Research Article

Efficient and Enhanced Diffusion of Vector Field for Active Contour Model

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Gradient vector flow (GVF) is an important external force field for active contour models. Various vector fields based on GVF have been proposed. However, these vector fields are obtained with many iterations and have difficulty in capturing the whole image area. On the other hand, the ability to converge to deep and complex concavity with these vector fields is also needed to improve. In this paper, by analyzing the diffusion equation of GVF, a normalized set is defined and a dynamically normalized constraint of vector fields is used for efficient diffusion, which makes the edge vector diffusing rapidly to the entire image region. In order to improve the ability to converge to concavity, an enhanced diffusion term is integrated into the original energy functional. With the dynamically normalized constraint and enhanced diffusion term, new vector fields of EDGVF (efficient and enhanced diffusion for GVF) and EDNGVF (efficient and enhanced diffusion of NGVF) are obtained. Experimental results demonstrate that vector fields with proposed method capture the entire image and are obtained with less iterations and computational times. In particular, EDNGVF greatly improves the ability to converge to concavity.

1. Introduction

Active contour model (ACM) is one of the most important methods for boundary extraction. It was firstly presented by Kass et al. [1]. There are two foundational types in active contour model, that is, parametric [1, 2] and geometric active contour models [3]. The former represents curves or surfaces explicitly in their parametric forms, while implicit approaches based on level set [4, 5] are adopted in geometric active contour. Compared with the parametric active contour, geometric active contour has advantages in dealing with complex image content. However, geometric active contour always brings much computational cost since curves in geometric active contour are implicitly represented as a level set of higher dimensional functions. In parametric active contour, curve deforms with the help of internal and external forces. Internal force, which is defined by the curve itself, is used to smooth the curve. External force computing from image data pushes the curves toward target boundary. Thus, external force plays a dominating role in converging to target

boundary. In this paper, we focus on the external force fields in parametric active contour model.

In parametric active contour, a curve is represented as $\nu(s) = (x(s), y(s))$, $s \in [0, 1]$. The curve is deformed to the object boundary by minimizing the following energy functional:

$$E_{\text{snake}} = \int_0^1 \frac{1}{2} (\alpha \nu_s^2(s) + \beta \nu_{ss}^2(s)) + E_{\text{ext}}(\nu(s)) ds, \quad (1)$$

where $\nu_s(s)$ and $\nu_{ss}(s)$ are the first and second derivatives of $\nu(s)$ with respect to arc length parameter s ; the parameters α and β are the weight constants. External energy E_{ext} is derived from an image, which is usually computed as follows:

$$E_{\text{ext}}(x, y) = -|\nabla [G_\sigma(x, y) * I(x, y)]|^2, \quad (2)$$

where $G_\sigma(x, y)$ is a two-dimensional Gaussian function with standard deviation σ ; $G_\sigma(x, y) = (1/2\pi\sigma^2)e^{-(x^2+y^2)/2\sigma^2}$. I is the gray intensity of an image. If an image is binary and the

background of the image is zero-valued, external energy is calculated as follows:

$$E_{\text{ext}}(x, y) = -I(x, y). \quad (3)$$

Minimizing the energy functional of (1) with calculus of variations [6], the Euler equation is obtained:

$$\alpha \gamma_{ss}(s) - \beta \gamma_{ssss}(s) - \nabla E_{\text{ext}}(s) = 0. \quad (4)$$

The first two terms are considered as the internal force, and $-\nabla E_{\text{ext}}(s)$ is viewed as the external force imposing on the deforming curve. Most of methods are concerned with the external force fields in parametric active contour models, such as gradient vector flow (GVF) vector field [7] and its improvements [8–15].

In this paper, based on the analysis of GVF, a dynamical constraint for the magnitude of vectors is performed by defining a special set. The unit difference of vector magnitude between the nonzero vectors and zero vectors makes the vectors rapidly diffuse to entire image region. On the other hand, by incorporating an enhanced diffusion term, the “equilibrium problems” [16–19] are greatly decreased.

2. Problem Analysis

GVF has a relative large capture range and can extract some U-shaped concavities compared with original vector field. However, GVF always fails to diffuse to entire image [15] and has difficulty in extracting deep concavity. Various methods modify the energy function of GVF to obtain new vector fields, such as GVF in the normal direction (NGVF). Unfortunately, these vector fields based on diffusion equation always need too many iterations (usually cost more than 300 iterations) and have difficulty in enlarging to entire image. For example, we take GVF. In GVF, vector field $V(x, y)$ is calculated by minimizing the following energy functional:

$$E_{\text{GVF}} = \int \mu |\nabla V|^2 + |\nabla f|^2 |V - \nabla f|^2 d\Omega, \quad (5)$$

where f is the edge mapping which is computed as $-E_{\text{ext}}$. When $|\nabla f|$ is small, the energy is dominated by sum of the squares of the partial derivatives of vector field V , yielding a slowly varying field. When $|\nabla f|$ is large, the second term dominates the integrand, and minimization is carried out by setting $V = \nabla f$. μ is a regularization parameter governing the tradeoff between the first term and the second term. Using the calculus of variations, GVF is obtained by solving the following Euler equation:

$$\mu \nabla^2 V - (V - \nabla f) |\nabla f|^2 = 0. \quad (6)$$

In homogeneous region ($|\nabla f| = 0$), the Laplacian operator (∇^2) makes the vectors smoothly diffuse to image region. The linear diffusion process [20] of vector field $V(x, y)$ in homogeneous region is represented as follows:

$$\begin{aligned} V_t &= \mu \nabla^2 V, \\ V(0) &= \nabla f. \end{aligned} \quad (7)$$

The solution is given as follows:

$$\begin{aligned} V(t) &= \mu G_{\sqrt{2t}} \otimes \nabla f, \quad t > 0 \\ V(0) &= \nabla f, \quad t = 0, \end{aligned} \quad (8)$$

where $G_{\sqrt{2t}}$ is the Gaussian function with the standard deviation $\sqrt{2t}$. According to (8), vectors near object boundaries have strong magnitudes, while the magnitudes of vectors gradually decrease to zero when moving away from the boundaries. The diffusion becomes weak with the increase of the number of iterations; it is difficult to capture entire image. An example for the capture ranges of GVF is shown on the left hand of Figure 1; it cannot capture entire image even with 300 iterations. Thus, the efficiency of generating vector field is to be improved. In order to show the efficiency of generating GVF, the capture ranges with the different iterations are shown on the left hand of Figure 2. The capture range of GVF with 40 iterations does not have obvious difference compared with the effect by 20 iterations. Furthermore, there are not enough strong forces which push contour to the concavity; even the critical points which stop the contour convergence to the concavity appear near the concavity, as shown in the first row of Figure 3.

3. Proposed Method

Based on the above section, the diffusion operator makes the vectors smoothly diffused, which stops the vectors diffusion to entire image region. In order to avoid the excessive smoothing, a dynamic constraint normalizing the set S of vectors is added in the diffusion equation. On the other hand, in order to decrease the critical points of vector field, a new energy functional term is integrated in the diffusion equation.

3.1. Dynamically Normalized Constraint for Efficient Diffusion. A vector $V(x, y)$ in the normalized function set S is defined as follows:

$$V(x, y) = \begin{cases} \frac{V(x, y)}{|V(x, y)|}, & V(x, y) \neq 0, \\ 0, & V(x, y) = 0. \end{cases} \quad (9)$$

Then, a new vector field is obtained by adding a dynamically normalized constraint in GVF:

$$V_t = \mu \nabla^2 V - (V - \nabla f) |\nabla f|^2 \quad V \in S. \quad (10)$$

Because of the defined S , the unit difference between nonzero vectors and zero vectors makes the gradient vector of edge map fast diffuse to homogeneous region. From the above equation, the defined set S is nonsmooth (the magnitude of vectors in S is 0 or 1), while the Laplacian operator ∇^2 in GVF has excessive smoothing effect. Therefore, Laplacian operator makes the vector field diffuse to other regions until the vector field enlarges the entire image region (the magnitude of every vector in vector field becomes to 1). As shown in Figure 2, the capture ranges of GVF and the above proposed vector field

FIGURE 1: Comparisons of capture ranges of vector fields. From left to right: GVF and proposed method.

FIGURE 2: Comparisons of capture ranges between GVF and vector fields with proposed method from 20 to 40 iterations. From left to right: GVF and proposed method. From top to bottom: vector fields by 20 and 40 iterations, respectively.

FIGURE 3: Comparison of GVF, NGVF, and EDNGVF vector fields. (a) From left to right: GVF force field; magnified GVF field at concave region and stationary point in GVF. (b) From left to right: the NGVF force field; magnified NGVF field at concave region and the saddle and stationary points in NGVF. (c) From left to right: the EDNGVF force field; magnified force field in concave region.

with 20 and 40 iterations, respectively, are compared. On the left hand of Figure 2, the capture range of GVF is limited and is not obviously changing from 20 to 40 iterations, while the capture range of vector field with proposed method enlarges

nearly entire image (except the corners of the image) with only 40 iterations, as shown on the right hand of Figure 2.

Proposed dynamically normalized constraint method has generality for diffusion equation. Similarly, it can be applied

to NGVF. NGVF is obtained based on anisotropic diffusion of $V = (u, v)$, which is computed by the following equation:

$$V_t = \mu V_{NN} - (V - \nabla f) |\nabla f|^2, \quad (11)$$

where μ is a constant and $\nabla^2 V = V_{NN} + V_{TT}$. V_{NN} and V_{TT} are obtained as follows:

$$\begin{aligned} V_{NN} &= \frac{V_x^2 V_{xx} + V_y^2 V_{yy} + 2V_x V_y V_{xy}}{V_x^2 + V_y^2}, \\ V_{TT} &= \frac{V_x^2 V_{yy} + V_y^2 V_{xx} - 2V_x V_y V_{xy}}{V_x^2 + V_y^2}. \end{aligned} \quad (12)$$

Then, a new vector field by adding the dynamically normalized constraint in NGVF is obtained by iterating the following equation:

$$V_t = \mu V_{NN} - (V - \nabla f) |\nabla f|^2 \quad V \in S, \quad (13)$$

where V_{NN} is the diffusion term; it has a similar effect of the Laplacian operator in GVF.

Unlike the damping diffusion of GVF or NGVF, because of the defined set S , edge gradient vectors keep nearly the same velocity diffusion to other regions until the magnitude of all the vectors becomes 1 (i.e., vector field captures entire image region). Meanwhile, the force field generated by concavity can generate enough force to push contour convergence to the concavity, and the ability to converge to boundary concavity is greatly enhanced.

3.2. The Energy Functional Term for Enhanced Diffusion. In order to decrease the critical points of vector field, the diffusion of vectors in homogeneous region should be enhanced. The enhanced diffusion term is defined as follows:

$$E = - \int |V| d\Omega. \quad (14)$$

The minimization of $\int \mu(-|V|)d\Omega$ means the maximization of $\int \mu(|V|)d\Omega$; that is, this term makes the magnitude of diffused vector $|V|$ large. This property of enhanced diffusion term can decrease the critical points since the magnitude of critical point is always zero-valued.

Based on the dynamically normalized constraint and enhanced diffusion terms, the energy functional of enhanced diffusion GVF is given:

$$E_{EDGVF} = \int \mu (|\nabla V|^2 - |V|) + |\nabla f|^2 |V - \nabla f|^2 d\Omega \quad V \in S. \quad (15)$$

Minimizing the above energy functional, a new vector field, called EDGVF (efficient and enhanced diffusion for GVF), is obtained by solving the following equation:

$$\mu \left(\nabla^2 V + \frac{V}{|V|} \right) - (V - \nabla f) |\nabla f|^2 = 0 \quad V \in S. \quad (16)$$

According to the above energy functional of equation, the effect of enhanced diffusion is analyzed as follows: the energy of $\int \mu |\nabla V|^2 d\Omega$ makes the vector diffusion smooth, and the vectors gradually decrease to 0 and the critical points may appear. Meanwhile, the minimization of $\int \mu(-|V|)d\Omega$ makes the magnitude of diffused vector $|V|$ large. This property of enhanced diffusion term could decrease the critical points since the magnitude of critical point is always zero-valued.

It has better effect in dealing with concavity when the enhanced diffusion term is integrated with anisotropic diffusion equations. For example, we take NGVF. In fact, Laplacian operator in the normal direction V_{NN} is got by minimizing ∞ -Laplacian energy functional [21, 22]. The ∞ -Laplacian energy functional is defined as follows:

$$\iint |G_\sigma * \nabla V|_{L^\infty} d\Omega. \quad (17)$$

Similar to L^2 , L^∞ is the infinity norm. Then, the enhanced diffusion term and dynamically normalized constraint are integrated with NGVF. Thus, the energy functional of enhanced diffusion for NGVF is defined as follows:

$$\begin{aligned} E_{EDNGVF} &= \iint \mu (|\nabla f|) (|G_\sigma * \nabla V|_{L^\infty} - |V|) \\ &\quad + |\nabla f|^2 |V - \nabla f|^2 d\Omega \quad V \in S. \end{aligned} \quad (18)$$

Different from NGVF, $\mu(|\nabla f|)$ is not a constant; $\mu(|\nabla f|) = \exp(-|\nabla f|/k)$; k is a constant. Near the boundaries, the anisotropic parameter $\mu(|\nabla f|)$ approaches 0; the boundaries are not distorted by diffusion effect.

With calculus of variation, EDNGVF (efficient and enhanced diffusion for NGVF) is calculated as follows:

$$\mu (|\nabla f|) \left(V_{NN} + \frac{V}{|V|} \right) - (V - \nabla f) |\nabla f|^2 = 0 \quad V \in S. \quad (19)$$

In homogeneous region, $\mu(|\nabla f|)$ approaches 1 and the diffusion of vector field is enhanced. This property improves the ability to extract complex concavity compared with NGVF. The minimization of $\int \mu(|\nabla f|)(-|V|)d\Omega$ makes the magnitude of vector nonzero, which enhances the diffusion of vector; meanwhile, the critical points in vector field are greatly decreased. As shown in Figure 3, the vector fields of GVF, NGVF, and EDNGVF at concavity are compared. The critical points appear in GVF and NGVF, which are shown in the first and second rows of Figure 3. Since the enhanced diffusion term improves the diffusion ability of vectors, the vectors generated by the deep concavity could diffuse enough, and the critical points disappear in EDNGVF, as shown in the final row of Figure 3. This property of EDNGVF makes the contour converge to the concavity.

4. Experimental Results

In this section, the performances with several methods and proposed method are compared. The edge map f used in snakes is normalized to the range $[0, 1]$; $\alpha = 0.2$ and $\beta = 0.1$ are set for all experiments. For each tested image,

the same initialization is employed for all tested methods. The parameter k for proposed method is set to 0.01 in all the experiments unless otherwise stated. The vector field with proposed method can capture entire image region, and the efficiency of generating vector field is improved. Otherwise, the proposed EDNGVF improves the performances in extracting the concavity, which is verified in this section.

4.1. Capture Range and Efficiency of Generating Vector Fields. In GVF force field, isotropic diffusion of GVF keeps the field diffusing smoothly. The anisotropic diffusion in NGVF has better effect in extracting concavity than isotropic diffusion in GVF. However, the diffusion of GVF and NGVF becomes weak as the number of iteration increases. Thus, GVF and NGVF have difficulty in capturing entire image region. As shown in the left column of Figure 4, GVF and NGVF cannot capture entire image region, while the vector fields with proposed method capture the whole image region, which are shown in the right column of Figure 4.

In addition, the proposed method speeds up the diffusion of vector fields; vector fields with proposed methods can be obtained with less iterations and times compared with GVF and NGVF. In order to test the generation efficiencies of force fields, the minimum numbers of iterations and costing times for enlarging the force fields to the same image region are compared. The images in Figures 5 and 6 are tested. As shown in Table 1, EDGVF and EDNGVF need from 50 to 120 iterations in test images. Meanwhile, GVF and NGVF cost more than 200 iterations in each tested image. For example, we take the image in the third row of Figure 5. GVF iterates 2000, while EDNGVF only needs 100 iterations. As shown in Table 1, the proposed vector field is generated with the least iterations. Furthermore, vector fields with proposed method cost the least times compared with GVF and NGVF. Proposed method introduces a normalized set and an enhanced diffusion energy term; these terms bring up some extra computational cost, but they greatly speed up the diffusion of vector field. The computational cost of computing the introduced terms is much lower than the cost of computing the diffusion equation in each iteration. As a whole, vector fields with proposed method cost less times compared with GVF and NGVF. As shown in Table 1, vector fields with proposed method cost less times in each tested image compared with GVF and NGVF.

4.2. Concavity Extracting. Because of the excessive smoothing of Laplacian operator in GVF, it stops the contour convergence to the deep concavity. NGVF and EDNGVF improve the ability to converge to the deep concavity compared with GVF. As shown in the first row of Figure 5(a), the size of image is $100 * 100$ pixels. The U-shaped concavity is 20-pixel long and 5-pixel wide. The contour with GVF snake stops at the entrance of concavity. NGVF only makes the diffusion along the normal direction of the isophotes; the smooth effect is decreased. The performance in dealing with deep concavity with NGVF is shown in the first row of Figure 5(b). EDNGVF is obtained by incorporating the enhanced diffusion term into NGVF; meanwhile, the normalized set is defined to greatly

TABLE 1: Comparative analysis made for iterations of generating GVF, NGVF, EDGVF, and EDNGVF force fields.

Image	Method	Iteration	Time (s)
Image 1	GVF	1800	1.17
	NGVF	400	0.53
	EDGVF	100	0.12
	EDNGVF	80	0.18
Image 2	GVF	800	0.48
	NGVF	220	0.36
	EDGVF	60	0.07
	EDNGVF	50	0.11
Image 3	GVF	2000	6.96
	NGVF	400	4.51
	EDGVF	120	1.02
	EDNGVF	100	1.24
Image 4	GVF	1400	1.15
	NGVF	200	0.81
	EDGVF	90	0.36
	EDNGVF	80	0.44
Image 5	GVF	1200	0.67
	NGVF	200	0.55
	EDGVF	70	0.08
	EDNGVF	50	0.11

enhance the diffusion effect. The performance with EDNGVF is shown in Figure 5(b). Both NGVF and EDNGVF succeed in converging to the U-shaped concavity.

For some images with complex concavity, the vectors generated by various edges in GVF and NGVF may have conflict components and some critical points appear, which is analyzed in [19]. As shown in Figure 3, GVF and NGVF suffer from “equilibrium problem.” While EDNGVF enhances the anisotropic diffusion, the conflict components in vector fields are greatly decreased. Thus, EDNGVF can be more fit to converge to complex concavity. As shown in Figure 5, the size of image with cross shaped concavity is $100 * 100$ pixels. Because of the complexity of shape, the diffusion of vectors near concavity may be restrained by other vectors. The evolutions of the contour with GVF and NGVF attain their convergence prematurely, which are shown in the second row of Figures 5(a) and 5(b). While the proposed method enhances its performance, the evolution of contour is shown in the second row of Figure 5(c) and the contour finally converges to the concavity completely.

The images with highly nonconvex boundaries are also tested. One takes, for example, the image with “hand” shape; it has a size of $202 * 188$ pixels. As shown in the third row of Figure 5(c), the hand is accurately extracted with EDNGVF. The evolutions with GVF and NGVF snakes are shown in the third row of Figures 5(a) and 5(b); both GVF and NGVF failed to extract object boundaries. The images in the first and second rows of Figure 6 are $126 * 110$ and $100 * 100$, respectively. The conflict components in GVF and NGVF stop the contour evolving to the concave region. Local minimum points and large conflict components are always

FIGURE 4: Comparisons of capture ranges of vector fields. From top to bottom: comparisons of GVF and EDNGVF and comparisons of NGVF and EDNGVF.

formed in GVF and NGVF, which makes the contour stop at the entrance of concave region, as shown in Figures 6(a) and 6(b). However, the conflict components in EDNGVF force field are avoided by introducing the enhanced diffusion term; the contour succeeds in attaining the deepest concavity with proposed method, which is shown in Figure 6(c). Therefore, for the U-shaped images, NGVF and proposed method have advantages over GVF, while proposed EDNGVF has better performance than NGVF and GVF in converging to some complex concavities, such as cross shaped concavity. The comparisons of GVF, NGVF, and EDNGVF show that proposed EDNGVF can be more fit to extract gray images with concavity.

Some gray images are also tested. Two images and the corresponding initial contours are shown in Figures 7(a) and 8(a); the sizes of these two images are 135×125 and 111×100 pixels, respectively. The comparisons of GVF, NGVF, and EDNGVF snakes are given. The same binary edge map is

performed for fair comparisons. For Figure 7, the initial contour crosses the object; EDNGVF snake completely extracts the boundary. Both GVF and NGVF suffer from local minimum. As for Figure 8, the initial contour is placed inside the object. Proposed method succeeds in extracting the boundary, as shown in Figure 8(d). Meanwhile, both GVF and NGVF snakes make the contour stop undesired locations, which are shown in Figures 8(b) and 8(c), respectively. These simulations for gray images also show that proposed method has better performances than GVF and NGVF.

4.3. Comparisons with the State-of-the-Art Methods. In this section, comparisons with some state-of-the-art methods are given and analyzed. The vector fields, normally biased GVF (NBGVF) and adaptive diffusion flow (ADF), are compared. Based on the diffusion equation, ADF and NBGVF have similar diffusion effect. Both of them could not capture entire image region since the diffusion becomes weak because of

FIGURE 5: Convergences of (a) GVF snake, (b) NGVF snake, and (c) EDNGVF snake.

the smooth effect of diffusion operator. On the other hand, the performances of converging to concavity are compared. NBGVF, ADF, and proposed method could extract some images with concavity. However, for some complex concavities, proposed method has advantages over NBGVF and ADF. For example, one just takes the first image of Figure 6. As shown in Figure 9, NBGVF and ADF failed to converge to the cross shaped concavity. While the performances of proposed method are shown in Figure 6, the proposed method make the contour succeed in converging to the concavity. Though NBGVF and ADF are obtained based on anisotropic diffusion similar to NGVF, some conflict components of vectors always appear in these vector fields. The enhanced diffusion term is incorporated in the proposed method; the saddle points are greatly decreased in EDNGVF vector field. Therefore, the proposed method has advantages in extracting objects with concavity compared with NBGVF and ADF.

5. Conclusion

In this paper, the efficient and enhanced diffusion terms for vector fields are incorporated in the energy function to obtain new vector fields. The proposed normalized set is used to keep the speed of diffusion to enlarge the capture range of vector field to the whole image region. On the other hand, the enhanced diffusion term is used to decrease critical points when it is integrated with anisotropic diffusion. Experimental results on synthetic and gray images show that proposed method greatly improves the generating efficiency of vector field and the performances in dealing with concavity, especially the highly nonconvex boundaries, such as cross shaped concavity. It is noted that proposed vector fields are also based on boundary information; they still yield unsatisfactory results when dealing with some images with complex content, such as images with low contrast. We expect

FIGURE 6: Convergences of (a) GVF snake, (b) NGVF snake, and (c) EDNGVF snake.

FIGURE 7: (a) Gray image with initial contour and convergences of (b) GVF snake, (c) NGVF snake, and (d) EDNGVF snake.

FIGURE 8: (a) Medical image with initial contour and convergences of (b) GVF snake, (c) NGVF snake, and (d) EDNGVF snake.

FIGURE 9: Results with ADF and NBGVF. From left to right: result with ADF and result with NBGVF.

to construct a vector field based on the region information to deal with the images with low contrast in the next task.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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