Research Article

A New Conjugate Gradient Algorithm with Sufficient Descent Property for Unconstrained Optimization

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A new nonlinear conjugate gradient formula, which satisfies the sufficient descent condition, for solving unconstrained optimization problem is proposed. The global convergence of the algorithm is established under weak Wolfe line search. Some numerical experiments show that this new WWPNNRP⁺ algorithm is competitive to the SWPPRP⁺ algorithm, the SWPHS⁺ algorithm, and the WWPDYHS⁺ algorithm.

1. Introduction

In this paper, we consider the following unconstrained optimization problem:

$$\min \{f(x) \mid x \in \mathbb{R}^n\}, \quad (1)$$

where $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a twice continuously differentiable function whose gradient is denoted by $g(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$. Its iterative formula is given by

$$x_{k+1} = x_k + t_k d_k, \quad (2)$$

where

$$d_k = \begin{cases} -g_k & \text{if } k = 1, \\ -g_k + \beta_k d_{k-1} & \text{if } k \geq 2, \end{cases} \quad (3)$$

and t_k is a step size which is computed by carrying out a line search, β_k is a scalar, and g_k denotes $g(x_k)$. There are at least six famous formulas for β_k , which are given below:

$$\beta_k^{\text{HS}} = \frac{g_k^T (g_k - g_{k-1})}{(g_k - g_{k-1})^T d_{k-1}},$$

$$\beta_k^{\text{FR}} = \frac{g_k^T g_k}{g_{k-1}^T g_{k-1}},$$

$$\beta_k^{\text{PRP}} = \frac{g_k^T (g_k - g_{k-1})}{g_{k-1}^T g_{k-1}},$$

$$\beta_k^{\text{CD}} = -\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}},$$

$$\beta_k^{\text{LS}} = -\frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T g_{k-1}},$$

$$\beta_k^{\text{DY}} = \frac{g_k^T g_k}{(g_k - g_{k-1})^T d_{k-1}}. \quad (4)$$

To establish the global convergence results of the above conjugate gradient (CG) methods, it is usually required that the step size t_k should satisfy some line search conditions, such as the weak Wolfe-Powell (WWP) line search

$$f(x_k + t_k d_k) - f(x_k) \leq \delta t_k g_k^T d_k, \quad (5)$$

$$g(x_k + t_k d_k)^T d_k \geq \sigma g_k^T d_k, \quad (6)$$

where $\delta \in (0, 1/2)$ and $\sigma \in (\delta, 1)$, and strong Wolfe-Powell (SWP) line search (5) and

$$|g(x_k + t_k d_k)^T d_k| \leq -\sigma g_k^T d_k, \quad (7)$$

where $\delta \in (0, 1/2)$ and $\sigma \in (\delta, 1)$. Wolfe-Powell is referred to as Wolfe.

Considerable attentions have been made on the global convergence behaviors for the above methods. Zoutendijk [1] proved that the FR method with exact line search is globally convergent. Al-Baali [2] extended this result to the strong Wolfe line search conditions. In [3], Dai and Yuan

proposed the DY method which produces a descent search direction at every iteration and converges globally provided that the line search satisfies the weak Wolfe conditions. In [4], Wei et al. discussed the global convergence of the PRP conjugate gradient method (CGM) with inexact line search for nonconvex unconstrained optimization. Recently, based on [5–7], Jiang et al. [8] proposed a hybrid CGM with

$$\beta_k^{\text{HJ}} = \frac{\|g_k\|^2 - \max\{0, (\|g_k\| / \|d_{k-1}\|) g_k^T d_{k-1}, (\|g_k\| / \|g_{k-1}\|) g_k^T g_{k-1}\}}{d_{k-1}^T (g_k - g_{k-1})}. \quad (8)$$

Under the Wolfe line search, the method possesses global convergence and efficient numerical performance.

On some studies of the conjugate gradient methods, the sufficient descent condition

$$g_k^T d_k \leq -c \|g_k\|^2, \quad c > 0 \quad (9)$$

is often used to analyze the global convergence of the nonlinear conjugate gradient method with the inexact line search techniques. For instance, Touati-Ahmed and Storey [9], Al-Baali [2], Gilbert and Nocedal [10], and Hu and Storey [11] hinted that the sufficient descent condition may be crucial for conjugate gradient methods. Unfortunately, this condition is hard to hold. It has been showed that the PRP method with the strong Wolfe Powell line search does not ensure this condition at each iteration. So, Grippo and Lucidi [12] managed to find some line searches which ensure the sufficient descent condition, and they presented a new line search which ensures this condition. The convergence of the PRP method with this line search had been established. Yu et al. [13] analyzed the global convergence of modified PRP CGM with sufficient descent property. Gilbert and Nocedal [10] gave another way to discuss the global convergence of the PRP method with the weak Wolfe line search. By using a complicated line search, they were able to establish the global convergence result of the PRP and HS methods by restricting the parameter β_k in (3), not allowed to be negative; that is,

$$\beta_k^+ = \max\{0, \beta_k^{\text{PRP}}\}, \quad (10)$$

which yields a globally convergent CG method, being also computationally efficient [14]. In spite of the numerical efficiency of the PRP method, as an important defect, the method lacks the following descent property:

$$g_k^T d_k \leq 0, \quad \forall k \geq 0, \quad (11)$$

even for uniformly convex objective functions [15]. This motivated the researchers to pay much attention to finding some extensions of the PRP method with descent property. In this context, Yu et al. [16] proposed a modified form of β_k^{PRP} as follows:

$$\beta_k^{\text{DPRP}} = \beta_k^{\text{PRP}} - C \frac{\|y_k\|^2}{\|g_k\|^4} g_{k+1}^T d_k, \quad (12)$$

with a constant $C \geq 1/4$, leading to a CG method with the sufficient descent property. Dai and Kou [17] propose a family of conjugate gradient methods and an improved Wolfe line search; meanwhile, to accelerate the algorithm, an adaptive restart along negative gradients method is introduced. Jiang and Jian [18] proposed two modified CGMs with disturbance factors based on a variant of PRP method; the two proposed methods not only generate sufficient descent direction at each iteration but also converge globally for nonconvex minimization if the strong Wolfe line search is used. A new hybrid conjugate gradient method was presented for unconstrained optimization. The proposed method can generate decent directions at every iteration; moreover, this property is independent of the steplength line search. Under the Wolfe line search, the proposed method possesses global convergence [19].

The main purpose of this paper is to design an efficient algorithm which possesses the properties of global convergence, sufficient descent, and good numerical results. In next section, we present a new CG formula and give its properties. In Section 3, the new algorithm and its global convergence result will be established. To test and compare the numerical performance of the proposed method, in the last part of this work, a large amount of medium-scale numerical experiments are reported by tables and performance profiles.

2. The Formula and Its Property

Because sufficient descent condition (9) is a very nice and important property to analyze the global convergence of the CG methods, we hope to find β_k such that d_k satisfies (9). In the following, we propose a sequence $\{\beta_k\}$ and prove that it has such property. Firstly, we give a definition of a *descent sequence* (or a *sufficient descent sequence*): a sequence $\{\beta_k\}$ is called a *descent sequence* (or a *sufficient descent sequence*) for the CG methods if there exists a constant $\tau \in (0, 1)$ (or $\tau \in [0, 1)$) such that, for all $k \geq 2$,

$$\beta_k g_k^T d_{k-1} \leq \tau \|g_k\|^2. \quad (13)$$

By using (3), we have, for all $k \geq 2$,

$$g_k^T d_k = -\|g_k\|^2 + \beta_k g_k^T d_{k-1}. \quad (14)$$

From the above discussion, we require that

$$-\|g_k\|^2 + \beta_k g_k^T d_{k-1} \leq 0. \quad (15)$$

The above inequality implies (13).

In [20], the authors proposed a variation of the FR formula:

$$\beta_k^{\text{VFR}}(\mu) = \frac{\mu_1 \|g_k\|^2}{\mu_2 |g_k^T d_{k-1}| + \mu_3 \|g_{k-1}\|^2}, \quad (16)$$

where $\mu_1 \in (0, +\infty)$, $\mu_2 \in [\mu_1 + \epsilon_1, +\infty)$, $\mu_3 \in (0, +\infty)$, and ϵ_1 is any given positive constant. It is easy to prove that $\{\beta_k^{\text{VFR}}\}$ is a descent sequence (with $\tau = \mu_1/\mu_2$) for CG methods if $g_k^T d_k \leq 0$. Formula (16) possesses the sufficient descent property and proved that there exist some nonlinear conjugate gradient formulae possessing the sufficient descent property without any line searches, where

$$\beta_k^{\text{WYL}} = \frac{g_k^T (g_k - \|g_k\| / \|g_{k-1}\|)}{g_{k-1}^T g_{k-1}}. \quad (17)$$

By restricting the parameter $\sigma \leq 1/4$ under the SWP line search condition, the WYL method possessed the sufficient descent condition [21].

In [22], the authors designed the following variation of the PRP formula which possesses the sufficient descent property without any line searches:

$$\beta_k^N(\mu) = \begin{cases} \frac{\|g_k\|^2 - |g_k^T d_{k-1}|}{\mu |g_k^T d_{k-1}| + \|g_{k-1}\|^2} & \text{if } \|g_k\|^2 \geq |g_k^T d_{k-1}|, \\ 0 & \text{Otherwise,} \end{cases} \quad (18)$$

in which $\mu \geq 1$.

Motivated by the ideas in [20, 22] without any line search and sufficient descent, and taking into account the good convergence properties of [10] and the good numerical performance in [14], we propose a class new formula about β_k as follows:

$$\beta_k^{\text{NPRP}} = \frac{\lambda \mu_1 \|g_k\|^2 + (1 - \lambda) \mu_1 (\|g_k\|^2 - |g_k^T d_{k-1}|)}{\mu_2 |g_k^T d_{k-1}| + \mu_1 \|g_{k-1}\|^2}, \quad (19)$$

where the definitions of μ_1, μ_2 are the same as those in formula (16); $\lambda \in (0, 1)$.

In order to ensure the nonnegative of the parameter β_k , we define

$$\beta_k^{\text{NPRP}^+} = \max \{0, \beta_k^{\text{NPRP}}\}. \quad (20)$$

Thus if a negative of β_k^{NPRP} occurs, this strategy will restart the iteration along the steepest direction.

The following two propositions show that the $\{\beta_k^{\text{NPRP}^+}\}$ is a descent sequence, so that d_k can make sufficient descent condition (9) hold.

Proposition 1. Suppose that β_k is defined by (19)-(20); then one has that

$$\beta_k^{\text{NPRP}^+} \leq \hat{\sigma} \frac{\|g_k\|^2}{|g_k^T d_{k-1}|}, \quad (21)$$

where $0 < \hat{\sigma} = \mu_1/\mu_2 < 1$.

Proof. It is clear that inequality (21) holds when $\beta_k^{\text{NPRP}^+} = 0$. Now we consider the case where $\beta_k^{\text{NPRP}^+} = \beta_k^{\text{NPRP}}$. So we have

$$\begin{aligned} \beta_k^{\text{NPRP}^+} &= \frac{\lambda \mu_1 \|g_k\|^2 + (1 - \lambda) \mu_1 (\|g_k\|^2 - |g_k^T d_{k-1}|)}{\mu_2 |g_k^T d_{k-1}| + \mu_1 \|g_{k-1}\|^2} \\ &\leq \frac{\lambda \mu_1 \|g_k\|^2 + (1 - \lambda) \mu_1 \|g_k\|^2}{\mu_2 |g_k^T d_{k-1}|} \leq \frac{\mu_1 \|g_k\|^2}{\mu_2 |g_k^T d_{k-1}|} \\ &= \hat{\sigma} \frac{\|g_k\|^2}{|g_k^T d_{k-1}|}. \end{aligned} \quad (22)$$

Hence $\beta_k^{\text{NPRP}^+}$ can make (21) hold. Furthermore $\{\beta_k^{\text{NPRP}^+}\}$ is a descent sequence without any line search. $\square$

Proposition 2. Suppose that β_k is defined by (19)-(20); then d_k satisfies the sufficient descent condition (9) for all $k \geq 1$, where $c = \lambda(1 - \mu_1/\mu_2)$.

Proof. For any $k > 1$, suppose that $g_{k-1}^T d_{k-1} < 0$.

If $\beta_k^{\text{NPRP}^+} = 0$, then $d_k = -g_k$. So we have

$$g_k^T d_k = -\|g_k\|^2 \leq -c \|g_k\|^2, \quad (23)$$

where $c = \lambda(1 - \mu_1/\mu_2)$.

Otherwise, from the definition of $\beta_k^{\text{NPRP}^+}$, we can obtain

$$\begin{aligned} g_k^T d_k &= g_k^T \left[-g_k + \left(\frac{\lambda \mu_1 \|g_k\|^2 + (1 - \lambda) \mu_1 (\|g_k\|^2 - |g_k^T d_{k-1}|)}{\mu_2 |g_k^T d_{k-1}| + \mu_1 \|g_{k-1}\|^2} \right) d_{k-1} \right] \\ &\leq -\|g_k\|^2 + \left(\frac{\lambda \mu_1 \|g_k\|^2}{\mu_2 |g_{k-1}^T d_{k-1}|} |g_k^T d_{k-1}| \right. \\ &\quad \left. + \frac{(1 - \lambda) \mu_1 \|g_k\|^2}{\mu_2 |g_k^T d_{k-1}|} |g_k^T d_{k-1}| \right) \leq -\|g_k\|^2 + \lambda \\ &\quad \cdot \frac{\mu_1}{\mu_2} \|g_k\|^2 + \|g_k\|^2 - \lambda \|g_k\|^2 \leq -\lambda \|g_k\|^2 + \lambda \\ &\quad \cdot \frac{\mu_1}{\mu_2} \|g_k\|^2 \leq -\lambda \left(1 - \frac{\mu_1}{\mu_2} \right) \|g_k\|^2 = -c \|g_k\|^2. \end{aligned} \quad (24)$$

For $g_1^T d_1 = -\|g_1\|^2 < 0$, we can deduce that d_k can make sufficient descent condition (9) hold for all $k \geq 1$. $\square$

By the proof of Proposition 2, we can know that the formula $\mu_1/\mu_2 < 1$ is necessary; otherwise, the sufficient descent condition can not be held.

3. Global Convergence

In this section, we propose an algorithm related to $\beta_k^{\text{NPRP}^+}$ and then we study the global convergence property of this algorithm. Firstly, we make the following two assumptions, which have been widely used in the literature to analyze the global convergence of the CG methods with the inexact line searches.

Assumption A. The level set

$$\Omega = \{x \in R^n \mid f(x) \leq f(x_1)\} \quad (25)$$

is bounded.

Assumption B. The gradient $g(x)$ is Lipschitz continuous; that is, there exists a constant $L > 0$ such that, for any $x, y \in \Omega$,

$$\|g(x) - g(y)\| \leq L \|x - y\|. \quad (26)$$

Now we give the algorithm.

Algorithm 3.

Step 0. Given $x_1 \in R^n$, set $d_1 = -g_1$; $k = 1$. If $g_1 = 0$, then stop. Otherwise go to Step 1.

Step 1. Find $t_k > 0$ satisfying weak Wolfe conditions (5) and (6).

Step 2. Let $x_{k+1} = x_k + t_k d_k$ and $g_{k+1} = g(x_{k+1})$. If $\|g_{k+1}\| = 0$, then stop. Otherwise go to Step 3.

Step 3. Compute $\beta_{k+1}^{\text{NPRP}^+}$ by formula (19) and (20). Then generate d_{k+1} by (3).

Step 4. Set $k = k + 1$; go to Step 0.

Since $\{f(x_k)\}$ is decreasing sequence, it is clear that the sequence $\{x_k\}$ is contained in Ω , and there exists a constant f^* , such that

$$\lim_{k \rightarrow \infty} f(x_k) = f^*. \quad (27)$$

By using Assumptions A and B, we can deduce that there exists $M > 0$ such that

$$\|g_k\| \leq M \quad \forall x \in \Omega. \quad (28)$$

The following important result was obtained by Zoutendijk [1] and Wolfe [23, 24].

Lemma 4. Suppose $f(x)$ is bounded below, and $g(x)$ satisfies the Lipschitz condition. Consider any iteration method of formula (2), where d_k satisfies $d_k^T g_k < 0$ and t_k is obtained by the weak Wolfe line search. Then

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < +\infty. \quad (29)$$

The following lemma was obtained by Dai and Yuan [25].

Lemma 5. Assume that a positive series $\{a_i\}$ satisfies the following inequality for all k :

$$\sum_{i=1}^k a_i \geq lk + c, \quad (30)$$

where $l > 0$ and c are constant. Then one has

$$\sum_{i=1}^k \frac{a_i^2}{i} = +\infty, \quad (31)$$

$$\sum_{k \geq 1} \frac{a_k^2}{\sum_{i=1}^k a_i} = +\infty.$$

Theorem 6. Suppose that Assumptions A and B hold; $\{x_k\}$ is a sequence generated by Algorithm 3. Then one has

$$\liminf_{k \rightarrow \infty} \|g_k\| = 0. \quad (32)$$

Proof. Equation (3) indicates that, for all $k \geq 2$,

$$d_k + g_k = \beta_k d_{k-1}. \quad (33)$$

Squaring both sides of (33), we obtain

$$\|d_k\|^2 = -\|g_k\|^2 - 2g_k^T d_k + \beta_k^2 \|d_{k-1}\|^2. \quad (34)$$

Suppose that $\beta_k^{\text{NPRP}^+} = \beta_k^{\text{NPRP}}$ in (20). Then,

$$\begin{aligned} \|d_k\|^2 &= -\|g_k\|^2 - 2g_k^T d_k + \left(\beta_k^{\text{NPRP}^+}\right)^2 \|d_{k-1}\|^2 \\ &= -\|g_k\|^2 - 2g_k^T d_k \\ &\quad + \left(\frac{\lambda \mu_1 \|g_k\|^2 + (1-\lambda) \mu_1 (\|g_k\|^2 - |g_k^T g_{k-1}|)}{\mu_2 |g_k^T d_{k-1}| + \mu_1 \|g_{k-1}\|^2} \right)^2 \\ &\quad \cdot \|d_{k-1}\|^2 \leq -\|g_k\|^2 - 2g_k^T d_k + \|g_k\|^4 \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4}. \end{aligned} \quad (35)$$

We have

$$h_k \leq h_{k-1} - \frac{1}{\|g_k\|^2} + \frac{2\gamma_k}{\|g_k\|^2}, \quad (36)$$

where $h_k = \|d_k\|^2 / \|g_k\|^4$ and $\gamma_k = -g_k^T d_k / \|g_k\|^2$.

Note that $h_1 = 1/\|g_1\|^2$ and $\gamma_1 = 1$. It follows from (36) that

$$h_k \leq -\sum_{i=1}^k \frac{1}{\|g_i\|^2} + 2 \sum_{i=1}^k \frac{|\gamma_i|}{\|g_i\|^2}. \quad (37)$$

Suppose that conclusion (32) does not hold. Then, there exists a positive scalar ϵ such that, for all $k \geq 1$,

$$\|g_k\| \geq \epsilon. \quad (38)$$

Thus, it follows from (28) and (38) that

$$h_k \leq -\frac{k}{M^2} + \frac{2}{\epsilon^2} \sum_{i=1}^k |\gamma_i|. \quad (39)$$

Further, we have

$$h_k \leq \frac{2}{\epsilon^2} \sum_{i=1}^k |\gamma_i|. \quad (40)$$

On the other hand, using $h_k \geq 0$, relation (39) implies that

$$\sum_{i=1}^k |\gamma_i| \geq \frac{\epsilon^2 k}{2M}. \quad (41)$$

Using Lemma 5 and (40), it follows that

$$\sum_{k \geq 1} \frac{(g_k^T d_k)^2}{\|d_k\|^2} = \sum_{k \geq 1} \frac{\gamma_k^2}{h_k} = +\infty, \quad (42)$$

which contradicts to Zoutendijk condition (29). This shows that (32) holds. The proof of the theorem is complete. $\square$

From the proof of the above theorem, we can conclude that any conjugate gradient method with the formula $\beta_k^{\text{NPRP}^+}$ and some certain step size technique which ensures that Zoutendijk condition (29) holds is globally convergent. In particular, the formula $\beta_k^{\text{NPRP}^+}$ with the weak Wolfe conditions can generate a globally convergent result.

4. Numerical Results

All methods above are tested on 56 test problems, where the former test problems 1–48 (from arwhead to woods) in Table 1 are taken from the CUTE library in Bongartz et al. [26] and the others are taken from Moré et al. [27];

$$\text{DYHS: } \beta_k = \max \{0, \min \{\beta_k^{\text{HS}}, \beta_k^{\text{DY}}\}\} \quad (43)$$

is generated by Grippo and Lucidi [12].

All codes were written in Matlab 7.5 and run on a HP with 1.87 GB RAM and Windows XP operating system. The parameters are $\sigma = 0.1$, $\delta = 0.01$, $u_1 = 1$, $u_2 = 2$, and $\lambda = 0.3$. Stop the iteration if criterion $\|g_k\| \leq \epsilon = 10^{-6}$ is satisfied.

In Table 1, “Name” denotes the abbreviation of the test problems, “ n ” denotes the dimension of the test problems, “Itr/NF/NG” denote the number of iteration, function evaluations, and gradient evaluations, respectively, and “Tcpu” denotes the computing time of CPU for computing a test problem (units: second).

On the other hand, to show the performance difference clearly between the hJHJ, hAN, hDY, and hHuS method, we adopt the performance profiles given by Dolan and Moré [28] to compare the performance according to Itr, NF, NG, and Tcpu, respectively. Benchmark results are generated by running a solver on a set $\mathcal{P}$ of problems and recording information of interest such as NF and Tcpu. Let $\mathcal{S}$ be the set of solvers in comparison. Assume that $\mathcal{S}$ consists of n_s

FIGURE 1: Performance profiles on Tpu.

FIGURE 2: Performance profile on NE.

solvers and $\mathcal{P}$ consists of n_p problems. For each problem $p \in \mathcal{P}$ and solver $s \in \mathcal{S}$, denote $t_{p,s}$ by the computing time (or the number of function evaluation, etc.) required to solve problem $p \in \mathcal{P}$ by solver $s \in \mathcal{S}$, and the comparison between different solvers is based on the performance ratio defined by

$$r_{p,s} = \frac{t_{p,s}}{\min \{t_{p,s} : s \in \mathcal{S}\}}. \quad (44)$$

Assume that a large enough parameter $r_M \geq r_{p,s}$ for all p, s is chosen, and $r_{p,s} = r_M$ if and only if solvers s do not solve problem p . Define

$$\rho_s(\tau) = \frac{1}{n_p} \text{size} \{p \in \mathcal{P} : \log r_{p,s} \leq \tau\}, \quad (45)$$

where $\text{size } A$ means the number of elements in set A ; then $\rho_s(\tau)$ is the probability for solver $s \in \mathcal{S}$ that a performance

TABLE 1: Numerical test report.

Name/ <i>n</i>	SWPHS ⁺ Itr/NF/NG/Tcpu	SWPPRP ⁺ Itr/NF/NG/Tcpu	WWPNRP ⁺ Itr/NF/NG/Tcpu	WWPDYHS Itr/NF/NG/Tcpu
arwhead 1000	10/293/98/0.078	204/5484/1606/1.462	6/92/6/0.026	23/275/32/0.073
arwhead 2000	16/441/139/0.184	261/6824/1825/2.753	6/97/6/0.035	23/325/43/0.135
arwhead 10000	31/952/301/1.554	366/11324/3139/17.892	8/154/13/0.219	8/138/12/0.206
chainwo 1000	11/329/91/0.058	128/3745/1170/0.632	26/305/43/0.053	32/373/44/0.062
chnrosn 20	3/72/13/0.004	3/72/13/0.004	3/72/17/0.004	3/72/13/0.004
chnrosnb 50	6/149/37/0.009	8/188/58/0.013	5/89/14/0.005	5/92/12/0.006
cosine 1000	13/368/153/0.236	3/113/38/0.159	30/121/32/0.048	1400/12416/1534/3.743
cragglvy 5000	1/1/2/0.026	1/1/2/0.026	1/1/2/0.026	1/1/2/0.026
cragglvy 10000	1/1/2/0.027	1/1/2/0.027	1/1/2/0.026	1/1/2/0.026
dixmaana 3000	1348/42287/19153/185.754	5/131/41/0.535	12/19/13/0.128	16/14/16/0.145
dixmaanb 3000	13/47/23/0.253	5/70/15/0.272	7/10/7/0.066	6/9/6/0.058
dixmaanc 3000	14/49/31/0.288	5/71/18/0.286	8/12/8/0.077	9/11/9/0.086
dixmaand 3000	144/4122/1686/17.815	5/72/21/0.289	11/16/11/0.109	12/13/12/0.116
dixmaani 3000	F/F/F/F	4/98/33/0.403	F/F/F/F	F/F/F/F
dixmaanjan 3000	F/F/F/F	6/69/23/0.305	1086/602/1676/12.069	1522/1420/2937/18.554
dixmaanl 3000	13/355/134/1.506	3/71/18/0.272	F/F/F/F	F/F/F/F
dqdrtic 1000	F/F/F/F	1804/55951/22587/6.841	205/1470/205/0.205	71/484/71/0.064
dqdrtic 3000	F/F/F/F	1805/56040/22893/13.054	182/1301/182/0.334	96/648/96/0.165
dqdrtic 5000	F/F/F/F	1805/56078/23060/19.535	142/1029/142/0.390	79/530/79/0.204
dqdrtic 6000	1/63/19/0.237	1/63/19/0.238	1/27/2/0.085	1/27/2/0.085
dqdrtic 15000	1/30/2/0.234	1/30/2/0.235	1/30/2/0.234	1/30/2/0.235
dqdrtic 20000	1/31/2/0.325	1/31/2/0.323	1/31/2/0.325	1/31/2/0.324
edensch 1000	1297/39852/18321/35.054	1202/37104/17643/21.938	34/181/39/0.171	58/263/77/0.265
engvall 3000	4/100/33/0.033	608/18698/7376/6.380	6/54/7/0.018	17/112/33/0.042
engvall 10000	4/100/28/0.092	580/17809/6994/17.446	7/58/10/0.056	19/146/45/0.155
engvall 20000	4/100/29/0.184	565/17366/6871/33.912	7/58/10/0.111	20/152/28/0.304
errinros 50	149/4029/1445/0.321	413/12363/4708/0.986	F/F/F/F	F/F/F/F
genhumps 2000	32/566/245/0.408	22/291/119/0.237	11/102/30/0.071	12/97/25/0.085
genhumps 15000	8/178/61/1.183	14/282/112/1.767	22/117/37/0.615	13/187/56/1.229
genhumps 20000	4/78/23/0.691	5/102/36/0.928	26/188/73/1.218	4/37/10/0.275
genrose 10000	1209/36119/12775/35.424	2/63/14/0.056	789/7297/792/7.213	263/2365/265/2.333
genrose 20000	1971/60100/21284/117.729	2/64/17/0.119	513/4754/515/9.346	297/2676/298/5.283
nondia 10000	1/41/10/0.028	1/41/10/0.028	1/21/2/0.015	1/21/2/0.013
nondia 15000	1/41/10/0.053	1/41/10/0.044	1/22/2/0.026	1/22/2/0.021
nondia 20000	1/42/9/0.063	1/42/9/0.061	1/22/2/0.029	1/22/2/0.029
nondquar 3000	2/63/11/0.208	140/4041/1715/17.728	472/896/660/6.186	662/956/1045/8.098
nondquar 15000	2/65/8/1.025	57/1419/507/29.773	196/630/301/16.855	822/1496/1395/55.029
penalty1 100	17/439/318/0.241	6/146/49/0.063	86/26/86/0.072	18/21/18/0.018
penalty1 1000	19/750/517/7.833	12/328/99/2.655	11/49/14/0.438	24/65/29/0.712
power1 10000	1/28/2/0.011	1/28/2/0.012	1/28/2/0.012	1/28/2/0.012
power1 15000	1/53/12/0.035	1/53/12/0.036	1/29/2/0.017	1/29/2/0.018
power1 20000	1/30/2/0.025	1/30/2/0.025	1/30/2/0.025	1/30/2/0.025
quartc 1000	1/54/17/0.038	1/54/17/0.037	1/22/2/0.013	1/22/2/0.013
quartc 6000	1/63/19/0.236	1/63/19/0.237	1/27/2/0.087	1/27/2/0.085
quartc 10000	1/29/2/0.152	1/29/2/0.152	1/29/2/0.152	1/29/2/0.151
srosenbr 5000	3/94/22/0.032	190/5870/2002/2.140	39/354/41/0.131	7/81/8/0.028
tridia 100	F/F/F/F	F/F/F/F	553/5057/553/0.372	348/3098/348/0.223
tridia 1000	F/F/F/F	F/F/F/F	1932/24283/1932/3.235	1443/17580/1443/2.335
bv 2000	16/358/161/20.822	434/13397/5234/752.704	4/11/4/0.636	707/2120/707/140.003
lin0 10000	1/90/16/86.643	1/90/16/86.659	1/65/2/58.180	1/65/2/58.186
lin1 3000	1/84/11/9.126	1/84/11/9.109	1/60/2/6.184	1/60/2/6.197
lin1 20000	1/93/18/122.849	1/93/18/122.946	1/68/2/82.817	1/68/2/82.707
pen1 1000	1/31/2/0.780	1/31/2/0.770	1/31/2/0.768	1/31/2/0.771
pen1 5000	1/38/2/17.416	1/38/2/17.415	1/38/2/17.425	1/38/2/17.415
vardim 1000	1/68/2/0.716	1/68/2/0.685	1/68/2/0.689	1/68/2/0.687
vardim 5000	1/84/2/19.569	1/84/2/20.066	1/84/2/19.628	1/84/2/19.720

FIGURE 3: Performance profile on NG.

FIGURE 4: Performance profile on Itr.

ratio $r_{p,s}$ is within a factor $\tau \in R^n$. The ρ_s is the (cumulative) distribution function for the performance ratio. The value of $\rho_s(1)$ is the probability that the solver will win over the rest of the solvers.

Based on the theory of the performance profile above, four performance figures, that is, Figures 1–4, can be generated according to Table 1. From the four figures, we can see that the NPRP is superior to the other three CGMs on the testing problems.

5. Conclusion

In this paper, we carefully studied the combination of the variations of the formulas β_k^{FR} and β_k^{PRP} . We have found that the new formula possesses the following features: (1) $\beta_k^{\text{NPRP}+}$ is a descent sequence without any line search; (2) the new method possesses the sufficient descent property and

converge globally; (3) the strategy will restart the iteration automatically along the steepest descent direction if a negative value of β_k^{NPRP} occurs; (4) the initial numerical results are promising.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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