Research Article

Methods for Fault Diagnosability Analysis of a Class of Affine Nonlinear Systems

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The fault diagnosability analysis for a given model, before developing a diagnosis algorithm, can be used to answer questions like “can the fault f_i be detected by observed states?” and “can it separate fault f_i from fault f_j by observed states?” If not, we should redesign the sensor placement. This paper deals with the problem of the evaluation of detectability and separability for the diagnosability analysis of affine nonlinear system. First, we used differential geometry theory to analyze the nonlinear system and proposed new detectability criterion and separability criterion. Second, the related matrix between the faults and outputs of the system and the fault separable matrix are designed for quantitative fault diagnosability calculation and fault separability calculation, respectively. Finally, we illustrate our approach to exemplify how to analyze diagnosability by a certain nonlinear system example, and the experiment results indicate the effectiveness of the fault evaluation methods.

1. Introduction

Analyzing the fault diagnosability of system, which is very important for optimizing sensors placement, should be solved firstly before the fault diagnosis. In theory, the more sensors it has the better it is for fault diagnosis and monitoring. However, the sensors placement is constrained by the cost of sensitive component and the installation space, such as the satellite system. Diagnosability assessment is the key to evaluate the performance that one can expect from a diagnoser at run time and to define the appropriate set of sensors to be included in the design of a system [1, 2].

The model-based approach to fault detection and isolation (FDI) has received a lot of attention. At first, the focus is on the linear system. Based on the transfer function method [3–5], the transfer function between residual and the fault is obtained. Through the transfer function, the diagnosability of faults is evaluated. In [6], the fault is detectable if the transfer function is not equal to zero. At the same time, the transfer function between residual and disturbance is used to determine whether the disturbance can be decoupled or not

[7]. In the case of the above conditions, the fault is regarded as separable, and then a set of unknown input observers are designed to realize fault isolation [8]. The FDI of nonlinear system is researched in [9, 10], and the FDI of uncertain linear systems is also concerned [11, 12]. However, compared with the rich results in FDI of linear and nonlinear systems, only a limited number of contributions about fault diagnosability analysis have been found, especially for the nonlinear system. In the field of linear continuous systems, diagnosability is formulated in terms of fault detectability and isolability which provides a survey of definitions from different points of view [13–15], and diagnosability which includes detectability and distinguishability are analyzed for discrete-event systems using labeled Petri net [16].

To the best of the authors' knowledge, up to now, almost no attention has been paid to the study of fault detectability criterion and fault separability criterion for nonlinear systems with faults occurring, though the diagnosability analysis and judgment of linear continuous system are focused [17]. But we notice that the differential geometry based method is one of the most important methods in robust fault diagnosis of

nonlinear system [18–21]. The problem of fault diagnosis is converted into residual generator problem [18]. A solution to the problem of fault detection and isolation is characterized in terms of properties of certain distributions, which can be considered as the nonlinear analog of the unobservability subspaces [19]. The same problem is solved and the sufficient and necessary conditions are given by some distributions named unobservability codistribution [20]. And the appropriate coordinate transformation based on the principle of differential geometry can be designed to make one subsystem only affected by one fault, and then the fault is detectable and separable.

In this paper, the methods, based on the principle of differential geometry, are proposed to analyze the fault detectability and separability (FDS) different to FDI. Usually, the attention is on the approach to FDI, and the conditions that make the faults be diagnosable are given based on the designed approaches. However, in this paper, the diagnosability of fault is regarded as the inherent attribute of the system and is not changed with the approach of diagnosis.

Affine nonlinear system is the most common system, such as induction/synchronous motor, rigid robot, space craft, and satellite. The main contributions of this paper are summarized as follows. For a class of affine nonlinear systems with faults occurring, (1) the definitions of detectability and separability are given and the criteria to judge detectability and separability are proposed and proved and (2) the detectability incidence matrix and separability incidence matrix are defined for quantitative fault diagnosability calculation and fault separability calculation, respectively.

2. System Model and Basic Properties

Consider the following affine nonlinear system:

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^m g_i(x) u_i, \\ y &= [h_1(x) \ \cdots \ h_l(x)],\end{aligned}\quad (1)$$

where $x \in \mathbf{R}^n$ is the state vector, u_i is the input ($i = 1, 2, \dots, m$), and $y \in \mathbf{R}^l$ is the output vector.

According to the principle of differential geometry, let us introduce the following lemmas which will be used in deriving our main results.

Lemma 1 (output invariance). *Given the affine nonlinear model (1), output $y_j = h_j(x)$ will be not affected by u_i (i.e., the output is invariant to the input), if and only if it has distribution Δ and satisfies the following conditions:*

- (1) distribution Δ is invariant under vector field $\{f, g_1, g_2, \dots, g_m\}$;
- (2) vector field $g_i \in \Delta \subset (\text{span}\{dh_j\})^\perp$.

Lemma 2 (transformation of coordinates). *Given the affine nonlinear model (1), if its distribution $\langle f, g_1, \dots, g_m \mid \text{span}\{dh_1, \dots, dh_l\}^\perp$ is d -dimensional and nonsingular, then $\forall x_0$ of the domain one can find a neighborhood U_0 and the*

transformation of local coordinates $\xi = (\xi_1, \xi_2) = \Phi(x)$ defined in U_0 . In the new coordinate system, (1) can be denoted by

$$\begin{aligned}\dot{\xi}_1 &= \eta_1(\xi) + \sum_{i=1}^m \psi_{1i}(\xi) u_i, \\ \dot{\xi}_2 &= \eta_2(\xi_2) + \sum_{i=1}^m \psi_{2i}(\xi_2) u_i, \\ y &= [\lambda_1(\xi_2), \dots, \lambda_l(\xi_2)],\end{aligned}\quad (2)$$

where ξ_1 is d -dimensional.

3. Fault Diagnosability Analysis

In the affine nonlinear system, its faults are regarded as the inputs of the system. The fault diagnosability evaluation includes two aspects: faults detection and faults separation.

3.1. Fault Detectability Judgment. Consider the following affine nonlinear system:

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^m g_i(x) u_i + \sum_{i=1}^s p_i(x) w_i, \\ y &= [h_1(x) \ \cdots \ h_l(x)],\end{aligned}\quad (3)$$

where w_i denotes the fault and $p_i(x)$ denotes its coupled mode ($i = 1, 2, \dots, s$).

Definition 3 (detectability). *Given the affine nonlinear model (3), if outputs y will be not influenced by w_i (i.e., y is invariant to w_i), then fault w_i will be not detected by y ; that is, w_i is undetectable; otherwise, w_i will be detected by y ; that is, w_i is detectable. The fact that the ability of faults will be detected by output is named as detectability.*

And then, we propose the following proposition as the criteria to judge detectability.

Proposition 4 (detectability judgment). *Given the affine nonlinear model (3), the necessary and sufficient condition for w_i being detectable is*

$$p_i \notin \langle f, g_1, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle, \quad (4)$$

where $\langle f, g_1, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle$ is the maximal distribution which is contained in $(\text{span}\{dh_1, \dots, dh_l\})^\perp$ and is invariant under the vector field $\{f, g_1, \dots, g_m, p_1, \dots, p_s\}$.

Proof. (1) The necessary condition, that is, if $p_i \notin \langle f, g_1, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle$, then it has that w_i is detectable.

Make a proof by contradiction, supposing that w_i is not detectable when

$$p_i \in \langle f, g_1, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle \quad (5)$$

then y will be not influenced by w_i according to Definition 3, and $\forall dh_j \in \{dh_1, \dots, dh_l\}$, it has

$p_i \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_j\}^\perp \rangle$ according to Lemma 1; that is,

$$p_i \in \left(\langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1\}^\perp \rangle \right. \\ \left. \cap \dots \cap \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_l\}^\perp \rangle \right). \quad (6)$$

According to the distribution algorithm $\Delta_1^\perp \cap \Delta_2^\perp = (\Delta_1 \cup \Delta_2)^\perp$ (where Δ_1 and Δ_2 are any distribution), we get the conclusion that

$$p_i \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1\} \cup \dots \cup \text{span}\{dh_l\}^\perp \rangle; \quad (7)$$

that is,

$$p_i \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle. \quad (8)$$

This is contradictory with the supposed equation (5). So, the necessary condition is true.

(2) The sufficient condition, that is, if w_i is detectable, then it has that

$$p_i \notin \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle. \quad (9)$$

Make a proof by contradiction, supposing that when w_i is detectable, it can get the conclusion that

$$p_i \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle \quad (10)$$

then, according to the distribution algorithm $(\Delta_1 \cup \Delta_2)^\perp = \Delta_1^\perp \cap \Delta_2^\perp$, $\forall dh_j \in \{dh_1, \dots, dh_l\}$, it has

$$p_i \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_j\}^\perp \rangle. \quad (11)$$

So, according to Lemma 1, for all $j = 1, \dots, l$, $y_j = h_j(x)$ (where $h_j \in \{h_1, \dots, h_l\}$) will be not influenced by w_i ; that is, w_i is not detectable, which is contradictory with the supposing that w_i is detectable. So, the sufficient condition is true.

This completes the proof of Proposition 4. $\square$

3.2. Evaluation Indicator of the Detectability. Here, we design the incidence matrix and quantitative fault diagnosability calculation method as the following definitions.

Definition 5 (detectability incidence matrix). For detectability incidence matrix $\Pi_{l \times s}$, its element is denoted by π_{ji} ($1 \leq j \leq l$, $1 \leq i \leq s$). If w_i will be detected by $y_j = h_j(x)$ (i.e., w_i is detectable), then set the value of π_{ji} as 1; else if w_i will be not detected by $y_j = h_j(x)$ (i.e., w_i is not detectable), then set the value of π_{ji} as 0.

Definition 6 (diagnosability calculation). DFD (Degree of Fault Detectability) is as follows:

$$\text{DFD} = \frac{n}{s} \times 100\%, \quad (12)$$

where n is the number of the nonzero column vectors in $\Pi_{l \times s}$ and s is the sum of the faults.

3.3. Fault Separability Judgment. First, we give the definition of faults separability. And then, we propose the following proposition as the criteria to judge separability.

Definition 7 (separability). For the detectable faults w_i and w_j of nonlinear system (3), there is the ability to separate w_i from w_j when the influence of outputs y affected by w_i is different from w_j , and this ability is named as separability.

For the detectable faults, we can judge their separability by the distribution (space) spanned by the fault vector field (i.e., the coupled mode of fault).

- (1) If the spanned distributions $\text{span}(p_i)$ and $\text{span}(p_j)$ are equivalent, then the influence of w_i on the outputs y is same as the influence of w_j ; that is, w_i and w_j are inseparable. Otherwise, w_i and w_j are separable.
- (2) When the distributions (space) $\text{span}(p_i)$ and $\text{span}(p_j)$ are not equivalent, the following proposition will be used to judge the separability of w_i and w_j .

Proposition 8 (separability judgment). *Given the affine nonlinear system (3) and its detectable faults w_i and w_j (fault vector fields are p_i and p_j), if $\text{span}\{p_i\} \neq \text{span}\{p_j\}$, then w_i and w_j are inseparable if all p (where $p \in \Delta$, $\Delta = \text{span}\{p_i\} \cup \text{span}\{p_j\} - \text{span}\{p_i\} \cap \text{span}\{p_j\}$) satisfy*

$$p \in \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle. \quad (13)$$

Otherwise, w_i and w_j are separable.

Proof. (1) The sufficient condition (proof by contradiction) is as follows. Let

$$\Delta = \text{span}\{p_i\} \cup \text{span}\{p_j\} - \text{span}\{p_i\} \cap \text{span}\{p_j\}. \quad (14)$$

all p (where $p \in \Delta$) satisfy

$$p \notin \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle \quad (15)$$

then the different influence on outputs y affected by w_i and w_j will be detected according to Lemma 1, so we can know that the faults w_i and w_j are separable according to Definition 7.

(2) The necessary condition (proof by contradiction) is as follows. According to Definition 7, if the detectable faults w_i and w_j are separable, then the influences on outputs y affected by w_i and w_j are different and detectable. Since the distribution $\Delta = \text{span}\{p_i\} \cup \text{span}\{p_j\} - \text{span}\{p_i\} \cap \text{span}\{p_j\}$ represents the different influences by w_i and w_j , according to Lemma 1, we can know that all p (where $p \in \Delta$) satisfy

$$p \notin \langle f, g_i, \dots, g_m, p_1, \dots, p_s \mid \text{span}\{dh_1, \dots, dh_l\}^\perp \rangle. \quad (16)$$

It is contradictory with (13). So, the fact that the faults w_i and w_j are separable is true.

This completes the proof of Proposition 8. $\square$

3.4. Evaluation Indicator of Separability. Similarly, we design the incidence matrix and quantitative fault separability calculation method as the following definitions.

Definition 9 (separability incidence matrix). For separability incidence matrix Γ_{s*s} , its element is denoted by Γ_{ij} ($1 \leq i \leq s$, $1 \leq j \leq s$). Γ_{ij} ($i \neq j$) is to express the separability of w_i and w_j . If w_i and w_j are separable, then set the value of Γ_{ij} as 1 or else the value as 0; and set $\Gamma_{ii} = 0$ particularly.

Definition 10 (separability calculation). DFS (Degree of Fault Separation) is as follows:

$$\text{DFS} = \frac{\sum_{j=1}^s \sum_{i=1}^s \Gamma_{ij}}{s(s-1)} \times 100\%, \quad (17)$$

where Γ_{ij} is the element of Γ_{s*s} and s is the sum of the detectable faults.

4. Illustration Example

The following affine nonlinear system is given as an example to demonstrate the fault diagnosability analysis methods outlined above:

$$\begin{aligned} \dot{x}_1 &= x_1 x_2 + x_3 e^{x_3} + x_1 u + x_1 w_2, \\ \dot{x}_2 &= -x_2 x_3 + w_1 + w_3, \\ \dot{x}_3 &= x_2 + u + x_2 w_1, \\ \dot{x}_4 &= x_2^2 + x_3 x_4 - x_2 x_3^2 + x_2 u, \\ y &= x_3, \end{aligned} \quad (18)$$

where w_1 , w_2 , and w_3 are the faults of the system and

$$\begin{aligned} f(x) &= \begin{bmatrix} x_1 x_2 + x_3 e^{x_3} \\ -x_2 x_3 \\ x_2 \\ x_2^2 + x_3 x_4 - x_2 x_3^2 \end{bmatrix}, \\ g(x) &= \begin{bmatrix} x_1 \\ 0 \\ 1 \\ x_2 \end{bmatrix}, \quad p_1(x) = \begin{bmatrix} 0 \\ 1 \\ x_2 \\ 0 \end{bmatrix}, \\ p_2(x) &= \begin{bmatrix} x_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_3(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (19)$$

4.1. Diagnosability Analysis

4.1.1. Analyze the Detectability of the Faults. After calculation, we obtain

$$\begin{aligned} p_1 &\notin \langle f, g, p_1, p_2 \mid \text{span}\{dh\} \rangle^\perp \\ p_2 &\in \langle f, g, p_1, p_2 \mid \text{span}\{dh\} \rangle^\perp \\ p_3 &\notin \langle f, g, p_1, p_2 \mid \text{span}\{dh\} \rangle^\perp. \end{aligned} \quad (20)$$

So, according to Proposition 4, w_1 is detectable, w_2 is not detectable, and w_3 is detectable.

Then, the detectability incidence matrix

$$\Pi_{I*s} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}. \quad (21)$$

So, the DFD is

$$\text{DFD} = \frac{n}{s} \times 100\% = \frac{2}{3} \times 100\% = 66.7\%. \quad (22)$$

4.1.2. Analyze the Separability of the Faults. Here, after calculation, we have $\text{span}\{p_1\} \neq \text{span}\{p_3\}$ and all p (where $p \in \Delta$) have

$$p \notin \langle f, g, p_1, p_2, p_3 \mid \text{span}\{dh_1\} \rangle^\perp, \quad (23)$$

where $\Delta = \text{span}\{p_1\} \cup \text{span}\{p_3\} - \text{span}\{p_1\} \cap \text{span}\{p_3\}$. So, w_1 and w_3 are separable according to Proposition 8. The separability incidence matrix

$$\Gamma_{2*2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (24)$$

So, the DFS of w_1 and w_3 is

$$\text{DFS} = \frac{\sum_{j=1}^s \sum_{i=1}^s \Gamma_{ij}}{s(s-1)} \times 100\% = 100\%. \quad (25)$$

4.2. Diagnosability Verification. According to Lemma 2, it can provide the transformation of local coordinates $\xi = (\xi_1, \xi_2) = \phi(x)$ by $\text{span}\{dx_1, dx_4\}$, where $\xi_1 = (\xi_{11} \ \xi_{12}) = (x_1 \ x_4)$ and $\xi_2 = (\xi_{21} \ \xi_{22}) = (x_3 \ x_2)$. Then, we get the following subsystem equations under the new coordinate system:

$$\begin{aligned} \dot{\xi}_{21} &= \xi_{22} + u + \xi_{22} w_1 \\ \dot{\xi}_{22} &= -\xi_{21} \xi_{22} + w_1 + w_3 \\ y &= \xi_{21}. \end{aligned} \quad (26)$$

Next, we design the state observer and get the residual error to verify the detectability and separability of the faults.

Let

$$\begin{aligned} \varepsilon_1 &= \frac{1}{2} \xi_{21}^2 + \xi_{22} \\ \varepsilon_2 &= \xi_{21}, \end{aligned} \quad (27)$$

and then subsystem (26) changed as

$$\begin{aligned} \begin{pmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \end{pmatrix} &= A \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{2} \varepsilon_2^2 \end{pmatrix} + \begin{pmatrix} \varepsilon_2 \\ 1 \end{pmatrix} u \\ &\quad + \begin{pmatrix} 1 + \varepsilon_1 \varepsilon_2 - \frac{1}{2} \varepsilon_2^3 \\ \varepsilon_1 - \frac{1}{2} \varepsilon_2^2 \end{pmatrix} w_1 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} w_3, \\ y &= C \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \varepsilon_2, \end{aligned} \quad (28)$$

FIGURE 1: The residual error when giving step fault w_1 at time 10~15 s.

where

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = [0 \quad 1]. \quad (29)$$

Design the state observer as follows:

$$\dot{\hat{\varepsilon}} = (A + GC) \begin{pmatrix} \hat{\varepsilon}_1 \\ \hat{\varepsilon}_2 \end{pmatrix} - Gy + \begin{pmatrix} 0 \\ -\frac{1}{2}y^2 \end{pmatrix} + \begin{pmatrix} y \\ 1 \end{pmatrix} u, \quad (30)$$

where $(A + GC)$ is Hurwitz; here, set $G = [-1 \quad -2]^T$.
Let

$$\begin{aligned} e &= \varepsilon - \hat{\varepsilon}, \\ r &= y - \varepsilon_2, \end{aligned} \quad (31)$$

and then

$$\begin{aligned} \dot{e} &= (A + GC) e + P_1 w_1 + P_3 w_3, \\ r &= e_2, \end{aligned} \quad (32)$$

where

$$P_1 = \begin{pmatrix} 1 + \varepsilon_1 y - \frac{1}{2}y^3 \\ \varepsilon_1 - \frac{1}{2}y^2 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (33)$$

The residual error output is shown in Figure 1 when giving step fault input w_1 at time 10~15 s into the system; and when giving step fault input w_3 at time 10~15 s, the residual error output is shown in Figure 2. In Figures 1 and 2, we can see that the residual error obviously changed at time 10~15 s. Therefore, the faults w_1 and w_3 are detectable, same as the above analyses of the detectability.

The residual error output is shown in Figure 3 when giving step fault input w_2 at time 10~15 s into the system; from Figure 3, we can see that fault w_2 is not detectable, same as the above analyses of the detectability.

When giving step fault input w_1 at time 10~15 s and step fault input w_3 at time 20~30 s, the residual error output is shown in Figure 4. In Figure 4, we can see that the residual error obviously changed at time 10~15 s and time 20~30 s separately. Therefore, the faults w_1 and w_3 are separable, same as the above analyses of the separability.

FIGURE 2: The residual error when giving step fault w_3 at time 10~15 s. FIGURE 3: The residual error when giving step fault w_2 at time 10~15 s. FIGURE 4: The residual error when giving step fault w_1 at time 10~15 s and step fault w_3 at time 20~30 s.

5. Conclusion

In this paper, we use the principle of differential geometry to analyze the problem of diagnosability of a class of affine nonlinear systems with faults. We first give the necessary and sufficient conditions for (1) detectability judgment, that is, the conditions which the system should satisfy whether the faults are detectable or not, and (2) separability judgment, that is, the conditions which the system should satisfy whether the faults are separable or not. And the measurement, that is, DFD (Degree of Fault Detectability) and DFS (Degree of Fault Separation), based on the defined DIM (detectability incidence matrix) and SIM (separability incidence matrix) is established to quantify diagnosability performance. Then, we have provided an example to analyze and test the faults diagnosability. However, the simple quantitative measure indexes, DFD and DFS, have not taken full advantage of the full information of DIM and SIM. More comprehensive

quantitative measure index will be designed in our future research.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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