Adaptive Synchronization via State Predictor on General Complex Dynamic Networks

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1. Introduction

In recent years, the synchronization of complex dynamical networks has received more and more attention. The synchronous research can be applied in many fields, such as biology, smart city, computer, and the traffic [1–23].

It is well known that the complex network has a lot of nodes; however, in order to save an increasing number of energies, the pinning control is introduced to study the synchronization of complex dynamical networks. So far, the pinning control is a main tool by controlling a small number of nodes to steer the whole network. In [2], the pinning control of a continuous-time complex dynamical network with general coupling topologies was researched. The speed of synchronization is a significant issue, so, in [6], a state predictor was introduced. In [7], the adaptive synchronization of complex dynamical networks with state predictor was studied; therefore, this paper studies the problem using the pinning control. This paper considers the adaptive synchronization of general complex dynamic networks via state predictor based on the fixed topology for nonlinear dynamical systems. Using Lyapunov stability properties, it is proved that the complex dynamical networks with state predictor are asymptotically stable. Moreover, it is also shown that the rate of convergence of complex dynamical networks with state predictor is faster than the complex dynamical networks without state predictor.

2. Preliminaries and Problem Statement

Consider a complex dynamical network described by

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} a_{ij} c_{ij}(t) \left[ x_j(t) - x_i(t) \right] + \gamma \sum_{j \in \mathcal{N}_i} a_{ij} c_{ij}(t) \left( \dot{x}_i^p(t) - \dot{x}_j^p(t) \right) + u_i(t),$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ ($i = 1, 2, \ldots, N$) is the state vector of the $i$th node at time $t$, where $t$ is the continuous time; $f_j : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function; $N_i$ represents the neighbor node of $i$; $a_{ij}$ typify the coupling weight between any two nodes, where $a_{ij} \geq 0$ and $a_{ii} = 0$; $c_{ij}(t)$ stands for the coupling strengths between node $i$ and node $j$;
define the matrix of the weighted coupling configuration of the system as

\[
U = \begin{bmatrix}
    a_{11}c_{11} & a_{12}c_{12} & \cdots & a_{1N}c_{1N} \\
    a_{21}c_{21} & a_{22}c_{22} & \cdots & a_{2N}c_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{N1}c_{N1} & a_{N2}c_{N2} & \cdots & a_{NN}c_{NN}
\end{bmatrix} \in \mathbb{R}^{N \times N},
\]  

(2)

with \(a_{ij}c_{ij} = -\sum_{j=1, j \neq i}^{N} a_{ij}c_{ij}\).

Introduce the state predictor as

\[
\dot{X}^p = -LX,
\]

(3)

where \(X^p = (\dot{x}_{1}^p, \dot{x}_{2}^p, \ldots, \dot{x}_{N}^p)^T\), \(y\) represents the impact factor of the state predictor.

Assumption 2

masarestated.

Introduce the state predictor as

\[
U = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1N} \\
    a_{21} & a_{22} & \cdots & a_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{N1} & a_{N2} & \cdots & a_{NN}
\end{bmatrix} \in \mathbb{R}^{N \times N},
\]

(2)

with \(a_{ij} \geq 0\). The control input is designed as

\[
u_i = -h_i \tilde{c}_i \left( x_i(t) - \bar{x}(t) \right),
\]

(5)

for \(\forall x, y \in \mathbb{R}^n\). And

\[
\Delta = \text{diag} \{ \delta_1, \ldots, \delta_n \},
\]

(10)

\[
P = \text{diag} \{ p_1, \ldots, p_n \}
\]

are positive constant matrices, for the constant \(\omega > 0\).

Lemma 3 (see [8]). For any vectors \(x, y \in \mathbb{R}^n\) and positive-definite matrix \(G \in \mathbb{R}^{n \times n}\), the following matrix inequality holds:

\[
2x^T y \leq x^T G x + y^T G^{-1} y.
\]

(11)

Lemma 4 (see [9]). Suppose that \(a\) and \(b\) are vectors; then for any positive-definite matrix \(E\), the following inequality holds:

\[
-2a^T b \leq \inf_{E > 0} \{ a^T E a + b^T E^{-1} b \}.
\]

(12)

Lemma 5 (see [10]). The following equation holds:

\[
\sum_{i=1}^{N} (x_i - \bar{x})^T P \sum_{j=1, j \neq i}^{N} a_{ij} \tilde{c}_i (x_i - x_j)
\]

\[
= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} \tilde{c}_i (x_i - x_j)^T P (x_i - x_j).
\]

(13)

Lemma 6 (see [18]). For a connected graph which is undirected, the Laplace matrix is positive semidefinite matrix, and the minimum nonzero eigenvalue is the algebraic connectivity of \(L\), as follows:

\[
\lambda_2(L) = \min_{x \neq 0, \|x\| = 1} \frac{x^T L x}{\|x\|^2}.
\]

(14)

Lemma 7 (see [18]). For a system which is similar to \(\dot{x}_i = u_i (i = 1, 2, \ldots, n)\), the evolution rate associated with the minimum nonzero eigenvalue \(\lambda_2\) describes the lower bound of convergence rate. Generally, the bigger the \(\lambda_2\) is, the faster the system converges.

3. Main Results

In the following, we will give the main result.

Theorem 8. Consider network (4) with the state predictor (3) and \(N\) nodes steered by adaptive control (8), under Assumption 2, and at least one node is selected to be controlled. Then, all nodes asymptotically synchronize with the given homogeneous stationary state:

\[
\lim_{t \to \infty} \|x_i(t) - \bar{x}(t)\| = 0.
\]

(15)

Proof. Let \(\xi_i(t) \equiv x_i(t) - \bar{x}(t)\). Construct the following Lyapunov function:

\[
V(t) = V_1(t) + V_2(t),
\]

(16)
where

\[ V_1(t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in E_i} \frac{|c_j(t) - m|^2}{2k_{ij}}. \]

Then

\[ V_1(t) = \sum_{i=1}^{N} \bar{x}_i^T P \left[ f(x_i(t)) + \sum_{j=1}^{N} a_{ij}c_{ij}(t)(x_j - x_i) \right. \]

\[ + \gamma \sum_{j \in E_i} \sum_{k \in E_i} a_{ijk}c_{ijk}(t)c_{ik}(t)(x_j - x_k) \]

\[ + \left. \sum_{j \in E_i} \sum_{p \in E_j} a_{ijp}c_{ijp}(t)c_{jp}(t) \right] \]

\[ + u_i(t) - f(\bar{x}(t)) \right] \]

\[ = \sum_{i=1}^{N} \bar{x}_i^T P \left[ f(x_i(t)) + \sum_{j=1}^{N} a_{ij}c_{ij}(t)(x_j - x_i) \right. \]

\[ - \gamma \sum_{j \in E_i} \sum_{k \in E_i} a_{ijk}c_{ijk}(t)c_{ik}(t)(x_j - x_k) \]

\[ - \sum_{j \in E_i} \sum_{p \in E_j} a_{ijp}c_{ijp}(t)c_{jp}(t) \]

\[ \times (x_j - x_p) \]

\[ - h_i c_i(t) - f(\bar{x}(t)) \right] \]

\[ \leq -\omega \sum_{i=1}^{N} \bar{x}_i^T \bar{x}_i + \sum_{i=1}^{N} \bar{x}_i^T P \Delta \bar{x}_i \]

\[ + \sum_{i=1}^{N} \bar{x}_i^T P \sum_{j=1}^{N} a_{ij}c_{ij}(t)(x_j - x_i) \]

\[ - \sum_{i=1}^{N} \bar{x}_i^T P h_i c_i(t) \bar{x}_i^T - \gamma \sum_{i=1}^{N} \bar{x}_i^T P \]

\[ \times \sum_{j \in E_i} \sum_{k \in E_i} a_{ijk}c_{ijk}(t)c_{ik}(t)(\bar{x}_i - \bar{x}_k) \]

\[ + \gamma \sum_{i=1}^{N} \bar{x}_i^T P \sum_{j \in E_i} \sum_{p \in E_j} a_{ijp}c_{ijp}(t)c_{jp}(t)(\bar{x}_j - \bar{x}_p) \]
×\sum_{j=1}^{N} a_{ij}c_{ij}(t) \left( x_j - x_i \right)

- \sum_{i=1}^{N} \sum_{j \in \mathcal{E}_i, k \in \mathcal{E}_j} \sum_{k=1}^{N} \tilde{x}_j^T P a_{kj} c_{kj}(t) c_{ik}(t) x_k

+ \frac{1}{2} \gamma \sum_{i=1}^{N} \sum_{j \in \mathcal{E}_i, k \in \mathcal{E}_j} \sum_{k=1}^{N} \tilde{x}_j^T P a_{kj} c_{kj}(t) c_{ik}(t) x_k

+ \frac{1}{2} \gamma \sum_{i=1}^{N} \sum_{j \in \mathcal{E}_i, k \in \mathcal{E}_j} \sum_{k=1}^{N} \tilde{x}_j^T p a_{kj} c_{kj}(t) c_{ip}(t) x_p

= -\omega \sum_{i=1}^{N} \tilde{x}_i^T x_i + \sum_{i=1}^{N} \tilde{x}_i^T P \Delta \tilde{x}_i

+ \sum_{i=1}^{N} \tilde{x}_i^T P a_{ii} \left( x_i - x_i \right)

- \sum_{i=1}^{N} \tilde{x}_i^T P a_{ij} \left( x_i - x_j \right).

(18)

where \( L_i = \sum_{j=1}^{N} a_{ij}c_{ij}(t) \), \( i, j = 1, 2, \ldots, N \).

Consider the following:

\[
\dot{V}_2(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( c_{ij} - m \right) \dot{c}_{ij} \frac{k_{ij}}{k_{ij}}
\]

\[
= \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathcal{E}_i} \left( c_{ij} - m \right) a_{ij} \left[ x_i - x_j \right]^T \left[ x_i - x_j \right]
\]

\[
= \sum_{i=1}^{N} \tilde{x}_i^T P \sum_{j \in \mathcal{E}_i} a_{ij} \left( x_i - x_j \right)
\]

\[
= \sum_{i=1}^{N} \tilde{x}_i^T P \sum_{j \in \mathcal{E}_i} a_{ij} \left( x_i - x_j \right).\]

\[
\dot{V}_1(t) + \dot{V}_2(t) \leq -\omega \sum_{i=1}^{N} \tilde{x}_i^T x_i + \sum_{i=1}^{N} \tilde{x}_i^T P \Delta \tilde{x}_i - \sum_{i=1}^{N} \tilde{x}_i^T P h_i c_i \tilde{x}_i
\]

\[
+ 2\gamma \sum_{i=1}^{N} \tilde{x}_i^T P \gamma a_{ij} \tilde{x}_j - m \sum_{i=1}^{N} \tilde{x}_i^T P \sum_{j \in \mathcal{E}_i} a_{ij} \left( \tilde{x}_i - \tilde{x}_j \right)
\]

\[
= \tilde{x}^T \left[ \left( I_N \otimes P \Delta + 2\gamma L_i^2 \otimes P \right) - \left( H \otimes P + mA \otimes P \right) \right] \tilde{x} - \omega \sum_{i=1}^{N} \tilde{x}_i^T x_i^T.
\]

(19)

Since the positive constant \( m \) is sufficiently large, \( \dot{V}(t) < 0 \).

Therefore, \( \lim_{t \to \infty} \| x_i(t) - \tilde{x}(t) \| = 0 \).

Theorem 9. Network (4) with the state predictor (3) is faster to achieve synchronization than the network without the state predictor.

Proof. For the system with state predictor, the main difference is whether the system contains \( \gamma \sum_{j \in \mathcal{E}_i} a_{ij} c_{ij}(t) (\tilde{x}_j(t) - \tilde{x}_j(t)) \).

We consider the minimum nonzero eigenvalue of state predictor. The Laplace matrix is positive semidefinite matrix, so there is a nonsingular matrix \( P \) that can make the Laplace matrix expressed as

\[
\gamma L_i^2 = P^{-1} \begin{bmatrix}
\gamma \lambda_1^2 & \cdots & \gamma \lambda_n^2
\end{bmatrix} P, \quad 0 < \lambda_2 \leq \cdots \leq \lambda_n. \quad (20)
\]

Obviously, under the same conditions, a system with state predictor has greater minimum nonzero eigenvalue. According to Lemma 7, network (4) with the state predictor (3) is faster to achieve synchronization than the network without the state predictor.

4. Simulations

In this section, a numerical simulation is given to illustrate the analytical results.

Consider a network with the undirected topology described as follows:

\[
A = \begin{bmatrix}
0 & 0.0984 & 0.0757 & 0.0570 \\
0.0984 & 0 & 0.1199 & 0.1396 \\
0.0757 & 0.1199 & 0 & 0.0581 \\
0.0570 & 0.1396 & 0.0581 & 0
\end{bmatrix},
\]

(21)

where each node is a Lorenz system:

\[
f(x(t)) = f(x^1, x^2, x^3) = \begin{bmatrix}
\dot{x}^1 = 10 \left( x^2 - x^1 \right) \\
\dot{x}^2 = 28x^1 - x^1 x^3 - x^2 \\
\dot{x}^3 = x^1 x^3 - 8 x^3
\end{bmatrix}.
\]

(22)

Figure 1 describes the error states on the x-axis, y-axis, and z-axis, respectively. From Figure 1, we can see that all nodes can synchronize with the synchronous state by degrees. In particular, under the same conditions, the network with a state predictor can be synchronized faster. It is shown in Figure 2.
5. Conclusion

In this paper, we have investigated the state predictor problem for synchronization of complex dynamical networks in fixed topology. By introducing local adaptive strategies for the coupling strengths, we have proved that the complex dynamical networks are asymptotically stable. It is obvious that the rate of convergence of the network with a state predictor is faster than the network without a state predictor.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References


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