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Research Article

Modelling of Cellular Networks with Traffic Overflow

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The paper proposes a new method for modelling multiservice cellular networks with traffic overflow. The proposed method employs a model of Erlang's Ideal Grading (EIG) with multiservice traffic and differentiated availability. The fundamental advantage of the proposed method, as compared to other relevant methods, is a major simplification in modelling systems with traffic overflow that results from the elimination of the necessity of a determination of the parameters of overflow traffic, that is, the average value and the variance. According to the proposed method, calculations in the overflow system can be reduced to calculations in a system composed of one grading only. The paper presents the method for determining availability in such a grading that models a system with traffic overflow. The results of analytical calculations were compared with the results of simulation experiments. The results of the research study confirm high accuracy of the proposed method.

1. Introduction

New technologies used in building mobile networks evolve very quickly. Not much long ago, UMTS networks (Universal Mobile Telecommunications System) [1, 2] and their extension HSPA (high speed packet access) [2, 3] were a novelty. Currently, more advanced 4G networks are being developed. 4G networks operate according to the LTE (long term evolution) standard [4] that enables data transmission to be enhanced in a way much exceeding hitherto existing possibilities of cellular networks. Mobility of users and the expanding available bitrates for data transmission make it possible for mobile networks to become universal and efficient multiservice access networks. The above is particularly apparent in highly urbanized areas where GSM networks (global system for mobile communications) 900/1800, UMTS, and LTE cooperate with one another within cellular networks of one operator.

In order to ensure optimum usage of resources of mobile networks that belong to a given operator, it is necessary to take advantage of various traffic management mechanisms [5]. One of such mechanisms is traffic overflow performed with the application of connection transfer procedures (HandOver (HO)) [6–8]. Connection handover ensures mobility of users on the one hand and makes it possible to utilize available network resources on the other hand. In mobile networks, each transfer of a connection (in the network of a given operator)—effected by lack of available resources in the cell in which a new call appears—can be treated as overflow [9]. In the existing cellular networks, the handover procedure can be executed between cells of the same system or between cells that belong to different systems (e.g., UMTS and GSM).

In 2G networks, the traffic overflow mechanism has been introduced to achieve optimum usage of GSM900 and GSM1800 network resources [6]. In the case when GSM900 and GSM1800 networks coexist in one area, handover of connections is used to control the admission process for new calls in such a way as to make resources of the GSM1800 network to be used first. Such a procedure results from the characteristics of both systems. A network operating in 900 MHz frequency range is characterized by large cell diameters (even up to 35 km), as well as high power of the transmitted signal. GSM1800 networks, in turn, have smaller cell diameters and lower power of the transmitted signal (as compared to GSM900). An application of a bigger number of GSM1800 cells would increase the involved costs in constructing the network and, at the same time, increase its capacity. From the point of view of optimal operation of the whole of the network, new calls should then be directed first to the GSM1800 network and then—in the case of

the lack of free resources—to the GSM900 network. In order to impose such a scenario for resource utilization—despite higher signal power in GSM900—each new call that appears in the GSM900 network is transferred to the GSM1800 network. It is only in the case when there are no free resources in the network that new calls remain in the GSM900 network (the HO mechanism will have no effect). Such an operation corresponds to traffic overflow in which traffic from many cells (or just one cell) in GSM1800 is transferred to one cell in GSM900.

The overflow mechanism is also used with the coexistence of 2G and 3G networks in a given area. In this case, calls are transferred from the 3G network to the 2G network based on the IRAT HO (Inter Radio Access Technology HandOver) connection transfer procedure [10]. Due to particular properties of the WCDMA (wideband code division multiple access) radio interface [10] used in 3G networks, the parameters deciding on the activation of the IRAT HO mechanism are signal power of a given RSCP (received signal code power) cell and the signal to noise ratio E_c/N_0. It is worthwhile to note that a handover of a connection from the 3G system to 2G can have influence upon a change in the parameters of some services; for example, data transmission speed in older generations of wireless systems can be lower. In turn, a transfer of a voice connection between 3G and 2G systems is not followed by a change in the quality parameters for this service. The traffic overflow mechanism has been also implemented in 4G systems in the form of the IRAT mechanism that manages handover connections between 3G and 2G networks [11].

The phenomenon of traffic overflow in telecommunications is well-known and has been studied since the mid-20th century. The mechanism was used for the first time in hierarchical networks in which resources (historically called link groups (defining network resources as the link group resulted from the structure of the first telecommunications networks in which the basic resource was the link, while the link corresponded to the physical pair of wires connecting, e.g., telephone exchanges. In the paper, the term "network resources" will be used to define resources of modern telecommunications networks, whereas, in the descriptions of analytical models of traffic overflow systems, the notion of the group, viewed as system resources expressed in allocation units, e.g., links, channels, basic bandwidth units [12], etc., is introduced due to historical reasons)) were divided into two groups: direct and alternative resources. New calls that appear in the network are offered first to direct resources (direct groups) and then, only in a situation when there are no free direct resources available, these calls are transferred to alternative resources (alternative groups). Since the nature of the traffic stream that overflows from direct resources and then is offered to alternative resources (called overflow traffic) is different than the nature of traffic streams offered to direct resources, the modelling process for such networks could not employ the models that were available at the time (e.g., Erlang model [12]). New methods for the analytical modelling of networks with single-service (single-rate) overflow traffic were proposed for the first time in the 1950s. The most important methods, summarized in Section 2.1, include equivalent random theory [13, 14] and Fredericks-Hayward method [15]. First methodologies for modelling of networks with multiservice (multirate) traffic, presented in Section 2.2, were proposed in [16–18].

All the methods for modelling traffic overflow systems that have been developed so far require first a determination of characteristics of overflow traffic, that is, the average value of overflow traffic intensity and the related variance. The present paper proposes a simpler analytical method for overflow systems calculation that is based on the Erlang's Ideal Grading model [19–22]. The method does not require calculations of the parameters of overflow traffic. To determine traffic characteristics of overflow systems it is sufficient to know the value of traffic offered to the system and the capacity of direct and alternative resources. The research studies carried out by the authors proved high accuracy of the above approach.

The paper is organized as follows. Section 2 describes the analytical network models with traffic overflow that are known from the literature of the subject. Section 3 proposes a new network model with traffic overflow. Section 4 presents a comparison of the results of the analytical calculations with the results of the simulations for different scenarios for traffic overflow in cellular networks. Section 5 sums up the paper.

2. Review of Research

2.1. Modelling of Single-Service Systems with Traffic Overflow. The first models of networks with traffic overflow were proposed in the 1950s. The literature of the subject includes a number of models of full-availability groups (full-availability group is the model of a resource with complete sharing policy [23, 24]) with overflow traffic, derived under the assumption that call streams and service streams of traffic offered to direct resources [13, 15, 25] have the exponential distribution. The best known methods include equivalent random theory method (ERT method) [13, 14] and Fredericks-Hayward method [15]. The basis for both methods is a determination of the parameters that describe the properties of overflow traffic, that is, the average value R of overflow traffic (the first moment of the probability distribution for the number of busy allocation units (as the allocation unit we consider, e.g., link/channel in circuit switching [23], bandwidth (bit rate) unit [24, 26, 27], or load factor (in wireless systems with soft capacity [12])) in the alternative resource with infinite capacity) and the second moment and the related variance σ^2. These parameters can be used for evaluation of "peakedness" of the overflow stream by the introduction of the peakedness coefficient Z that is equal to the ratio between the variance σ^2 and the average value of overflow traffic R:

$$Z = \frac{\sigma^2}{R}. \tag{1}$$

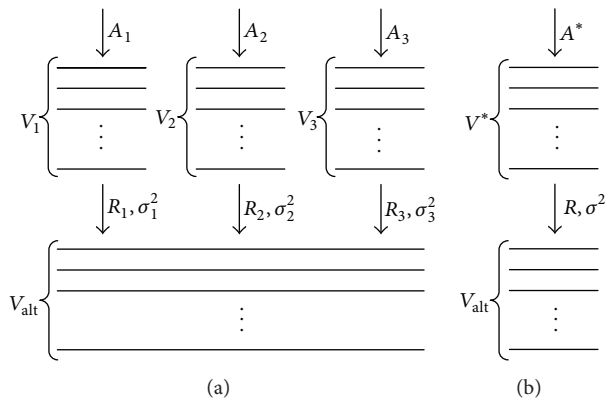

Figure 1: Graphic presentation of the ERT method: (a) overflow scheme for real network and (b) overflow scheme for equivalent system.

In both the ERT method and Fredericks-Hayward method, the parameters R and σ^2 are determined on the basis of Riordan formula [13]:

$$R = AE_V(A),$$
$$\sigma^2 = R\left[\frac{A}{V+1-A+R} + 1 - R\right], \tag{2}$$

where A is the traffic offered to the direct resource and V is the capacity of the direct resource.

The most common form of overflow in telecommunications networks is when call streams from a number of direct resources overflow to one alternative resource. If we assume that PCT1 (pure chance traffic of Type 1 (PCT1) [23], i.e., traffic generated by infinite number of sources, the arrival rate is constant and independent of the number of occupied sources (Poisson streams)) streams that are offered to these resources are statistically independent, then traffic streams that overflow from these resources will also be independent. If this is the case, then the parameters of the total overflow traffic can be determined:

$$R = \sum_{s=0}^{v} R_s, \qquad \sigma^2 = \sum_{s=0}^{v} \sigma_s^2, \tag{3}$$

where v is the number of direct resources, R_s is the average value of traffic that overflows from the resource s, and σ_s^2 is the variance of traffic that overflows from the resource s.

In the ERT method all direct resources are replaced by one equivalent resource with the capacity V^*, to which such equivalent traffic A^* is offered that the parameters of overflow traffic (R and σ^2) from such a resource are the same as those of overflow traffic from all direct resources. The parameters A^* and V^* of the equivalent resource can be determined on the basis of the known values R and σ^2 after providing the solution to the Riordan system of equations [13], written in the following form:

$$R = A^* E_{V^*}(A^*),$$
$$\sigma^2 = R\left[\frac{A^*}{V^*+1-A^*+R} + 1 - R\right]. \tag{4}$$

Equivalent traffic A^* requires $V^* + V_{\text{alt}}$ allocation units (Figure 1) for servicing calls with the assigned blocking probability E. The required capacity of an alternative resource can be finally determined on the basis of Erlang-B formula:

$$E = E_{(V^*+V_{\text{alt}})}(A^*). \tag{5}$$

It is worthwhile to mention at this point that in this method the so-called overflow scheme was used for the graphical representation of a network with traffic overflow. The overflow scheme shows in a simplified way and independently of the network technology the dependencies between direct and alternative resources. Such an exemplary overflow scheme, used in the ERT method, is presented in Figure 1.

The other method for modelling of single-service systems with traffic overflow, the so-called Fredericks-Hayward method [15], was proposed in the 1980s. The basic foundation for this method is a decomposition of the alternative resource with the capacity V_{alt}, to which overflow traffic with the parameters R and Z is offered, into Z identical component resources with the following parameters:

(i) capacity of a single component resource:

$$V_e = \frac{V_{\text{alt}}}{Z}, \tag{6}$$

(ii) traffic offered to a single component resource:

$$R_e = \frac{R}{Z}, \tag{7}$$

(iii) peakedness coefficient Z_e for traffic offered to each of the component resources:

$$Z_e = \frac{\sigma_e^2}{R_e} = \frac{(1/Z)^2\sigma^2}{R/Z} = 1. \tag{8}$$

The value of the coefficient Z_e equal to 1 means that traffic offered to each of the component resource is Erlang traffic, that is, PCT1 traffic. Therefore, the blocking probability for each of the resources formed as a result of decomposition, described by the parameters $V_e = V/Z$, $R_e = R/Z$, and

$Z_e = 1$, can be modelled on the basis of Erlang-B Formula [15]. Since this probability is the same in each of the resources formed as a result of the decomposition, it is then possible to adopt that it also describes the blocking probability in a resource after a decomposition with the parameters V_{alt}, R, and Z. Ultimately, we get

$$E\left(R, V_{\text{alt}}, Z\right) \approx E\left(\frac{R}{Z}, \frac{V_{\text{alt}}}{Z}, 1\right) \approx E_{V_{\text{alt}}/Z}\left(\frac{R}{Z}\right), \tag{9}$$

where E is the blocking probability determined on the basis of Erlang-B formula.

Equation (9) is a modified Erlang-B formula that takes into account the peakedness of the overflow call stream offered to the alternative resource. In traffic theory, the formula is called the Fredericks-Hayward formula. This approach makes use of the Erlang model only, which, in consequence, is followed by a significant simplification as compared to the ERT method.

Fredericks-Hayward method and the ERT method that both make it possible to determine traffic properties of systems that are characterized by the exponential service time and service calls generated according to the Poisson distribution triggered further work on single-service systems with overflow traffic. The author of [28] attempts to approach the problem of modelling resources with overflow traffic with the assumption of hyperexponential service time, whereas single-service systems with overflow traffic generated by a finite number of traffic sources (pure chance traffic of type 2 (PCT2) (pure chance traffic of type 2 (PCT2); the traffic generated by a finite number of sources. Each idle source has its own arrival rate and the total arrival rate is proportional to the number of simultaneous idle sources) traffic [23]) are considered in, for example, [29–31].

2.2. Modelling of Multiservice Systems with Traffic Overflow. First studies in modelling systems with multiservice traffic began in the 1980s [32–35]. Over time, the problem of a determination of the occupancy distribution and the blocking probability in the full-availability group with multiservice traffic with the known value of the parameter Z [36–38] was handled. One of the first methods enabling modelling systems with multiservice traffic overflow was the methods that are based on MMPP processes (*Markov-modulated Poisson process*), and proposed in [39, 40], among others. The accuracy of these methods is accompanied, however, with high computational complexity of the process of the determination of variances for overflow traffic streams. The first effective methodology for modelling multiservice systems with traffic overflow that made it possible to both analyse and dimension such systems was proposed in [17, 41]. In the works [17, 41] the traffic overflow scheme presented in Figure 2 was considered. In the considered network with traffic overflow, each of the direct resources s ($s = 1, \ldots, v$) and the alternative resource were offered a call from the set $\mathbf{M} = \{1, 2, \ldots, m\}$ of traffic classes. Each direct resource is offered a maximum of m traffic classes of PCT1 type. Calls of class c demand t_c allocation units to set up a connection. The intensity of PCT1 traffic of class c offered to the resource s is equal to $A_{c,s}$.

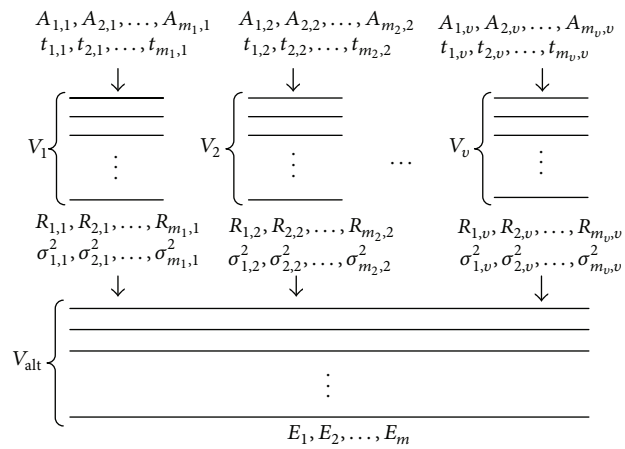

Figure 2: Overflow scheme for multiservice overflow traffic networks.

In the case when a given resource did not service calls of a given traffic class, it was assumed that the traffic intensity for this stream was equal to zero. Traffic streams of individual classes, not serviced in direct resources, overflow to an alternative resource. The blocking probability $E_{c,s}$ for calls of class c in the direct resource s can be determined on the basis of the Kaufman-Roberts recursion [34, 35]:

$$\begin{aligned} n\left[P_n\right]_{V_s} &= \sum_{c=1}^{m} A_{c,s} t_c \left[P_{n-t_c}\right]_{V_s}, \\ E_{c,s} &= \sum_{n=V_s-t_c+1}^{V_s} \left[P_n\right]_{V_s}, \end{aligned} \tag{10}$$

where $[P_n]_{V_s}$ is the occupancy probability of n allocation units in a resource with the capacity V_s allocation units and $[P_n]_{V_s} = 0$ for $n < 0$.

The variance of each of call streams is determined approximatively by carrying out a decomposition of each real resource s into m_s fictitious component resources with the capacities $V_{c,s}$ (m_s is the number of traffic classes offered to the resource s). The capacity of the fictitious component resource $V_{c,s}$ is defined, after [17], as this part of the real resource V_s which is not occupied by calls of the remaining classes:

$$V_{c,s} = V_s - \sum_{l=1; l\neq c}^{m_s} Y_{l,s} t_l, \tag{11}$$

where $Y_{c,s}$ determines the average number of calls of class c that are serviced in the resource s:

$$Y_{c,s} = A_{c,s}\left(1 - E_{c,s}\right). \tag{12}$$

Each fictitious resource will service calls of one class exclusively, which will make it possible to apply Riordan formulas in order to determine the following:

(i) the average value of the intensity of traffic of class c that overflows from the resource s:

$$R_{c,s} = A_{c,s} - Y_{c,s}, \tag{13}$$

(ii) the variance $\sigma_{c,s}^2$ for particular call streams that overflow to the alternative resource:

$$\sigma_{c,s}^2 = R_{c,s}\left[\frac{A_{c,s}}{V_{c,s}/t_c + 1 - A_{c,s} + R_{c,s}} + 1 - R_{c,s}\right], \quad (14)$$

where the quotient $V_{c,s}/t_c$ normalizes the system to a single-service case,

(iii) the peakedness coefficient for class c overflow traffic stream:

$$Z_c = \frac{\sigma_c^2}{R_c}, \quad (15)$$

where the parameters of the total traffic of class c offered to the alternative resource are calculated as follows:

$$R_c = \sum_{s=1}^{v} R_{c,s}, \qquad \sigma_c^2 = \sum_{s=1}^{v} \sigma_{c,s}^2. \quad (16)$$

An alternative method for a determination of the capacity of fictitious groups is proposed in [18, 42]. In this method, the capacity of the fictitious group $V_{c,s}$ is determined as such a capacity that leads to obtaining the blocking value $E_{c,s}$, on the assumption of offered traffic $A_{c,s}$ and given calls of class c being equal to 1 BBU:

$$E_{(V_{c,s},A_{c,s})} = E_{c,s}. \quad (17)$$

To determine the blocking probability of calls in the resource servicing multiservice overflow traffic, the Hayward method described in Section 2.1 was generalized in [43]. In the case of the multiservice systems, appropriate values of peakedness coefficients are introduced [17] into the Kaufman-Roberts model [34, 35] that originally described multiservice systems without traffic overflow (i.e., systems with $Z_c = 1$ for all c):

$$\begin{aligned} n\left[P_n\right]_{V_{\text{alt}}/Z} &= \sum_{c=1}^{m} \frac{R_c}{Z_c} t_c \left[P_{n-t_c}\right]_{V_{\text{alt}}/Z}, \\ E_{c,\text{alt}} &= \sum_{n=V_{\text{alt}}/Z-t_c+1}^{V_{\text{alt}}/Z} \left[P_n\right]_{V_{\text{alt}}/Z}, \end{aligned} \quad (18)$$

where Z is the so-called collective peakedness coefficient. In [17], the authors propose to define the coefficient Z as the weighted mean of the coefficients Z_c for individual call streams [17]:

$$Z = \sum_{c=1}^{m} Z_c k_c, \quad \text{where: } k_c = \frac{R_c t_c}{\sum_{l=1}^{m} R_l t_l}. \quad (19)$$

Formula (19) adopts that the contribution of the peakedness coefficient for the stream of class c in the collective peakedness coefficient Z is directly proportional to the value of traffic offered to the alternative resource by the class c.

Another general approach to the analysis of multiservice, hierarchical networks with overflow traffic were proposed in [16]. It is based on the process in which the overflow traffic was approximated by ON/OFF and MMPP traffic. This method is characterized by high computational complexity which greatly hinders its practical application.

3. New Approach to Modelling of Overflow Systems

All the methods presented in earlier sections are based in their operation on the full-availability group model. It is necessary in these models to determine values of additional parameters (R_c, σ_c^2) describing traffic that overflows to the alternative resource. In this section we propose a method for a determination of the blocking probability in systems with multiservice overflow traffic that is based on the Erlang's Ideal Grading (EIG) model. The principal advantage of the proposed method is that it is not necessary to determine the mean value and the variance of overflow traffic. To determine traffic characteristics of overflow systems it is sufficient to know the value of traffic offered to the system, the capacity of direct and alternative resources, and the value of the so-called availability parameter.

The proposed method relates to the remark given by the authors of the book on the life and lifetime achievements of Agner Krarup Erlang [44], who noticed that a simple EIG approximates other nonfull-availability groups (gradings) very well, including those that correspond to the simplest symmetrical overflow schemes (overflow scheme symmetry is understood here as the identical structure of direct resources, i.e., the same capacity and the same offered traffic). The remark of the authors of the book [44] passed unnoticed and has never been addressed by scientific community that dealt with research pertaining to traffic overflow. The reason for the above is the asymmetry of overflow systems, that is, different capacity of direct resources and, in consequence, different availability of resources of an overflow scheme for individual call streams offered to direct resources. It was only the construction of the generalized EIG model [45] with different values of offered traffic and different availabilities for different call classes that made the problem of approximation of asymmetrical overflow schemes by such groups very interesting. Reference [46] attempts for the first time to apply a model of EIG for the analysis of simplified multiservice overflow system in which only one call class is offered to given direct resources. The study showed that the use of the Erlang's Ideal Grading to calculate the blocking probability in networks with traffic overflow provided the accuracy of calculations comparable to others available but more complex methods described in the literature of the subject. Further on in Section 3 a generalized model of multiservice networks with traffic overflow will be proposed in which direct resources can service a number of traffic classes. To enhance the presentation of the basic assumptions of the proposed method for modelling systems with multiservice overflow traffic, the basic assumption of Erlang's Ideal Grading will be outlined in Section 3.1.

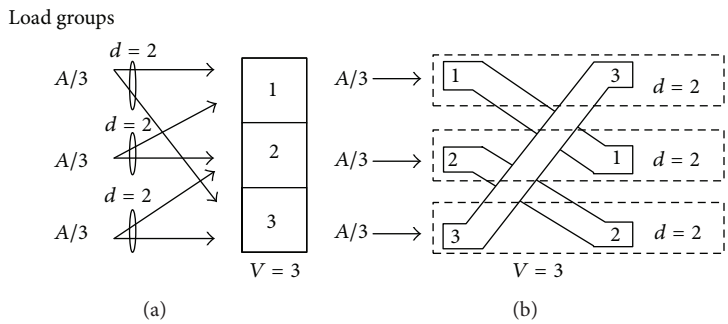

Figure 3: Erlang's Ideal Grading (a) offered traffic distribution and (b) concept of availability [22].

3.1. Erlang's Ideal Grading. Erlang's Ideal Grading (EIG) for single-service traffic was defined by A. K. Erlang [19, 47], who introduced a formula for a determination of the occupancy distribution in the grading in question. EIG, schematically presented in Figure 3(a), is described by three parameters: A, V, and d, where A is the average traffic offered to the grading, V is the grading capacity, and d is the so-called availability. Availability determines resources that are available for a given group of inputs, the so-called load group. The number of load groups g in EIG is equal to the number of possible ways of choosing the d allocation units from their general number V:

$$g = \binom{V}{d}. \tag{20}$$

Further assumption is that traffic offered by calls is distributed uniformly to all load groups.

Figure 3 shows Erlang's Ideal Grading with the following parameters: $V = 3$, $d = 2$, and $g = 3$. Due to the uniform distribution of traffic with the intensity A to all load groups, traffic offered by each load group is $A/3$.

In 1993, in [20] a model for Erlang's Ideal Grading with multiservice traffic was proposed. Soon afterwards, in [21, 45], this model was expanded for the case in which each class can have a different value of the availability parameter and then for the case of noninteger values of the availability parameter. An exemplary model of the multiservice EIG with the capacity $V = 3$ with different values of the availability parameter ($d_1 = 2$, $t_1 = 1$, $g_1 = 3$, $d_2 = 3$, $t_2 = 3$, and $g_2 = 1$) is shown in Figure 4. The grading under scrutiny was offered streams of two traffic classes with the intensities A_1 and A_2, respectively. Traffic A_1 was distributed uniformly into $g_1 = 3$ load groups, whereas traffic A_2 was distributed into $g_2 = 1$ load group.

According to [45], the occupancy distribution in such a grading can be described by the following formula:

$$n\left[P_n\right]_V = \sum_{c=1}^{m} A_c t_c \sigma_c\left(n - t_c\right)\left[P_{n-t_c}\right]_V, \tag{21}$$

where A_c is the traffic intensity of class c traffic, t_c is the number of allocation units demanded by a call of class c, m is the number of traffic classes, and $\sigma_c(n)$ is the conditional probability of transition for a class c traffic stream in occupancy state n:

$$\sigma_c(n) = 1 - \sum_{x=d_c-t_c+1}^{k_c} P_{V,d_c}(n, x), \tag{22}$$

where $k_c = n$ if $(d_c - t_c + 1) \le n < d_c$, and $k_c = d_c$ if $n \ge d_c$.

Probability $P_{V,d_c}(n, x)$ is the probability of occupancy of x allocation units in a given load group, under the condition that in the whole of the grading n allocation units are occupied. The probability $P_{V,d_i}(n, x)$ is described by a hypergeometric distribution:

$$P_{V,d_c}(n, x) = \frac{\binom{d_c}{x}\binom{V-d_c}{n-x}}{\binom{V}{n}}, \tag{23}$$

where d_c is the availability for calls of class c expressed in allocation units.

The blocking probability for calls of class c in the considered model is determined by the following formula:

$$E_c = \sum_{n=d_c-t_c+1}^{V} \left[1 - \sigma_c(n)\right]\left[P_n\right]_V. \tag{24}$$

3.2. New Network Model with Traffic Overflow. Let us consider now a new approach to modelling systems with multiservice traffic overflows. The presented multiservice traffic overflow scheme shown in Figure 5 will be used to demonstrate the operation of the method. The overflow system presented in Figure 5(a) is composed of v direct resources and an alternative resource.

The considered network with traffic overflow services calls from the set $\mathbf{M} = \{1, 2, \ldots, m\}$ of traffic classes. Each direct resource is offered the maximum of m traffic classes of PCT1 type. A call of class c appearing at the input of resource s demands t_c allocation units to set up a connection. The assumption is that calls of class c demand the same number of allocation units in each of the direct resources and in the alternative resource. The capacity of the alternative resource is equal to V_{alt} allocation units, whereas the capacity of the direct resource s is V_s allocation units.

The intensity of PCT1 traffic of class c offered to the resource s is equal to $A_{c,s}$. In the case when a given resource

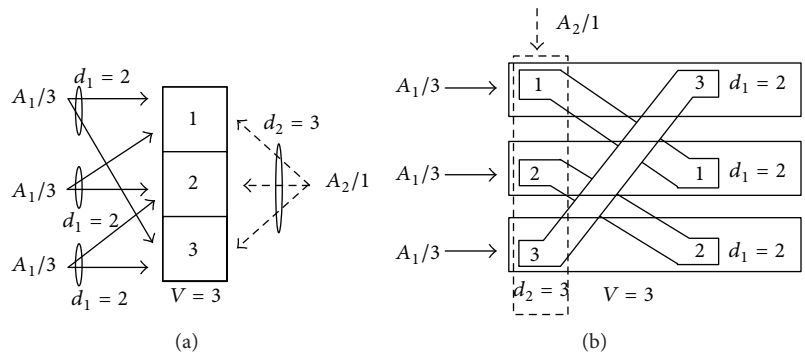

Figure 4: Erlang's Ideal Grading with different availability (a) offered traffic distribution, (b) concept of availability [22].

does not service calls of a given traffic class, it is adopted that the traffic intensity for this stream is equal to zero. Traffic streams of individual classes, not serviced in direct resources, overflow to an alternative resource. The alternative resource can also service traffic that is offered directly—not only overflow traffic from direct resources (Figure 5(a)). The intensity of PCT1 traffic of class c offered directly to the alternative resource is equal to $A_{c,\text{alt}}$.

In the proposed method, calculations of the blocking probability in the overflow system composed of many full-availability groups (both direct and alternative resources) can be reduced to calculations of this probability in a system that is composed of one group only: Erlang's Ideal Grading (Figure 5(b)). According to the proposed method, the blocking probability in the overflow system is determined from the following dependence:

$$\begin{aligned}
&\{[E_{1,1},\ldots,E_{c,1},\ldots,E_{m,1}],\ldots,[E_{1,s},\ldots,E_{c,s},\ldots,E_{m,s}],\\
&\quad\ldots,[E_{1,\text{alt}},\ldots,E_{c,\text{alt}},\ldots,E_{m,\text{alt}}]\}\\
&\quad= f\{[(A_{1,1},t_1,d_{1,1}),\ldots,(A_{c,1},t_c,d_{c,1}),\ldots,\\
&\qquad(A_{m,1},t_m,d_{m,1})],\ldots,\\
&\qquad[(A_{1,s},t_1,d_{1,s}),\ldots,\\
&\qquad(A_{c,s},t_c,d_{c,s}),\ldots,(A_{m,s},t_m,d_{m,s})],\ldots,\\
&\qquad[(A_{1,v},t_1,d_{1,v}),\ldots,(A_{c,v},t_c,d_{c,v}),\ldots,\\
&\qquad(A_{m,v},t_m,d_{m,v})],\\
&\qquad[(A_{1,\text{alt}},t_1,d_{1,\text{alt}}),\ldots,(A_{c,\text{alt}},t_c,d_{c,\text{alt}}),\ldots,\\
&\qquad(A_{m,\text{alt}},t_m,d_{m,\text{alt}})],V\},
\end{aligned} \tag{25}$$

where V is the total capacity of Erlang's Ideal Grading approximating the overflow system, $d_{c,s}$ is the availability for calls of class c, offered to direct resource s, in EIG approximating the overflow system, $d_{c,\text{alt}}$ is the availability for calls of class c, offered directly to the alternative resource, in EIG approximating the overflow system, $A_{c,s}$ is the traffic intensity offered to direct resource s by calls of class c, $A_{c,\text{alt}}$ is the traffic intensity offered directly to the alternative resource by calls of class c, t_c is the number of allocation units demanded by calls of class c, and f is the function that defines the dependence between the values of blocking of particular traffic classes offered to the network and the traffic intensity of offered streams and the availability.

At the first stage of defining the function f that makes a determination of the values for blocking in call streams offered to a network with traffic overflow possible, the occupancy distribution in the nonfull-availability group (grading) that approximates the network with traffic overflow is determined on the basis of the modified formula (21):

$$\begin{aligned}
n\left[P_n\right]_V &= \sum_{s=1}^{v}\sum_{c=1}^{m} A_{c,s}t_c\sigma_{c,s}(n-t_c)\left[P_{n-t_c}\right]_V\\
&\quad+\sum_{c=1}^{m} A_{c,\text{alt}}t_c\sigma_{c,\text{alt}}(n-t_c)\left[P_{n-t_c}\right]_V.
\end{aligned} \tag{26}$$

In (26), $\sigma_{c,s}(n)$ is the conditional probability of transition for a class c traffic stream, offered to the direct resource s, in occupancy state n:

$$\sigma_{c,s}(n) = 1 - \sum_{d_{c,s}-t_c+1}^{k_{c,s}} P_{V,d_c}(n,x), \tag{27}$$

where $k_{c,s} = n$ if $(d_{c,s} - t_c + 1) \le n < d_{c,s}$, and $k_{c,s} = d_{c,s}$ if $n \ge d_{c,s}$.

Probability $P_{V,d_{c,s}}(n,x)$ is the occupancy probability of x allocation units in resources available for calls of class c offered to the direct resource s, determined under the condition that there are n busy allocation units in the whole resources:

$$P_{V,d_{c,s}}(n,x) = \frac{\binom{d_{c,s}}{x}\binom{V-d_{c,s}}{n-x}}{\binom{V}{n}}, \tag{28}$$

where $d_{c,s}$ is the availability—expressed in allocation units – for calls of class c offered to direct resource s.

The value of the parameter $\sigma_{c,\text{alt}}$ in Formula (26) can be determined on the basis of Formulas (27) and (28), by replacing the index of direct resource "s" with the index of alternative resource "alt."

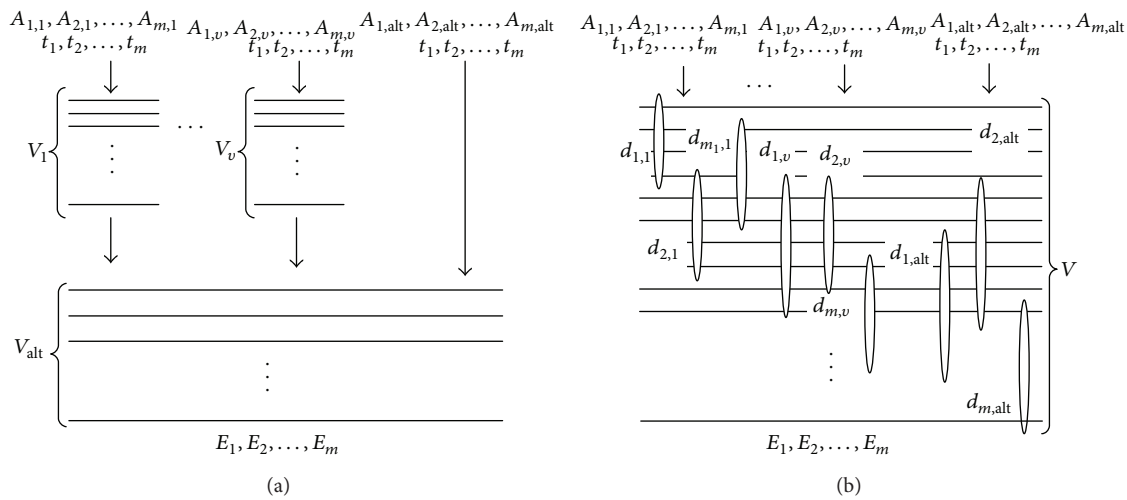

Figure 5: Presentation of the method for modelling systems with traffic overflow with the application of Erlang's Ideal Grading (a) overflow scheme for real network and (b) equivalent grading.

The proposed model assumes that the capacity of Erlang's Ideal Grading used for the calculations is determined as the sum of the capacities of the resources in the system under consideration, that is, the capacity of all direct resources and the alternative resource:

$$V = \sum_{s=1}^{v} V_s + V_{\text{alt}}. \tag{29}$$

Such an approach results in the approximating Erlang's Ideal Grading to be corresponding, in terms of its capacity, to the overflow system (considered group of cells) for which the calculations are to be made. Moreover, offered traffic to a single allocation unit is the same in both systems (real and its analytical model).

The values of the availability parameters for individual call classes are determined in the following way: a call of class c that appears at the input of the direct resource s can be serviced only when it can be serviced entirely by the direct resource s or, if the resource is busy, by the alternative resource. Hence, the value of the availability parameter will be the sum of availabilities in the direct resource s (at the input of which the considered call class appears) and in the alternative resource. In the case when the resource capacity is not an integer multiple of the number of allocation units demanded by a call of class c, it is assumed that the availability in the direct resource will be defined by the number of allocation units that corresponds to the maximum number of calls of class c that can be serviced in this resource, that is, $\lfloor V_s/t_c \rfloor t_c$. Availability in the alternative resource is determined in a similar way, that is, on the basis of the maximum number of allocation units that calls of class c can occupy in the alternative resource:

$$d_{c,\text{alt}} = \left\lfloor \frac{V_{\text{alt}}}{t_c} \right\rfloor t_c. \tag{30}$$

Taking into consideration the availability for class c calls (offered to the direct resource s) in the direct resource s and the alternative resource, the total availability will be determined on the basis of the following formula:

$$d_{c,s} = \left\lfloor \frac{V_s}{t_c} \right\rfloor t_c + \left\lfloor \frac{V_{\text{alt}}}{t_c} \right\rfloor t_c. \tag{31}$$

Having determined the capacity and the availability that approximates the overflow system by the EIG, we are in position to determine the occupancy distribution in Erlang's Ideal Grading on the basis of formulas (26) and (27) and then, the blocking probability for calls of class c:

$$E_{c,s} = \sum_{n=d_{c,s}-t_c+1}^{V} \left[1 - \sigma_{c,s}(n)\right] \left[P_n\right]_V. \tag{32}$$

The sequence of calculations in the proposed method can be written in the form of the Overflow-EIG method:

Overflow-EIG Method

(1) Define capacity V of the approximating Erlang's Ideal Grading (formula (29)).

(2) Determine availabilities $d_{c,s}$ for call classes that are offered to individual direct resources (formula (31)).

(3) Determine availabilities $d_{c,\text{alt}}$ for call classes that are offered directly to the alternative resource (formula (30)).

(4) Determine the occupancy distribution in Erlang's Ideal Grading (formula (26)).

(5) Determine the blocking probability in Erlang's Ideal Grading (formula (32)) that approximates the blocking probability of the overflow system.

Observe that the analytical model of the network with multiservice traffic overflow (each of direct resources services many traffic classes) proposed in the paper is simplified to

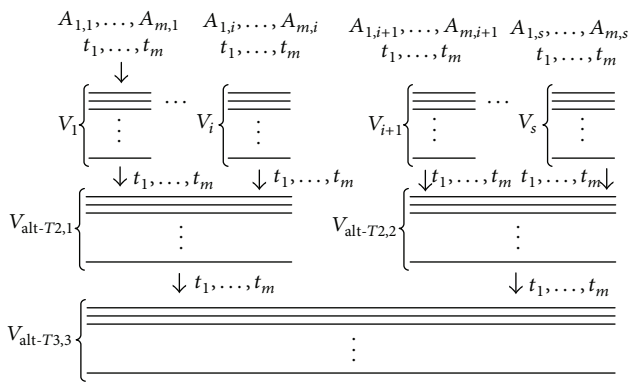

Figure 6: Scheme of three-tier overflow.

the model considered in [46] if each of the direct resources will be offered only one traffic stream.

The method for modelling networks with multiservice overflow traffic proposed in the paper is presented for the case of systems in which direct resources are offered Poisson call streams. It should be stressed, however, that the method can be easily adopted for modelling systems with traffic overflow in which direct resources are offered multiservice BPP (Bernoulli-Poisson-Pascal) call streams [23]. To achieve that, one can make use of the results of studies devoted to modelling (determination of occupancy distributions and the blocking probability) of systems with BPP traffic, such as [48–50].

The proposed method can be easily adopted for modelling of multitier overflow systems. An example of a scheme for traffic overflow in three-tier systems is shown in Figure 6. Traffic of class c that overflows from the direct resource V_s ($s = 1, 2, \ldots, v$) is directed to one of the two alternative resources of the second tier, that is, $V_{\text{alt-T2,1}}$ and $V_{\text{alt-T2,2}}$. If the resources do not currently have free allocation units, then overflow traffic is directed to an alternative resource in the third tier $V_{\text{alt-T3,3}}$.

In the considered case, the availability for traffic A_c includes all resources that can service this traffic, and formula (31) can be written in the following way:

$$d_{c,s} = \left\lfloor \frac{V_s}{t_c} \right\rfloor t_c + \left\lfloor \frac{V_{\text{alt-T2},x}}{t_c} \right\rfloor t_c + \left\lfloor \frac{V_{\text{alt-T3}}}{t_c} \right\rfloor t_c, \qquad (33)$$

where x is the number of allocation units of the second tier to which traffic from resource s overflows.

4. Numerical Results for Modelling Cellular Networks with Traffic Overflow

The presented method for modelling multiservice systems with overflow traffic is an approximate method. To evaluate the accuracy of the proposed solution, the results of the analytical calculations of the blocking probability in cellular networks with traffic overflow have been compared with simulation data. The study was conducted for many scenarios of traffic overflow that occur in currently used mobile 2G, 2.5G, 3G, and 4G networks. The studies on systems with traffic overflow, for the scenarios under consideration, were carried out at the call level. In the simulation model, each cell was represented as an object with a given capacity and offered traffic. Depending on the current load of cells to which calls generated by subscribers were directly offered, these calls were either serviced by the cells or directed to the alternative cell. The analysis of the system at the call level makes it possible to consider traffic overflow regardless of the technology employed to construct a mobile network. A detailed description of the methods for modelling mobile systems at the call level in wireless networks of different technologies is presented in, for example, [12]. The obtained results of the blocking probability for individual traffic classes are presented in Figures 8–16 in relation to the value of the average traffic offered to an allocation unit of the system:

$$a = \frac{\sum_{s=1}^{v} \sum_{c=1}^{m} A_{c,s} t_c}{V_{\text{alt}} + \sum_{s=1}^{v} V_s}. \qquad (34)$$

The diagram for the case of traffic overflow that was considered first is presented in Figure 7. The scheme relates to traffic overflow between cells of the GSM900 system. Figure 7(a) shows the idea of the handover of connections between the cells of the considered GSM900 system, whereas Figure 7(b) shows the overflow scheme that was used for analytical calculations according to the proposed method. In the considered system, the area serviced by the base station was divided into three sectors with the angle of the coverage area equal to 120 degrees. To enhance the effectiveness of the operation of the network, the base station managed yet another cell, the so-called omnidirectional cell that serviced the whole of the area. This cell admits for service only those calls that cannot be serviced in one of the three sectors. This means that the omnidirectional cell does not have its own traffic, that is, calls appearing directly in this cell.

The results for the blocking probability obtained for the considered case of traffic overflow (Figure 7) are shown in Figure 8. The assumption was that all cells (sectors) operated in the same technology (GSM900) and that they serviced only voice connections that demanded $t = 1$ allocation unit. The capacities of the individual sectors and the omnidirectional cell were identical and were equal to $V_1 = V_2 = V_3 = V_{\text{alt}} = 16$ allocation units. Another assumption was that traffic offered to individual sectors had the same value. Since the capacities of the sectors and the intensities of offered traffic were identical, the blocking probability for calls appearing in each cell is the same—Figure 8 shows then the results for calls offered to one, selected sector.

Figure 9 presents the results for the blocking probability obtained also for the scheme shown in Figure 7, and this time with the assumption that each cell services two classes of traffic: voice connections ($t_1 = 1$ allocation units) and data transmission ($t_2 = 2$ allocation units). Similarly as in the earlier case, the capacity of each sector and the omnidirectional cell was $V_1 = V_2 = V_3 = V_{\text{alt}} = 16$ allocation units. Since the capacities of the sectors are identical; therefore, availabilities for particular call classes in each of the sectors are the same. This means that the blocking probability $E_{1,1}$, $E_{1,2}$, and $E_{1,3}$ is identical, which

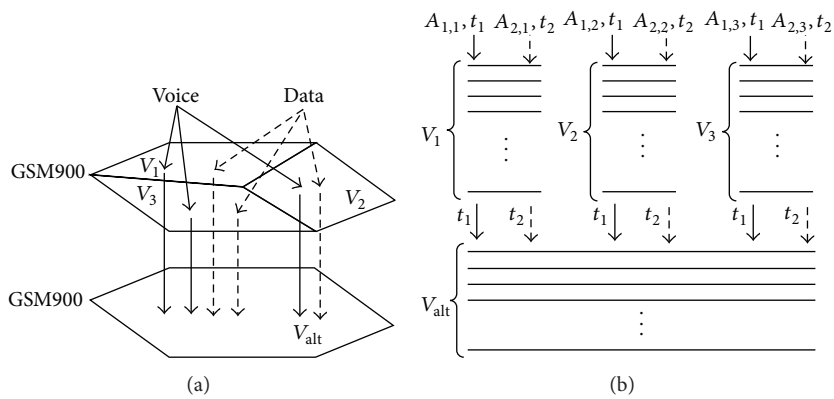

Figure 7: Traffic overflow between cell of the GSM900 system: (a) traffic overflow between cells and (b) overflow scheme ($A_{x,y}$ is traffic generated by calls of class x in cell y and t_x is the number of resources demanded by calls of class x).

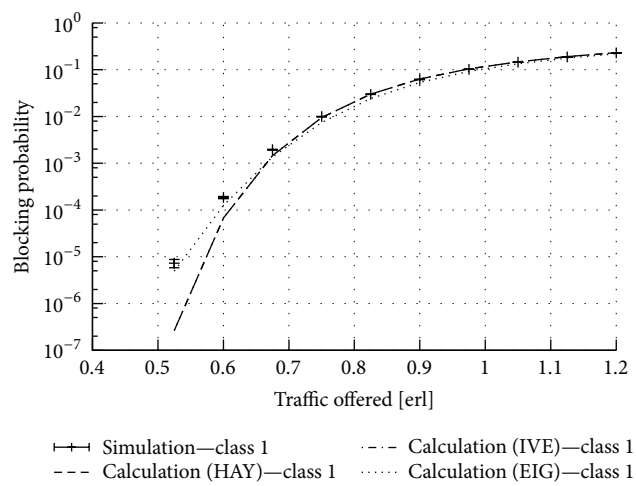

Figure 8: Blocking probability in mobile system GSM900 with overflow traffic, only voice traffic.

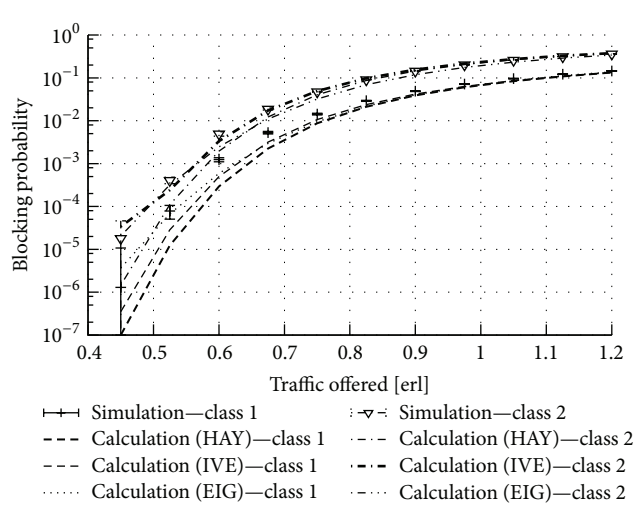

Figure 9: Blocking probability in mobile system GSM900 with overflow traffic, two traffic classes: voice and data.

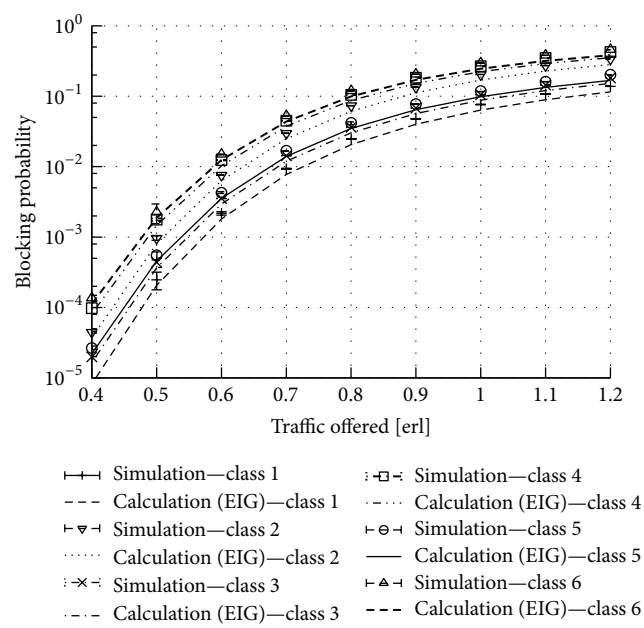

Figure 10: Blocking probability in mobile system GSM900 with overflow traffic, various capacities of direct resources.

is indicated in Figure 9 as the blocking probability of class 1. Similarly, traffic $A_{2,1}$, $A_{2,2}$, and $A_{2,3}$ is characterized by identical blocking probability, indicated in Figure 9 as the blocking probability for calls of class 2.

Figure 9 shows the results obtained during the simulation study and on the basis of the three analytical methods: the Overflow-EIG method proposed in the paper, generalized Hayward method (HAY) [17] and the method that takes into consideration a modification to the way the determination of the capacity of fictitious groups is executed (IVE) (17). It is noticeable that for each of the offered traffic classes the proposed Overflow-EIG (EIG) method is characterized by the greatest accuracy.

Figure 10 shows the results with the assumption that the capacities of GSM900 cells (direct resources) have different

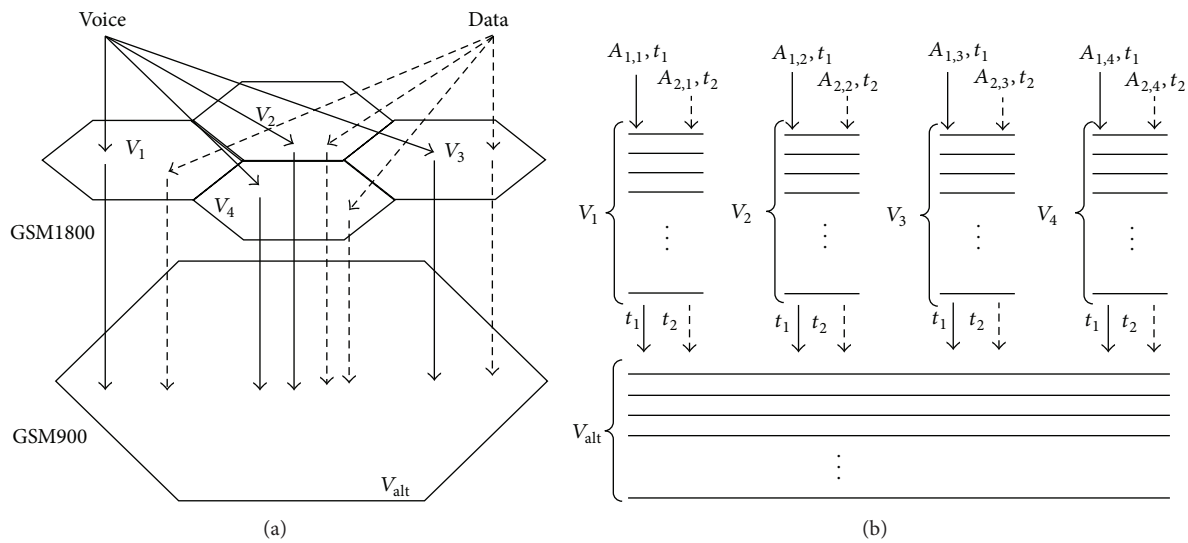

Figure 11: Overflow between cells of systems GSM1800 and GSM900: (a) traffic overflow between cells and (b) overflow scheme ($A_{x,y}$ is traffic generated by calls of class x in cell y and t_x is the number of demanded resources by calls of class x).

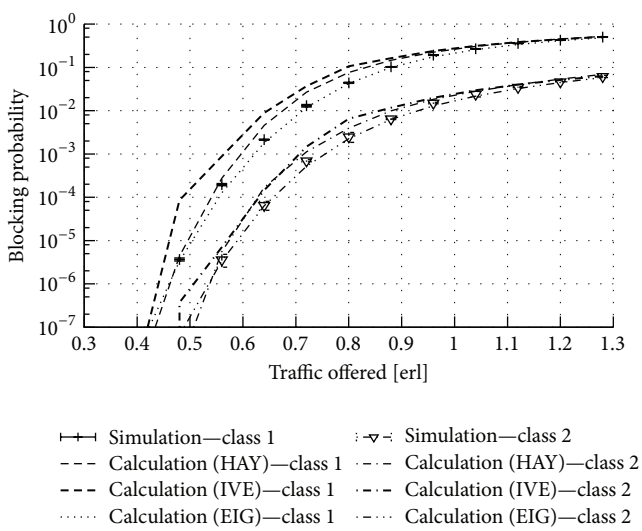

Figure 12: Blocking probability in mobile system with overflow traffic between cells of GSM1800 and GSM900 systems.

capacities that are, respectively, equal to $V_1 = 16$, $V_2 = 10$, and $V_3 = 8$ allocation units. The capacity of the alternative cell is equal to $V_{alt} = 16$ allocation units. The presented results show that the accuracy of the proposed method does not depend on the capacities of direct resources. In the case of a coexistence of the systems: GSM900 and GSM1800 within the network of one operator, there is still a possibility for connections to be transferred between cells. As the example presented in Figure 11 shows, the cell GSM900 (with higher power range than cells of GSM1800) is considered as the alternative resource to which calls from occupied cells of GSM1800 (operating in the area also covered by the GSM900 cell) are directed.

The results of the blocking probability for the considered network with traffic overflow between cells of the GSM1800 and GSM900 systems (according to the overflow scheme presented in Figure 11) are shown in Figure 12. The presented results were obtained for a network combined of 4 GSM1800 cells (direct resources) and one GSM900 cell (alternative resources). An additional assumption was that the GSM900 cell, servicing overflow traffic from 4 GSM1800 cells, had no traffic of its own. In the considered example, each cell serviced two classes of call: voice connections ($t_1 = 4$ allocation units) and data transmission ($t_2 = 1$ allocation unit). The capacity of each of the cells was 32 allocation units. The results obtained in the simulation study were compared with the data provided by the methods. Overflow-EIG generalized Hayward method (HAY) [17] and the method that includes a modification to the way of the determination of the capacity of fictitious groups (IVE) [42]. It is clearly observable that also in this case the proposed Overflow-EIG method is characterized by the greatest accuracy for each of the offered traffic classes. Consequently, in Figures 13–17 the results obtained on the basis of Overflow-EIG method are compared with simulation results only.

The next considered case was a network with traffic overflow between the systems: UMTS and GSM (Figure 13). These systems differ in terms of available packet transmission speed, but assuming that the GSM system is enhanced by the EDGE, mechanism, redirecting packet transmission to this system is possible (in the boundary case, only voice connections can be transferred). The assumption is that calls transferred to the GSM system are first directed to the GSM1800 system. If there are no free resources, calls are transferred to the GSM900 system. Figure 14 shows the results for the considered type of overflow, that is, between 4 cells of the UMTS system and one GSM1800 cell and one GSM900 cell. The assumption was that the alternative resource, that is, the GSM cells, did not service its own traffic.

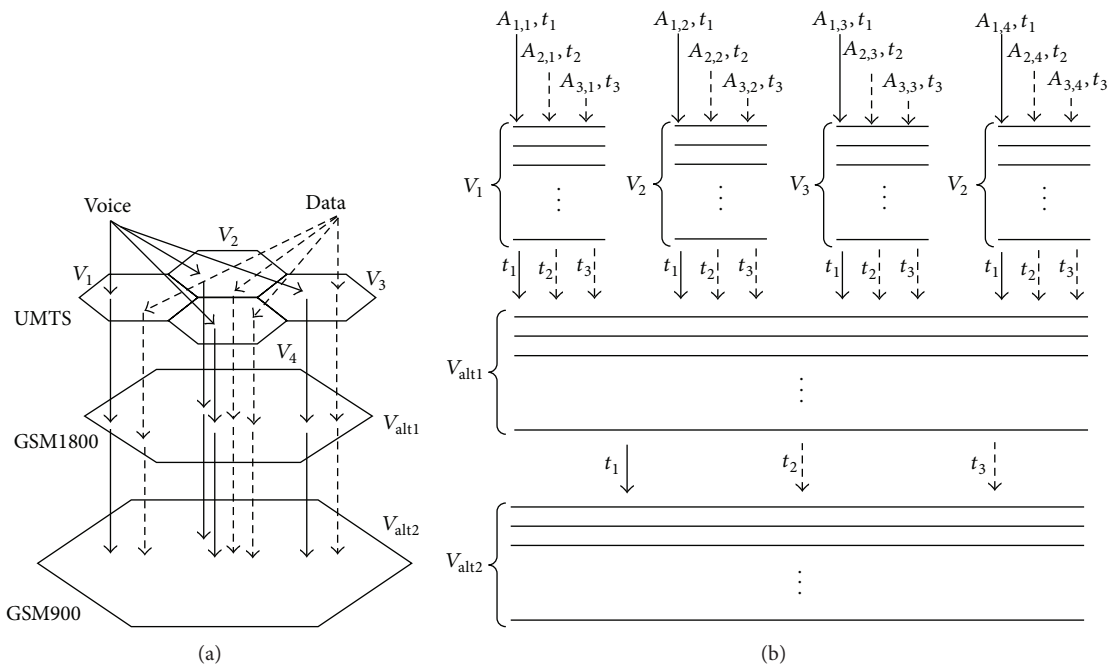

Figure 13: Traffic overflow between cells of the UMTS system and GSM1800 and GSM900: (a) traffic overflow between cells and (b) overflow scheme ($A_{x,y}$ is traffic generated by calls of class x in cell y and t_x is the number of demanded resources by calls of class x).

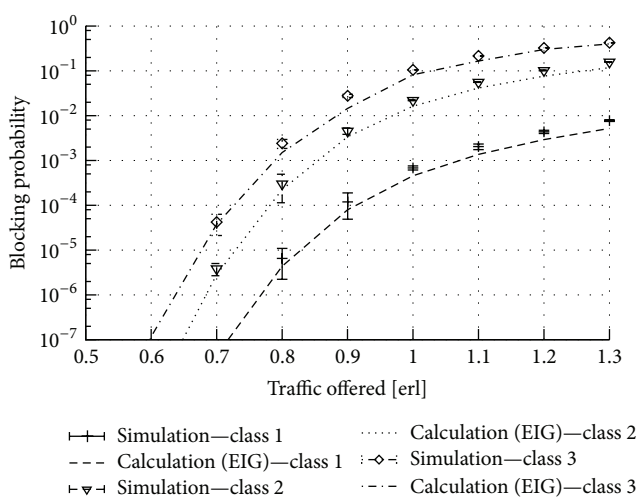

Figure 14: Blocking probability in mobile system with overflow traffic: overflow between cells of the UMTS system and GSM1800 and GSM900.

The capacity of the UMTS cells was equal to 140 allocation units, whereas the capacity of the GSM cell was 32 allocation units. Each UMTS cell serviced three classes of calls that demanded 1, 4, and 10 allocation units, respectively. Since the capacities of all cells were identical, then availabilities for particular classes of calls offered to each of the cells were also identical. This means that the blocking probability for calls of a given traffic class depends only on the size of demanded resources and not on the cell to which these calls are offered. Therefore, Figure 14 shows only blocking probabilities for 3 traffic classes.

The last considered case of traffic overflow involves traffic overflow between the systems: LTE and UMTS. We assume that these two systems are comparable as far as data transmission possibilities are concerned. Traffic overflow in this particular case relates to all types of connections (Figure 15). In this scenario, overflow of voice connections to the GSM system is omitted. The results of the blocking probability for calls of particular traffic classes, in the network with traffic overflow between the LTE and the UMTS systems, are presented in Figure 16. Connections from 3 LTE cells were directed to one UMTS cell that serviced additionally its own traffic. The system serviced four classes of calls: voice connection class ($t_1 = 1$ allocation units) and three call classes related to data transmission ($t_2 = 5$ allocation units, $t_3 = 10$ allocation units, and $t_4 = 30$ allocation units, resp.). The capacities of the LTE cells were identical and were equal to $V_1 = V_2 = V_3 = 600$ allocation units, whereas the capacity of the UMTS cell was equal to 200 allocation units. The cells 1 and 2 were offered four traffic classes and the cell 3 was offered three traffic classes (Figure 15). Traffic offered directly to the UMTS cell was lower by 80% than traffic offered to the LTE cells.

In the last part of our research we have considered the network with traffic overflow between the LTE and the UMTS systems (Figure 15), under the assumption that the LTE cells have different capacities: $V_1 = V_2 = 300$ allocation units, $V_3 = 400$ allocation units, and $V_{alt} = 200$ allocation units. The results of the blocking probability for calls of particular traffic classes are presented in Figure 17. Connections from 3 LTE cells were directed to one UMTS cell

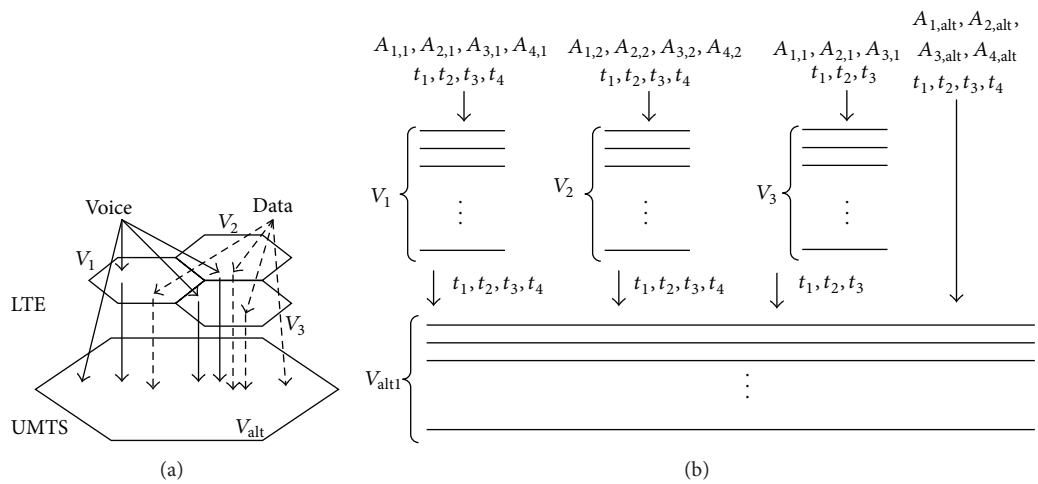

Figure 15: Overflow between cells of the systems: LTE and UMTS, (a) traffic overflow between cells and (b) overflow scheme ($A_{x,y}$ is traffic generated by calls of class x in cell y and t_x is the number of demanded resources by calls of class x).

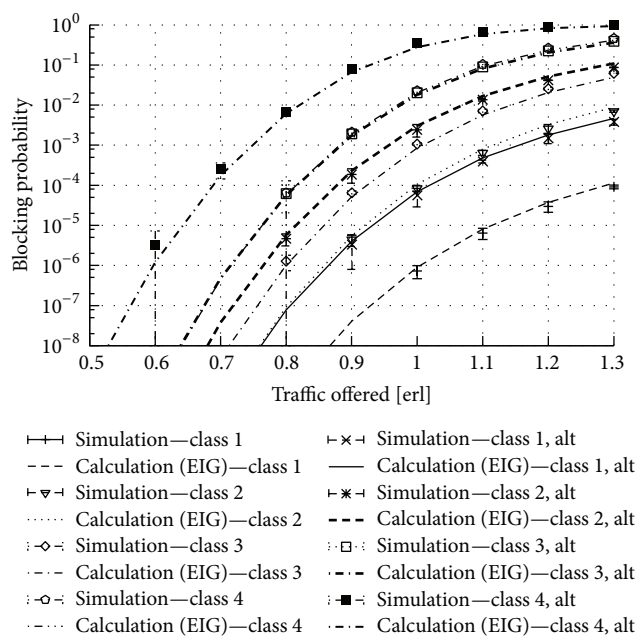

Figure 16: Blocking probability in mobile system with overflow traffic: overflow between cells of the LTE system and UMTS.

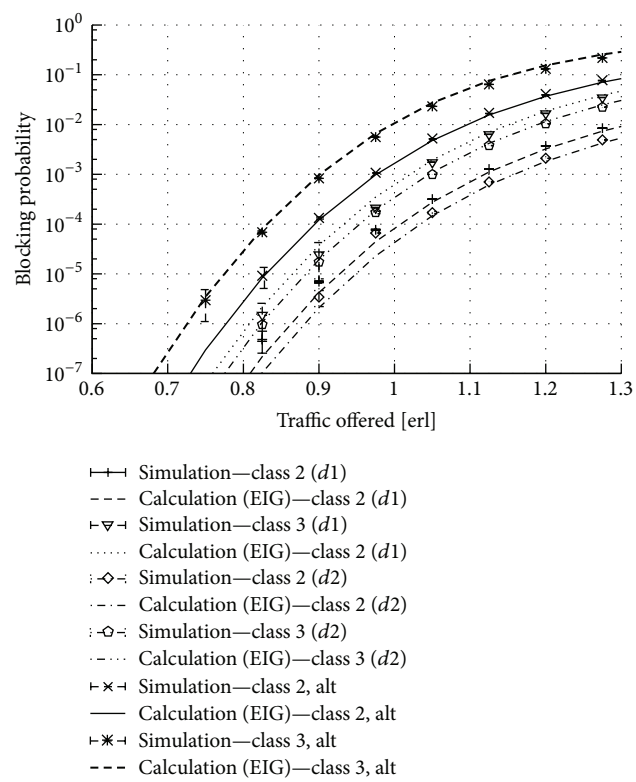

Figure 17: Blocking probability in mobile system with overflow traffic: overflow between cells of the LTE system (different capacities of LTE cells) and UMTS; ($d1$) is blocking probability in resources V_1 and V_2, ($d2$) is blocking probability in resource V_3.

that serviced additionally its own traffic. The system serviced four classes of calls, as in the case of the previous considered case. The presented results indicate that the differences in availabilities for particular traffic classes do not influence the accuracy of the proposed method.

The analysis of the presented results allows us to state that for each of the considered traffic overflow scheme the presented method ensures high accuracy of calculations, which is sufficient for practical purposes in engineering applications at the stage of analysis and dimensioning of present-day cellular networks.

5. Conclusions

This paper proposes a simple analytical method for a determination of traffic characteristics of mobile systems with traffic overflow, both single service and multiservice. The proposed

method is based on the Erlang's Ideal Grading model with different availabilities and makes it possible to determine the blocking probability in overflow systems based only on the information on the capacity of individual network resources (cells) and the value of offered traffic. The proposed method is characterised by the complexity of order $\Theta(mV^2)$, whereas the method is based on Fredericks-Hayward method by the complexity of order $\Theta(mV)$. The higher complexity of the proposed method is compensated by the lack of necessity to determine the approximate variances.

The paper considers a number of cases that involved traffic overflow in mobile networks, that is, both between cells (sectors of cells) of different systems (GSM, UMTS, and LTE), and between cells (sectors of cells) of one system. For each of the considered overflow scenarios, blocking probabilities obtained on the basis of the proposed analytical method are evaluated. These results are then compared with the data obtained in the simulations, which confirms very high accuracy of the proposed model. The model, far more simpler than any earlier developed methods, can be thus used for solving practical problems in design and optimization of modern cellular networks.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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