Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 384659, 5 pages
http://dx.doi.org/10.1155/2015/384659

Research Article

Multiobjective Shape Optimization for Deployment and Adjustment Properties of Cable-Net of Deployable Antenna

Guoqiang You and Pei Yang

Xi'an FANYI University, Taiyigong Street, Xi'an 710105, China

Correspondence should be addressed to Guoqiang You; simonyouyou@163.com

Received 26 April 2015; Accepted 11 August 2015

Academic Editor: Sergio Preidikman

Based on structural features of cable-net of deployable antenna, a multiobjective shape optimization method is proposed to help to engineer antenna's cable-net structure that has better deployment and adjustment properties. In this method, the multiobjective optimum mathematical model is built with lower nodes' locations of cable-net as variables, the average stress ratio of cable elements and strain energy as objectives, and surface precision and natural frequency of cable-net as constraints. Sequential quadratic programming method is used to solve this nonlinear mathematical model in conditions with different weighting coefficients, and the results show the validity and effectiveness of the proposed method and model.

1. Introduction

Cable-net structure has been widely used in space deployable antenna for light weight, high precision, and small stowed size [1–6]. The deployable antenna studied in this paper mainly consists of three parts: reflector surface cable-net system, rigid beam system, and center truss. The antenna is stowed in rocket before it is launched and deployed when the rocket is in its ideal orbit location. In antenna's deployment state, the cable-net system of antenna can form a long-span reflector with a specific shape, and for structural design of cable-net system in this state, the most important thing is to reduce the difference between cable-net reflector's actual shape and its ideal shape to improve antenna's surface precision. In previous optimization studies for antenna's cable-net system, many try to find a set of feasible prestresses that can make sure the antenna has the optimum surface precision and dynamic performance; others studied optimization methods that can improve antenna's surface precision by adjusting lengths of cable elements [7]. But all these optimization methods are proposed on the same assumption that the antenna has been in its deployment state. And deployment and adjustment properties of cable-net system are not taken into account in these methods.

Firstly, the optimum cable-net prestresses obtained on assumption that the antenna has been in its deployment state may not make sure it is easier for the antenna to be deployed. And if the antenna cannot be deployed smoothly, it will simply not work, so it is necessary to take a look at what kinds of cable-net prestresses that can ensure a high surface precision and make the antenna easier to be deployed. Secondly, there is much lower electrical energy for antenna cable-net system's adjustment in space compared to that on earth, so the cable-net structure should be designed to make sure it can be easily adjusted in conditions with lower electrical energy. Thirdly, for antenna's cable-net system, conventional optimization methods with feasible prestresses as objective cannot improve its deployment and adjustment properties, so the structural design method that can attain these goals should be studied carefully.

Based on the above considerations, a new design formulation with a multiobjective optimum mathematical model is proposed here to optimize cable-net's shape to improve antenna's deployment and adjustment properties. Because the optimum mathematical model is complex and nonlinear, sequential quadratic programming method is used to solve it. And in this multiobjective optimization, we can change

weighting coefficients of objectives to meet different demands for practical necessity.

2. Multiobjective Optimum Mathematical Model

2.1. Objectives. Structure of cable-net deployable antenna studied in this paper is shown in Figure 1. Because the structure of antenna's cable-net system is axisymmetric and it consists of the same forty-eight pieces of single cable-net and hoop cables, we just studied one piece of it in the optimization below in order to simplify the scale of the problem. The single cable-net shown in Figure 2 consists of upper cable elements, lower cable elements, and vertical cable elements, and the numbers in Figure 2 represent the nodes serial numbers. The static equilibrium equations at any node i can be written as

$$\sum F_{xi} = 0, \qquad \sum F_{yi} = 0, \tag{1}$$

where F_{xi} is cable elements' horizontal forces at node i and F_{yi} is cable elements' vertical forces at node i.

As shown in Figure 2, the angles θ_1, θ_2 between upper cable elements K_1, K_2 on either side of node i and horizontal direction are very small, so the vertical forces of K_1, K_2 are very small too, which will lead to a very small tension in vertical cable element K_3. It is clear that when single cable-net is in its static equilibrium state, tensions in vertical cable elements must be much less than tensions in upper cable elements and lower cable elements. Uneven prestress distribution of cable-net system can make the antenna hard to be deployed in conditions with lower electrical energy in space. In order to solve this problem, we use an average stress ratio $\overline{\sigma}$ that can calculate differences of cable elements' prestresses to measure antenna's deployment property. And it can be written as

$$\overline{\sigma} = \frac{\sum_{i=1}^{n} (\sigma_i - \sigma_{\min}) / n}{\sigma_{\min}}, \tag{2}$$

where n is the total number of cable elements of single cable-net, σ_i is the prestress of cable element i, and $\sigma_{\min}$ is the smallest cable element's prestress in single cable-net.

If it is considered that all cable elements have the same cross-sectional area, the smaller the average stress ratio is the more easily the antenna will be deployed, so we set the minimum average stress ratio as one objective.

In space, lower electrical energy cannot provide large cable tensions for antenna deployment, and it cannot provide enough energy for antenna adjustment as well. So in order to make the cable-net system easier to be adjusted, it is needed to get larger structural deformations in conditions with small adjusting force to increase cable-net system's adjustment property. We choose strain energy E reflecting relationship between strain and stress as the measurement for the above problem. It can be written as [8]

$$E = \mathbf{U}^{\mathbf{T}}\mathbf{K}\mathbf{U} = \frac{1}{2}\sum_{i=1}^{n} T_i \cdot \Delta_i, \tag{3}$$

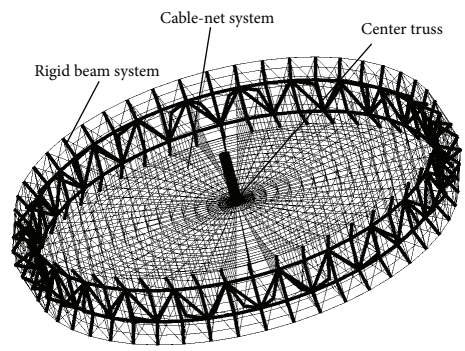


Figure 1: Structure of cable-net deployable antenna.

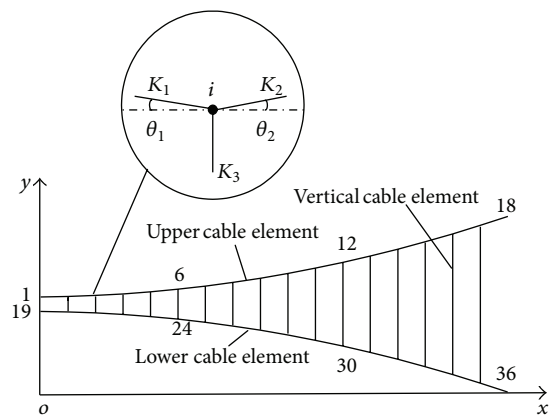


Figure 2: Structure of single cable-net.

where $\mathbf{U}$ is structural displacement matrix, $\mathbf{K}$ is structural stiffness matrix, T_i is tension of cable element i, and Δ_i is length change of cable element i.

The larger cable-net's strain energy is the smaller its rigidity is, and this represents a better adjustment property for the cable-net system. So the maximum strain energy is set as another objective here.

2.2. Constraints. High surface precision can make sure deployable antenna works well in space [9–11], so it is necessary for the cable-net system to have the specific shape in its deployment state. For this reason the surface nodes' root mean square error reflecting the differences between actual shape and ideal shape of upper cable-net is chosen as one constraint for the optimization.

There may be vibrations when deployable antenna is deploying or deflecting in space. In order to protect antenna from damage by these vibrations, it is needed to make sure antenna's natural frequency is higher than excited frequency. So we choose antenna's natural frequency as another constraint here.

2.3. Variables. The above objectives are closely related to shape of cable-net system, so conventional optimization with only the cable elements' cross-sectional area as variables is not suitable for the problem here. For this reason we choose cable

nodes locations as the new variables. Because upper cable nodes are all on the surface of antenna's reflector, and their locations cannot be freely changed, only locations of lower cable nodes are set as the variables.

As shown in Figure 2, there is the same horizontal spacing between each lower cable node of single cable-net, and when it is assumed that lower cable nodes can only move to its left side in optimization, the new horizontal coordinate of lower cable node i can be written as

$$x_i = (i-1)\cdot u + p_i \cdot u \quad 0 \le p_i \le 1, \tag{4}$$

where u is the horizontal spacing between each lower cable node before optimization and p_i is the variable—the horizontal distance lower cable node i moves to the left.

We can change single cable-net shape by adjusting p_i of every lower cable node to optimize the above two objectives.

2.4. Optimum Mathematical Model. Based on the above considerations, we establish the optimum mathematical model below:

$$\begin{aligned} &\text{Find} \quad p_1, p_2, \ldots, p_{16} \\ &\text{Min} \quad \overline{\sigma}, E \\ &\text{s.t.} \quad D = \sqrt{\frac{\sum_{i=1}^{m} \delta_i^2}{m}} \le [D] \\ &\qquad \omega \ge [\omega] \\ &\qquad 0 \le p_i \le 1, \end{aligned} \tag{5}$$

where D is the surface precision, root mean square of displacements of upper cable nodes, δ_i is the displacement of upper cable node i, m is the total number of upper cable nodes of single cable-net, $[D]$ is the upper limit of surface precision, ω is the natural frequency of single cable-net, and $[\omega]$ is the lower limit of natural frequency.

3. Objectives Function

Dimension and order are very different between the average stress ratio $\overline{\sigma}$ and structural strain energy E, so it will be inefficient to separately optimize these two different objectives, and for this reason we first change them as follows:

$$\begin{aligned} f_1 &= \frac{\overline{\sigma}}{c_1}, \\ f_2 &= \frac{c_2}{E}, \end{aligned} \tag{6}$$

where c_1 is equal to the optimal average stress ratio obtained with $\overline{\sigma}$ as the single objective for the above optimum mathematical model and c_2 is equal to the optimal value of strain energy with E as the single objective. These two values can help f_1 and f_2 obtain similar values from the given range of variables to make multiobjective optimization easier.

We use weighting coefficients method [12–14] to deal with the multiobjective problem here. In this method, all the objectives are multiplied by different weighting coefficients separately, and then they are added together to form a single objective that can be solved by conventional optimization methods:

$$f = w_1 f_1 + w_2 f_2, \tag{7}$$

where $w_1 \ge 0$, $w_2 \ge 0$ and $w_1 + w_2 = 1$.

4. An Example

The optimization for single cable-net shown in Figure 2 is given here as an example. In the single cable-net structure, there are 18 upper cable nodes, 18 lower cable nodes, 17 upper cable elements, 17 lower cable elements, and 18 vertical cable elements—50 cable elements in total. Single cable-net's length in the x direction is 8.3 m, and it is evenly divided into 17 parts. Single cable-net's length in the y direction is 2.9789 m, and all cable nodes' locations are determined by the equation: $z = 0.0192305x^2 + 0.10005$, and the 4 cable nodes at two ends of structure have no displacements in the x, y, and z directions. Each cable element is round in cross section, and its elastic modulus, diameter, and density are 1.3×10^{11} N/m^2, 1 mm, and 1.44×10^3 kg/m^3. Electrical energy is lower in space; it cannot provide large pretension for cable-net, so each cable element pretension is just set at 50 N for calculations below. The constraints are $D \le 1$ mm and $\omega \ge 4.3$ Hz. The 16 horizontal distances p_1–p_{16} of lower cable nodes are considered as variables, and their initial values are p_1–$p_4 = 0.2$, p_5–$p_{14} = 0.8$, and p_{15}-$p_{16} = 0.2$. Based on the above conditions, we can establish the multiobjective optimum mathematical model and use sequential quadratic programming method [15, 16] to solve it.

First, we solve the optimum mathematical model in conditions with $w_1 = 1$ and $w_1 = 0$. $w_1 = 1$ indicates a minimum average stress ratio and $w_1 = 0$ indicates a maximum structural strain energy, and in these two cases, we set $c_1 = 1$ and $c_2 = 1$ temporarily for the optimization. As the optimum result in condition with $w_1 = 1$, the average stress ratio of single cable-net drops to 32.80 that compares with its initial value 61.52. The optimum structure obtained in this case is shown in Figure 3. And in this figure, we can observe that gradients of vertical cable elements on the left side are larger than those on the right side, which is caused by the smaller curvature of upper cable elements on the left side. When the angle between upper cable element and the x direction is small, in order to meet the demand for minimum average stress ratio, the vertical cable element under it will slant to the left to obtain a larger angle between the y direction and itself. In condition with $w_1 = 0$, the optimum result is that strain energy of single cable-net is 0.1025 N·m that compares with its initial value 0.09007 N·m. And at this point, all variables are equal or almost equal to 1. The optimum structure in condition with $w_1 = 0$ is similar to Figure 2, and this suggests that the single cable-net structure will have the best adjustment property when all vertical cable elements are strictly vertical.

We can also adjust weighting coefficients w_1 and w_2 to meet different demands in engineering. For example,

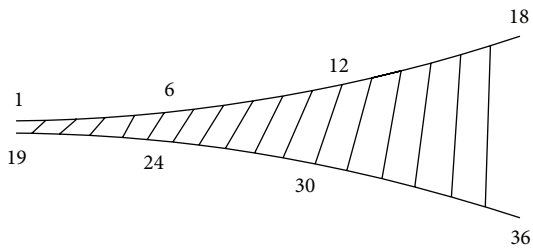


Figure 3: The optimum structure in condition with $w_1 = 1$.

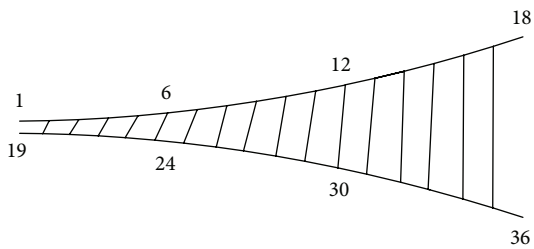
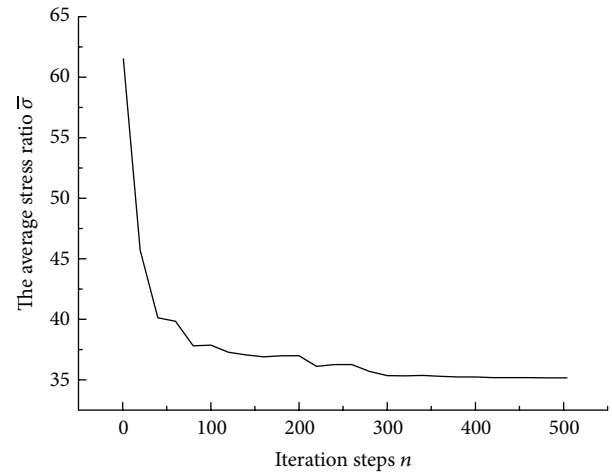
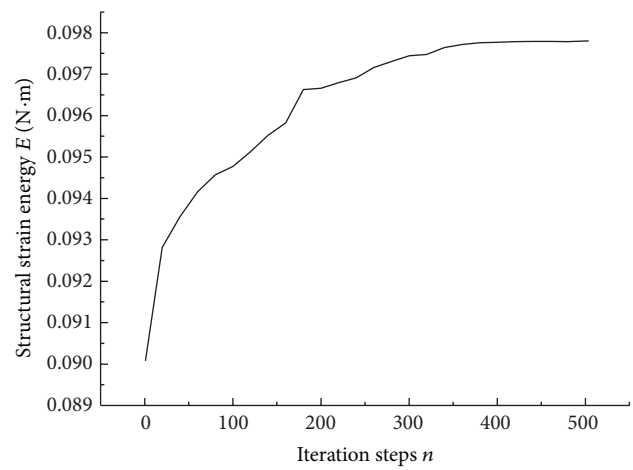


Figure 4: The optimum structure in condition with $w_1 = 0.4$.

we again solve the optimum mathematical model with $w_1 = 0.4$, $w_2 = 0.6$, $c_1 = 32.80$, and $c_2 = 0.1025$. This time the optimum average stress ratio and strain energy of single cable-net are 35.1572 and 0.09780 N·m, the optimum structure is shown in Figure 4, and the iteration processes of objectives are shown in Figure 5. As shown in Figure 4, several vertical cable elements on the left side slope to the left, and this suggests that the gradients of left vertical cable elements where upper cable elements' curvatures are smaller than others have more influences on the average stress ratio.



(a) The iteration process of the average stress ratio $\overline{\sigma}$



(b) The iteration process of strain energy E

Figure 5: The optimum results in condition with $w_1 = 0.4$.

5. Conclusions

(1) In this paper, we establish the multiobjective optimum mathematical model of the single cable-net first. Then we normalize objectives and combine them to form a single objective by using weighting coefficients method. Later on we use sequential quadratic programming method to solve this single objective problem and obtain the optimum results in conditions with different weighting coefficients. These results suggest that the average stress ratio is sensitive to the nodes locations of lower cable elements on the left side of the structure.

(2) Conventional optimization method with only cable elements' prestresses as variables cannot improve deployable antenna's deployment and adjustment properties to suit conditions like lower electrical energy in space, so optimizing cable-net's shape to get higher deployment and adjustment properties will be a good way for deployable antenna. And the optimization processes of objectives show the validity and effectiveness of the proposed method and model.

Conflict of Interests

The authors have no conflict of interests to declare.

Acknowledgment

Financial supports by Xi'an FANYI University (BK001), Shaanxi Province Department of Education (14JK2040), are gratefully acknowledged.

References

[1] A. V. Lopatin and E. V. Morozov, "Modal analysis of the thin-walled composite spoke of an umbrella-type deployable space antenna," *Composite Structures*, vol. 88, no. 1, pp. 46–55, 2009.

[2] A. Meguro, K. Shintate, M. Usui, and A. Tsujihata, "In-orbit deployment characteristics of large deployable antenna reflector onboard Engineering Test Satellite VIII," *Acta Astronautica*, vol. 65, no. 9-10, pp. 1306–1316, 2009.

[3] T. H. Legg, "A proposed new design for a large radio telescope," *Astronomy and Astrophysics Supplement*, vol. 130, no. 2, pp. 369–379, 1998.

[4] T. Takano, M. Natori, K. Miyoshi, and T. Inoue, "Characteristics verification of a deployable onboard antenna of 10 m maximum diameter," *Acta Astronautica*, vol. 51, no. 11, pp. 771–778, 2002.

[5] H. Tanaka, "Design optimization studies for large-scale contoured beam deployable satellite antennas," *Acta Astronautica*, vol. 58, no. 9, pp. 443–451, 2006.

[6] K. Ando, J. Mitsugi, and Y. Senbokuya, "Analyses of cable-membrane structure combined with deployable truss," *Computers & Structures*, vol. 74, no. 1, pp. 21–39, 2000.

[7] G. Tibert, *Deployment Tensegrity Structure for Space Applications*, Royal Institute of Technology Department of Mechanics, Stockholm, Sweden, 2002.

[8] G. P. Steven, Q. Li, and Y. M. Xie, "Multicriteria optimization that minimizes maximum stress and maximizes stiffness," *Computers & Structures*, vol. 80, no. 27–30, pp. 2433–2448, 2002.

[9] H. Tanaka and M. C. Natori, "Shape control of space antennas consisting of cable networks," *Acta Astronautica*, vol. 55, no. 3–9, pp. 519–527, 2004.

[10] C. S. Wang, B. Y. Duan, F. S. Zhang, and M. Zhu, "Analysis of performance of active phased array antennas with distorted plane error," *International Journal of Electronics*, vol. 96, no. 5, pp. 549–559, 2009.

[11] N. Mathers and L. Thompson, "Using inflatable antennas for portable satellite-based personal communications systems," *Acta Astronautica*, vol. 61, no. 7-8, pp. 659–663, 2007.

[12] S. Paul and S. Das, "Simultaneous feature selection and weighting—an evolutionary multi-objective optimization approach," *Pattern Recognition Letters*, vol. 65, pp. 51–59, 2015.

[13] C. S. Wang, B. Y. Duan, and Y. Y. Qiu, "On distorted surface analysis and multidisciplinary structural optimization of large reflector antennas," *Structural and Multidisciplinary Optimization*, vol. 33, no. 6, pp. 519–528, 2007.

[14] R. V. Grandhi, G. Bharatram, and V. B. Venkayya, "Multi-objective optimization of large-scale structures," *AIAA Journal*, vol. 31, no. 7, pp. 1329–1337, 1993.

[15] S. H. R. Pasandideh, S. T. A. Niaki, and A. R. Nia, "A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model," *Expert Systems with Applications*, vol. 38, no. 3, pp. 2708–2716, 2011.

[16] A. Belloufi, M. Assas, and I. Rezgui, "Optimization of turning operations by using a hybrid genetic algorithm with sequential quadratic programming," *Journal of Applied Research and Technology*, vol. 11, no. 1, pp. 88–94, 2013.