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Corrigendum

Corrigendum to "Observer-Based Adaptive Iterative Learning Control for a Class of Nonlinear Time Delay Systems with Input Saturation"

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The purpose of this paper is to correct the errors in the paper titled "Observer-Based Adaptive Iterative Learning Control for a Class of Nonlinear Time Delay Systems with Input Saturation" [1]. In the adaptive learning law of $\widehat{W}_k(t)$ in (44) of [1], the variable z_k was used. However in fact, z_k is difficult to be obtained as the states are unmeasurable. The correction and some consequent modifications in the technical derivations are detailed as follows, while the main results are unchanged.

By Young's inequality, we can have

$$\begin{aligned}
&-2z_k^T PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\quad = -2z_k^T CC^T \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\qquad - 2z_k^T \delta I_n \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\quad \le -2z_{1,k} C^T \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\qquad + \frac{z_k^T PP^T z_k}{\left\|CC^T + \delta I_n\right\|^2} + \delta^2 l \widetilde{W}_k^T \widetilde{W}_k,
\end{aligned} \tag{1}$$

where $\delta > 0$ is a small positive constant.

Then (27) in [1] should be updated to

$$\begin{aligned}
&\dot{V}_{z_k} + \dot{V}_{U_k} \\
&\quad \le z_k^T \left(A^T P + PA + \frac{n+3}{\lambda} PP^T + \frac{PP^T}{\left\|CC^T + \delta I_n\right\|^2} \right) z_k \\
&\qquad - 2z_{1,k} C^T \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\qquad + \frac{\lambda}{(1-\kappa)} \sum_{j=1}^{n} \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda l \varepsilon_W^2 + \lambda \varepsilon_0^2 \\
&\qquad + \delta^2 l \widetilde{W}_k^T \widetilde{W}_k \\
&\quad \le -z_k^T Q z_k - 2z_{1,k} C^T \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\qquad + \frac{\lambda}{(1-\kappa)} \sum_{j=1}^{n} \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda l \varepsilon_W^2 + \lambda \varepsilon_0^2 \\
&\qquad + \delta^2 l \widetilde{W}_k^T \widetilde{W}_k \\
&\quad \le -\lambda_{\min}(Q) \|z_k\|^2 \\
&\qquad - 2z_{1,k} C^T \left(CC^T + \delta I_n\right)^{-1} PB\widetilde{W}_k^T \phi(\widehat{x}_k) \\
&\qquad + \frac{\lambda}{(1-\kappa)} \sum_{j=1}^{n} \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda l \varepsilon_W^2 + \lambda \varepsilon_0^2 \\
&\qquad + \delta^2 l \widetilde{W}_k^T \widetilde{W}_k,
\end{aligned} \tag{2}$$

where using updated inequality (12) of [1]

$$A^T P + PA + \frac{n+3}{\lambda} PP^T + \frac{PP^T}{\left\|CC^T + \delta I_n\right\|^2} < -Q. \tag{3}$$

Consequently, the derivative of V_k should be revised as

$$\begin{aligned}
\dot{V}_k &\le -\lambda_{\min}(Q) \|z_k\|^2 - 2z_{1,k} C^T \left(CC^T + \delta I_n\right)^{-1} \\
&\quad \cdot PB\widetilde{W}_k^T \phi(\widehat{x}_k) + \delta^2 l \widetilde{W}_k^T \widetilde{W}_k + s_k \left[\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} \right.
\end{aligned}$$

$$+\left[\Lambda^T \quad 1\right] K_0 z_{1,k} + K e_{sk} - y_d^{(n)} + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k + \frac{b\tanh^2(s_k/\eta(t))\,\Xi(y_k)}{s_k}\Bigg] + \Bigg[1 - b\tanh^2\left(\frac{s_k}{\eta(t)}\right)\Bigg]\Xi(y_k) - K s_k^2. \tag{4}$$

In order to update $\widehat{W}_k(t)$ in the absence of z_k, we design the following differential-difference type learning law:

$$\begin{aligned}(1-\gamma_1)\dot{\widehat{W}}_k &= -\gamma_1\widehat{W}_k - \gamma_1\alpha_1\widehat{W}_k + \gamma_1\widehat{W}_{k-1} + 2q_1 z_{1,k} C^T\left(CC^T + \delta I_n\right)^{-1} PB\phi(\widehat{x}_k),\\ \widehat{W}_k(0) &= \widehat{W}_{k-1}(T),\\ \widehat{W}_0(t) &= 0,\\ & t\in[0,T].\end{aligned} \tag{5}$$

The adaptive learning law for $\widehat{W}_{2,k}(t)$ is not changed, but some notations are updated, which is specified by

$$\begin{aligned}(1-\gamma_2)\dot{\widehat{W}}_{2,k} &= -\gamma_2\widehat{W}_{2,k} + \gamma_2\widehat{W}_{2,k-1} + q_2 b\tanh^2\left(\frac{s_k}{\eta(t)}\right)\phi_2(y_k),\\ \widehat{W}_{2,k}(0) &= \widehat{W}_{2,k-1}(T),\\ \widehat{W}_{2,0}(t) &= 0,\\ & t\in[0,T],\end{aligned} \tag{6}$$

where $0<\gamma_1,\gamma_2<1$, and $\alpha_1>0$ are design parameters.

By substituting the controller back into (4), (46)-(47) of [1] should be replaced by

$$\begin{aligned}\dot{V}_k \le{}& -\lambda_{\min}(Q)\|z_k\|^2 - 2q_1 z_{1,k} C^T\left(CC^T+\delta I_n\right)^{-1} PB\widetilde{W}_k^T\phi(\widehat{x}_k)\\ &+ \delta^2 l\widetilde{W}_k^T\widetilde{W}_k - b\widetilde{W}_{2,k}^T\phi_2(y_k)\tanh^2\left(\frac{s_k}{\eta(t)}\right)\\ &+\left[1-b\tanh^2\left(\frac{s_k}{\eta(t)}\right)\right]\Xi(y_k) - K s_k^2,\end{aligned} \tag{7}$$

$$\begin{aligned}&2q_1 z_{1,k} C^T\left(CC^T+\delta I_n\right)^{-1} PB\widetilde{W}_k^T\phi(\widehat{x}_k) + b\widetilde{W}_{2,k}^T\phi_2(y_k)\tanh^2\left(\frac{s_k}{\eta(t)}\right) \le -\dot{V}_k\\ &- \lambda_{\min}(Q)\|z_k\|^2 + \left[1-b\tanh^2\left(\frac{s_k}{\eta(t)}\right)\right]\Xi(y_k) - K s_k^2 + \delta^2 l\widetilde{W}_k^T\widetilde{W}_k.\end{aligned} \tag{8}$$

Accordingly, the Lyapunov-like CEF is updated to

$$\begin{aligned}E_k(t) ={}& \frac{\gamma_1}{2q_1}\int_0^t \widetilde{W}_k^T\widetilde{W}_k\,d\sigma + \frac{(1-\gamma_1)}{2q_1}\widetilde{W}_k^T\widetilde{W}_k\\ &+ \frac{\gamma_2}{2q_2}\int_0^t \widetilde{W}_{2,k}^T\widetilde{W}_{2,k}\,d\sigma + \frac{(1-\gamma_2)}{2q_2}\widetilde{W}_{2,k}^T\widetilde{W}_{2,k}.\end{aligned} \tag{9}$$

The difference of $E_k(t)$ should be changed to

$$\begin{aligned}\Delta E_k(t) &= E_k(t) - E_{k-1}(t)\\ &= \frac{\gamma_1}{2q_1}\int_0^t\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]d\sigma\\ &\quad+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\ &\quad+\frac{\gamma_2}{2q_2}\int_0^t\left[\widetilde{W}_{2,k}^T\widetilde{W}_{2,k} - \widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right]d\sigma\\ &\quad+\frac{(1-\gamma_2)}{2q_2}\left[\widetilde{W}_{2,k}^T\widetilde{W}_{2,k} - \widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right].\end{aligned} \tag{10}$$

By using adaptive learning law (5) and inequality $2\widetilde{W}_k^T\widetilde{W}_k \ge \widetilde{W}_k^T\widetilde{W}_k - W^{*T}W^*$, we may have

$$\begin{aligned}&\frac{\gamma_1}{2q_1}\int_0^t\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]d\sigma + \frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\ &= \frac{\gamma_1}{2q_1}\int_0^t\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]d\sigma + \frac{(1-\gamma_1)}{q_1}\int_0^t\widetilde{W}_k^T\dot{\widetilde{W}}_k\,d\sigma\\ &\quad+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0) - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\ &= \int_0^t 2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_k^T\phi(\widehat{x}_k)\,d\sigma\\ &\quad-\frac{\gamma_1}{q_1}\int_0^t\widetilde{W}_k^T\left(\alpha_1\widehat{W}_k + \widehat{W}_k - \widehat{W}_{k-1}\right)d\sigma\\ &\quad+\frac{\gamma_1}{2q_1}\int_0^t\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]d\sigma\\ &\quad+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0) - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\ &= \int_0^t 2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_k^T\phi(\widehat{x}_k)\,d\sigma\\ &\quad-\frac{\alpha_1\gamma_1}{q_1}\int_0^t\widetilde{W}_k^T\widehat{W}_k\,d\sigma - \frac{\gamma_1}{q_1}\int_0^t\widetilde{W}_k^T\left(\widetilde{W}_k - \widetilde{W}_{k-1}\right)d\sigma\\ &\quad+\frac{\gamma_1}{2q_1}\int_0^t\left[\widetilde{W}_k^T\widetilde{W}_k - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]d\sigma\end{aligned}$$

$$
\begin{aligned}
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\
&=\int_0^t 2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_k^T\phi(\hat{x}_k)\,\mathrm{d}\sigma\\
&-\frac{\alpha_1\gamma_1}{q_1}\int_0^t\widetilde{W}_k^T\widetilde{W}_k\mathrm{d}\sigma\\
&-\frac{\gamma_1}{2q_1}\int_0^t\left(\widetilde{W}_k-\widetilde{W}_{k-1}\right)^T\left(\widetilde{W}_k-\widetilde{W}_{k-1}\right)\mathrm{d}\sigma\\
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\
&\le\int_0^t 2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_k^T\phi(\hat{x}_k)\,\mathrm{d}\sigma\\
&-\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\widetilde{W}_k^T\widetilde{W}_k\mathrm{d}\sigma+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\\
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right].
\end{aligned}
\tag{11}
$$

Recalling (8), inequality (B.8) of [1] is changed to

$$
\begin{aligned}
&\Delta E_k(t)\\
&\le\int_0^t 2z_k^TPB\widetilde{W}_k^T\phi(\hat{x}_k)\,\mathrm{d}\sigma\\
&+\int_0^t b\tanh^2\left(\frac{s_k}{\eta}\right)\widetilde{W}_{2,k}^T\phi_2(y_k)\,\mathrm{d}\sigma\\
&+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma-\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\widetilde{W}_k^T\widetilde{W}_k\mathrm{d}\sigma\\
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\
&+\frac{(1-\gamma_2)}{2q_2}\left[\widetilde{W}_{2,k}^T(0)\widetilde{W}_{2,k}(0)-\widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right]\\
&\le -V_k(t)+V_k(0)-K\int_0^t s_k^2\mathrm{d}\sigma\\
&-\lambda_{\min}(Q)\int_0^t\left\|z_k\right\|^2\mathrm{d}\sigma\\
&-\int_0^t\left(\frac{\alpha_1\gamma_1}{2q_1}-l\delta^2\right)\widetilde{W}_k^T\widetilde{W}_k\mathrm{d}\sigma\\
&+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\\
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\
&+\frac{(1-\gamma_2)}{2q_2}\left[\widetilde{W}_{2,k}^T(0)\widetilde{W}_{2,k}(0)-\widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right].
\end{aligned}
\tag{12}
$$

Choose suitable design parameters such that $\alpha_1\gamma_1/2q_1-l\delta^2>0$. Then it follows from (12) that

$$
\begin{aligned}
&\Delta E_k(t)\\
&\le -V_k(t)+V_k(0)-K\int_0^t s_k^2\mathrm{d}\sigma\\
&-\lambda_{\min}(Q)\int_0^t\left\|z_k\right\|^2\mathrm{d}\sigma+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\\
&+\frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0)-\widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]\\
&+\frac{(1-\gamma_2)}{2q_2}\left[\widetilde{W}_{2,k}^T(0)\widetilde{W}_{2,k}(0)-\widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right].
\end{aligned}
\tag{13}
$$

Consequently, (B.9) of [1] is updated to

$$
\begin{aligned}
\Delta E_k(T)&\le -V_k(T)-K\int_0^T s_k^2\mathrm{d}\sigma\\
&-\lambda_{\min}(Q)\int_0^T\left\|z_k\right\|^2\mathrm{d}\sigma\\
&+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\\
&\le -K\int_0^T s_k^2\mathrm{d}\sigma-\lambda_{\min}(Q)\int_0^T\left\|z_k\right\|^2\mathrm{d}\sigma\\
&+\frac{\alpha_1\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma.
\end{aligned}
\tag{14}
$$

By using adaptive learning law $(1-\gamma_1)\dot{\widehat{W}}_1=-\gamma_1\widehat{W}_1-\gamma_1\alpha_1\widehat{W}_1+2q_1z_{1,1}C^T(CC^T+\delta I_n)^{-1}PB\phi(\hat{x}_1)$, it is clear that

$$
\begin{aligned}
&\frac{\gamma_1}{2q_1}\widetilde{W}_1^T\widetilde{W}_1+\frac{(1-\gamma_1)}{q_1}\widetilde{W}_1^T\dot{\widetilde{W}}_1\\
&=\frac{\gamma_1}{2q_1}\widetilde{W}_1^T\widetilde{W}_1-\frac{\gamma_1}{q_1}\widetilde{W}_1^T\widehat{W}_1-\frac{\alpha_1\gamma_1}{q_1}\widetilde{W}_1^T\widehat{W}_1\\
&\quad+2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_k^T\phi(\hat{x}_k)\\
&=\frac{\gamma_1}{2q_1}\left[\widetilde{W}_1^T\widetilde{W}_1-2\widetilde{W}_1^T\widehat{W}_1+\widehat{W}_1^T\widehat{W}_1\right]-\frac{\gamma_1}{2q_1}\widehat{W}_1^T\widehat{W}_1\\
&\quad-\frac{\alpha_1\gamma_1}{q_1}\widetilde{W}_1^T\widehat{W}_1\\
&\quad+2z_{1,1}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_1^T\phi(\hat{x}_1)\\
&\le\frac{\gamma_1}{2q_1}\left[\widehat{W}_1-\widetilde{W}_1\right]^T\left[\widehat{W}_1-\widetilde{W}_1\right]-\frac{\alpha_1\gamma_1}{2q_1}\widetilde{W}_1^T\widetilde{W}_1\\
&\quad+\frac{\alpha_1\gamma_1}{2q_1}W^{*T}W^*\\
&\quad+2z_{1,1}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_1^T\phi(\hat{x}_1)
\end{aligned}
$$

$$\begin{aligned}
&\leq (1+\alpha_1)\frac{\gamma_1}{2q_1}W^{*T}W^* \\
&\quad + 2z_{1,1}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_1^T\phi(\hat{x}_1) \\
&\quad - \frac{\alpha_1\gamma_1}{2q_1}\widetilde{W}_1^T\widetilde{W}_1.
\end{aligned} \tag{15}$$

Thus $\dot{E}_1(t)$ is changed to

$$\begin{aligned}
\dot{E}_1(t) &\leq (1+\alpha_1)\frac{\gamma_1}{2q_1}W^{*T}W^* + \frac{\gamma_2}{2q_2}W_2^{*T}W_2^* \\
&\quad + 2z_{1,k}C^T\left(CC^T+\delta I_n\right)^{-1}PB\widetilde{W}_1^T\phi(\hat{x}_1) \\
&\quad + b\tanh^2\left(\frac{s_1}{\eta(t)}\right)\widetilde{W}_{2,1}^T\phi_2(y_1) - \frac{\alpha_1\gamma_1}{2q_1}\widetilde{W}_1^T\widetilde{W}_1 \\
&\leq -\dot{V}_1 - \lambda_{\min}(Q)\|z_1\|^2 - Ks_1^2 \\
&\quad + (1+\alpha_1)\frac{\gamma_1}{2q_1}W^{*T}W^* + \frac{\gamma_2}{2q_2}W_2^{*T}W_2^* \\
&\quad - \left(\frac{\alpha_1\gamma_1}{2q_1} - l\delta^2\right)\widetilde{W}_1^T\widetilde{W}_1 \\
&\leq -\dot{V}_1 - \lambda_{\min}(Q)\|z_1\|^2 - Ks_1^2 \\
&\quad + (1+\alpha_1)\frac{\gamma_1}{2q_1}W^{*T}W^* + \frac{\gamma_2}{2q_2}W_2^{*T}W_2^*.
\end{aligned} \tag{16}$$

Denote $c_{\max} = \max_{t\in[0,T]}\{(1+\alpha_1)(\gamma_1/2q_1)W^{*T}(t)W^*(t) + (\gamma/2q_2)W_2^{*T}W_2^*\}$. The integral of $\dot{E}_1(t)$ over $[0,t]$ is updated as follows:

$$\begin{aligned}
E_1(t) - E_1(0) &\leq -V_1(t) + V_1(0) \\
&\quad - \lambda_{\min}(Q)\int_0^t \|z_1\|^2\,d\sigma \\
&\quad - \int_0^t Ks_1^2(\sigma)\,d\sigma + t\cdot c_{\max}.
\end{aligned} \tag{17}$$

According to the new definition of CEF (9), $E_1(0)$ should be computed as follows:

$$\begin{aligned}
E_1(0) &= \frac{(1-\gamma_1)}{2q_1}\widetilde{W}_1^T(0)\widetilde{W}_1(0) \\
&\quad + \frac{(1-\gamma_2)}{2q_2}\widetilde{W}_{2,1}^T(0)\widetilde{W}_{2,1}(0) \\
&= \frac{(1-\gamma_1)}{2q_1}\|W^*\|^2 + \frac{(1-\gamma_2)}{2q_2}\|W_2^*\|^2.
\end{aligned} \tag{18}$$

Hence $E_1(T)$ is bounded by

$$\begin{aligned}
E_1(T) &\leq T\cdot c_{\max} + \frac{(1-\gamma_1)}{2q_1}\|W^*\|^2 + \frac{(1-\gamma)}{2q_2}\|W_2^*\|^2 \\
&< \infty.
\end{aligned} \tag{19}$$

We choose $\alpha_1 = \Delta_k$, with $\{\Delta_k\}$ being a convergent series, which is defined by $\Delta_k = q/k^l$, where l and q are design parameters, $q(\in R) > 0$, $l(\in Z_+) \geq 2$. Δ_k has the following property.

Property 1 (see S. Zhu, M. X. Sun, and X. X. He, "Iterative learning control of strict-feedback nonlinear time-varying systems," Acta Automatica Sinica, vol. 36, no. 3, pp. 454–458, 2010). $\lim_{k\to\infty}\sum_{j=1}^k \Delta_j \leq 2q$.

Using (14), it is followed by

$$\begin{aligned}
E_k(T) &= E_1(T) + \sum_{j=2}^k \Delta E_j(T) \\
&\leq E_1(T) - K\sum_{j=2}^k\int_0^T s_j^2\,d\sigma \\
&\quad - \lambda_{\min}(Q)\sum_{j=2}^k\int_0^T \|z_j\|^2\,d\sigma \\
&\quad + \frac{\gamma_1}{2q_1}\int_0^T \|W^*(\sigma)\|\,d\sigma\sum_{j=2}^k\Delta_k.
\end{aligned} \tag{20}$$

According to Property 1, we know $\sum_{j=1}^k \Delta_k \leq \lim_{k\to\infty}\sum_{j=1}^k \Delta_k \leq 2q$, which implies the boundedness of $E_k(T)$.

In the derivation of finiteness of $E_k(t)$, the changes are specified as follows.

Separate $E_k(t)$ into two parts:

$$\begin{aligned}
E_k^1(t) &= \frac{\gamma_1}{2q_1}\int_0^t \widetilde{W}_k^T\widetilde{W}_k\,d\sigma + \frac{\gamma_2}{2q_2}\int_0^t \widetilde{W}_{2,k}^T\widetilde{W}_{2,k}\,d\sigma, \\
E_k^2(t) &= \frac{(1-\gamma_1)}{2q_1}\widetilde{W}_k^T\widetilde{W}_k + \frac{(1-\gamma_2)}{2q_2}\widetilde{W}_{2,k}^T\widetilde{W}_{2,k}.
\end{aligned} \tag{21}$$

The boundedness of $E_k^1(T)$ and $E_k^2(T)$ is guaranteed $\forall k \in N$. Thus, there exist two positive constants M_1 and M_2 satisfying

$$\begin{aligned}
E_k^1(t) &\leq E_k^1(T) \leq M_1 < \infty, \\
E_k^2(T) &\leq M_2.
\end{aligned} \tag{22}$$

Consequently,

$$E_k(t) = E_k^1(t) + E_k^2(t) \leq M_1 + E_k^2(t). \tag{23}$$

On the other hand, it follows from (13) that

$$\begin{aligned}
&\Delta E_{k+1}(t) \\
&\quad < \frac{\Delta_k\gamma_1}{2q_1}\int_0^t \|W^*(\sigma)\|\,d\sigma \\
&\quad + \frac{(1-\gamma_1)}{2q_1}\left[\widetilde{W}_k^T(0)\widetilde{W}_k(0) - \widetilde{W}_{k-1}^T\widetilde{W}_{k-1}\right]
\end{aligned}$$

$$+\frac{(1-\gamma_2)}{2q_2}\left[\widetilde{W}_{2,k}^T(0)\widetilde{W}_{2,k}(0)-\widetilde{W}_{2,k-1}^T\widetilde{W}_{2,k-1}\right]$$
$$\leq \frac{\Delta_k\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma+M_2-E_k^2(t). \tag{24}$$

Combining (23) and (24) results in

$$\begin{aligned}E_{k+1}(t)&=E_k(t)+\Delta E_{k+1}(t)\\&\leq M_1+M_2+\frac{\Delta_k\gamma_1}{2q_1}\int_0^t\left\|W^*(\sigma)\right\|\mathrm{d}\sigma.\end{aligned} \tag{25}$$

Since we have proved the boundedness of $E_1(t)$, the finiteness of $E_k(t)$ can be deduced by induction method.

Finally, we give the necessary revisions for the proof of convergence of tracking errors.

We can obtain from (20) that

$$\begin{aligned}&\sum_{j=2}^k\int_0^T s_j^2\mathrm{d}\sigma\\&\leq\frac{\left(E_1(T)-E_k(T)+(\gamma_1/2q_1)\int_0^T\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\sum_{j=2}^k\Delta_k\right)}{K},\\&\sum_{j=2}^k\int_0^T\left\|z_j\right\|^2\mathrm{d}\sigma\\&\leq\frac{\left(E_1(T)-E_k(T)+(\gamma_1/2q_1)\int_0^T\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\sum_{j=2}^k\Delta_k\right)}{\lambda_{\min}(Q)}.\end{aligned} \tag{26}$$

Considering Property 1 and taking the limitation of the above two inequalities yield

$$\begin{aligned}&\lim_{k\to\infty}\sum_{j=2}^k\int_0^T s_j^2(\sigma)\,\mathrm{d}\sigma\\&\quad\leq\frac{\left(E_1(T)+(\gamma_1q/q_1)\int_0^T\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\right)}{K},\\&\lim_{k\to\infty}\sum_{j=2}^k\int_0^T\left\|z_j\right\|^2\mathrm{d}\sigma\\&\quad\leq\frac{\left(E_1(T)+(\gamma_1q/q_1)\int_0^T\left\|W^*(\sigma)\right\|\mathrm{d}\sigma\right)}{\lambda_{\min}(Q)}.\end{aligned} \tag{27}$$

Similarly, according to the convergence of the sum of series, we can obtain the convergence of errors. The other parts are not changed.

References

[1] J.-m. Wei, Y.-a. Hu, and M.-m. Sun, "Observer-based adaptive iterative learning control for a class of nonlinear time delay systems with input saturation," *Mathematical Problems in Engineering*, vol. 2015, Article ID 645161, 19 pages, 2015.