using a functional link network (FLN) in [8]. In [9, 10], an adaptive tracking control was proposed for the NSV using Takagi-Sugeno fuzzy models. In [11, 12], a robust back-stepping control was studied for a flexible air-breathing hypersonic

vehicle in the presence of input constraint and aerodynamic uncertainty. The disturbance observer-based control method was proposed for uncertain flight dynamics system in [13–16]. In the above-mentioned control approaches, sliding mode control (SMC) has been extensively studied for handling systems with large uncertainties, nonlinearities, and bounded external disturbances. Due to remarkable robust control performance, the SMC has been widely applied in many practical systems [17–23].

In order to alleviate chattering, a new approach called sliding mode control with unidirectional auxiliary surfaces (UAS-SMC) has been proposed to constrain the system states [24–27]. The unidirectional auxiliary surfaces in UAS-SMC are used to construct a positive invariant set. It implies that once the system states come into this positive invariant set, they would never come out again. However, the UAS-SMC scheme has a specific disadvantage. Its drawback is that the convergence speed of UAS-SMC is slower than that of the conventional SMC. This drawback prevents the UAS-SMC strategy from being applied to some high-frequency systems such as the angular rate loop of flight control systems.

Based on the above analysis, the key breakthroughs can be concluded as follows: The aim of this paper is to first propose UAS-TSMC with suitably high convergence speed. The linear switching surface in UAS-SMC strategy is replaced by a terminal switching surface in UAS-TSMC to improve the convergence speed. And the nonlinear unidirectional auxiliary surfaces in UAS-TSMC can be proven to stay in a positive invariant set. Secondly, by combining the nonlinear disturbance observer with the adaptive terminal sliding control via unidirectional auxiliary surfaces (UAS-TSMC-NDO), a composite UAS-TSMC-NDO method is constructed to handle the external disturbances, and it can make the system more stable, alleviate chattering, and strengthen robustness.

This work is motivated by the above discussion. A novel UAS-TSMC-NDO control scheme is proposed for the NHV, which has been applied to the tracking control of attitude angles. The organization of the paper is as follows: Section 2 details the problem statement, and Section 3 gives the design of terminal sliding mode control with unidirectional auxiliary surfaces based on the disturbance observer. Simulation results are presented to demonstrate the effectiveness of our proposed approaches given in Section 4, followed by the conclusion remarks in Section 5.

2. Problem Statement

The attitude mathematic model for hypersonic vehicles developed at NASA Langley Research Center is given by [28]

$$\begin{aligned}\dot{\alpha} &= \frac{1}{MV \cos \beta} + [-L + Mg \cos \gamma \cos \mu - T_x \sin \alpha \\ &\quad + T_z \cos \alpha] + q - \tan \beta (p \cos \alpha + r \sin \alpha), \\ \dot{\beta} &= \frac{1}{MV} [-T_x \sin \beta \cos \alpha + T_y \cos \beta - T_z \sin \alpha \sin \beta] \\ &\quad + \frac{1}{MV} [Y \cos \beta + Mg \cos \gamma \sin \mu] - r \cos \alpha + p \sin \alpha,\end{aligned}$$

$$\begin{aligned}\dot{\mu} &= \sec \beta (p \cos \alpha + r \sin \alpha) + \frac{1}{MV} [L \tan \gamma \sin \mu \\ &\quad + L \tan \beta] + \frac{1}{MV} [-Mg \cos \gamma \cos \mu \tan \beta \\ &\quad + (Y + T_y) \cos \beta \cos \mu \tan \gamma] \\ &\quad + \frac{1}{MV} [(T_x \sin \alpha - T_z \cos \alpha) (\tan \gamma \sin \mu + \tan \beta)] \\ &\quad - \frac{1}{MV} [(T_x \cos \alpha + T_z \sin \alpha) \tan \gamma \cos \mu \sin \beta], \\ \dot{p} &= I_{qr}^p q r + \dot{I}_p^p p + g_l^p (l_A + l_T), \\ \dot{q} &= I_{pr}^q p r + \dot{I}_q^q q + g_m^q (m_A + m_T), \\ \dot{r} &= I_{pq}^r p q + \dot{I}_r^r r + g_n^r (n_A + n_T),\end{aligned}\tag{1}$$

where α is the angle of attack, β is the sideslip angle, μ is the bank angle, p is the roll rate, q is the pitch rate, and r is the yaw rate.

According to the singular perturbation theory, the six equations can be divided into the fast loop and the slow loop, respectively. In order to study robust attitude control for the NHV, the kinematic and dynamic attitudes of the hypersonic vehicle are described as the following multi-input and multioutput (MIMO) affine nonlinear system [29]:

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2, \tag{2a}$$

$$\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u + \psi(t), \tag{2b}$$

$$y = x_1, \tag{2c}$$

where $x_1 = [\alpha, \beta, \mu]^T$ is the slow loop state, the attitude angle vector. $x_2 = [p, q, r]^T$ is the fast loop state, the body-axis angular rate vector. $\psi(t)$ is the compounded external disturbance. u is the system control input and y is the system output. The detailed expressions of $f_1(x_1)$, $f_2(x_1, x_2)$, $g_1(x_1)$, and $g_2(x_1, x_2)$ can be found in [29].

In this paper, the attitude control objective is that the robust control schemes are designed to track a given desired output y_d of the attitude motion. In the attitude control design, we define

$$e_1 = y - y_d, \tag{3}$$

$$e_2 = x_2 - \pi_1, \tag{4}$$

where e_1 is the attitude angle tracking error vector, e_2 is the attitude angular rate tracking error vector, and $\pi_1 \in R^3$ is a command signal which will be designed.

In order to proceed with the design of robust attitude control scheme for attitude motions (2a), (2b), and (2c) of the NHV, the following assumptions are required.

Assumption 1. For the compounded disturbance $\psi(t)$, there exists an unknown positive constant ϑ such that $\|\psi(t)\| \leq \vartheta$.

Assumption 2. For all $t > 0$, there exist Y_i such that $\|y_d^{(i)}(t)\| \leq Y_i$, $i = 1, 2$.

3. Design of Terminal Sliding Mode Control with Unidirectional Auxiliary Surfaces

In this section, a terminal sliding mode control with unidirectional auxiliary is developed. First, a terminal sliding mode control algorithm is designed to stabilize the slow loop system and ensure that the unidirectional auxiliary sliding mode converges to the equilibrium surfaces asymptotically. Then, the robust adaptive control will be designed based on disturbance observer that guarantees the fast loop system will remain stable in the presence of uncertainties and external disturbances effecting the NHV.

3.1. Slow Loop Terminal Sliding Mode Controller Design. Considering the slow loop dynamic (3) of the NHV and differentiating e_1 yielded

$$\begin{aligned} \dot{e}_1 &= \dot{y} - \dot{y}_d = f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d \\ &= f_1(x_1) + g_1(x_1)e_2 - \dot{y}_d + g_1(x_1)\pi_1. \end{aligned} \quad (5)$$

The detailed design of UAS-TSMC is given as follows.

Step 1. The terminal switching surfaces for system (5) are given by

$$\begin{aligned} S_{1i} &= e_{1i} + \xi_{12i} \cdot \left(\int e_{1i} + \xi_{3i} \int e_{1i}^{n/m} \right) = 0, \\ S_{2i} &= e_{1i} + \xi_{22i} \cdot \left(\int e_{1i} + \xi_{3i} \int e_{1i}^{n/m} \right) = 0, \end{aligned} \quad (6)$$

where $\xi_{12i} > \xi_{22i} > 0$, $\xi_{3i} > 0$ are the designed constants and m and n are positive odd integers, which satisfy the following condition:

$$m > n. \quad (7)$$

Defining $z = [z_{2i}, z_{1i}]^T$, $z_{2i} = e_{1i}$, and $z_{1i} = \int e_{1i} + \xi_{3i} \int e_{1i}^{n/m}$, the terminal switching surfaces in (6) can be rewritten as

$$\begin{aligned} S_{1i} &= z_{2i} + \xi_{12i} \cdot z_{1i} = 0, \\ S_{2i} &= z_{2i} + \xi_{22i} \cdot z_{1i} = 0. \end{aligned} \quad (8)$$

Step 2. Based on the terminal switching surface S_{1i} , S_{2i} , number $0i, \dots, 3i$ subspace can be defined as in Figure 1.

As shown in Figure 1, the appropriate points $P_{S_{1i+}}$, $P_{S_{1i-}}$, $P_{S_{2i+}}$, and $P_{S_{2i-}}$ should be selected on terminal switching surfaces S_{1i} , S_{2i} , where points $P_{S_{1i+}}$, $P_{S_{2i-}}$ are located in the fourth quadrant and points $P_{S_{1i-}}$, $P_{S_{2i+}}$ are located in the second quadrant. The lines $P_{S_{1i-}}P_{S_{2i-}}$, $P_{S_{1i+}}P_{S_{2i-}}$, $P_{S_{1i-}}P_{S_{2i+}}$, and $P_{S_{1i+}}P_{S_{2i+}}$ defined in Figure 2 are UAS H_{0i} , H_{1i} , H_{2i} , and H_{3i} , respectively. For the simplicity of discussion, we define that the UAS H_{ki} , $k \in \{0, 1, 2, 3\}$, is designed by points $P_{S_{1i}}$, $P_{S_{2i}}$ where $P_{S_{1i}} \in \{P_{S_{1i+}}, P_{S_{1i-}}\}$, $P_{S_{2i}} \in \{P_{S_{2i+}}, P_{S_{2i-}}\}$. The expression of UAS H_{ki} is given as follows:

$$H_{ki}(z) = \omega_{ki1} \cdot z_{2i} + \omega_{ki2} \cdot z_{1i} + M_i, \quad (9)$$

where $H_{ki}(P_{S_{1i}}) = 0$, $H_{ki}(P_{S_{2i}}) = 0$, $\omega_{ki1} \neq 0$, $M_i > 0$.

FIGURE 1: The UAS H_{0i} , H_{1i} , H_{2i} , and H_{3i} .

FIGURE 2: The point p in number k_i subspace.

Remark 3. These UAS H_{0i} , H_{1i} , H_{2i} , H_{3i} should constitute a convex quadrilateral which is given by

$$Q_i = \{(z_{2i}, z_{1i}) \mid H_{ki}(z) \geq 0, k = 0, 1, 2, 3\}. \quad (10)$$

Hence, the system states e_i in set Q_i imply that the system states satisfy the state constraints. This convex quadrilateral can be proved to be a positively invariant set [30] with control equation (9).

Step 3. The UAS in Figure 1 would be used to design control input π_1 when the states are moving in number $0i, \dots, 3i$ subspace. The compact form of current UAS H_i is given by

$$H_i(z) = \Omega_{i1} \cdot z_{2i} + \Omega_{i2} \cdot z_{1i} + M_i, \quad (11)$$

where

$$\begin{aligned} \Omega_{i1} &= \begin{cases} \omega_{0i1} & S_{1i} < 0, S_{2i} < 0 \\ \omega_{1i1} & S_{1i} < 0, S_{2i} \geq 0 \\ \omega_{2i1} & S_{1i} \geq 0, S_{2i} < 0 \\ \omega_{3i1} & S_{1i} \geq 0, S_{2i} \geq 0, \end{cases} \\ \Omega_{i2} &= \begin{cases} \omega_{0i2} & S_{1i} < 0, S_{2i} < 0 \\ \omega_{1i2} & S_{1i} < 0, S_{2i} \geq 0 \\ \omega_{2i2} & S_{1i} \geq 0, S_{2i} < 0 \\ \omega_{3i2} & S_{1i} \geq 0, S_{2i} \geq 0. \end{cases} \end{aligned} \quad (12)$$

Step 4. The UAS-TSMC control law for system (5) is designed as

$$\pi_1 = g_1(x_1)^{-1} \left(-f_1(x_1) - \Omega_{i1}^{-1} \cdot \Omega_{i2} \cdot e_1 - \xi_{3i} \Omega_{i1}^{-1} \cdot \Omega_{i2} \cdot e_1^{n/m} + \dot{y}_d - g_1(x_1) \cdot e_2 + \Omega_{i1}^{-1} \cdot N \right), \quad (13)$$

where $N > \sup\{-\Omega_{i1} \cdot \psi\}$ is the approaching law.

Remark 4. π_1 is a nonsingular sliding mode surface, which does not produce singularities. But it may cause singularity when we compute the time derivative of π_1 . Therefore, in the design of the controller, we add the saturation function in order to overcome the singularity problem [31].

Definition 5 (see [30]). The set Q_i is said to be positively invariant for system (5) with control law in (13). If for all $e_1(t) \in Q_i$ the solution $e_1(t) \in Q_i$ for $t > 0$. If $e_1(0) \in Q_i$ implies $e_1(t) \in Q_i$ for all $t \in R$, we say that Q_i is invariant.

Remark 6. As shown in Figure 2, the set $Q_i = \{(z_{2i}, z_{1i}) \mid H_{ki}(z) \geq 0, k = 0, 1, 2, 3\}$ is a convex set. For the point $p(t) = [z_{2i}(t), z_{1i}(t)]^T \notin Q_i$, if the point $p(t)$ locates in number k_i subspace where $k \in \{0, 1, 2, 3\}$, there exists $H_{ki}(p(t)) < 0$.

Lemma 7. For the point $p = [z_{1i}, z_{2i}]^T$ in number k_i subspace where $k \in \{0, 1, 2, 3\}$, the UAS $H_{ki}(z)$ in No. k_i subspace is designed by points p_{S1i}, p_{S2i} where $H_{ki}(p_{S1i}) = 0$ and $H_{ki}(p_{S2i}) = 0$. The expression of UAS $H_{ki}(z)$ is written as

$$H_{ki}(z) = \omega_{ki1} \cdot z_{2i} + \omega_{ki2} \cdot z_{1i} + M_i, \quad (14)$$

where $M_i > 0$. Then, for point p , there exists the fact that $M_i - H_{ki}(p) \geq 0$ and $p = [0, 0]^T$ when $H_{ki}(p) = M_i$.

Proof. Define the coordinates of points p_{S1i} and p_{S2i} as follows:

$$\begin{aligned} p_{S1i} &= [z_{12}, z_{11}]^T, \\ p_{S2i} &= [z_{22}, z_{21}]^T. \end{aligned} \quad (15)$$

As shown in Figure 2, it is noted that, for the point $p = [z_{1i}, z_{2i}]^T$ in number k_i subspace, there exists $p = k_1 p_{S1i} + k_2 p_{S2i}$, $k_1, k_2 \geq 0$. Then, the coordinate of point p can be expressed as follows:

$$p = [k_1 \cdot z_{12} + k_2 \cdot z_{22}, k_1 \cdot z_{11} + k_2 \cdot z_{21}]^T. \quad (16)$$

In (14) with (16), one has

$$\begin{aligned} H_{ki}(p) &= \omega_{ki1} \cdot (k_1 \cdot z_{12} + k_2 \cdot z_{22}) + M_i + \omega_{ki2} \\ &\quad \cdot (k_1 \cdot z_{11} + k_2 \cdot z_{21}) \\ &= k_1 \cdot (\omega_{ki1} \cdot z_{12} + \omega_{ki2} \cdot z_{11} + M_i) \\ &\quad - (k_1 + k_2) \cdot M_i + k_2 \\ &\quad \cdot (\omega_{ki1} \cdot z_{22} + \omega_{ki2} \cdot z_{21} + M_i). \end{aligned} \quad (17)$$

From the condition $H_{ki}(p_{S1i}) = 0$, $H_{ki}(p_{S2i}) = 0$, we have

$$\begin{aligned} H_{ki}(p_{S1i}) &= \omega_{ki1} \cdot z_{12} + \omega_{ki2} z_{11} + M_i = 0, \\ H_{ki}(p_{S2i}) &= \omega_{ki1} \cdot z_{22} + \omega_{ki2} z_{21} + M_i = 0. \end{aligned} \quad (18)$$

Substituting (18) into (17), we obtain

$$H_{ki}(p) = -(k_1 + k_2) M_i + M_i. \quad (19)$$

Hence, the first conclusion is as follows: $M_i - H_{ki}(p) = (k_1 + k_2) \cdot M_i \geq 0$.

When $H_{ki}(p) = M_i$, there exists $-(k_1 + k_2)M_i = 0$ from (19). Since $k_1, k_2 \geq 0$ and $-(k_1 + k_2)M_i = 0$, we can conclude $k_1 = 0, k_2 = 0$. Then, we can obtain the second conclusion as follows:

$$\begin{aligned} p &= [z_{2i}, z_{1i}]^T \\ &= [k_1 \cdot z_{12} + k_2 \cdot z_{22}, k_1 \cdot z_{11} + k_2 \cdot z_{21}]^T = [0, 0]^T. \end{aligned} \quad (20)$$

This completes the proof. $\square$

Theorem 8. Considering system (5) with the control law in (13), system (5) is asymptotically stable. And the set $Q_i = \{(z_{2i}, z_{1i}) \mid H_{ki}(z) \geq 0, k = 0, 1, 2, 3\}$ can be guaranteed as a positively invariant set [30].

Proof. See the Appendix. $\square$

3.2. Fast Loop Terminal Sliding Mode Disturbance Observer Design. In this section, a robust flight control approach will be developed based on the combination of nonlinear disturbance observer and terminal sliding mode control with unidirectional auxiliary surfaces. For control of external disturbances and model uncertainties in the fast loop, they cannot be directly used to design the robust sliding mode controller. To efficiently handle them, the nonlinear disturbance observer is proposed to estimate them.

In order to estimate the external disturbance ψ of the near space hypersonic vehicle system, the general nonlinear near space hypersonic vehicle fast loop dynamic error model in (4) will be used. Considering (4) and differentiating e_2 yielded

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{\pi}_1 \\ &= f_2(x_1, x_2) + g_2(x_1, x_2)u + \psi(t) - \dot{\pi}_1. \end{aligned} \quad (21)$$

The disturbance observer is proposed as follows [32]:

$$\begin{aligned} \hat{\psi} &= z + P(e_2), \\ \dot{z} &= -L(e_2)z \\ &\quad - L(e_2)(P(e_2) + f(x_1, x_2) + g(x_1, x_2)u), \end{aligned} \quad (22)$$

where z is an auxiliary variable of the proposed disturbance observer, $\hat{\psi}$ is the output of the nonlinear disturbance

observer, $P(e_2) = [P_1(e_2), P_2(e_2), \dots, P_n(e_2)] \in R^n$, and $L(e_2)$ is an appropriate dimension gain matrix which is given by

$$L(e_2) = \frac{\partial P(e_2)}{\partial e_2} = \begin{bmatrix} \frac{\partial P_1(e_2)}{\partial e_{21}} & \frac{\partial P_1(e_2)}{\partial e_{22}} & \dots & \frac{\partial P_1(e_2)}{\partial e_{2n}} \\ \frac{\partial P_2(e_2)}{\partial e_{21}} & \frac{\partial P_2(e_2)}{\partial e_{22}} & \dots & \frac{\partial P_2(e_2)}{\partial e_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n(e_2)}{\partial e_{21}} & \frac{\partial P_n(e_2)}{\partial e_{22}} & \dots & \frac{\partial P_n(e_2)}{\partial e_{2n}} \end{bmatrix}. \quad (23)$$

Define the estimate error $e = \psi - \hat{\psi}$. Since there is no prior information about the derivative of the disturbance ψ , it is reasonable to suppose that $\dot{\psi} = 0$, which implies that the disturbance varies slowly relative to the observer dynamics.

Differentiating e , we have

$$\dot{e} = \dot{\psi} - \dot{\hat{\psi}} \approx -\dot{\hat{\psi}}. \quad (24)$$

Considering (18) and differentiating with respect to time yielded

$$\begin{aligned} \dot{\hat{\psi}} &= \dot{z} + L(e_2) \dot{x}_2 \\ &= -L(e_2) z \\ &\quad - L(e_2) (P(e_2) + f(x_1, x_2) + g(x_1, x_2) u) \\ &\quad + L(e_2) \dot{x}_2 = L(e_2) (\psi - \hat{\psi}) = L(e_2) e. \end{aligned} \quad (25)$$

Thus, we have $\dot{e} = -L(e_2)e$. It is apparent that the choice of $L(e_2)$ must make the disturbance observer error converge to zero.

According to the design of terminal sliding mode with unidirectional auxiliary surfaces, the robust adaptive control of the hypersonic vehicle based on estimated disturbance observation can be surmised in the following theorem.

Theorem 9. For NHV dynamic error model (4), the disturbance observer is designed as (22), and there exists a terminal sliding mode with unidirectional auxiliary surfaces (26), under the two terminal switching surfaces (28), (29), the adaptive law (27) and the NDO-based robust flight control law (30). Then the robust stability of the control system in the presence of the parameter uncertainties and external disturbances is guaranteed:

$$H = \Omega_1 e_2 + \Omega_2 \left(\int e_2 + \int e_2^{n/m} \right) + M, \quad (26)$$

$$\dot{\hat{\eta}} = \lambda \delta, \quad (27)$$

$$S_1 = e_2 + \xi_1 \left(\int e_2 + \int e_2^{n/m} \right), \quad (28)$$

$$S_2 = e_2 + \xi_2 \left(\int e_2 + \int e_2^{n/m} \right), \quad (29)$$

$$\begin{aligned} u &= g_2(x_1, x_2)^{-1} (-f_2(x_1, x_2)) \\ &\quad + \Omega_1^{-1} (-\Omega_2 \cdot e_2^{n/m} + \lambda \delta + \hat{\eta} + \dot{\pi}_1), \end{aligned} \quad (30)$$

$$\delta = \Omega_1 e_2 + \Omega_2 \left(\int e_2 + \int e_2^{n/m} \right), \quad (31)$$

where, $-\Omega_1 \psi \leq \eta$, $\eta > 0$, $M > 0$, $\lambda > 0$ are designed constants and m and n are positive odd integers and satisfy $1 < m/n < 2$.

Proof. Consider the following Lyapunov function:

$$V = \frac{1}{2} \delta^T \delta + \frac{1}{2\lambda} \tilde{\eta}^T \tilde{\eta}. \quad (32)$$

Differentiating V with respect to time yields

$$\begin{aligned} \dot{V} &= -\delta^T \dot{\delta} - \frac{1}{\lambda} \tilde{\eta}^T \dot{\tilde{\eta}} = -\delta^T (\Omega_1 \dot{e}_2 + \Omega_2 \dot{e}_2 + \Omega_2 e_2^{n/m}) \\ &\quad - \frac{1}{\lambda} \tilde{\eta}^T \dot{\tilde{\eta}} = -\delta^T (\Omega_1 \dot{e}_2 + \Omega_2 \dot{e}_2 + \Omega_2 e_2^{n/m}) - \frac{1}{\lambda} \tilde{\eta}^T \dot{\tilde{\eta}} \\ &= -\delta^T [\Omega_1 (\Omega_1^{-1} (-\Omega_2 e_2 - \Omega_2 e_2^{n/m} + \lambda \delta + \hat{\eta} \\ &\quad + \dot{\pi}_1) + \psi - \dot{\pi}_1) + \Omega_2 e_2 + \Omega_2 e_2^{n/m}] - \tilde{\eta}^T \delta \\ &\leq -\delta^T \lambda \delta - \delta^T \hat{\eta} - \delta^T \Omega_1 \psi - \delta^T \tilde{\eta} = -\delta^T \lambda \delta - \delta^T (\hat{\eta} \\ &\quad + \Omega_1 \psi + \tilde{\eta}) \leq -\delta^T \lambda \delta - \delta^T (\tilde{\eta} - \eta + \tilde{\eta}) \\ &\leq \lambda_{\min} \|\delta\|^2 < 0. \end{aligned} \quad (33)$$

Thus, the proof is completed. $\square$

4. Simulation Study

To illustrate the effectiveness of the proposed robust control scheme, the simulations are given for the near space hypersonic vehicles. The initial conditions are chosen as follows: $M = 13680$ kg, $V(0) = 3000$ m/s, $X(0) = 1000$ m, $Y(0) = 1000$ m, $Z(0) = -30$ km, $m = 9$, $n = 7$, the attitude angle $\alpha(0) = 0$ deg, $\beta(0) = 0$ deg, $\gamma(0) = 0$ deg, the attitude angle rate $p(0) = 0$ deg/s, $q(0) = 3$ deg/s, and $r(0) = 0$ deg/s. In order to guarantee the stability of the near space hypersonic vehicles attitude, the reference signals of the attitude angle are chosen as $\alpha_c = 4$ deg, $\beta_c = 0$ deg, and $\gamma_c = -2$ deg. The disturbances are considered as follows:

$$\begin{aligned} d_1(t) &= 200000 \sin(\pi t) \text{ Nm}, \\ d_2(t) &= 5000000 \sin(\pi t) \text{ Nm}, \\ d_3(t) &= 1000000 \sin(\pi t) \text{ Nm}. \end{aligned} \quad (34)$$

For the purpose of comparison for the effectiveness of TSMC between the proposed approach presented in this paper and an existing method, the corresponding simulation

FIGURE 3: Simulation results with TSMC.

results are depicted in Figures 3–5. From Figure 3, it is shown that control torques responses using TSMC method cannot meet system performance requirements. From Figure 4, under the designed UAS-TSMC attitude control scheme,

attitude angle rate and attitude angle become unstable quickly. The result shows that there exists seriously error during most of the simulation time. Furthermore, in Figure 5, the UAS-TSMC with NDO algorithm can make the system

FIGURE 4: Simulation results with UAS-TSMC.

stable and indicates that all of the responses of attitude angle and angular velocity can track the desired objectives well. Thus, the robust flight control approach based on the UAS-TSMC with NDO can satisfy precision and robustness requirements in the presence of uncertainties and large external disturbances.

5. Conclusion

In this paper, a novel robust sliding mode control with unidirectional auxiliary surface approach has been developed to solve the tracking control problem for the NHV with functional uncertainty and external disturbance. UAS-TSMC

FIGURE 5: Simulation results with UAS-TSMC-NDO.

is employed to increase the robustness of the control system and improves the control performance. Meanwhile, the nonlinear disturbance observer has been developed to estimate uncertainties and external disturbances. Finally, the proposed method has been applied to near space hypersonic

vehicles attitude dynamics, and simulation results have been given to illustrate the effectiveness of the proposed control approach. Studying attitude tracking control and accommodation approaches for NHVs with functional uncertainty and external disturbance will be investigated in our future work.

FIGURE 6: The point z between number k_i and j_i subspaces.

Appendix

Proof of Theorem 8. (1) The proof of the asymptotical stability for system (5) is given in this part.

For system (5), we consider the following Lyapunov function candidate:

$$V_i(z) = \frac{1}{2} \cdot \left[M_i - \frac{H_i(z)}{M_i} \right]^2. \quad (\text{A.1})$$

Firstly, it would be proved that $V_i \geq 0$ and there exists $z = [z_{2i}, z_{1i}]^T = [0, 0]^T$ when $V_i = 0$. From the definition of $V_i(\cdot)$, it follows that $V_i(\cdot) \geq 0$ and there exists $H_i(z) = M_i$ when $V_i = 0$. Based on the discussion given in Lemma 7, it is noted that $H_{ki}(z) = H_k(z)$ when z is located in number k_i subspace where $k \in \{0, 1, 2, 3\}$. Then, the condition $H_{ki}(z) = M_i$ implies $z = [z_{2i}, z_{1i}]^T$. Hence, there exists $z = [z_{2i}, z_{1i}]^T = [0, 0]^T$ when $V_i = 0$.

Secondly, the conclusion $\dot{V}_i < 0$ when $z \neq [0, 0]^T$ is given as follows.

It follows from (11) and (13) that the time derivative of $H_i(z)$ is given as

$$\begin{aligned} \dot{H}_i(z) &= \Omega_{i1} \cdot \dot{z}_{2i} + \Omega_{i2} \cdot \dot{z}_{1i} \\ &= \Omega_{i1} \cdot \dot{e}_{1i} + \Omega_{i2} \cdot (\dot{e}_{1i} + \xi_{3i} \cdot e_{1i}^{n/m}) = N. \end{aligned} \quad (\text{A.2})$$

Since $N = \dot{H}_i(z)$, the derivative of $H_i(z)$ should be a positive value.

From the Lyapunov function $V_i(\cdot)$ in (A.1), the derivative of $V_i(\cdot)$ is expressed as

$$\dot{V}_i = -(M_i - H_i(z)) \cdot \frac{\dot{H}_i(z)}{M_i^2}. \quad (\text{A.3})$$

Based on Lemma 7, the conclusion is obtained that $H_{ki}(z) = H_i(z)$ when z is located in number k_i subspace and $M_i - H_{ki}(z) \geq 0$. As $z = [0, 0]^T$ when $H_{ki}(z) = M_i$, it implies that $M_i - H_{ki}(z) > 0$ when $z \neq [0, 0]^T$. Then, there exists the fact that $M_i - H_{ki}(z) > 0$ when $z \neq [0, 0]^T$. From $M_i - H_{ki}(z) > 0$ and the positive derivative of $H_{ki}(z)$, the derivative of $V_i(\cdot)$ should be a negative value when $z \neq [0, 0]^T$.

Thirdly, the proof for the continuity of Lyapunov function $V_i(\cdot)$ in (A.1) is given in this part.

As shown in Figure 6, the point z on switching surface is selected to show the continuity of Lyapunov function $V_i(\cdot)$ in

(A.1) when the point z is switched between number k_i and j_i , $k, j \in \{0, 1, 2, 3\}$, $k_i \neq j_i$ subspace.

The point p_{Sli} is one of the points which is used to design the UAS H_{ki} and H_{ji} . Define the coordinates of point p_{Sli} as $p_{Sli} = [z_{12}, z_{11}]^T$. From (18), the expressions of UAS H_{ki} and H_{ji} can be given as follows:

$$\begin{aligned} H_{ki}(z) &= \omega_{ki1} \cdot z_{2i} + \omega_{ki2} \cdot z_{1i} + M_i, \\ H_{ji}(z) &= \omega_{ji1} \cdot z_{2i} + \omega_{ji2} \cdot z_{1i} + M_i. \end{aligned} \quad (\text{A.4})$$

Since the point p_{Sli} is located on the UAS H_{ki} and H_{ji} , (A.5) can be written as

$$\begin{aligned} H_{ki}(p_{Sli}) &= \omega_{ki1} \cdot z_{12} + \omega_{ki2} \cdot z_{11} + M_i = 0, \\ H_{ji}(p_{Sli}) &= \omega_{ji1} \cdot z_{12} + \omega_{ji2} \cdot z_{11} + M_i = 0. \end{aligned} \quad (\text{A.5})$$

For the point z in number k_i subspace, there exists $z = k_1 \cdot p_{Sli} = [k_1 \cdot z_{12}, k_1 \cdot z_{11}]^T$, $k_1 \geq 0$, as given in Figure 6. By substituting the coordinate of point z into the UAS H_{ki} , we have

$$\begin{aligned} H_{ki}(z) &= \omega_{ki1} \cdot k_1 \cdot z_{12} + \omega_{ki2} \cdot k_1 \cdot z_{11} + M_i \\ &= k_1 (\omega_{ki1} \cdot z_{12} + \omega_{ki2} \cdot z_{11} + M_i) - k_1 \cdot M_i + M_i. \end{aligned} \quad (\text{A.6})$$

Clearly, there exists $H_{ki}(z) = -k_1 \cdot M_i + M_i$ by considering (A.5) and (A.6).

Then, the Lyapunov function $V_i(\cdot)$ in number k_i subspace can be rewritten as follows:

$$\begin{aligned} V_{ki}(z) &= \frac{1}{2} \cdot \left[\frac{(M_i - H_{ki}(z))}{M_i} \right]^2 \\ &= \frac{1}{2} \cdot \left[\frac{(M_i - H_{ki}(z))}{M_i} \right]^2 = \frac{1}{2} \cdot (k_1)^2. \end{aligned} \quad (\text{A.7})$$

For the point z in number j_i subspace, there exists $z = k_1 \cdot p_{Sli} = [k_1 \cdot z_{12}, k_1 \cdot z_{11}]^T$, $k_1 \geq 0$, as given in Figure 6. By substituting the coordinate of point z into the UAS H_{ji} , it is obtained that

$$\begin{aligned} H_{ji}(z) &= \omega_{ji1} \cdot k_1 \cdot z_{12} + \omega_{ji2} \cdot k_1 \cdot z_{11} + M_i \\ &= k_1 (\omega_{ji1} \cdot z_{12} + \omega_{ji2} \cdot z_{11} + M_i) - k_1 \cdot M_i + M_i. \end{aligned} \quad (\text{A.8})$$

Clearly, there exists $H_{ji}(z) = -k_1 \cdot M_i + M_i$ by considering (A.5) and (A.8).

Then the Lyapunov function $V_i(\cdot)$ in number j_i subspace can be rewritten as follows:

$$\begin{aligned} V_{ji}(z) &= \frac{1}{2} \cdot \left[\frac{(M_i - H_{ji}(z))}{M_i} \right]^2 \\ &= \frac{1}{2} \cdot \left[\frac{(M_i - H_{ji}(z))}{M_i} \right]^2 = \frac{1}{2} \cdot (k_1)^2. \end{aligned} \quad (\text{A.9})$$

Hence, when the point $\mathbf{z}$ is switched between number k_i and j_i subspaces, there exists $V_{k_i}(\mathbf{z}) = V_{j_i}(\mathbf{z})$. Then, the continuity of Lyapunov function $V_i(\cdot)$ in (A.1) is proved.

From the above demonstration, it is noted that the Lyapunov function $V_i(\cdot)$ in (A.1) is continuous; $V_i \geq 0$ and there exists $\mathbf{z} = [z_{1i}, z_{2i}]^T = [0, 0]^T$ when $V_i = 0$; $\dot{V}_i < 0$ when $\mathbf{z} \neq [0, 0]^T$. Hence, system (5) with the control law in (13) is asymptotically stable and $\mathbf{z} = [z_{2i}, z_{1i}]^T$ converges to $\mathbf{z} = [0, 0]^T$. And the state $\mathbf{z} = [z_{2i}, z_{1i}]^T$ is defined as

$$\begin{aligned} z_{2i} &= e_{1i}, \\ z_{1i} &= \int e_{1i} + \xi_{3i} \int e_{1i}^{n/m}. \end{aligned} \quad (\text{A.10})$$

It is clear that $[e_{2i}, e_{1i}]^T = [0, 0]^T$ when $\mathbf{z} = [z_{2i}, z_{1i}]^T = [0, 0]^T$. Thus, we obtain that system (5) converges to $\mathbf{z} = [0, 0]^T$.

(2) Demonstration for the positively invariant set $Q_i = \{(z_{2i}, z_{1i}) \mid H_{k_i}(z) > 0, k_i = 0, 1, 2, 3\}$ is given in this part. This conclusion is supported with the reduction to absurdity.

Considering the initial point $\mathbf{z}(0) = [z_{2i}(0), z_{1i}(0)]^T \in Q_i$ in number k_i subspace, we suppose that there exists $\mathbf{z}(t_1) = [z_{2i}(t_1), z_{1i}(t_1)]^T \notin Q_i$ in number j_i subspace, where $t_1 > 0$. Since point $\mathbf{z}(0) \in Q_i = \{(z_{2i}, z_{1i}) \mid H_{k_i}(z) \geq 0, k_i = 0, 1, 2, 3\}$ in number k_i subspace and $M_i > 0$, it is noted that the Lyapunov function $V_i(\cdot)$ in (A.1) can be expressed as follows:

$$V_i(z(0)) = \frac{1}{2} \left[\frac{(M_i - H_{k_i}(z(0)))^2}{M_i} \right] \leq \frac{1}{2}. \quad (\text{A.11})$$

Since point $\mathbf{z}(t_1) = [z_{2i}(t_1), z_{1i}(t_1)]^T \notin Q_i$ in number j_i subspace, there exists $H_{j_i}(\mathbf{z}(t_1)) < 0$ from Remark 6. Clearly, the conclusion in (A.2) is obtained as

$$\begin{aligned} H_{j_i}(\mathbf{z}(t_1)) &< 0, \\ M_i &> 0, \\ V_i(z(t_1)) &= \frac{1}{2} \left[\frac{(M_i - H_{j_i}(z(t_1)))^2}{M_i} \right] \leq \frac{1}{2}. \end{aligned} \quad (\text{A.12})$$

It follows from (A.11) and (A.12) that the continuous Lyapunov function $V_i(\cdot)$ has the result $V_i(\mathbf{z}(0)) < V_i(\mathbf{z}(t_1))$, $0 < t_1$. Hence, there should exist a point $\mathbf{z}(t_2)$, $0 < t_2 < t_1$, where $\dot{V}_i(\mathbf{z}(t_2)) > 0$. However, from the above discussion in Part (1), it is noted that $\dot{V}_i < 0$. Then, we find the contradiction since the derivative of Lyapunov function $V_i(\cdot)$ cannot be positive and negative at the same time on the point $\mathbf{z}(t_2)$. Hence, the assumption that there exists $\mathbf{z}(t_1) \notin Q_i$ is incorrect, and $\mathbf{z}(0) \in Q_i$ implies $\mathbf{z}(t) \in Q_i$ for all $t > 0$. From Definition 5, the set $Q_i = \{(z_{2i}, z_{1i}) \mid H_{k_i}(\mathbf{z}) \geq 0, k_i = 0, 1, 2, 3\}$ is a positively invariant set. This completes the proof. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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