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Research Article

Semiglobal Stabilization via Output-Feedback for a Class of Nontriangular Nonlinear Systems with an Unknown Coefficient

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This paper is devoted to the semiglobal stabilization via output-feedback for a class of uncertain nonlinear systems. Remark that the systems in question contain an unknown control coefficient which inherently depends on the system output and allow larger-than-two order growing unmeasurable states which is the obstruction of global stabilization via output-feedback. By introducing a recursive reduced-order observer and combining with saturated state estimate, a desired output-feedback controller is explicitly constructed for the systems. Under the appropriate choice of design parameters, the controller can make the closed-loop system semiglobally attractive and locally exponentially stable at the origin. A simulation example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

In this paper, we consider the semiglobal stabilization via output-feedback for a class of nonlinear systems described by

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(t, x), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= g(t, x)\, u + \phi_n(t, x), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}}$ is the system state with the initial value $x(t_0) = x_0$; $u \in \mathbf{R}$ and $y \in \mathbf{R}$ are the input and output of the system, respectively; $\phi_i : [t_0, +\infty) \times \mathbf{R}^n \to \mathbf{R}$, $i = 1, \ldots, n$, and $g : [t_0, +\infty) \times \mathbf{R}^n \to \mathbf{R}$ are unknown but continuous functions, called *nonlinearities* and *control coefficient* of the system, respectively. In what follows, suppose that only the output y is measurable.

Rigorously speaking, the control objective of this paper is, for any given constant $\rho_0 > 0$ (may be arbitrarily large), to seek a dynamic output-feedback controller for system (1) as follows:

$$\begin{aligned} \dot{\eta} &= \psi(\eta, y), \quad \eta \in \mathbf{R}^{n-1}, \\ u &= \varphi(\eta, y), \end{aligned} \tag{2}$$

where $\psi(\cdot)$ and $\varphi(\cdot)$ are continuous functions satisfying $\psi(0,0) = 0$ and $\varphi(0,0) = 0$, such that the closed-loop system is

(i) *semiglobally attractive*; that is, by starting from the compact set $\Gamma^x_{\rho_0} \times \Gamma^\eta_{\rho_0}$, all the trajectories of the closed-loop system converge to the origin, where $\Gamma^x_{\rho_0} = \{x \mid x \in [-\rho_0, \rho_0] \times \cdots \times [-\rho_0, \rho_0] \subset \mathbf{R}^n\}$ and $\Gamma^\eta_{\rho_0} = \{\eta \mid \eta \in [-\rho_0, \rho_0] \times \cdots \times [-\rho_0, \rho_0] \subset \mathbf{R}^{n-1}\}$;

(ii) *locally exponentially stable*; that is, the closed-loop system is locally exponentially stable at the origin $(x, \eta) = (0, 0)$.

As in [1, 2], we introduce the dilation $\delta_\varepsilon(x) = (x_1, \varepsilon x_2, \ldots, \varepsilon^{n-1} x_n)$ with ε being any positive constant. Based on this, the following assumptions are made on the nonlinearities and the control coefficient of system (1).

Assumption 1. There exist known positive continuous functions $g_0(y)$ and $g_1(y)$ such that, for any $\varepsilon \geq 1$,

$$g_0(y) \leq g(t, \delta_\varepsilon(x)) \leq g_1(y). \tag{3}$$

Assumption 2. There exist a known constant $\sigma \in [0,1)$ and known nonnegative continuous function $c_i(x)$, $i = 1,\dots,n$, such that, for any $\varepsilon \geq 1$,

$$\phi_i(t,\delta_\varepsilon(x)) \leq \varepsilon^{i-1+\sigma} c_i(x)\left(|x_1| + \cdots + |x_n|\right), \quad i = 1,\dots,n. \tag{4}$$

For nonlinear systems, semiglobal stabilization via output-feedback has attracted a great deal of attention (see, e.g., [3–13] and references therein), since it has a control objective meeting the needs of practical application and, compared to the global case, it requires rather weak restrictions on the system nonlinearities. We would like to stress that our problem is nontrivial and rather difficult to solve, which is mainly due to the generality of Assumptions 1 and 2. On the one hand, system (1) allows additional unknowns and nonlinearities in the control coefficient. Assumption 1 shows that the control coefficient of system (1) is unknown and inherently depends on the system output, essentially different from [7–10, 14–16] where the control coefficients are known/unknown constants or known functions of the system output. On the other hand, system (1) can be of nontriangular structure and has inherent nonlinearities. Assumption 2 indicates that, for any $i = 1,\dots,n$, $\phi_i(\cdot)$ inherently depends on all the states, rather than merely on $x_1,\dots,x_i$ like [8, 10, 11]. Also, due to the presence of $c_i(\cdot)$'s, system (1) allows more serious nonlinearities than those in [10, 11, 17]. Particularly, in [10], the system nonlinearities $\psi_i(\cdot)$'s are required to satisfy

$$|\psi_i(t,\eta,u)| \leq \mu_{\psi_i}(y)\sum_{j=1}^{i}|\eta_j| + \nu_{\psi_i}(y)\sum_{j=2}^{i-1}|\eta_j|^{(i-1)/(j-1)}, \quad i = 1,\dots,n, \tag{5}$$

where $\mu_{\psi_i}(\cdot)$, $i = 1,\dots,n$, and $\nu_{\psi_i}(\cdot)$, $i = 3,\dots,n$, are known nonnegative smooth functions and $\nu_{\psi_1}(\cdot) = \nu_{\psi_2}(\cdot) \equiv 0$. This is a special case of Assumption 2 with $c_i(x) \geq \mu_{\psi_i}(y) + \nu_{\psi_i}(y)\sum_{j=1}^{i}|x_j|^{(i-1)/(j-1)-1}$, $i = 1,\dots,n$. Therefore, one can see that the nonlinearities $\phi_i(\cdot)$'s of system (1) permit larger-than-two order growing unmeasurable states, which is the obstruction of global stabilization via output-feedback.

In this paper, a semiglobal stabilization scheme via output-feedback is proposed for uncertain nontriangular nonlinear system (1) with serious nonlinearities and the unknown control coefficient depending on the system output. Specifically, a state-feedback controller is first constructed for a nominal system (where $\phi_i(\cdot) \equiv 0$, $i = 1,\dots,n$), which ensures that the closed-loop system (corresponding to the nominal system) is globally exponentially stable. Then, a recursive reduced-order observer is introduced to recover the unmeasurable states of system (1). Based on these and combining with saturated state estimate [3, 4], a semiglobal output-feedback stabilizer is explicitly constructed for system (1). By appropriately choosing design parameters, the controller can guarantee that the closed-loop system (corresponding to system (1)) is semiglobally attractive and locally exponentially stable at the origin.

The remainder of this paper is organized as follows. Section 2 presents semiglobal output-feedback control design for system (1). Section 3 provides the main results and the rigorous performance analysis of the closed-loop system. Section 4 gives a numerical example to illustrate effectiveness of the proposed method. Section 5 addresses some concluding remarks. This paper ends with an appendix that collects two proofs of important propositions.

2. Semiglobal Output-Feedback Control

The section is to design a semiglobal stabilizer via output-feedback for system (1) under Assumptions 1 and 2.

To achieve this, we introduce the coordinate transformation

$$z_i = \frac{x_i}{L^{i-1}}, \quad i = 1,\dots,n, \qquad v = \frac{u}{L^n}, \tag{6}$$

which changes system (1) into the following:

$$\begin{aligned} \dot{z}_i &= Lz_{i+1} + \frac{\phi_i(t,\delta_L(z))}{L^{i-1}}, \quad i = 1,\dots,n-1, \\ \dot{z}_n &= Lg(t,\delta_L(z))v + \frac{\phi_n(t,\delta_L(z))}{L^{n-1}}, \\ y &= z_1, \end{aligned} \tag{7}$$

where $z = [z_1,\dots,z_n]^{\mathrm{T}}$ and $L \geq 1$ is a design parameter to be determined later.

For simplicity, we denote $\tilde{g}(t,z) = g(t,\delta_L(z))$ and $\tilde{\phi}_i(t,z) = \phi_i(t,\delta_L(z))/L^{i-1}$, $i = 1,\dots,n$. By Assumption 2, it can be verified that, for $i = 1,\dots,n$,

$$\tilde{\phi}_i(t,z) \leq L^{\sigma} c_i(z)\left(|z_1| + \cdots + |z_n|\right). \tag{8}$$

Moreover, it is worth stressing that the semiglobal stabilization of system (1) is implied by that of system (7). In the sequel, we turn to the controller design of system (7).

We first establish the following proposition, which gives a state-feedback controller for system (7) without considering the nonlinearities $\tilde{\phi}_i(\cdot)$'s.

Proposition 3. *Under Assumption 1, consider the following nominal system:*

$$\begin{aligned} \dot{z}_i &= Lz_{i+1}, \quad i = 1,\dots,n-1, \\ \dot{z}_n &= L\tilde{g}(t,z)\tilde{v}. \end{aligned} \tag{9}$$

There exist positive constants k_i, $i = 1,\dots,n$, such that system (9) is globally exponentially stabilized by the state-feedback controller:

$$\tilde{v}(z) = -\frac{k_n}{g_0(z_1)}\left(z_n + k_{n-1}z_{n-1} + \cdots + k_1 z_1\right). \tag{10}$$

Proof. Choose constants k_i, $i = 1,\dots,n-1$, such that polynomial $p(s) = s^{n-1} + k_{n-1}s^{n-2} + \cdots + k_2 s + k_1$ is Hurwitz. Then, we rewrite system (9) as

$$\begin{aligned} \dot{z}_{[n-1]} &= L(A_1 - B_1K_1)z_{[n-1]} + LB_1(z_n + K_1 z_{[n-1]}), \\ \dot{z}_n &= L\tilde{g}(t,z)\tilde{v}, \end{aligned} \tag{11}$$

where $z_{[n-1]} = [z_1, \ldots, z_{n-1}]^{\mathrm{T}}$, $K_1 = [k_1, \ldots, k_{n-1}]$,

$$A_1 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbf{R}^{(n-1)\times(n-1)}, \qquad B_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbf{R}^{(n-1)\times 1}. \tag{12}$$

Noting that $A_1 - B_1K_1$ is a Hurwitz matrix, there exists a positive definite matrix such that

$$(A_1 - B_1K_1)^{\mathrm{T}} P_1 + P_1 (A_1 - B_1K_1) \le -2I_{n-1}. \tag{13}$$

Then, we define the Lyapunov function

$$V_z(z) = z_{[n-1]}^{\mathrm{T}} P_1 z_{[n-1]} + \frac{(z_n + K_1 z_{[n-1]})^2}{2}, \tag{14}$$

whose derivative along system (11) is as follows:

$$\begin{aligned} \dot{V}_z(z) \le & -2L\|z_{[n-1]}\|^2 + 2Lz_{[n-1]}^{\mathrm{T}} P_1 B_1 (z_n + K_1 z_{[n-1]}) \\ & + L(z_n + K_1 z_{[n-1]})\, \tilde{g}(t,z)\, \tilde{v} \\ & + (z_n + K_1 z_{[n-1]}) \\ & \cdot K_1 L((A_1 - B_1K_1) z_{[n-1]} + B_1 (z_n + K_1 z_{[n-1]})) \\ \le & -2L\|z_{[n-1]}\|^2 + L(2\|P_1B_1\| + \|K_1(A_1 - B_1K_1)\|) \\ & \cdot \|z_{[n-1]}\| \, |z_n + K_1 z_{[n-1]}| + Lk_{n-1}(z_n + K_1 z_{[n-1]})^2 \\ & + L\tilde{g}(t,z)(z_n + K_1 z_{[n-1]})\, \tilde{v}. \end{aligned} \tag{15}$$

Using Young's inequality, we have

$$\begin{aligned} & (2\|P_1B_1\| + \|K_1(A_1 - B_1K_1)\|)\, \|z_{[n-1]}\| \, |z_n + K_1 z_{[n-1]}| \\ & \quad \le \|z_{[n-1]}\|^2 + C|z_n + K_1 z_{[n-1]}|^2, \end{aligned} \tag{16}$$

where $C = (2\|P_1B_1\| + \|K_1(A_1 - B_1K_1)\|)^2/4$. Substituting the above estimation into (15) yields

$$\begin{aligned} \dot{V}_z(z) \le & -L\|z_{[n-1]}\|^2 + L(k_{n-1} + C)(z_n + K_1 z_{[n-1]})^2 \\ & + L\tilde{g}(t,z)(z_n + K_1 z_{[n-1]})\, \tilde{v}. \end{aligned} \tag{17}$$

Choose $k_n \ge k_{n-1} + C + 1$ and construct the state feedback controller

$$\tilde{v}(z) = -\frac{k_n}{g_0(z_1)} (z_n + K_1 z_{[n-1]}). \tag{18}$$

Then, by Assumption 1, we derive

$$\dot{V}_z(z) \le -L\|z_{[n-1]}\|^2 - L(z_n + K_1 z_{[n-1]})^2, \tag{19}$$

which, together with (14), implies that the closed-loop system consisting of (9) and (10) is globally exponentially stable.

This completes the proof. □

Based on controller (10) of nominal system (9), we construct a desired output-feedback controller for system (7). Motivated by [8, 18, 19], we introduce the following recursive reduced-order observer to recover unmeasurable states z_i, $i = 2, \ldots, n$, of system (7):

$$\begin{aligned} \dot{\eta}_2 &= -L(\ell_2\eta_2 + \ell_2^2 y), & \hat{z}_2 &= \eta_2 + \ell_2\hat{z}_1, \\ \dot{\eta}_3 &= -L(\ell_3\eta_3 + \ell_3^2\hat{z}_2), & \hat{z}_3 &= \eta_3 + \ell_3\hat{z}_2, \\ &\vdots & &\vdots \\ \dot{\eta}_n &= -L(\ell_n\eta_n + \ell_n^2\hat{z}_{n-1}), & \hat{z}_n &= \eta_n + \ell_n\hat{z}_{n-1}, \end{aligned} \tag{20}$$

where $\hat{z}_1 = z_1$ and $\ell_i > 1$, $i = 2, \ldots, n$, are the gains to be determined later. For the later use, we denote $\hat{z} = [\hat{z}_1, \ldots, \hat{z}_n]^{\mathrm{T}}$ and $\eta = [\eta_2, \ldots, \eta_n]^{\mathrm{T}}$.

Furthermore, using saturated state estimate, we construct the following output-feedback controller for system (7):

$$\begin{aligned} v(\hat{z}) = & -\frac{k_n}{g_0(\hat{z}_1)} (\mathbf{sat}(\hat{z}_n) + k_{n-1}\mathbf{sat}(\hat{z}_{n-1}) \\ & + \cdots + k_2\mathbf{sat}(\hat{z}_2) + k_1\hat{z}_1), \end{aligned} \tag{21}$$

where k_i, $i = 1, \ldots, n$, are the same as those in Proposition 3; the saturation function $\mathbf{sat} : \mathbf{R} \to [-M, M]$ is defined as

$$\mathbf{sat}(a) = \begin{cases} -M, & a < -M, \\ a, & |a| \le M, \\ M, & a > M, \end{cases} \tag{22}$$

$$M \ge \max_{z\in\Omega_z} \|z\|_\infty, \tag{23}$$

with $\Omega_z = \{z \in \mathbf{R}^n \mid V_z(z) \le 1 + \max_{z\in\Gamma_{\rho_0}^z} V_z(z)\}$, $\Gamma_{\rho_0}^z = \{z \mid z \in [-\rho_0, \rho_0] \times \cdots \times [-\rho_0, \rho_0] \subset \mathbf{R}^n\}$, and $\|z\|_\infty$ standing for the ∞-norm of vector z.

3. Main Results

This section is devoted to the performance analysis of the closed-loop system consisting of (7), (20), and (21) and summarizes the main results of this paper.

Define $e_i = z_i - \ell_i z_{i-1} - \eta_i$, $i = 2, \ldots, n$, and denote $e = [e_2, \ldots, e_n]^{\mathrm{T}}$. Then, by (7) and (20), we deduce

$$\begin{aligned} \dot{e}_i &= Lz_{i+1} + \tilde{\phi}_i(\cdot) - L\ell_i e_i - \ell_i\tilde{\phi}_{i-1}(\cdot) \\ &\quad - L\ell_i^2(z_{i-1} - \hat{z}_{i-1}), \quad i = 2, \ldots, n-1, \\ \dot{e}_n &= L\tilde{g}(t,z)\, v(\hat{z}) + \tilde{\phi}_n(\cdot) - L\ell_n e_n - \ell_n\tilde{\phi}_{n-1}(\cdot) \\ &\quad - L\ell_n^2(z_{n-1} - \hat{z}_{n-1}). \end{aligned} \tag{24}$$

Define the Lyapunov function $V_e(e) = \sum_{i=2}^{n}(e_i^2/2)$ and choose the same Lyapunov function $V_z(z)$ used in Proposition 3. On the set $\Gamma_M^z \times \mathbf{R}^{n-1} = \{(z,\eta) \mid z \in [-M,M] \times \cdots \times [-M,M] \subset \mathbf{R}^n,\ \eta \in \mathbf{R}^{n-1}\}$, there hold Propositions 4 and 5 (whose detailed proofs are given in Appendices A and B, resp.), which will play a key role in the later performance analysis.

Proposition 4. *For system (7) on the set $\Gamma_M^z \times \mathbf{R}^{n-1}$, there exist constants $c > 0$ and $d_1 > 0$, such that*

$$\begin{aligned}\dot{V}_z(z)\big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} &\le -\frac{cL}{2}\|z\|^2 + L^\sigma d_1\|z\|^2 \\ &\quad + L\min\left\{\widehat{\alpha}_n e_n^2 + \sum_{i=2}^{n-1}\widehat{\alpha}_i(\ell_{i+1},\ldots,\ell_n)e_i^2, \widehat{\alpha}_n\right\},\end{aligned} \tag{25}$$

where $\widehat{\alpha}_n \ge 1$ is a constant independent of ℓ_i's and $\widehat{\alpha}_i(\ell_{i+1},\ldots,\ell_n) \ge \widehat{\alpha}_n$, $i = 2,\ldots,n-1$, are certain polynomial functions of their arguments.

Proposition 5. *For system (24) on the set $\Gamma_M^z \times \mathbf{R}^{n-1}$ and constant $c > 0$ given in Proposition 4, there holds*

$$\begin{aligned}\dot{V}_e(e)\big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} &\le \frac{cL}{4}\|z\|^2 - L\left(\ell_n - \widetilde{\alpha}_n - \frac{2}{c}\right)e_n^2 \\ &\quad - L\sum_{i=2}^{n-1}\left(\ell_i - \widetilde{\alpha}_i(\ell_{i+1},\ldots,\ell_n) - \frac{2}{c}\right)e_i^2 \\ &\quad + \sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i\widetilde{\phi}_{i-1}(\cdot)\right|,\end{aligned} \tag{26}$$

where $\widetilde{\alpha}_n > 0$ is a constant independent of ℓ_i's and $\widetilde{\alpha}_i(\ell_{i+1},\ldots,\ell_n) > 0$, $i = 2,\ldots,n-1$, are certain polynomial functions of their arguments.

In view of (25) and (26), we recursively determine the observer gains ℓ_i's as follows:

$$\begin{aligned}\ell_n(\ell) &= \ell\widehat{\alpha}_n + \widetilde{\alpha}_n + \frac{2}{c}, \\ \ell_i(\ell) &= \ell\widehat{\alpha}_i(\ell_{i+1},\ldots,\ell_n) + \widetilde{\alpha}_i(\ell_{i+1},\ldots,\ell_n) + \frac{2}{c}, \\ &\qquad i = n-1,\ldots,2,\end{aligned} \tag{27}$$

such that

$$\dot{V}_e(e) \le \frac{cL}{4}\|z\|^2 - L\ell W_e(e) + \sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i\widetilde{\phi}_{i-1}(\cdot)\right|, \tag{28}$$

where ℓ is a positive constant to be determined later and $W_e(e) = \widehat{\alpha}_n e_n^2 + \sum_{i=2}^{n-1}\widehat{\alpha}_i(\ell_{i+1},\ldots,\ell_n)e_i^2$.

Now, we are ready to address the main results of this paper, which are summarized in the following theorem.

Theorem 6. *Consider system (1) under Assumptions 1 and 2. For any given constant $\rho_0 > 0$ (may be arbitrarily large), there exist appropriate L and ℓ_i, $i = 2,\ldots,n$, depending on ρ_0, such that the closed-loop system, consisting of (1), (20), and (21), is locally exponentially stable, and, by starting from the given compact set $\Gamma_{\rho_0}^x \times \Gamma_{\rho_0}^\eta$, all the trajectories of the closed-loop system converge to the origin.*

Proof. Motivated by [5, 6, 13], for the closed-loop system consisting of (7), (20), and (21), we choose Lyapunov function

$$V_{z,\eta}(z,\eta) = V_z(z) + \frac{\ln(1+V_e(e))}{\ln(1+\mu(\ell))}, \tag{29}$$

where $\mu(\ell) = (1/2)\sum_{i=2}^{n}(2\rho_0 + \ell_i(\ell)\rho_0)^2 \ge \max_{\Gamma_{\rho_0}^z\times\Gamma_{\rho_0}^\eta} V_e(e)$. From the definitions of $V_z(z)$ and $V_e(e)$, we see that $V_{z,\eta}(z,\eta)$ is positive definite with respect to (z,η). Moreover, by (25) and (28), we obtain that, on the set $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned}\dot{V}_{z,\eta}(z,\eta)\big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} \le &-\frac{cL}{2}\|z\|^2 + L^\sigma d_1\|z\|^2 \\ &+ L\min\{W_e(e),\widehat{\alpha}_n\} \\ &+ \frac{L}{\ln(1+\mu(\ell))(1+V_e(e))} \\ &\cdot\left(\frac{c}{4}\|z\|^2 - \ell W_e(e)\right) \\ &+ \frac{1}{\ln(1+\mu(\ell))(1+V_e(e))} \\ &\cdot\sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i\widetilde{\phi}_{i-1}(\cdot)\right|.\end{aligned} \tag{30}$$

From $\widehat{\alpha}_i(\cdot) \ge \widehat{\alpha}_n \ge 1$, $i = 2,\ldots,n-1$, it follows that

$$\begin{aligned}\frac{W_e(e)}{1+V_e(e)} &\ge \frac{\widehat{\alpha}_n(W_e(e)/\widehat{\alpha}_n)}{1+W_e(e)/\widehat{\alpha}_n} \ge \frac{\widehat{\alpha}_n}{2}\min\left\{\frac{W_e(e)}{\widehat{\alpha}_n},1\right\} \\ &\ge \frac{1}{2}\min\{W_e(e),\widehat{\alpha}_n\},\end{aligned} \tag{31}$$

which implies that, on the set $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned}\dot{V}_{z,\eta}(z,\eta)\big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} \le &-\frac{cL}{2}\|z\|^2 + L^\sigma d_1\|z\|^2 \\ &+ \frac{cL}{4\ln(1+\mu(\ell))}\|z\|^2 \\ &- L\left(\frac{\ell}{2\ln(1+\mu(\ell))} - 1\right) \\ &\cdot\min\{W_e(e),\widehat{\alpha}_n\} \\ &+ \frac{1}{\ln(1+\mu(\ell))} \\ &\cdot\sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i\widetilde{\phi}_{i-1}(\cdot)\right|.\end{aligned} \tag{32}$$

From the choice of $\ell_i(\ell)$'s, we see that $\ell_i(\ell)$'s are polynomial functions of ℓ. Then, we can find $\ell > 1$ to be sufficiently large such that

$$\frac{\ell}{2\ln(1+\mu(\ell))} > 2, \qquad \ln(1+\mu(\ell)) > 1, \tag{33}$$

which, together with (32), implies that, on the set $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned}\dot{V}_{z,\eta}(z,\eta)\big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} &\le -\frac{cL}{4}\|z\|^2 + L^\sigma d_1\|z\|^2 - L\min\{W_e(e),\widehat{\alpha}_n\} \\ &\quad + \sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i(\ell)\widetilde{\phi}_{i-1}(\cdot)\right|.\end{aligned} \tag{34}$$

Moreover, define the compact set $\Omega = \{(z,\eta) \mid V_{z,\eta}(z,\eta) \le 1+\max_{z\in\Gamma_{\rho_0}^z} V_z(z)\}$. Then, it is easy to verify the following relation:

$$\Gamma_{\rho_0}^z \times \Gamma_{\rho_0}^\eta \subset \Omega \subset \Gamma_M^z \times \mathbf{R}^{n-1}. \tag{35}$$

We now estimate the last term in the right-hand side of (34) on the compact set Ω. By (8) and Young's inequality and noting the boundedness of $c_i(z)$'s and $W_e(e)$ on Ω, there exist positive constants d_2 and d_3 depending on ℓ_i's, such that, on the set Ω,

$$\begin{aligned}&\sum_{i=2}^{n}|e_i|\left|\widetilde{\phi}_i(\cdot) - \ell_i(\ell)\widetilde{\phi}_{i-1}(\cdot)\right| \\ &\quad\le L^\sigma\sum_{i=2}^{n}\left|c_i(z) - \ell_i(\ell)c_{i-1}(z)\right||e_i|\sum_{j=1}^{n}|z_j| \\ &\quad\le L^\sigma d_2\|z\|^2 + L^\sigma d_3\min\{W_e(e),\widehat{\alpha}_n\}.\end{aligned} \tag{36}$$

Substituting this into (34), we obtain that, on the set Ω,

$$\begin{aligned}\dot{V}_{z,\eta}(z,\eta)\big|_{\Omega} \le &-\left(\frac{cL}{4} - L^\sigma d_1 - L^\sigma d_2\right)\|z\|^2 \\ &-(L - L^\sigma d_3)\min\{W_e(e),\widehat{\alpha}_n\}.\end{aligned} \tag{37}$$

By $0 \le \sigma < 1$, we can choose $L \ge 1$ satisfying

$$L^{1-\sigma} \ge \max\left\{\frac{d_1+d_2+1}{4c}, d_3+1\right\}. \tag{38}$$

Then, we obtain that, on the set Ω,

$$\dot{V}_{z,\eta}(z,\eta)\big|_{\Omega} \le -L^\sigma\|z\|^2 - L^\sigma\min\{W_e(e),\widehat{\alpha}_n\}. \tag{39}$$

By the definitions of η_i's and $W_e(e)$, we derive $\dot{V}_{z,\eta}(z,\eta) < 0$ on $\Omega \setminus \{(0,0)\}$, which implies that, by starting from $\Gamma_{\rho_0}^z \times \Gamma_{\rho_0}^\eta$, $(z(t),\eta(t))$ remains in the compact set Ω, and

$$\lim_{t\to+\infty}(z(t),\eta(t)) = (0,0). \tag{40}$$

By $W_e(0) = 0$ and the continuity of $W_e(e)$ on Ω, there exists a neighborhood $\Omega_0 \subseteq \Omega$ of the origin, such that $W_e(e) \le \widehat{\alpha}_n$ for all $(z,\eta) \in \Omega_0$. Then, by (39), we obtain that, on the set Ω_0,

$$\dot{V}_{z,\eta}(z,\eta)\big|_{\Omega_0} \le -L^\sigma\|z\|^2 - L^\sigma W_e(e). \tag{41}$$

By the definition of $V_e(e)$ and $W_e(e)$, we deduce that, on the set Ω_0,

$$W_e(e) \ge \mu_1 V_e(e) \ge \mu_2\frac{\ln(1+V_e(e))}{\ln(1+\mu(\ell))} \tag{42}$$

with some positive constants μ_1 and μ_2. Substituting this into (41) and noting the definitions of $V_z(z)$, we have that, on the set Ω_0,

$$\dot{V}_{z,\eta}(z,\eta)\big|_{\Omega_0} \le -L^\sigma\mu_3 V_{z,\eta}(z,\eta), \tag{43}$$

with positive constant μ_3. Therefore, the closed-loop system consisting of (7), (20), and (21) is locally exponentially stable.

By (6), (35) and noting $L \ge 1$, there holds

$$x \in \Gamma_{\rho_0}^x, \quad \eta \in \Gamma_{\rho_0}^\eta \Longrightarrow z \in \Gamma_{\rho_0}^z, \quad \eta \in \Gamma_{\rho_0}^\eta \Longrightarrow (z,\eta) \in \Omega. \tag{44}$$

Furthermore, by the invertibility of (6), we arrive at the fact that the closed-loop system, consisting of (1), (20), and (21), is locally exponentially stable, and, by starting from $\Gamma_{\rho_0}^x \times \Gamma_{\rho_0}^\eta$, the trajectories $(x(t),\eta(t))$ converge to the origin.

This completes the proof. □

4. A Simulation Example

In this section, we consider the following nonlinear system to illustrate the correctness and effectiveness of the theoretical results:

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1^{2/3}x_3^{1/3}, \\ \dot{x}_2 &= x_3 + x_1\ln(1+x_2^2), \\ \dot{x}_3 &= g(t,x)u + x_1, \\ y &= x_1,\end{aligned} \tag{45}$$

with $2(x_1^2+1) \le g(t,x) \le 4(x_1^2+1)$. It can be verified that system (45) satisfies Assumption 2 with $\sigma = 2/3$, $c_1(x) = 1$, and $c_2(x) = 2(1+x_2^2)$.

According to the design procedure of the state feedback controller, we choose $k_1 = 0.8$ and $k_2 = 1.2$ and then, by solving the matrix inequality (13), yield

$$P_1 = \begin{pmatrix} 1.6875 & -0.5 \\ -0.5 & 0.75 \end{pmatrix}. \tag{46}$$

Furthermore, by Proposition 3, we choose $k_3 = 5$.

Let $\rho_0 = 3$. Then, by the above P_1, (14), and (23), we choose $M = 6$. Moreover, from (33) and (38), we choose $\ell = 1.5$ and $L = 2$ and, in terms of (27), recursively determine $\ell_2 = 10$ and $\ell_3 = 8$. Thus, by the design procedure in Section 2, we construct the following observer:

$$\begin{aligned} \dot{\eta}_2 &= -2\left(10\eta_2 + 100y\right), \qquad \widehat{z}_2 = \eta_2 + 10\widehat{z}_1, \\ \dot{\eta}_3 &= -2\left(8\eta_3 + 64\widehat{z}_2\right), \qquad \widehat{z}_3 = \eta_3 + 8\widehat{z}_2, \end{aligned} \tag{47}$$

and design the following output-feedback controller:

$$v(\widehat{z}) = -\frac{5}{2\left(\widehat{z}_1^2 + 1\right)}\left(\mathbf{sat}\left(\widehat{z}_3\right) + 1.2\mathbf{sat}\left(\widehat{z}_2\right) + 0.8\widehat{z}_1\right). \tag{48}$$

Let $g(t,x) = (3 - \sin x_2)(x_1^2 + 1)$, and let the initial values of the states be $x(0) = [1, -0.5, -0.5]^{\mathrm{T}}$ and $\eta(0) = [1, -1]^{\mathrm{T}}$. Using MATLAB, Figures 1–3 are obtained to exhibit the trajectories of the closed-loop system states and the controller, from which the effectiveness of the design method is indeed demonstrated.

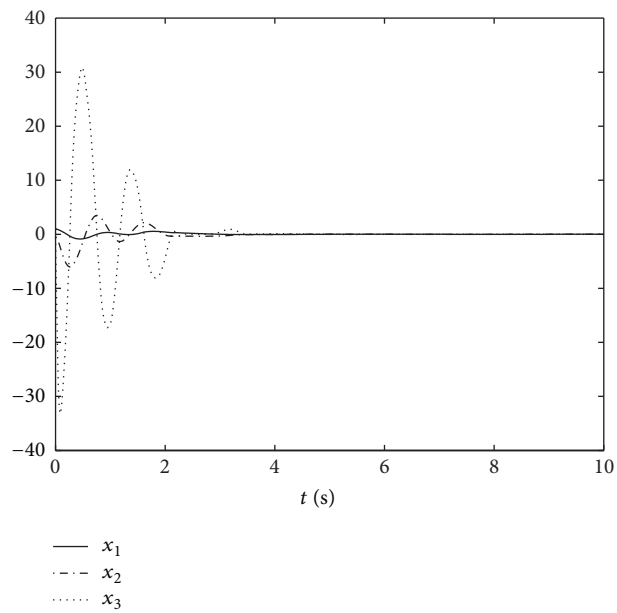

Figure 1: Trajectory of states x_1, x_2, and x_3.

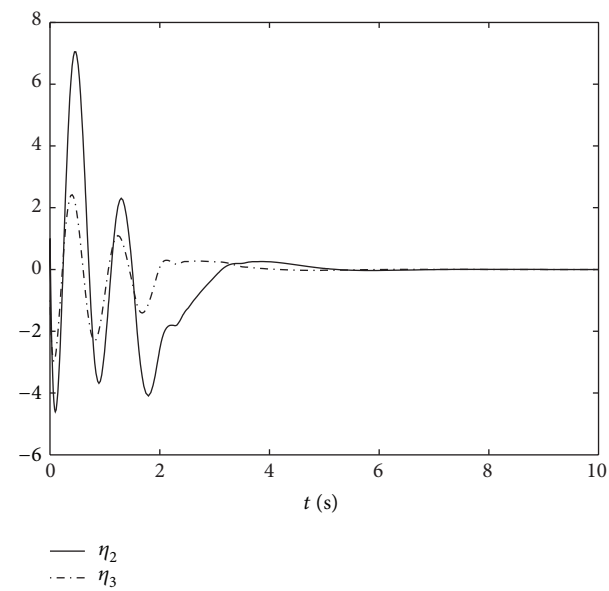

Figure 2: Trajectories of states η_2 and η_3.

5. Concluding Remarks

In this paper, the semiglobal stabilization via output-feedback has been investigated for a class of uncertain nontriangular nonlinear systems. Essentially different from the existing related works, the control coefficient of the system is unknown and inherently depends on the system output, and, hence, the scope of the nonlinear systems is considerably broadened. By introducing a recursive reduced-order observer and combining with saturated state estimate, a semiglobal output-feedback controller has been constructed for the uncertain system. Under the appropriate choice of design parameters, the controller can guarantee that the closed-loop system achieves semiglobal attractivity and locally exponential stability. Along this direction, another interesting research problem is how to design a semiglobal finite-time stabilizer via output-feedback for system (1).

Appendices

The appendix provides the rigorous proofs of Propositions 4 and 5, which are collected here for the sake of compactness.

A. The Proof of Proposition 4

By (7) and (19), we have

$$\begin{aligned} \dot{V}_z(z) \le & -L\left\|z_{[n-1]}\right\|^2 - L\left(z_n + K_1 z_{[n-1]}\right)^2 \\ & + L\widetilde{g}(t,z)\left|z_n + K_1 z_{[n-1]}\right|\left|v(\widehat{z}) - \widetilde{v}(z)\right| \\ & + \left|z_{[n-1]}^{\mathrm{T}} P_1 \widetilde{\phi}_{[n-1]}(\cdot)\right| \\ & + \left|z_n + K_1 z_{[n-1]}\right|\left|\widetilde{\phi}_n(\cdot) + K_1 \widetilde{\phi}_{[n-1]}(\cdot)\right|. \end{aligned} \tag{A.1}$$

In what follows, we estimate the right-hand side of (A.1). By $\|K_1\|^2\|z_{[n-1]}\|^2 \ge (K_1 z_{[n-1]})^2$, we obtain

$$\begin{aligned} & \left\|z_{[n-1]}\right\|^2 + \left(z_n + K_1 z_{[n-1]}\right)^2 \\ & \quad \ge \frac{1}{2}\left\|z_{[n-1]}\right\|^2 + \frac{1}{2\left\|K_1\right\|^2}\left(K_1 z_{[n-1]}\right)^2 + \left(z_n + K_1 z_{[n-1]}\right)^2 \\ & \quad \ge \frac{1}{2}\left\|z_{[n-1]}\right\|^2 + \frac{1}{2 + 4\left\|K_1\right\|^2} z_n^2 \\ & \quad \ge c\left\|z\right\|^2, \end{aligned} \tag{A.2}$$

where $c = 1/(2 + 4\|K_1\|^2)$.

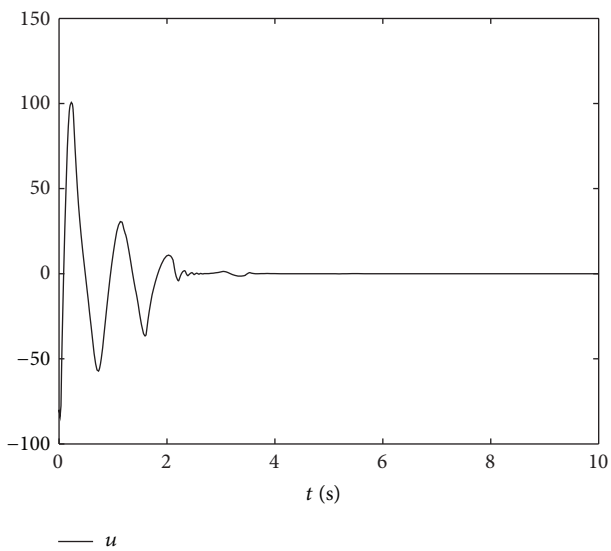

Figure 3: Trajectories of control u.

By the definitions of z_i's and e_i's, we have

$$z_i - \widehat{z}_i = e_i + \ell_i \left(z_{i-1} - \widehat{z}_{i-1}\right), \quad i = 2, \ldots, n, \tag{A.3}$$

which implies

$$\begin{gathered} \left|z_2 - \widehat{z}_2\right| = \left|e_2\right|, \\ \left|z_i - \widehat{z}_i\right| \le \left|e_i\right| + \ell_i \left|e_{i-1}\right| + \cdots + \ell_i \cdots \ell_3 \left|e_2\right|, \quad i = 3, \ldots, n. \end{gathered} \tag{A.4}$$

Using (10) and (21) yields

$$\left|v\left(\widehat{z}\right) - \widetilde{v}\left(z\right)\right| \le \frac{k_n \overline{k}}{g_0\left(z_1\right)} \sum_{i=2}^{n} \left|z_i - \mathbf{sat}\left(\widehat{z}_i\right)\right|, \tag{A.5}$$

where $\overline{k} = \max\{1, k_1, \ldots, k_{n-1}\}$. By this and (A.4) and noting that, for any $i = 2, \ldots, n$, $|z_i - \mathbf{sat}(\widehat{z}_i)| \le \min\{z_i - \widehat{z}_i, 2M\}$ on $\Gamma_M^z \times \mathbf{R}^{n-1}$, we can find a positive constant $\lambda_1 > 1$ such that, on $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned} &\left|v\left(\widehat{z}\right) - \widetilde{v}\left(z\right)\right|\big|_{\Gamma_M^z \times \mathbf{R}^{n-1}} \\ &\quad \le \frac{k_n \overline{k} \lambda_1}{g_0\left(z_1\right)} \min\left\{\left|e_n\right| + \sum_{i=2}^{n-1} \gamma_i\left(\ell_{i+1}, \ldots, \ell_n\right)\left|e_i\right|, 1\right\}, \end{aligned} \tag{A.6}$$

where $\gamma_i(\ell_{i+1}, \ldots, \ell_n) = 1 + \ell_{i+1} + \sum_{j=i+2}^{n}(\ell_{i+1} \cdots \ell_j)$, $i = 2, \ldots, n-1$. Then, by Assumption 1, we derive that, on $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned} &\widetilde{g}\left(t, z\right)\left|z_n + K_1 z_{[n-1]}\right|\left|v\left(\widehat{z}\right) - \widetilde{v}\left(z\right)\right| \\ &\quad \le \overline{k} g_1\left(z_1\right) \sum_{i=1}^{n} \left|z_i\right|\left|v\left(\widehat{z}\right) - \widetilde{v}\left(z\right)\right| \\ &\quad \le \frac{c}{2}\left\|z\right\|^2 + \min\left\{W_e\left(e\right), \widehat{\alpha}_n\right\}, \end{aligned} \tag{A.7}$$

where $\widehat{\alpha}_i(\ell_{i+1}, \ldots, \ell_n) = (n\overline{k}^4 \lambda_1^2 \lambda_2^2 / 2c)\gamma_i^2(\ell_{i+1}, \ldots, \ell_n)$, $i = 2, \ldots, n-1$, and $\widehat{\alpha}_n = n\overline{k}^4 \lambda_1^2 \lambda_2^2 / 2c$ with $\lambda_2 = \max_{z \in \Gamma_M^z}(k_n g_1(z_1)/g_0(z_1))$.

Moreover, by (8), there exists a positive constant d_1 independent of ℓ_i's, such that, on $\Gamma_M^z \times \mathbf{R}^{n-1}$,

$$\begin{aligned} &\left|z_{[n-1]}^{\mathrm{T}} P_1 \widetilde{\phi}_{[n-1]}\left(\cdot\right)\right| \\ &\quad + \left|z_n + K_1 z_{[n-1]}\right|\left|\widetilde{\phi}_n\left(\cdot\right) + K_1 \widetilde{\phi}_{[n-1]}\left(\cdot\right)\right|\big|_{\Gamma_M^z \times \mathbf{R}^{n-1}} \\ &\quad \le L^{\sigma} d_1 \left\|z\right\|^2. \end{aligned} \tag{A.8}$$

By substituting (A.2), (A.7), and (A.8) into (A.1), we conclude that Proposition 4 holds.

B. The Proof of Proposition 5

By system (24), we have

$$\begin{aligned} \dot{V}_e\left(e\right) \le &-L\sum_{i=2}^{n} \ell_i e_i^2 + L\sum_{i=2}^{n-1}\left|e_i\right|\left|z_{i+1}\right| \\ &+ L\sum_{i=2}^{n} \ell_i^2 \left|e_i\right|\left|z_{i-1} - \widehat{z}_{i-1}\right| \\ &+ L\widetilde{g}\left(t, z\right)\left|e_n v\left(\widehat{z}\right)\right| + \sum_{i=2}^{n}\left|e_i\right|\left|\widetilde{\phi}_i\left(\cdot\right) - \ell_i \widetilde{\phi}_{i-1}\left(\cdot\right)\right|. \end{aligned} \tag{B.1}$$

By (A.4) and Young's inequality, the second and third terms in the right-hand side of (B.1) satisfy

$$\begin{aligned} &\sum_{i=2}^{n-1}\left|e_i\right|\left|z_{i+1}\right| \le \frac{c}{8}\sum_{i=2}^{n-1} z_{i+1}^2 + \frac{2}{c}\sum_{i=2}^{n-1} e_i^2 \le \frac{c}{8}\left\|z\right\|^2 + \frac{2}{c}\sum_{i=2}^{n-1} e_i^2, \\ &\sum_{i=2}^{n} \ell_i^2 \left|e_i\right|\left|z_{i-1} - \widehat{z}_{i-1}\right| \\ &\quad \le \sum_{i=2}^{n} \ell_i^2 \left|e_i\right|\left(\left|e_{i-1}\right| + \ell_{i-1}\left|e_{i-2}\right| + \cdots + \ell_{i-1} \cdots \ell_3 \left|e_2\right|\right) \\ &\quad \le \widetilde{\beta}_n e_n^2 + \sum_{i=2}^{n-1} \widetilde{\beta}_i\left(\ell_{i+1}, \ldots, \ell_n\right) e_i^2, \end{aligned} \tag{B.2}$$

where $\widetilde{\beta}_n = n - 2$ and $\widetilde{\beta}_i(\ell_{i+1}, \ldots, \ell_n) = i - 2 + (1/4)\ell_{i+1}^4 + (1/4)\sum_{j=i+2}^{n}(\ell_{i+1}^2 \cdots \ell_{j-1}^2 \ell_j^4)$, $i = 2, \ldots, n-1$.

By (A.3) and (A.4), we have that

$$\begin{gathered} \left|\widehat{z}_2\right| \le \left|z_2\right| + \left|e_2\right|, \\ \left|\widehat{z}_i\right| \le \left|z_i\right| + \left|e_i\right| + \ell_i\left|e_{i-1}\right| + \cdots + \ell_i \cdots \ell_3\left|e_2\right|, \quad i = 3, \ldots, n, \end{gathered} \tag{B.3}$$

which, together with Young's inequality and Assumption 1, implies

$$\begin{aligned}
&\widetilde{g}(t,z)\left|e_n v(\widehat{z})\right|\Big|_{\Gamma_M^z\times\mathbf{R}^{n-1}} \\
&\quad\le \frac{k_n g_1(z_1)}{g_0(z_1)}\left|e_n\right| \\
&\qquad\cdot\left|\mathbf{sat}(\widehat{z}_n)+k_{n-1}\mathbf{sat}(\widehat{z}_{n-1})+\cdots+k_2\mathbf{sat}(\widehat{z}_2)+k_1\widehat{z}_1\right| \\
&\quad\le \lambda_2\overline{k}\left|e_n\right|\sum_{i=1}^{n}\left|\widehat{z}_i\right| \\
&\quad\le \frac{c}{8}\|z\|^2+\widehat{\beta}_n e_n^2+\sum_{i=2}^{n-1}\widehat{\beta}_i(\ell_{i+1},\ldots,\ell_n)e_i^2,
\end{aligned} \tag{B.4}$$

where $\lambda_2 = \max_{z\in\Gamma_M^z}(k_n g_1(z_1)/g_0(z_1))$, $\overline{k} = \max\{1,k_1,\ldots,k_{n-1}\}$, $\widehat{\beta}_n = n(n-1)\overline{k}\lambda_2/2 + 2\overline{k}^2\lambda_2^2/c$, and $\widehat{\beta}_i(\ell_{i+1},\ldots,\ell_n) = 2\overline{k}^2\lambda_2^2/c + \ell_{i+1}^2 + \sum_{j=i+2}^{n}(\ell_{i+1}^2\cdots\ell_j^2)$, $i = 2,\ldots,n-1$.

Substituting (B.2) and (B.4) into (B.1), we conclude that (26) holds with $\widetilde{\alpha}_n = \widehat{\beta}_n + \widetilde{\beta}_n$, and $\widetilde{\alpha}_i(\ell_{i+1},\ldots,\ell_n) = \widehat{\beta}_i(\ell_{i+1},\ldots,\ell_n) + \widetilde{\beta}_i(\ell_{i+1},\ldots,\ell_n)$, $i = 2,\ldots,n-1$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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