Research Article

Dynamics Modeling and $\mathcal{L}_1$ Adaptive Control of a Transport Aircraft for Heavyweight Airdrop

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The longitudinal nonlinear aircraft model with cargo extraction is derived using theoretical mechanics and flight mechanics. Furthermore, the nonlinear model is approximated by a semilinear time-varying system with the cargo disturbances viewed as unknown nonlinearities, both matched and unmatched types. On this basis, a novel autopilot inner-loop based on the LQR and $\mathcal{L}_1$ adaptive theory is developed to reject the unknown nonlinear disturbances caused by the cargo and also to accommodate uncertainties. Analysis shows that the controller can guarantee robustness in the presence of fast adaptation, without exciting control signal oscillations and gain scheduling. The overall control system is completed with the outer-loop altitude-hold control based on a PID controller. Simulations are conducted under the condition that one transport aircraft performs maximum load airdrop mission at the height of 82 ft, using single row single platform mode. The results show the good performance of the control scheme, which can meet the airdrop mission performance indexes well, even in the presence of $\pm 20\%$ aerodynamic uncertainties.

1. Introduction

Heavyweight airdrop can deliver heavy equipment (>3 ton) to the desired region accurately, which is an essential capability of a large transport aircraft. During the standard airdrop operations, materials and goods are released at altitudes of 50–1500 ft at aircraft velocities between 120 and 150 knots to avoid enemy radar detection and antiaircraft artillery counteraction [1, 2]. In the process of natural disaster relief, the heavyweight airdrop can be used to deliver supplies to the emergency line accurately [3, 4]; for example, airdrop plays an important role in 5.12 Wenchuan earthquake's primary saving. The research and application around heavyweight airdrop have important military as well as civilian significance.

Current research of the heavyweight airdrop system mainly focuses on the development of precision airdrop systems [3, 4], turbulent-flow structures study behind the aircraft's empennage region [1, 5, 6], flight quality evaluations [6, 7], and control law design of the aircraft [8–10]. However, for flight quality evaluations and control law design, academics just begin a preliminary study. During the airdrop process, the continuous movement and sudden delivery of the heavy cargo can exert large disturbances on the aircraft thus

leading to considerable deviation of the aircraft dynamics from the trim position. Large variation of flight states can affect precision allocation of the payloads and even threat flight safety. To hold the flight states, a forward force is required, followed by an abrupt change in the direction of the applied force. The manipulation is quite sophisticated and does not leave a large margin for operation errors [6, 7]. Therefore, the design of an aircraft controller for the heavyweight airdrop mode is indispensable, and it is also a challenging task due to the large and sudden disturbances, strong coupling between the cargo and aircraft dynamics, and multiple uncertainties.

References [9, 10] design an attitude control law of a class of aircrafts with variable weight and variable center of gravity by means of the inverse dynamics and balanced type autopilot control strategy. Reference [11] develops a full dimension heavyweight airdrop control law based on the dynamic inversion control method. Although these approaches can improve the performance of the system from different aspects, one shortcoming is that as model-based control methods, accurate knowledge of the plant dynamics should be available.

This is not the case with the heavyweight airdrop flight control project, since there always exist some unmodeled dynamics and unknown external disturbances such as atmospheric disturbance [12–14] and ground effect [15, 16]. Furthermore, aerodynamic coefficients obtained from wind tunnel tests, augmented by computational fluid dynamics results, always contain a certain degree of uncertainty. The problem of model deficiencies can be dealt with by closing the control loop with a robust controller, for instance, combining feedback linearization with sliding mode control [14, 17, 18] or backstepping sliding mode control [19]. These methods devise control laws based on the bounds on the unknown nonlinearities. However, the bounds of the complex nonlinearities, which are composed of aircraft-cargo dynamics coupled with aerodynamic perturbations, are difficult to obtain. Therefore, the methods mentioned in [14, 17–19] tend to be conservative and sometimes lead to high-gain feedback. This might further introduce the high frequency control activity problem.

In these cases, adaptive control approaches are called for. These methods can make use of online parameter update laws to adapt the large and sudden disturbances and also to adapt model uncertainties. The model reference adaptive control (MRAC) architecture, including direct and indirect type, has been widely applied for developing autopilots for aircrafts and weapon systems [20, 21]. However, the conventional MRAC approach fails to meet the guaranteed transient performance and robustness in the presence of fast adaptation [22, 23]. Higher adaptation gains are required for achieving faster estimation. However, larger adaptation gains might lead to control signal oscillations in conventional adaptive control approaches. The recently developed $\mathcal{L}_1$ adaptive control approach [23–25] addressed this problem via designing a low-pass filter in the input channel. The low-pass filter filters out the high frequencies in the control signals excited by large adaptation gains, which guarantees the trade-off between adaptation and robustness. Briefly, the $\mathcal{L}_1$ adaptive control can guarantee transient performance and robustness in the presence of fast adaptation without exciting control signal oscillations [26, 27].

In this paper, a novel longitudinal flight control system based on the $\mathcal{L}_1$ adaptive architecture is developed for the heavyweight airdrop mode. Following [14, 17, 18], the control system is implemented with dual-loop structure, including an outer-loop controller for altitude hold and an inner-loop controller for attitude and airspeed control. In the inner-loop, a baseline controller based on the LQR method is designed firstly to provide nominal system performance. On this basis, the $\mathcal{L}_1$ adaptive control law is augmented to reject the unknown nonlinear disturbances caused by the cargo and also to accommodate other unknown disturbances and uncertainties. The outer-loop controller is designed using the classical PID control law to hold the flight altitude and also to generate a pitch angle command for the inner-loop. The structure diagram of the control system is presented in Figure 1. To verify the performance and robustness of the control system, two simulation scenarios are designed. In scenario 1, the aircraft responses during the airdrop process with and without control laws are compared and analyzed

to verify the control performance. Other than scenario 1, $\pm 20\%$ aerodynamic coefficients uncertainties are taken into consideration in scenario 2 without gains retuning of the control gains. To be sure, all the simulations are conducted with the longitudinal nonlinear aircraft model of the airdrop process although the $\mathcal{L}_1$ adaptive controller is derived via the semilinear time-varying system. The results show the good performance of the control scheme which can meet the airdrop mission performance indexes [17, 28] well, even in the presence of $\pm 20\%$ aerodynamic coefficients uncertainties.

The paper is organized as follows. The aircraft-cargo coupled model with cargo extraction is derived in the following section. In Section 3, the autopilot inner-loop based on the $\mathcal{L}_1$ adaptive control is developed. Section 4 analyzes the performance and robustness of the $\mathcal{L}_1$ adaptive controller. Simulations and discussions are presented in Section 5 and conclusions follow in Section 6.

2. Aircraft-Cargo Coupled Model with Cargo Extraction

In the aircraft model derivation, some fundamental assumptions are adopted as follows: (1) the aircraft is viewed as a rigid body; (2) the cargo is considered as a particle; and (3) the cargo moves along the rail system on the cargo deck, which coincides with the aircraft longitudinal body axis. Coordinate systems for modeling are illustrated in Figure 2. They are the body-fixed frame $Ox_b y_b z_b$ and the track-axes frame $Ox_k y_k z_k$. As shown in Figure 2, m_b is the mass of the aircraft, m_c is the mass of the cargo, $\mathbf{F}$ is the aerodynamic force vector, $\mathbf{M}$ is the aerodynamic moment vector, $\mathbf{F}_c$ is the disturbance force vector in which the cargo acts on the aircraft, $\mathbf{M}_c$ is the disturbance moment vector caused by the cargo, $\mathbf{T}$ is the engine thrust vector, $\mathbf{F}_p$ is the extraction force vector, and $\mathbf{N}_c$ is the resultant force vector in which the aircraft reacts on the cargo with $\mathbf{N}_c = -\mathbf{F}_c$.

In application of the Newton's second law, the aircraft dynamics is obtained as

$$m_b \frac{d\mathbf{V}}{dt} = \mathbf{F} + m_b \mathbf{g} + \mathbf{F}_c + \mathbf{T}, \quad (1)$$

$$\frac{d}{dt} (\mathbf{I}\boldsymbol{\Omega}) = \mathbf{M} + \mathbf{M}_c,$$

where $\mathbf{V}$ is the airspeed vector, $\boldsymbol{\Omega}$ is the angular velocity vector of the aircraft, and $\mathbf{I}$ represents the moment of inertia matrix. The cargo affects the longitudinal dynamics of the aircraft but does not influence its lateral dynamics. Then, (1) can be expanded in the form

$$\begin{aligned} m_b \frac{dV}{dt} &= T \cos \alpha - D - m_b g \sin \gamma + \mathbf{F}_c|_x^k, \\ m_b V \frac{d\gamma}{dt} &= T \sin \alpha + L - m_b g \cos \gamma - \mathbf{F}_c|_z^k, \\ I_y \frac{dq}{dt} &= M_y + M_c, \end{aligned} \quad (2)$$

where γ is the climb angle, q is the pitch rate, $*|_x^k$ and $*|_z^k$ denote the components of $*$ in track- x and track- z ,

FIGURE 1: Structure diagram of the control system.

FIGURE 2: Coordinate systems definition and force analysis.

respectively, I_y is the pitch moment inertia, and M_y is the pitch aerodynamic moment with

$$M_y = \bar{Q} S c_A \left[C_{m0} + C_{m\alpha} (\alpha - \alpha_0) + C_{m\dot{\alpha}} \frac{\dot{\alpha} c_A}{2V} + C_{mq} \frac{q c_A}{2V} + C_{m\delta_e} \delta_e \right], \quad (3)$$

where $\bar{Q}$ is the dynamic pressure, S is the wing area, δ_e is the elevator deflection, and c_A is the mean aerodynamic chord. The drag force and lift force are given by

$$\begin{aligned} D &= \bar{Q} S \left[C_{D0} + C_{D\alpha} (\alpha - \alpha_0) + C_{D\delta_e} \delta_e \right], \\ L &= \bar{Q} S \left[C_{L0} + C_{L\alpha} (\alpha - \alpha_0) + C_{L\delta_e} \delta_e \right], \end{aligned} \quad (4)$$

and the engine thrust is found by

$$T = T_m \delta_p, \quad (5)$$

where δ_p is the throttle opening ranging from 0 to 100% and T_m is the maximal thrust.

In noninertial coordinate system, the absolute acceleration of the cargo is

$$\mathbf{a}_c = \mathbf{a}_q + \mathbf{a}_r + \mathbf{a}_g, \quad (6)$$

where $\mathbf{a}_q$, $\mathbf{a}_g$, and $\mathbf{a}_r$ denote the implicated acceleration vector, the Coriolis acceleration vector, and the relative acceleration vector, respectively, with

$$\begin{aligned} \mathbf{a}_q &= \frac{d\mathbf{V}}{dt} + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r}_{cb} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{cb}), \\ \mathbf{a}_g &= 2\boldsymbol{\Omega} \times \frac{\tilde{d}\mathbf{r}_{cb}}{dt^2}, \\ \mathbf{a}_r &= \frac{\tilde{d}^2\mathbf{r}_{cb}}{dt^2}, \end{aligned} \quad (7)$$

where $\mathbf{r}_{cb}$ is the position vector that is c with respect to O and $\tilde{d}(\cdot)/dt$ denotes the relative derivative operator. Then, $\mathbf{a}_c$ can be expressed in the body-fixed frame as

$$\begin{aligned} a_{c|x}^b &= \frac{dV}{dt} \cos \alpha + V \frac{d\gamma}{dt} \sin \alpha + q^2 r_{cb} - \frac{d^2 r_{cb}}{dt^2}, \\ a_{c|z}^b &= \frac{dV}{dt} \sin \alpha - V \frac{d\gamma}{dt} \cos \alpha + \frac{dq}{dt} r_{cb} + 2q \frac{dr_{cb}}{dt}, \end{aligned} \quad (8)$$

where $*|_x^b$ and $*|_z^b$ denote the components of $*$ in body- x and body- z , respectively. The dynamic equation of the cargo can be expressed in the body-fixed frame as

$$\begin{aligned} m_c \mathbf{a}_c|_x^b &= \mathbf{N}_c|_x^b - m_c g \sin \theta - F_p \cos \varphi_p, \\ m_c \mathbf{a}_c|_z^b &= \mathbf{N}_c|_z^b + m_c g \cos \theta - F_p \sin \varphi_p, \\ \mathbf{N}_c|_x^b &= -\mu \mathbf{N}_c|_z^b, \end{aligned} \quad (9)$$

where μ is the friction coefficient of rolling between the cargo and the roller on the floor. Using coordinates transformation, we have

$$\begin{aligned} \mathbf{F}_c|_x^k &= -\mathbf{N}_c|_x^b \cos \alpha - \mathbf{N}_c|_z^b \sin \alpha, \\ \mathbf{F}_c|_z^k &= \mathbf{N}_c|_x^b \sin \alpha - \mathbf{N}_c|_z^b \cos \alpha, \\ M_c &= -r_{cb} \mathbf{N}_c|_z^b. \end{aligned} \quad (10)$$

The extraction force points to the direction of the airflow. Hence, $\varphi_p = \alpha$. Then, together with (2)~(10), we can obtain the aircraft dynamics of the form

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t, \mathbf{x}) + \mathbf{G}(t, \mathbf{x}) \mathbf{u}(t), \quad (11)$$

where $\mathbf{x} = [V \ \alpha \ q \ \theta]^T$, $\mathbf{F}(t, \mathbf{x}) = [f_1 \ f_2 \ f_3 \ f_4]^T$, $\mathbf{u}(t) = [\delta_e \ \delta_p]^T$, and $\mathbf{G}(t, \mathbf{x}) = \begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ g_{12} & g_{22} & g_{32} & g_{42} \end{bmatrix}^T$ with f_i ($i = 1, 2, 3, 4$) and g_{ij} ($i = 1, 2, 3, 4$; $j = 1, 2$) being provided in Appendix A.

Since the aircraft should keep in straight level flight condition before the cargo is unlocked [7–11, 14, 17–19], thus system (11) can be approximated by a linear time-varying system of the form

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}^T \mathbf{x}(t), \end{aligned} \quad (12)$$

where $\mathbf{x} = [\Delta V \ \Delta \alpha \ \Delta q \ \Delta \theta]^T$, $\mathbf{u}(t) = [\Delta \delta_e \ \Delta \delta_p]^T$, $\mathbf{x}_0 = \mathbf{x}(0) = \mathbf{0}$, $\mathbf{A} \in \mathbb{R}^{4 \times 4}$ and $\mathbf{B} \in \mathbb{R}^{4 \times 2}$ are the system matrix and control input matrix, and $\mathbf{y}(t) = [\Delta V \ \Delta \theta]^T$ is the regulated output.

While the cargo is unlocked, two changes will be introduced to system (12) in the following. One is the perturbation of $\mathbf{A}$ as the flight condition deviates far away from the predefined operating point. The other is the disturbance moment caused by the cargo, which is similar to a strong downward force exerts on the aircraft tail. Then, the linearized model of the aircraft with cargo extraction can be depicted as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B} [\mathbf{u}(t) + \bar{\mathbf{f}}_1(t, \mathbf{x}(t))] + \bar{\mathbf{f}}_2(t, \mathbf{x}(t)), \\ \mathbf{y}(t) &= \mathbf{C}^T \mathbf{x}(t), \end{aligned} \quad (13)$$

where $\mathbf{B}\bar{\mathbf{f}}_1(t, \mathbf{x}(t)) + \bar{\mathbf{f}}_2(t, \mathbf{x}(t))$ stands for the resultant unknown nonlinearities including model perturbation, disturbance moment caused by the cargo, and some other uncertainties such as the ground effect. $\bar{\mathbf{f}}_1 : \mathbb{R} \times \mathbb{R}^4 \rightarrow \mathbb{R}^2$ is the matched component of the unknown nonlinearities, whereas $\bar{\mathbf{f}}_2 : \mathbb{R} \times \mathbb{R}^4 \rightarrow \mathbb{R}^2$ is the unmatched component.

Assumption 1 (uniform boundedness of $\bar{\mathbf{f}}_i(t, \mathbf{0})$). There exists $b_i > 0$ such that $\|\bar{\mathbf{f}}_i(t, \mathbf{0})\|_\infty \leq b_i$, $i = 1, 2$.

Assumption 2 (semiglobal uniform boundedness of partial derivatives). For $i = 1, 2$ and an arbitrary $\delta > 0$, there exist positive constants $d_{\bar{f}_{xi}}(\delta) > 0$ and $d_{\bar{f}_{ti}}(\delta) > 0$ independent of time such that, for all $\|\mathbf{x}(t)\|_\infty < \delta$, the partial derivatives of $\bar{\mathbf{f}}_i(t, \mathbf{x})$ are continuous and bounded:

$$\begin{aligned} \left\| \frac{\partial \bar{\mathbf{f}}_i(t, \mathbf{x})}{\partial \mathbf{x}} \right\|_\infty &\leq d_{\bar{f}_{xi}}(\delta), \\ \left\| \frac{\partial \bar{\mathbf{f}}_i(t, \mathbf{x})}{\partial t} \right\|_\infty &\leq d_{\bar{f}_{ti}}(\delta). \end{aligned} \quad (14)$$

Before giving the $\mathcal{L}_1$ adaptive controller architecture, some lemmas are presented which will be used in the subsequent controller development and analysis.

Lemma 3 (see [29]). If $\|\mathbf{x}_\tau\|_{\mathcal{L}_\infty} < \rho$ and $\|\dot{\mathbf{x}}_\tau\|_{\mathcal{L}_\infty} < d_x$ for $\tau \geq 0$, where ρ and d_x are some positive constants, then there exist continuous $\theta_i(t)$ and $\sigma_i(t)$ with derivative such that for $t \in [0, \tau]$

$$\bar{\mathbf{f}}_i(t, \mathbf{x}) = \theta_i(t) \|\mathbf{x}(t)\|_\infty + \sigma_i(t), \quad i = 1, 2, \quad (15)$$

where $\|\theta_i(t)\|_\infty < \theta_{\rho i}$, $\|\dot{\theta}_i(t)\|_\infty < d_{\theta i}$, $\|\sigma_i(t)\|_\infty < \sigma_{bi}$, and $\|\dot{\sigma}_i(t)\|_\infty < d_{\sigma i}$ with $\theta_{\rho i} \triangleq d_{f_{xi}}(\rho)$, $\sigma_{bi} \triangleq b_i + \varsigma_i$, in which $\varsigma_i \in \mathbb{R}^+$ are arbitrary numbers, and $d_{\theta i}$ and $d_{\sigma i}$ are computable bounds.

Lemma 4 (see [30]). For a stable proper multi-input multioutput system $\mathbf{G}(s)$, with input $\mathbf{R}(t) \in \mathbb{R}^m$ and output $\mathbf{y}(t) \in \mathbb{R}^n$, one has

$$\|\mathbf{y}_\tau\|_{\mathcal{L}_\infty} \leq \|\mathbf{G}(s)\|_{\mathcal{L}_1} \|\mathbf{r}_\tau\|_{\mathcal{L}_\infty}, \quad \forall \tau > 0. \quad (16)$$

3. Autopilot Inner-Loop Design

Consider the following control law:

$$\mathbf{u}(t) = \mathbf{u}_{\text{lin}}(t) + \mathbf{u}_{\text{ad}}(t), \quad (17)$$

where $\mathbf{u}_{\text{lin}}(t)$ is the baseline controller that yields the desired performance and $\mathbf{u}_{\text{ad}}(t)$ is the adaptive augmentation. Design $\mathbf{u}_{\text{lin}}(t) = -\mathbf{K}_m^T \mathbf{x}(t)$, where $\mathbf{K}_m^T \in \mathbb{R}^{2 \times 4}$ is obtained using the LQR approach to render $\mathbf{A}_m = \mathbf{A} - \mathbf{B}\mathbf{K}_m^T$ Hurwitz. From the control law in (17), we can obtain partially closed-loop dynamics of system (13):

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_m \mathbf{x}(t) + \mathbf{B} [\mathbf{u}_{\text{ad}}(t) + \bar{\mathbf{f}}_1(t, \mathbf{x}(t))] \\ &\quad + \bar{\mathbf{f}}_2(t, \mathbf{x}(t)), \\ \mathbf{y}(t) &= \mathbf{C}^T \mathbf{x}(t). \end{aligned} \quad (18)$$

The control specifications are given via the following ideal system:

$$\begin{aligned} \dot{\mathbf{x}}_m(t) &= \mathbf{A}_m \mathbf{x}_m(t) + \mathbf{B}\mathbf{K}_g \mathbf{r}(t), \\ \mathbf{y}_m(t) &= \mathbf{C}^T \mathbf{x}_m(t), \end{aligned} \quad (19)$$

where $\mathbf{r}(t)$ is the reference signal vector and $\mathbf{K}_g$ is a feedforward prefilter for achieving the desired transient and steady-state tracking of commands $\mathbf{r}(t)$. Design $\mathbf{K}_g$ as

$$\mathbf{K}_g = \lim_{s \rightarrow 0} [\mathbf{C}^T \mathbf{H}(s)]^{-1} = [\mathbf{C}^T \mathbf{H}(0)]^{-1} \quad (20)$$

with $\mathbf{H}(s) = (s\mathbf{I} - \mathbf{A}_m)^{-1} \mathbf{B}$.

Remark 5. The selection of the matrices $\mathbf{Q}$ and $\mathbf{R}$ is an interesting optimization problem which is still a research challenge [31]. Readers can refer to [32, 33] for detailed discussions. In the current work, the $\mathbf{Q}$ and $\mathbf{R}$ are obtained in a trial and error way.

Remark 6. By design $\mathbf{K}_g = \lim_{s \rightarrow 0} [\mathbf{C}^T \mathbf{H}(s)]^{-1}$, the diagonal elements of the desired transfer matrix $\mathbf{C}^T (s\mathbf{I} - \mathbf{A}_m)^{-1} \mathbf{B} \mathbf{K}_g$ have DC gain equal to one, while the off-diagonal elements have zero DC gain.

Next, we will design the adaptive augmentation $\mathbf{u}_{\text{ad}}(t)$ for system (18) based on the $\mathcal{L}_1$ adaptive theory.

3.1. State Predictor and Adaptation Laws. Consider the following state predictor:

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= \mathbf{A}_m \hat{\mathbf{x}}(t) + \mathbf{B} [\mathbf{u}_{\text{ad}}(t) + \hat{\boldsymbol{\theta}}_1(t) \|\mathbf{x}(t)\|_{\infty} + \hat{\boldsymbol{\sigma}}_1(t) \\ &\quad + \hat{\boldsymbol{\theta}}_2(t) \|\mathbf{x}(t)\|_{\infty} + \hat{\boldsymbol{\sigma}}_2(t)], \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0, \\ \hat{\mathbf{y}}(t) &= \mathbf{C}^T \hat{\mathbf{x}}(t), \end{aligned} \quad (21)$$

where $\hat{\boldsymbol{\theta}}_1(t)$, $\hat{\boldsymbol{\sigma}}_1(t)$, $\hat{\boldsymbol{\theta}}_2(t)$, and $\hat{\boldsymbol{\sigma}}_2(t)$ are the adaptive parameters governed by

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}_1(t) &= \Gamma \text{Proj}(\hat{\boldsymbol{\theta}}_1(t), -(\hat{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B})^T \|\mathbf{x}(t)\|_{\infty}), \\ \hat{\boldsymbol{\theta}}_1(0) &= \hat{\boldsymbol{\theta}}_{10}, \\ \dot{\hat{\boldsymbol{\sigma}}}_1(t) &= \Gamma \text{Proj}(\hat{\boldsymbol{\sigma}}_1(t), -(\hat{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B})^T), \\ \hat{\boldsymbol{\sigma}}_1(0) &= \hat{\boldsymbol{\sigma}}_{10}, \\ \dot{\hat{\boldsymbol{\theta}}}_2(t) &= \Gamma \text{Proj}(\hat{\boldsymbol{\theta}}_2(t), -(\hat{\mathbf{x}}^T(t) \mathbf{P})^T \|\mathbf{x}(t)\|_{\infty}), \\ \hat{\boldsymbol{\theta}}_2(0) &= \hat{\boldsymbol{\theta}}_{20}, \\ \dot{\hat{\boldsymbol{\sigma}}}_2(t) &= \Gamma \text{Proj}(\hat{\boldsymbol{\sigma}}_2(t), -(\hat{\mathbf{x}}^T(t) \mathbf{P})^T), \quad \hat{\boldsymbol{\sigma}}_2(0) = \hat{\boldsymbol{\sigma}}_{20}, \end{aligned} \quad (22)$$

where $\tilde{\mathbf{x}}(t) \triangleq \hat{\mathbf{x}}(t) - \mathbf{x}(t)$ is the prediction error, $\Gamma \in \mathbb{R}^+$ is the adaptation gain, and $\mathbf{P} = \mathbf{P}^T > 0$ is the solution of the algebraic equation $\mathbf{A}_m^T \mathbf{P} + \mathbf{P} \mathbf{A}_m = -\mathbf{Q}$ for some $\mathbf{Q} = \mathbf{Q}^T > 0$. $\text{Proj}(\cdot, \cdot)$ denotes the projection operator [34] which ensures that $\|\hat{\boldsymbol{\theta}}_i(t)\|_{\infty} \leq \theta_{\rho i}$ and $\|\hat{\boldsymbol{\sigma}}_i(t)\|_{\infty} \leq \sigma_{bi}$.

3.2. Control Law. The adaptive control signal is generated through the following feedback system:

$$\mathbf{u}_{\text{ad}}(s) = -\mathbf{K} \mathbf{D}(s) \hat{\boldsymbol{\eta}}(s), \quad (23)$$

where $\hat{\boldsymbol{\eta}}(s)$ is the Laplace transform of $\hat{\boldsymbol{\eta}}(t)$:

$$\hat{\boldsymbol{\eta}}(t) = \mathbf{u}_{\text{ad}}(t) + \hat{\boldsymbol{\eta}}_1(t) + \hat{\boldsymbol{\eta}}_{2m}(t) - \mathbf{K}_g \mathbf{r}(t) \quad (24)$$

with

$$\begin{aligned} \hat{\boldsymbol{\eta}}_i(t) &= \hat{\boldsymbol{\theta}}_i(t) \|\mathbf{x}(t)\|_{\infty} + \hat{\boldsymbol{\sigma}}_i(t), \quad i = 1, 2, \\ \hat{\boldsymbol{\eta}}_{2m}(s) &= \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \hat{\boldsymbol{\eta}}_2(s), \\ \mathbf{H}_m(s) &= \mathbf{C}^T (s\mathbf{I} - \mathbf{A}_m)^{-1} \mathbf{B}, \\ \mathbf{H}_{um}(s) &= \mathbf{C}^T (s\mathbf{I} - \mathbf{A}_m)^{-1}. \end{aligned} \quad (25)$$

Furthermore, $\mathbf{K} \in \mathbb{R}^{2 \times 2}$ is the feedback gain matrix, and $\mathbf{D}(s)$ is a 2×2 strictly proper transfer matrix leading to a strictly proper stable transfer function:

$$\mathbf{C}_f(s) = \mathbf{K} \mathbf{D}(s) [\mathbf{I} + \mathbf{K} \mathbf{D}(s)]^{-1} \quad (26)$$

with DC gain $\mathbf{C}_f(0) = \mathbf{I}$. The choice of $\mathbf{D}(s)$ needs to ensure that $\mathbf{C}_f(s) \mathbf{H}_m^{-1}(s)$ is proper and stable. For the proofs of stability and performance bounds, the choice of $\mathbf{K}$ and $\mathbf{D}(s)$ also needs to ensure that there exists a constant $\rho_r \in \mathbb{R}^+$ such that $\mathcal{L}_1$ -norm condition

$$\begin{aligned} &\|\mathbf{G}_m(s)\|_{\mathcal{L}_1} + \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} \ell_0 \\ &< \frac{\rho_r - \|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{K}_g\|_{\mathcal{L}_1} \|\mathbf{r}\|_{\mathcal{L}_{\infty}}}{L_{1\rho_r} \rho_r + b_{\max}}, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \mathbf{G}_m(s) &= \mathbf{H}(s) [\mathbf{I} - \mathbf{C}_f(s)], \\ \mathbf{G}_{um}(s) &= (s\mathbf{I} - \mathbf{A}_m)^{-1} - \mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \end{aligned} \quad (28)$$

with $\mathbf{H}(s)$ and $\mathbf{H}_m(s)$ defined in (20) and (25), respectively. And

$$b_{\max} \triangleq \max \left\{ b_1, \frac{b_2}{\ell_0} \right\}, \quad \ell_0 \triangleq \frac{L_{2\rho_r}}{L_{1\rho_r}}, \quad (29)$$

where b_1, b_2 are introduced in Assumption 1, and $L_{i\rho_r}$ for $i = 1, 2$ are given by

$$L_{i\rho_r} \triangleq \frac{\bar{\rho}(\rho_r)}{\rho_r} d_{\bar{f}_{xi}}[\bar{\rho}(\rho_r)], \quad \bar{\rho}(\rho_r) \triangleq \rho_r + \bar{\varepsilon}, \quad (30)$$

where $d_{\bar{f}_{xi}}(\cdot)$ are introduced in Assumption 2 and $\bar{\varepsilon} \in \mathbb{R}^+$ is an arbitrary small constant. The above design procedure (i.e., (17)–(26)) yields the autopilot inner-loop architecture as shown in Figure 3.

To streamline the subsequent analysis, we need to introduce some notations. Let ρ introduced in Lemma 3 be defined as

$$\rho \triangleq \rho_r + \bar{\varepsilon} \quad (31)$$

and let ε be given by

$$\varepsilon \triangleq \frac{\|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1}}{1 - \|\mathbf{G}_m(s)\|_{\mathcal{L}_1} L_{1\rho_r} - \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} L_{2\rho_r}} \varepsilon_0 + \beta, \quad (32)$$

where $\varepsilon_0, \beta \in \mathbb{R}^+$ are arbitrary small constants such that $\varepsilon \leq \bar{\varepsilon}$. Next, define

$$\rho_u \triangleq \rho_{ur} + \varepsilon_u \quad (33)$$

FIGURE 3: Autopilot inner-loop architecture based on the LQR and $\mathcal{L}_1$ adaptive theory.

with ρ_{ur} and ε_u being defined as

$$\begin{aligned} \rho_{ur} &\triangleq \|C_f(s)\|_{\mathcal{L}_1} (L_{1\rho_r} \rho_r + b_1) \\ &+ \|C_f(s) H_m^{-1}(s) H_{um}(s)\|_{\mathcal{L}_1} (L_{2\rho_r} \rho_r + b_2) \\ &+ \|C_f(s) K_g\|_{\mathcal{L}_1} \|\mathbf{r}\|_{\mathcal{L}_\infty}, \\ \varepsilon_u &\triangleq (\|C_f(s)\|_{\mathcal{L}_1} L_{1\rho_r} \\ &+ \|C_f(s) H_m^{-1}(s) H_{um}(s)\|_{\mathcal{L}_1} L_{2\rho_r}) \varepsilon \\ &+ \|C_f(s) H_m^{-1}(s) C^T\|_{\mathcal{L}_1} \varepsilon_0. \end{aligned} \quad (34)$$

4. Performance Analysis of the Controller

4.1. Closed-Loop Reference System. In this section, we define the closed-loop reference system that the $\mathcal{L}_1$ adaptive controller tracks using the ideal nonadaptive case of the adaptive controller:

$$\begin{aligned} \dot{\mathbf{x}}_{\text{ref}}(t) &= \mathbf{A}_m \mathbf{x}_{\text{ref}}(t) + \mathbf{B} [\mathbf{u}_{\text{ref}}(t) + \bar{\mathbf{f}}_1(t, \mathbf{x}_{\text{ref}}(t))] \\ &+ \bar{\mathbf{f}}_2(t, \mathbf{x}_{\text{ref}}(t)), \quad \mathbf{x}_{\text{ref}}(0) = \mathbf{x}_0, \\ \mathbf{u}_{\text{ref}}(s) &= -\mathbf{C}_f(s) \\ &\cdot [\boldsymbol{\eta}_{1\text{ref}}(s) + \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \boldsymbol{\eta}_{2\text{ref}}(s) - \mathbf{K}_g \mathbf{r}(s)], \\ \mathbf{y}_{\text{ref}} &= \mathbf{C}^T \mathbf{x}_{\text{ref}}(t), \end{aligned} \quad (35)$$

where $\boldsymbol{\eta}_{1\text{ref}}(s)$ and $\boldsymbol{\eta}_{2\text{ref}}(s)$ denote the Laplace transforms of $\boldsymbol{\eta}_{i\text{ref}}(t) \triangleq \bar{\mathbf{f}}_i(t, \mathbf{x}_{\text{ref}}(t))$ for $i = 1, 2$.

Theorem 7. For system (35), if the $\mathcal{L}_1$ -norm condition in (27) holds, one has

$$\|\mathbf{x}_{\text{ref}}\|_{\mathcal{L}_\infty} < \rho_r \quad (36)$$

$$\|\mathbf{u}_{\text{ref}}\|_{\mathcal{L}_\infty} < \rho_{ur}. \quad (37)$$

Proof. From system (35), it follows that

$$\begin{aligned} \mathbf{x}_{\text{ref}}(s) &= \mathbf{G}_m(s) \boldsymbol{\eta}_{1\text{ref}}(s) + \mathbf{G}_{um}(s) \boldsymbol{\eta}_{2\text{ref}}(s) \\ &+ \mathbf{H}(s) \mathbf{C}_f(s) \mathbf{K}_g \mathbf{r}(s). \end{aligned} \quad (38)$$

Provided that the bound in (36) is not true, since $\mathbf{x}_{\text{ref}}(0) = \mathbf{x}_0 = \mathbf{0}$, then there must exist time τ such that

$$\begin{aligned} \|\mathbf{x}_{\text{ref}}(t)\|_{\infty} &< \rho_r, \quad \forall t \in [0, \tau), \\ \|\mathbf{x}_{\text{ref}}(\tau)\|_{\infty} &= \rho_r \end{aligned} \quad (39)$$

which implies that

$$\|\mathbf{x}_{\text{ref}\tau}\|_{\mathcal{L}_\infty} = \rho_r. \quad (40)$$

From Assumptions 1 and 2, together with (30), it follows that

$$\|\boldsymbol{\eta}_{i\text{ref}\tau}\|_{\mathcal{L}_\infty} \leq L_{i\rho_r} \rho_r + b_i, \quad i = 1, 2. \quad (41)$$

From (38) and (41), based on Lemma 4, we have

$$\begin{aligned} \|\mathbf{x}_{\text{ref}\tau}\|_{\mathcal{L}_\infty} &\leq \|\mathbf{G}_m(s)\|_{\mathcal{L}_1} (L_{1\rho_r} \rho_r + b_1) \\ &+ \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} (L_{2\rho_r} \rho_r + b_2) \\ &+ \|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{K}_g\|_{\mathcal{L}_1} \|\mathbf{r}\|_{\mathcal{L}_\infty}. \end{aligned} \quad (42)$$

Moreover, it follows from the $\mathcal{L}_1$ -norm condition in (27) that

$$\begin{aligned} &(\|\mathbf{G}_m(s)\|_{\mathcal{L}_1} + \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} \ell_0) (L_{1\rho_r} \rho_r + b_{\max}) \\ &+ \|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{K}_g\|_{\mathcal{L}_1} \|\mathbf{r}\|_{\mathcal{L}_\infty} < \rho_r. \end{aligned} \quad (43)$$

From (42) and (43), we obtain

$$\|\mathbf{x}_{\text{ref}\tau}\|_{\mathcal{L}_\infty} < \rho_r \quad (44)$$

which contradicts (40), thus proving the bound in (36). Also, this further means that the bounds in (41) hold all the time with strict inequality; that is,

$$\|\boldsymbol{\eta}_{i\text{ref}}\|_{\mathcal{L}_\infty} < L_{i\rho_r} \rho_r + b_i, \quad i = 1, 2. \quad (45)$$

Together with the definition of ρ_{ur} in (34), we can obtain the bound in (37). $\square$

4.2. Equivalent Linear Time-Varying System. Since $\mathbf{x}_0 = \mathbf{x}(0) = \mathbf{0}$ and $\mathbf{u}(0) = \mathbf{0}$, thus there always exists time τ such that $\|\mathbf{x}_\tau\|_{\mathcal{L}_\infty} \leq \rho$ and $\|\mathbf{u}_\tau\|_{\mathcal{L}_\infty} \leq \rho_u$. Then, based on Lemma 3, system (18) can be rewritten over $[0, \tau]$ as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_m \mathbf{x}(t) + \mathbf{B} [\mathbf{u}_{\text{ad}}(t) + \boldsymbol{\theta}_1(t) \|\mathbf{x}(t)\|_\infty + \boldsymbol{\sigma}_1(t)] \\ &\quad + \boldsymbol{\theta}_2(t) \|\mathbf{x}(t)\|_\infty + \boldsymbol{\sigma}_2(t), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ \mathbf{y}(t) &= \mathbf{C}^T \mathbf{x}(t),\end{aligned}\quad (46)$$

where $\boldsymbol{\theta}_1(t) \in \mathbb{R}^2$, $\boldsymbol{\sigma}_1(t) \in \mathbb{R}^2$, $\boldsymbol{\theta}_2(t) \in \mathbb{R}^4$, and $\boldsymbol{\sigma}_2(t) \in \mathbb{R}^4$ are unknown time-varying vectors.

4.3. Prediction Error Analysis. From systems (21) and (46), we can derive the prediction error dynamics:

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}_m \tilde{\mathbf{x}}(t) + \mathbf{B} \tilde{\boldsymbol{\eta}}_1(t) + \tilde{\boldsymbol{\eta}}_2(t), \\ \tilde{\mathbf{x}}(0) &= \mathbf{0},\end{aligned}\quad (47)$$

where $\tilde{\boldsymbol{\eta}}_i(t) = \hat{\boldsymbol{\eta}}_i(t) - \boldsymbol{\eta}_i(t)$ with $\hat{\boldsymbol{\eta}}_i(t)$ being introduced in (25) and $\boldsymbol{\eta}_i(t) = \boldsymbol{\theta}_i(t) \|\mathbf{x}(t)\|_\infty + \boldsymbol{\sigma}_i(t)$, $\tilde{\boldsymbol{\theta}}_i(t) = \hat{\boldsymbol{\theta}}_i(t) - \boldsymbol{\theta}_i(t)$, and $\tilde{\boldsymbol{\sigma}}_i(t) = \hat{\boldsymbol{\sigma}}_i(t) - \boldsymbol{\sigma}_i(t)$, $i = 1, 2$.

Let

$$\begin{aligned}\lambda &\triangleq 8 \left[(\theta_{b_1}^2 + \sigma_{b_1}^2) + 2(\theta_{b_2}^2 + \sigma_{b_2}^2) \right] + 8 \\ &\quad \cdot \frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{Q})} \left[\theta_{b_1} d_{\theta_1} + \sigma_{b_1} d_{\sigma_1} + 2(\theta_{b_2} d_{\theta_2} + \sigma_{b_2} d_{\sigma_2}) \right]\end{aligned}\quad (48)$$

with $\mathbf{P}, \mathbf{Q}$ being introduced in (22).

The following theorem will show that the prediction error $\tilde{\mathbf{x}}(t)$ can be systematically reduced through enlarging the adaptation gain.

Theorem 8. Consider the system in (18) with the $\mathcal{L}_1$ adaptive controller defined via (21)–(23), subject to the $\mathcal{L}_1$ -norm condition in (27); if the adaptation gain is chosen such that

$$\Gamma > \frac{\lambda}{\lambda_{\min}(\mathbf{P}) \varepsilon_0^2}, \quad (49)$$

then we have

$$\|\tilde{\mathbf{x}}_\tau\|_{\mathcal{L}_\infty} < \varepsilon_0, \quad (50)$$

where $\varepsilon_0 \in \mathbb{R}^+$ is an arbitrary small constant.

Proof. Choose the following Lyapunov function:

$$\begin{aligned}v(t) &= \tilde{\mathbf{x}}^T(t) \mathbf{P} \tilde{\mathbf{x}}(t) \\ &\quad + \frac{1}{\Gamma} \left[\sum_{i=1}^2 \left(\tilde{\boldsymbol{\theta}}_i^T(t) \tilde{\boldsymbol{\theta}}_i(t) + \tilde{\boldsymbol{\sigma}}_i^T(t) \tilde{\boldsymbol{\sigma}}_i(t) \right) \right].\end{aligned}\quad (51)$$

First, we prove that $v(t) \leq \lambda/\Gamma$. Taking the time derivative of $v(t)$, together with the adaptation laws in (22), yields

$$\begin{aligned}\dot{v}(t) &= \tilde{\mathbf{x}}^T(t) (\mathbf{A}_m^T \mathbf{P} + \mathbf{P} \mathbf{A}_m) \tilde{\mathbf{x}}(t) + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B} \tilde{\boldsymbol{\eta}}_1(t) \\ &\quad + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \tilde{\boldsymbol{\eta}}_2(t) \\ &\quad + \frac{2}{\Gamma} \left[\sum_{i=1}^2 \left(\tilde{\boldsymbol{\theta}}_i^T(t) \dot{\tilde{\boldsymbol{\theta}}}_i(t) + \tilde{\boldsymbol{\sigma}}_i^T(t) \dot{\tilde{\boldsymbol{\sigma}}}_i(t) \right) \right] \\ &\quad - \frac{2}{\Gamma} \left[\sum_{i=1}^2 \left(\tilde{\boldsymbol{\theta}}_i^T(t) \dot{\boldsymbol{\theta}}_i(t) + \tilde{\boldsymbol{\sigma}}_i^T(t) \dot{\boldsymbol{\sigma}}_i(t) \right) \right] \\ &= \tilde{\mathbf{x}}^T(t) (\mathbf{A}_m^T \mathbf{P} + \mathbf{P} \mathbf{A}_m) \tilde{\mathbf{x}}(t) \\ &\quad + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B} \tilde{\boldsymbol{\theta}}_1(t) \|\mathbf{x}(t)\|_\infty + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B} \tilde{\boldsymbol{\sigma}}_1(t) \\ &\quad + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \tilde{\boldsymbol{\theta}}_2(t) \|\mathbf{x}(t)\|_\infty + 2\tilde{\mathbf{x}}^T(t) \mathbf{P} \tilde{\boldsymbol{\sigma}}_2(t) \\ &\quad + 2\tilde{\boldsymbol{\theta}}_1^T(t) \text{Proj}(\hat{\boldsymbol{\theta}}_1(t), -(\tilde{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B})^T \|\mathbf{x}(t)\|_\infty) \\ &\quad + 2\tilde{\boldsymbol{\theta}}_2^T(t) \text{Proj}(\hat{\boldsymbol{\theta}}_2(t), -(\tilde{\mathbf{x}}^T(t) \mathbf{P})^T \|\mathbf{x}(t)\|_\infty) \\ &\quad + 2\tilde{\boldsymbol{\sigma}}_1^T(t) \text{Proj}(\hat{\boldsymbol{\sigma}}_1(t), -(\tilde{\mathbf{x}}^T(t) \mathbf{P} \mathbf{B})^T) \\ &\quad + 2\tilde{\boldsymbol{\sigma}}_2^T(t) \text{Proj}(\hat{\boldsymbol{\sigma}}_2(t), -(\tilde{\mathbf{x}}^T(t) \mathbf{P})^T) \\ &\quad - \frac{2}{\Gamma} \left[\sum_{i=1}^2 \left(\tilde{\boldsymbol{\theta}}_i^T(t) \dot{\tilde{\boldsymbol{\theta}}}_i(t) + \tilde{\boldsymbol{\sigma}}_i^T(t) \dot{\tilde{\boldsymbol{\sigma}}}_i(t) \right) \right] \\ &\leq -\tilde{\mathbf{x}}^T(t) \mathbf{Q} \tilde{\mathbf{x}}(t) \\ &\quad + \frac{2}{\Gamma} \left[\sum_{i=1}^2 \left(\left| \tilde{\boldsymbol{\theta}}_i^T(t) \dot{\tilde{\boldsymbol{\theta}}}_i(t) \right| + \left| \tilde{\boldsymbol{\sigma}}_i^T(t) \dot{\tilde{\boldsymbol{\sigma}}}_i(t) \right| \right) \right].\end{aligned}\quad (52)$$

From Lemma 3, $\boldsymbol{\theta}_i(t)$ and $\boldsymbol{\sigma}_i(t)$, $i = 1, 2$, in system (48) are considered such that

$$\begin{aligned}\|\boldsymbol{\theta}_i(t)\|_\infty &< \theta_{\rho i}, \\ \|\dot{\boldsymbol{\theta}}_i(t)\|_\infty &< d_{\theta i}, \\ \|\boldsymbol{\sigma}_i(t)\|_\infty &< \sigma_{b i}, \\ \|\dot{\boldsymbol{\sigma}}_i(t)\|_\infty &< d_{\sigma i}, \\ &\forall t \in [0, \tau]\end{aligned}\quad (53)$$

which implies that

$$\begin{aligned}\dot{v}(t) &\leq -\tilde{\mathbf{x}}^T(t) \mathbf{Q} \tilde{\mathbf{x}}(t) \\ &\quad + \frac{8}{\Gamma} \left[\theta_{\rho 1} d_{\theta 1} + \sigma_{b 1} d_{\sigma 1} + 2(\theta_{\rho 2} d_{\theta 2} + \sigma_{b 2} d_{\sigma 2}) \right].\end{aligned}\quad (54)$$

The projection operator ensures the following inequality:

$$\begin{aligned} \max_{t \in (0, \tau)} \left(\sum_{i=1}^2 \left(\tilde{\theta}_i^T(t) \tilde{\theta}_i(t) + \tilde{\sigma}_i^T(t) \tilde{\sigma}_i(t) \right) \right) \\ \leq 8 \left[(\theta_{b_1}^2 + \sigma_{b_1}^2) + 2 (\theta_{b_2}^2 + \sigma_{b_2}^2) \right]. \end{aligned} \quad (55)$$

If there exists a time $t_1 \in [0, \tau]$ such that $v(t_1) > \lambda/\Gamma$, then we obtain

$$\begin{aligned} \tilde{\mathbf{x}}^T(t_1) \mathbf{P} \tilde{\mathbf{x}}(t_1) \\ > \frac{8 \lambda_{\max}(\mathbf{P})}{\Gamma \lambda_{\min}(\mathbf{Q})} \left[\theta_{b_1} d_{\theta_1} + \sigma_{b_1} d_{\sigma_1} + 2 (\theta_{b_2} d_{\theta_2} + \sigma_{b_2} d_{\sigma_2}) \right] \end{aligned} \quad (56)$$

thus yielding

$$\begin{aligned} \tilde{\mathbf{x}}^T(t_1) \mathbf{Q} \tilde{\mathbf{x}}(t_1) \\ \geq \frac{\lambda_{\min}(\mathbf{Q})}{\lambda_{\max}(\mathbf{P})} \tilde{\mathbf{x}}^T(t_1) \mathbf{P} \tilde{\mathbf{x}}(t_1) \\ > \frac{8}{\Gamma} \left[\theta_{b_1} d_{\theta_1} + \sigma_{b_1} d_{\sigma_1} + 2 (\theta_{b_2} d_{\theta_2} + \sigma_{b_2} d_{\sigma_2}) \right]. \end{aligned} \quad (57)$$

From (54) and (57), we have $\dot{v}(t_1) < 0$. Notice that $(0) \leq \lambda/\Gamma$; hence

$$v(t) \leq \frac{\lambda}{\Gamma} \quad \text{with } \forall t \in [0, \tau]. \quad (58)$$

Moreover, since $\lambda_{\min}(\mathbf{P}) \|\tilde{\mathbf{x}}(t)\|_2^2 \leq \tilde{\mathbf{x}}^T(t) \mathbf{P} \tilde{\mathbf{x}}(t) \leq v(t)$, thus for $t \in [0, \tau]$

$$\|\tilde{\mathbf{x}}(t)\|_\infty^2 \leq \|\tilde{\mathbf{x}}(t)\|_2^2 \leq \frac{\lambda}{\lambda_{\min}(\mathbf{P}) \Gamma} \quad (59)$$

which implies that

$$\|\tilde{\mathbf{x}}_\tau\|_{\mathcal{L}_\infty} \leq \sqrt{\frac{\lambda}{\lambda_{\min}(\mathbf{P}) \Gamma}} \quad (60)$$

which, together with (49), yields the bound in (50). $\square$

4.4. Stability and Performance Bounds Analysis. In the following theorem, we will analyze the performance bounds of the actual closed-loop adaptive system with respect to the reference system.

Theorem 9. Consider the system in (18) with the $\mathcal{L}_1$ adaptive controller defined via (21)–(23), subject to the $\mathcal{L}_1$ -norm condition in (27). If the adaptation gain is chosen to satisfy the bound in (49), then it can be concluded that

$$\begin{aligned} \|\mathbf{x}\|_{\mathcal{L}_\infty} &< \rho, \\ \|\mathbf{u}_{ad}\|_{\mathcal{L}_\infty} &< \rho_u, \end{aligned} \quad (61)$$

$$\|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty} < \varepsilon_0,$$

$$\|\mathbf{x}_{ref} - \mathbf{x}\|_{\mathcal{L}_\infty} < \varepsilon, \quad (62)$$

$$\|\mathbf{u}_{ref} - \mathbf{u}_{ad}\|_{\mathcal{L}_\infty} < \varepsilon_u,$$

$$\|\mathbf{y}_{ref} - \mathbf{y}\|_{\mathcal{L}_\infty} < \|\mathbf{C}^T\|_\infty \varepsilon, \quad (63)$$

where ρ , ρ_u , ε , and ε_u are defined in (31)–(34).

Proof. Provided that the bounds in (62) are not true, since $\|\mathbf{x}_{ref}(0) - \mathbf{x}(0)\|_\infty = 0$, $\|\mathbf{u}_{ref}(0) - \mathbf{u}_{ad}(0)\|_\infty = 0$, then there must exist τ such that $\|\mathbf{x}_{ref}(\tau) - \mathbf{x}(\tau)\|_\infty = \varepsilon$ or $\|\mathbf{u}_{ref}(\tau) - \mathbf{u}_{ad}(\tau)\|_\infty = \varepsilon_u$, and furthermore $\|\mathbf{x}_{ref}(t) - \mathbf{x}(t)\|_\infty < \varepsilon$ and $\|\mathbf{u}_{ref}(t) - \mathbf{u}_{ad}(t)\|_\infty < \varepsilon_u$ with $\forall t \in [0, \tau]$. Then we obtain that at least one of the following equalities holds:

$$\begin{aligned} \|(\mathbf{x}_{ref} - \mathbf{x})_\tau\|_{\mathcal{L}_\infty} &= \varepsilon, \\ \|(\mathbf{u}_{ref} - \mathbf{u}_{ad})_\tau\|_{\mathcal{L}_\infty} &= \varepsilon_u. \end{aligned} \quad (64)$$

From Theorem 7 and (64), it follows that

$$\begin{aligned} \|\mathbf{x}_\tau\|_{\mathcal{L}_\infty} &< \rho, \\ \|\mathbf{u}_{ad\tau}\|_{\mathcal{L}_\infty} &< \rho_u. \end{aligned} \quad (65)$$

Additionally, while the adaptation gain is chosen as (49), Theorem 8 implies that

$$\|\tilde{\mathbf{x}}_\tau\|_{\mathcal{L}_\infty} < \varepsilon_0. \quad (66)$$

Next, let $\tilde{\boldsymbol{\eta}}(t) = \tilde{\boldsymbol{\eta}}_1(t) + \tilde{\boldsymbol{\eta}}_{2m}(t)$, where the Laplace transform of $\tilde{\boldsymbol{\eta}}_{2m}(t)$ is $\tilde{\boldsymbol{\eta}}_{2m}(s) = \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \tilde{\boldsymbol{\eta}}_2(s)$. Then it follows from (23) and the definition of $\mathbf{C}_f(s)$ in (28) that

$$\begin{aligned} \mathbf{u}_{ad}(s) &= -\mathbf{C}_f(s) \left[\boldsymbol{\eta}_1(s) + \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \boldsymbol{\eta}_2(s) \right. \\ &\quad \left. - \mathbf{K}_g \mathbf{r}(s) + \tilde{\boldsymbol{\eta}}(s) \right]. \end{aligned} \quad (67)$$

Substituting (67) into system (18) yields

$$\begin{aligned} \mathbf{x}(s) &= \mathbf{G}_m(s) \boldsymbol{\eta}_1(s) + \mathbf{G}_{um}(s) \boldsymbol{\eta}_2(s) \\ &\quad - \mathbf{H}(s) \mathbf{C}_f(s) \tilde{\boldsymbol{\eta}}(s) + \mathbf{H}(s) \mathbf{C}_f(s) \mathbf{K}_g \mathbf{r}(s). \end{aligned} \quad (68)$$

Then from (38) and (68), we get

$$\begin{aligned} \mathbf{x}_{ref}(s) - \mathbf{x}(s) &= \mathbf{G}_m(s) [\boldsymbol{\eta}_{1ref}(s) - \boldsymbol{\eta}_1(s)] \\ &\quad + \mathbf{G}_{um}(s) [\boldsymbol{\eta}_{2ref}(s) - \boldsymbol{\eta}_2(s)] \\ &\quad - \mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T \tilde{\mathbf{x}}(s). \end{aligned} \quad (69)$$

Hence,

$$\begin{aligned} \|(\mathbf{x}_{ref} - \mathbf{x})_\tau\|_{\mathcal{L}_\infty} \\ \leq \|\mathbf{G}_m(s)\|_{\mathcal{L}_1} \|\boldsymbol{\eta}_{1ref} - \boldsymbol{\eta}_1\|_{\mathcal{L}_\infty} \\ + \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} \|(\boldsymbol{\eta}_{2ref} - \boldsymbol{\eta}_2)_\tau\|_{\mathcal{L}_\infty} \\ + \|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1} \|\tilde{\mathbf{x}}_\tau\|_{\mathcal{L}_\infty}. \end{aligned} \quad (70)$$

Since, for $t \in [0, \tau]$,

$$\boldsymbol{\eta}_{iref}(t) - \boldsymbol{\eta}_i(t) = \bar{\mathbf{f}}_i(t, \mathbf{x}_{ref}(t)) - \bar{\mathbf{f}}_i(t, \mathbf{x}(t)), \quad i = 1, 2 \quad (71)$$

which, together with Assumption 2, leads to

$$\begin{aligned} \|(\boldsymbol{\eta}_{iref} - \boldsymbol{\eta}_i)_\tau\|_{\mathcal{L}_\infty} &\leq d_{\bar{f}_{xi}} [\bar{\rho}(\rho_r)] \|(\mathbf{x}_{iref} - \mathbf{x}_i)_\tau\|_{\mathcal{L}_\infty} \\ &< L_{i\rho_r} \|(\mathbf{x}_{iref} - \mathbf{x}_i)_\tau\|_{\mathcal{L}_\infty}. \end{aligned} \quad (72)$$

It follows from (70) and (72) that

$$\begin{aligned} & \|(\mathbf{x}_{\text{ref}} - \mathbf{x})_{\tau}\|_{\mathcal{L}_{\infty}} \\ & < \|\mathbf{G}_m(s)\|_{\mathcal{L}_1} L_{1\rho_r} \|(\mathbf{x}_{\text{ref}} - \mathbf{x})_{\tau}\|_{\mathcal{L}_{\infty}} \\ & + \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} L_{2\rho_r} \|(\mathbf{x}_{\text{ref}} - \mathbf{x})_{\tau}\|_{\mathcal{L}_{\infty}} \\ & + \|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1} \|\tilde{\mathbf{x}}_{\tau}\|_{\mathcal{L}_{\infty}}. \end{aligned} \quad (73)$$

The $\mathcal{L}_1$ -norm condition in (27) and the bound in (66) lead to

$$\begin{aligned} & \|(\mathbf{x}_{\text{ref}} - \mathbf{x})_{\tau}\|_{\mathcal{L}_{\infty}} \\ & < \frac{\|\mathbf{H}(s) \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1}}{1 - \|\mathbf{G}_m(s)\|_{\mathcal{L}_1} L_{1\rho_r} - \|\mathbf{G}_{um}(s)\|_{\mathcal{L}_1} L_{2\rho_r}} \varepsilon_0. \end{aligned} \quad (74)$$

Further, combining (74) with (32) yields

$$\|(\mathbf{x}_{\text{ref}} - \mathbf{x})_{\tau}\|_{\mathcal{L}_{\infty}} < \varepsilon - \beta < \varepsilon. \quad (75)$$

On the other aspect, from (35) and (67), we obtain

$$\begin{aligned} & \mathbf{u}_{\text{ref}}(s) - \mathbf{u}_{\text{ad}}(s) \\ & = -\mathbf{C}_f(s) [\boldsymbol{\eta}_{1\text{ref}}(s) - \boldsymbol{\eta}_1(s)] \\ & - \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) [\boldsymbol{\eta}_{2\text{ref}}(s) - \boldsymbol{\eta}_2(s)] \\ & + \mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T \tilde{\mathbf{x}}(s). \end{aligned} \quad (76)$$

The bounds in (66), (72), and (75), along with the definition of ε_u in (34), yield

$$\begin{aligned} & \|(\mathbf{u}_{\text{ref}} - \mathbf{u}_{\text{ad}})_{\tau}\|_{\mathcal{L}_{\infty}} < \left(\|\mathbf{C}_f(s)\|_{\mathcal{L}_1} L_{1\rho_r} \right. \\ & + \left. \|\mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \mathbf{C}^T\|_{\mathcal{L}_1} L_{2\rho_r} \right) (\varepsilon - \beta) \\ & + \|\mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1} \varepsilon_0 < \varepsilon_u. \end{aligned} \quad (77)$$

Obviously, the bounds in (75) and (77) contradict (64), which proves the bounds in (62). Thus, the bounds in (65) hold uniformly, which further implies that the bound in (66) also holds uniformly. This proves the bounds in (61). And the bound in (63) follows from the fact that $\mathbf{y}_{\text{ref}}(t) - \mathbf{y}(t) = \mathbf{C}^T [\mathbf{x}_{\text{ref}}(t) - \mathbf{x}(t)]$. $\square$

Remark 10. Note that if $\mathbf{C}_f(s) = \mathbf{I}_{2 \times 2}$, the $\mathcal{L}_1$ adaptive controller degenerates into the MRAC architecture. In that case $\|\mathbf{C}_f(s) \mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1}$ in (34) becomes $\|\mathbf{H}_m^{-1}(s) \mathbf{C}^T\|_{\mathcal{L}_1}$, which is not bounded. Thus, for the control signal in the MRAC architecture, one cannot reduce the bound of the control signal in (62) by increasing the adaptation gain.

Remark 11. It follows from Theorem 9 that the tracking error between $\mathbf{y}(t)$ and $\mathbf{y}_{\text{ref}}(t)$, as well as $\mathbf{u}(t)$ and $\mathbf{u}_{\text{ref}}(t)$, is uniformly bounded by a constant. One can achieve arbitrary close tracking performance by increasing Γ . Moreover, fast

adaptation can be obtained and the amount of gain scheduling can be reduced in the presence of high adaptation gains. From (23)–(26), we have

$$\mathbf{u}_{\text{ad}}(s) = -\mathbf{C}_f(s) \bar{\mathbf{r}}(s) \quad (78)$$

with $\bar{\mathbf{r}}(s) = \hat{\boldsymbol{\eta}}_1(s) + \hat{\boldsymbol{\eta}}_{2m}(s) - \mathbf{K}_g \mathbf{r}(s)$. The ability of the $\mathcal{L}_1$ control architecture to tolerate large adaptive gains is enabled through the low-pass filter $\mathbf{C}_f(s)$ in the feedback path that filters out the high frequencies in $\bar{\mathbf{r}}(s)$ excited by the large Γ . However, for the conventional MRAC strategy, large adaptation gains will lead to control signal oscillations.

Remark 12. We consider the following ideal control signal:

$$\mathbf{u}_{\text{id}}(s) = -\boldsymbol{\eta}_1(s) - \mathbf{H}_m^{-1}(s) \mathbf{H}_{um}(s) \boldsymbol{\eta}_2(s) + \mathbf{K}_g \mathbf{r}(s) \quad (79)$$

which leads to the desired system output response as shown in (19):

$$\mathbf{y}_{\text{id}}(s) = \mathbf{C}^T (s\mathbf{I} - \mathbf{A}_m)^{-1} \mathbf{B} \mathbf{K}_g \mathbf{r}(s) \quad (80)$$

by canceling the unknown nonlinearities exactly. In system (35), $\mathbf{u}_{\text{ref}}(t)$ is filtered by $\mathbf{C}_f(s)$. Obviously, the closed-loop reference system has a different response compared with the desired system output response in (80). As is discussed in [35, 36], the response of $\mathbf{y}_{\text{ref}}(t)$ can be made as close as desired to (80) by increasing the bandwidth of the low-pass filter $\mathbf{C}_f(s)$.

5. Simulations and Discussion

Take a certain type of transport aircraft for example. We simulate maximum load airdrop mission of the aircraft at the height of 82 ft ($m_c = 8,000$ kg), using single row single platform mode. The cargo is locked in the c.g. of the aircraft initially. The trim condition of the aircraft is $H_0 = 82$ ft, $V_0 = 229$ ft/s, $\alpha_0 = \theta_0 = 5.9813$ deg, the throttle opening is $\delta_p = 34\%$, the elevator deflection is $\delta_e = 0$ deg, the flap deflection is 25 deg, and the deflection of the horizontal stabilizer is -5.4093 deg. δ_e and δ_p are chosen as the control input with operations ranging from -25 deg to 25 deg and 0% to 100% , respectively. The extraction force is given by $F_p = m_c g \lambda$ [37], where λ is the parachute extraction ratio and set $\lambda = 0.2$.

By the linearized aircraft dynamics at the initial trim point, we use LQR control approach to render

$$\begin{aligned} \mathbf{A}_m &= \begin{bmatrix} -0.0022 & 8.1520 & 1.2717 & -2.8904 \\ -0.0039 & -0.5151 & 0.9844 & -0.0845 \\ 0.0019 & -1.3164 & -3.8647 & -4.2088 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} 4.4212 & 1.6137 \\ -0.0540 & -0.0014 \\ -2.6891 & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned} \quad (81)$$

and set the adaptation gain $\Gamma = 20000$, $\mathbf{D}(s) = (1/s)\mathbf{I}_{2 \times 2}$, and $\mathbf{K} = \text{diag}(10, 10)$. The outer-loop altitude-hold controller parameters are selected as $K_P = 0.013$, $K_I = 0.003$, and $K_D = 0.012$. For sure, the control law is implemented on the nonlinear model in (11) in the following simulations.

For implementation of airdrop missions, we have consulted mission performance requirements both at home and abroad. Following [17, 28], with the requirements of mission completeness and flight safety, the indexes parameters can be given as

- (i) $|\Delta H| \leq 45 \text{ ft}$ with H_0 denoting the initial trim altitude and $H_{\min} > 20 \text{ ft}$,
- (ii) $|\Delta V| \leq 0.13V_0$,
- (iii) $|\Delta \theta| \leq 5 \text{ deg}$ and $\theta > 2 \text{ deg}$,
- (iv) $\alpha_{\max} \leq 0.7\alpha_s$ with α_s denoting the critical angle of attack.

Remark 13. As is discussed above, the inner-loop tracking performance can be improved by increasing the bandwidth (i.e., $\mathbf{K}$) of the low-pass filter $\mathbf{C}_f(s)$. However, while $\mathbf{K} \rightarrow \infty$, then $\mathbf{C}_f(s) \rightarrow \mathbf{I}_{2 \times 2}$, and the $\mathcal{L}_1$ adaptive controller degenerates into the MRAC architecture. On the other hand, if $\mathbf{K}$ is too small, the control signal filtered by $\mathbf{C}_f(s)$ cannot cancel uncertainties perfectly. By lots of tests, we conclude that the components of $\mathbf{K}$ belong to the range of $[5 \ 16]$. The following experiments are conducted with $\mathbf{K} = \text{diag}(10, 10)$.

5.1. Control Performance Analysis. Simulation results of the dropping process with and without control laws are depicted in Figure 4. The blue solid lines in Figure 4 show that the heavy cargo moving inside and sudden dropping out impact the flight characteristics seriously. This process is similar to a strong downward force exerts on the aircraft tail, which makes the flight altitude rise continuously, the airspeed lose severely, and the maximum angle of attack is up to 10 deg. Taking about 2.8 seconds the cargo is dropped out. Almost at the same time, the angle of attack decreases suddenly and the airspeed keeps on losing which shows a trend of divergence. These phenomena are insufferable for the airdrop indexes thus exerting serious threats on the aircraft safety and mission performance.

Successful implementation of the control laws for the airdrop operation can be observed from the red dotted lines in Figure 4. The altitude increment is controlled in the range of 6 ft (i.e., $|\Delta H| \leq 6 \text{ ft}$) and after the cargo has dropped out 7 seconds the aircraft altitude is well maintained at the trim position. Besides, owing to the loss of heavy weight, the final angle of attack and pitch angle become smaller compared with the trim position. But the change value is less than 5 deg and the final pitch angle is greater than 2 deg, which could meet the requirements of the indexes. Furthermore, the elevator deflection and the throttle opening are well within the practicable range. In summary, the pitch-up motion caused by the cargo is completely suppressed through appropriately configuring the elevator and throttle.

5.2. System Robustness Analysis. The aircraft model, during the dropping process, may be further complicated by a multitude of uncertainties, such as the ground effect. So it is imperative for the flight control system to accommodate various system uncertainties. To verify system robustness, the following three cases are simulated and compared.

Case 1. Hypothesize that the aircraft motion model is accurate.

Case 2. Hypothesize that the aerodynamic coefficients exist with +20% uncertainty.

Case 3. Hypothesize that the aerodynamic coefficients exist with -20% uncertainty.

While using the same controller parameters in Section 5.1, the results are shown in Figure 5. We can see from the plots that the altitude increment is controlled in the range of 8 ft, which can be stabilized at the trim position after the cargo dropping out 7 seconds. The responses of the aircraft can meet the mission performance indexes for Cases 1 to 3.

We further test the robustness of the controller against time-varying uncertainties. Let the aerodynamic coefficients uncertainty be subjects to the following distribution:

$$\Delta \sim U(-0.2, 0.2), \quad (82)$$

where $U(\cdot)$ stands for the uniform distribution. We repeat the simulation fifty times using the Monte Carlo method, and the results are depicted in Figure 6. While the aerodynamic coefficients uncertainty varies randomly within $[-20\%, 20\%]$, the aircraft's altitude is controlled in the range of $[82 \text{ ft}, 88 \text{ ft}]$, the velocity increment is stabilized less than 0.03 ft/s, and the pitch angle as well as the angle of attack is converged within $[3.5 \text{ deg}, 6.5 \text{ deg}]$. All the states meet the airdrop mission performance indexes strictly, which verifies the robustness of the control method against time-varying uncertainties.

6. Conclusions

Heavyweight airdrop is facing a control system with large external disturbances and strong time-varying uncertainties. To achieve good stability and robustness characteristics, this paper developed a novel flight control system with dual-loop structure, including an outer-loop for altitude hold and an inner-loop for pitch attitude and airspeed control. The autopilot inner-loop is designed based on the LQR method and $\mathcal{L}_1$ adaptive theory, which can overcome the drawbacks of failing to meet guaranteed robustness with fast adaptation of conventional adaptive control approaches. Analysis shows that the controller can guarantee tracking performance and robustness in the presence of fast adaptation, without exciting control signal oscillations and gain scheduling. In view of mission performance, we have verified the delivery aircraft responses satisfy the airdrop technical indexes requirements well. The results will benefit future airdrop mission.

FIGURE 4: Aircraft responses contrast of the dropping process with and without control laws.

FIGURE 5: Aircraft responses with $\pm 20\%$ aerodynamic coefficients uncertainty.

Appendices

A. Expressions of f_i ($i=1, 2, 3$) and g_{ij} ($i=1, 2, 3$; $j=1, 2$) in System (11)

Consider

$$f_1(\mathbf{x}) = \frac{1}{m_b + m_c} \left[-m_c r_{cb} \sin \alpha f_3(\mathbf{x}) + T_0 \cos \alpha - \bar{Q}S(C_{D0} + C_{D\alpha}(\alpha - \alpha_0)) - m_b g \sin \gamma \right],$$

$$- \left(\Lambda_1 \cos \alpha - \Lambda_2 \sin \alpha + F_p \right),$$

$$f_2(\mathbf{x}) = \frac{1}{\Lambda_5} \left[-T_0 \sin \alpha - \bar{Q}S(C_{L0} + C_{L\alpha}(\alpha - \alpha_0)) \right.$$

$$+ m_b g \cos \gamma + \Lambda_1 \sin \alpha + \Lambda_2 \cos \alpha$$

$$\left. - \frac{\bar{Q}S c_A m_c r_{cb} \cos \alpha}{\Lambda_3} (C_{m0} + C_{m\alpha}(\alpha - \alpha_0)) \right]$$

FIGURE 6: Aircraft responses in the presence of time-varying aerodynamic coefficients uncertainty.

$$\begin{aligned}
& + C_{mq} \frac{q c_A}{2V} \Big) + \frac{m_c r_{cb}^2 \cos \alpha}{\Lambda_3} (\Lambda_2 - F_p \sin \alpha) \\
& + \frac{m_c^2 r_{cb}^2 \cos \alpha \sin \alpha}{(m_b + m_c) \Lambda_3} \overline{QS} (C_{D0} + C_{D\alpha} (\alpha - \alpha_0)) \\
& + \frac{m_c^2 r_{cb}^2 \cos^2 \alpha}{(m_b + m_c) \Lambda_3} \overline{QS} (C_{L0} + C_{L\alpha} (\alpha - \alpha_0)) \\
& + \frac{m_c^2 r_{cb}^2 \cos \alpha}{(m_b + m_c) \Lambda_3} (m_b g \sin \gamma \sin \alpha \\
& - m_b g \cos \gamma \cos \alpha - \Lambda_2 + F_p \sin \alpha) + (m_b \\
& + m_c) V q \Big], \\
f_3(\mathbf{x}) &= \frac{\overline{QS} c_A}{\Lambda_3} \left(C_{m0} + C_{m\alpha} (\alpha - \alpha_0) + C_{mq} \frac{q c_A}{2V} \right) \\
& + \frac{r_{cb}}{\Lambda_3} (\Lambda_2 - F_p \sin \alpha) + \frac{m_c r_{cb}}{(m_b + m_c) \Lambda_3} \left[\overline{QS} \sin \alpha \right. \\
& \cdot (C_{D0} + C_{D\alpha} (\alpha - \alpha_0)) + \overline{QS} \cos \alpha (C_{L0} \\
& + C_{L\alpha} (\alpha - \alpha_0)) + m_b g \sin \gamma \sin \alpha - m_b g \cos \gamma \\
& \cdot \cos \alpha - \Lambda_2 + F_p \sin \alpha \Big] + \frac{\overline{QS} c_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3} f_2(\mathbf{x}), \\
f_4(\mathbf{x}) &= q,
\end{aligned}$$

$$\begin{aligned}
g_{11} &= -\frac{1}{m_b + m_c} \left(m_c r_{cb} \sin \alpha \frac{\overline{QS} c_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3} g_{21} + m_c r_{cb} \right. \\
& \cdot \sin \alpha \Lambda_4 + \overline{QS} C_{D\delta_e} \Big),
\end{aligned}$$

$$\begin{aligned}
g_{12} &= \frac{1}{m_b + m_c} \left(T_m \cos \alpha - m_c r_{cb} \sin \alpha \frac{\overline{QS} c_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3} \right. \\
& \cdot g_{22} \Big),
\end{aligned}$$

$$g_{21} = -\frac{\overline{QS} C_{L\delta_e} + m_c r_{cb} \cos \alpha \Lambda_4}{\Lambda_5},$$

$$g_{22} = -\frac{T_m \sin \alpha}{\Lambda_5},$$

$$g_{31} = \frac{\overline{QS} c_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3} g_{21} + \Lambda_4,$$

$$g_{32} = \frac{\overline{QS} c_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3} g_{22},$$

$$g_{41} = g_{42} = 0,$$

$$\Lambda_1 = m_c g \sin \theta + m_c q^2 r_{cb} - m_c \frac{d^2 r_{cb}}{dt^2},$$

$$\Lambda_2 = m_c g \cos \theta - 2m_c q \frac{dr_{cb}}{dt},$$

$$\begin{aligned}
\Lambda_3 &= I_y + m_c r_{cb}^2 - \frac{m_c^2 r_{cb}^2}{m_b + m_c}, \\
\Lambda_4 &= \frac{m_c r_{cb}}{(m_b + m_c) \Lambda_3} (\overline{QSC}_{D\delta_e} \sin \alpha + \overline{QSC}_{L\delta_e} \cos \alpha) \\
&\quad + \frac{\overline{QSC}_A C_{m\delta_e}}{\Lambda_3}, \\
\Lambda_5 &= (m_b + m_c) V + m_c r_{cb} \cos \alpha \frac{\overline{QSC}_A^2 C_{m\dot{\alpha}}}{2V \Lambda_3}.
\end{aligned} \tag{A.1}$$

B. Definition of $\mathcal{L}_\infty$ -Norm and $\mathcal{L}_1$ -Norm

For a signal $\xi(t) \in \mathbb{R}^n$, $\mathcal{L}_\infty$ -norms are defined as

$$\|\xi\|_{\mathcal{L}_\infty} \triangleq \max_{1 \leq i \leq n} \left\{ \sup_{\tau \geq 0} |\xi_i(\tau)| \right\} < \infty, \tag{B.1}$$

where ξ_i is the i th component of ξ .

And $\mathcal{L}_1$ -norms are defined as

$$\|\xi\|_{\mathcal{L}_1} \triangleq \int_0^\infty \|\xi(\tau)\| d\tau < \infty, \tag{B.2}$$

where $\|\cdot\|$ denotes any of the vector norms.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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