Research Article

Application of the Preview Control Method to the Optimal Tracking Control Problem for Continuous-Time Systems with Time-Delay

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This paper considers the application of the preview control method to the optimal tracking control problem for a class of continuous-time systems with state and input delays. First, through a transformation, the system is transformed into a nondelayed one. Then, the tracking problem of the time-delay system is transformed into one of a nondelayed system via processing of the reference signal. We then apply preview control theory to derive an augmented system for the nondelayed system and design a controller with preview function assuming that the reference signal is previewable. Finally, we obtain the optimal control law of the augmented error system and thus obtain that of the original system by letting the preview length of the reference signal go to zero. Numerical simulations are provided to illustrate the effectiveness and validity of our conclusions.

1. Introduction

As we all know, a time-delay phenomenon exists widely in practical engineering, and it is usually the main source of instability and performance degradation in various control systems. Therefore, considerable research has been devoted to study and analysis of time-delay systems. It is a common practice to transform the time-delay system into a nondelayed system in research work of control theory. Actually, discrete time-delay systems can eliminate delay by using the discrete lifting technique [1]. As for continuous time-delay systems, these usually can be converted to nondelayed systems by a transformation. References [2, 3] introduced state-delay systems according to this insight, and in the literature [4–6] they have studied input-delay systems. Reference [7] has discussed systems with state and input delays.

In many industrial applications, optimal tracking control [8–11], as the combination of optimal control and tracking control, aims at looking for an optimal control law to minimize the given performance index function, that is to say, to make the system output track the reference signal in

an optimal way. Thus, optimal tracking control has been a goal pursued unrelentingly in many fields. Preview control is a control technique to improve the performance of a closed-loop system via sufficient utilization of the known future information on the reference signals or disturbances. Preview control theory has attracted academic attention since it was proposed 50 years ago [12–16]. In the existing literature, the most extensive research into preview control centers on the linear quadratic optimal control problem with preview compensation [17–21].

We notice that [7] studied sliding mode control and the regulator problem, but the method of eliminating time-delay is universally applicable. Furthermore, a preview control system will be a normal control system when the preview length goes to zero. Based on these observations, the contribution of this paper is to apply preview control theory to the optimal tracking control problem for continuous-time control systems that are subject to state and input delays. First, according to a transformation in the literature [7], the time-delay system is transformed into the form of a nondelayed system. Second, by dealing with the reference signal, the

tracking problem of the time-delay system is converted to the tracking problem of the nondelayed system. Then, taking advantage of the methods of preview control theory to derive an augmented error system for the nondelayed system, a controller with preview action for the augmented error system can be obtained, assuming that the reference signal is previewable. Finally, we propose an optimal control law for the augmented error system when the preview length of the reference signal goes to zero and obtain the optimal control law of the original time-delay system. Numerical simulations show the effectiveness and validity of the proposed conclusions in this paper.

The paper is organized as follows. Section 2 proposes the problem and states the basic assumptions. In Section 3, a transformation is introduced that transforms the time-delay system into the form of a nondelayed system. An augmented error system is derived and the optimal controller is presented in Section 4. The existence conditions of the controller are discussed in Section 5. Section 6 provides numerical simulations and Section 7 is the brief conclusion.

2. Problem Statement and Basic Assumptions

Consider a continuous-time control system with state and input delays described as

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + A_1 x(t - \beta) + B_1 u(t - \tau), \\ y(t) &= Cx(t), \\ x(\theta) &= x_0(t), \quad \theta \in [-\beta, 0],\end{aligned}\quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input control vector, $y(t) \in R^p$ is the output vector, the matrices $A_0, A_1 \in R^{n \times n}$, $B_1 \in R^{n \times r}$, and $C \in R^{p \times n}$ are known real constant matrices with appropriate dimensions, and $\beta, \tau > 0$ denotes the constant time-delay, which appears in the state and input vectors, respectively.

We have the following assumptions for system (1).

Assumption 1. A_0 is invertible.

Assumption 2. The reference signal $y_d(t) \in R^p$ is a piecewise-continuously differentiable function.

Assumption 3. The matrix $\begin{bmatrix} A_0 + MA_1 & NB_1 \\ C & 0 \end{bmatrix}$ is of full row rank; the pair $(A_0 + MA_1, NB_1)$ is stabilizable; the pair $(C, A_0 + MA_1)$ is detectable. Here,

$$\begin{aligned}M &= [A_1 A_0^{-1} (e^{A_0 \beta} - I) + e^{A_0 \beta}]^{-1}, \\ N &= [A_1 A_0^{-1} (e^{A_0 \tau} - I) + e^{A_0 \tau}]^{-1}.\end{aligned}\quad (2)$$

Define the subtraction of the reference signal $y_d(t)$ from the output vector $y(t)$ as the tracking error:

$$e(t) = y(t) - y_d(t). \quad (3)$$

The objective of this paper is to design a controller to make the output vector $y(t)$ accurately track the reference signal $y_d(t)$ without static errors; namely,

$$\lim_{t \rightarrow \infty} e(t) = 0. \quad (4)$$

For this, we introduce the quadratic performance index:

$$J = \int_0^\infty [e^T(t) Q_e e(t) + \dot{u}^T(t) R \dot{u}(t)] dt, \quad (5)$$

where $Q_e \in R^{p \times p}$ and $R \in R^{r \times r}$ are both positive definite matrices.

As has been pointed out in [16], introducing the derivative of the input control vector $\dot{u}(t)$ into the quadratic performance index J can create the controller with integral action, which helps the system eliminate static errors.

3. System Transformation

First, we eliminate the time-delay of system (1) through a transformation. Utilizing the transformation in the literature [7],

$$\begin{aligned}z(t) &= x(t) + \int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta \\ &\quad + \int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta,\end{aligned}\quad (6)$$

where A, B satisfy

$$\begin{aligned}A &= A_0 + e^{-A\beta} A_1, \\ B &= e^{-A\tau} B_1\end{aligned}\quad (7)$$

and the state equation of system (1) is converted to the nondelayed form:

$$\dot{z}(t) = Az(t) + Bu(t). \quad (8)$$

Now, we solve A and B from (7). Treat β as a variable. Left multiply $e^{A\beta}$ on both sides of (7), which by transposition gives

$$e^{A\beta} A - e^{A\beta} A_0 = A_1; \quad (9)$$

that is,

$$\frac{d}{d\beta} (e^{A\beta}) - (e^{A\beta}) A_0 = A_1. \quad (10)$$

Then, right multiplying $e^{-A_0\beta}$ on both sides, we obtain

$$\frac{d}{d\beta} (e^{A\beta}) e^{-A_0\beta} + (e^{A\beta}) \frac{d}{d\beta} (e^{-A_0\beta}) = A_1 e^{-A_0\beta}; \quad (11)$$

namely,

$$\left[\frac{d}{d\beta} (e^{A\beta} e^{-A_0\beta}) \right] = A_1 e^{-A_0\beta}. \quad (12)$$

Integrate $[0, \beta]$ on both sides and get

$$e^{A\beta} e^{-A_0\beta} - I = A_1 \int_0^\beta e^{-A_0\sigma} d\sigma. \quad (13)$$

Therefore,

$$e^{A\beta} e^{-A_0\beta} - I = A_1 A_0^{-1} (I - e^{-A_0\beta}). \quad (14)$$

Thus, we have

$$e^{A\beta} = A_1 A_0^{-1} (e^{A_0\beta} - I) + e^{A_0\beta}. \quad (15)$$

Then, substituting (15) into the first formula of (7) obtains A :

$$A = A_0 + [A_1 A_0^{-1} (e^{A_0\beta} - I) + e^{A_0\beta}]^{-1} A_1. \quad (16)$$

Similarly, substituting (15) into the second formula of (7) gives B :

$$B = e^{-A\tau} B_1 = [A_1 A_0^{-1} (e^{A_0\tau} - I) + e^{A_0\tau}]^{-1} B_1. \quad (17)$$

According to $z(t) = x(t) + \int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta + \int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta$, we know $z(t)$ is reasonable to be the state vector of (8). Considering the output equation $y(t) = Cx(t)$ of system (1) and the relationship of $x(t)$ and $z(t)$, we can take $w(t) = Cz(t)$ as the output equation of system (8).

Hence, time-delay system (1) is reduced to the following nondelayed system:

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t), \\ w(t) &= Cz(t). \end{aligned} \quad (18)$$

Then, we make a transformation to the reference signal. Let

$$\begin{aligned} r(t) &= y_d(t) + C \int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta \\ &\quad + C \int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta. \end{aligned} \quad (19)$$

Treating (18) as the control system equation and (19) as the reference signal, we determine that the tracking error of system (1) remains identical with the tracking error of system (18); that is,

$$e(t) = y(t) - y_d(t) = w(t) - r(t). \quad (20)$$

Thus, the performance index (5) can be directly applied to system (18). Note that $\int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta$ and $\int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta$, as the integrations for historical values of the state vector and the input control vector, respectively, are both known at each instant of time t ; therefore, $r(t)$ is a known vector. It is shown that taking $r(t)$ as the reference signal of system (18) is reasonable.

Now, the problem is converted to designing the optimal control law of system (18) under performance index (5), where $e(t) = w(t) - r(t)$ is the tracking error.

4. Augmented Error System and Optimal Control

At first, the methods of preview control theory are adopted deriving an augmented error system that transforms the tracking problem into a regulator problem.

We derive tracking error $e(t)$ and get

$$\dot{e}(t) = \dot{w}(t) - \dot{r}(t) = C\dot{z}(t) - \dot{r}(t). \quad (21)$$

Differentiating both sides of (8), we have

$$\frac{d}{dt} \dot{z}(t) = A\dot{z}(t) + B\dot{u}(t). \quad (22)$$

Combining (21) and (22) gives

$$\dot{X}(t) = \tilde{A}X(t) + \tilde{B}\dot{u}(t) - \tilde{D}\dot{r}(t), \quad (23)$$

where $X(t)$, $\tilde{A}$, $\tilde{B}$, and $\tilde{D}$ are, respectively, given by

$$\begin{aligned} X(t) &= \begin{bmatrix} e(t) \\ \dot{z}(t) \end{bmatrix}, \\ \tilde{A} &= \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, \\ \tilde{B} &= \begin{bmatrix} 0 \\ B \end{bmatrix}, \\ \tilde{D} &= \begin{bmatrix} I \\ 0 \end{bmatrix}. \end{aligned} \quad (24)$$

According to the output equation of system (18), we now take $e(t) = [I \ 0]X(t)$ as the output equation and get

$$\begin{aligned} \dot{X}(t) &= \tilde{A}X(t) + \tilde{B}\dot{u}(t) - \tilde{D}\dot{r}(t), \\ e(t) &= \tilde{C}X(t), \end{aligned} \quad (25)$$

where $\tilde{C} = [I \ 0] \in R^{p+n}$.

To utilize the research results of preview control, we assume that the reference signal $r(t)$ is previewable and the preview length is l_r . After obtaining the control input and letting $l_r \rightarrow 0^+$, we can obtain the solution of the problem in this paper.

Equations (25) are the needed augmented error system. Our basic ideas are as follows: applying the idea of optimal preview control, designing the controller of system (25), and then obtaining the controller of system (1). It is easy to see that if we can design a state feedback to make the closed-loop system of system (25) asymptotically stable, we will have $\lim_{t \rightarrow \infty} X(t) = 0$. Hence, the error signal $e(t)$ as a partial component of the state vector $X(t)$ will satisfy (4).

$y_d(t)$ is a piecewise-continuously differentiable function; so $r(t)$ is piecewise-continuously differentiable.

According to the state and input vectors of (25), the performance index function (5) can be denoted as follows:

$$J = \int_0^\infty \left[X^T(t) \tilde{Q} X(t) + \dot{u}^T(t) R \dot{u}(t) \right] dt, \quad (26)$$

where $\tilde{Q} = \tilde{C}^T Q_e \tilde{C} = \begin{bmatrix} Q_e & 0 \\ 0 & 0 \end{bmatrix}$.

Meanwhile, noting that the form of system (25) and the performance index function are similar to those mentioned in the literature [22], we obtain the following theorem by using similar derivations of [22].

Theorem 4. Suppose $(\tilde{A}, \tilde{B})$ is stabilizable and $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable. The optimal control input of system (25) to minimize performance index function (26) is given by

$$\dot{u}(t) = -R^{-1} \tilde{B}^T P X(t), \quad (27)$$

where

$$\tilde{A}_c = \tilde{A} - \tilde{B} R^{-1} \tilde{B}^T P \quad (28)$$

and P is a positive semidefinite matrix satisfying the algebraic Riccati equation

$$\tilde{A}^T P + P \tilde{A} - P \tilde{B} R^{-1} \tilde{B}^T P + \tilde{Q} = 0. \quad (29)$$

Proof. According to the results of [22], when $(\tilde{A}, \tilde{B})$ is stabilizable, $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable, and l_r is the preview length of the reference signal $r(t)$, the optimal preview control input of system (25) is

$$\dot{u}(t) = -R^{-1} \tilde{B}^T P X(t) + R^{-1} \tilde{B}^T g(t, l_r), \quad (30)$$

where

$$g(t, l_r) = \int_t^{t+l_r} \exp[\tilde{A}_c^T(s-t)] P \tilde{D} \dot{r}(s) ds \quad (31)$$

is the desired output preview compensation of system (25) and $\tilde{A}_c$ and P are described by (28) and (29), respectively.

Because $r(t)$ is not previewable, $\dot{r}(t)$ is not previewable. Therefore, $g(t, l_r)$ is unsolvable. To overcome the difficulty, we let $l_r \rightarrow 0^+$ utilize the property of integration and get

$$\lim_{l_r \rightarrow 0^+} g(t, l_r) = 0. \quad (32)$$

Making $l_r \rightarrow 0^+$ on both sides of (30), we give (27) immediately because $g(t, l_r)$ is only related to l_r . The proof is complete. $\square$

Returning to system (1), we get the following.

Theorem 5. Suppose $(\tilde{A}, \tilde{B})$ is stabilizable and $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable. Let $x(t) = 0$, [namely, $x_0(t) \equiv 0$ in system (1)] $u(t) = 0$, and $y_d(t) = 0$, for $t \leq 0$; the optimal control input of system (1) to minimize performance index (5) is given by

$$\begin{aligned} u(t) = & -K_e \int_0^t e(\sigma) d\sigma - K_x \left[x(t) \right. \\ & + \int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta \\ & \left. + \int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta \right], \end{aligned} \quad (33)$$

where P is the unique positive semidefinite solution of Riccati equation (29), $K_e = R^{-1} \tilde{B}^T P_e$, $K_x = R^{-1} \tilde{B}^T P_x$, and $P = \begin{bmatrix} P_e & P_x \end{bmatrix}$.

The method of proof is similar to that in [22] and is omitted here.

5. Existence Conditions of the Controller

In this section, we discuss conditions of the original system (1) that can make $(\tilde{A}, \tilde{B})$ stabilizable and $(\tilde{Q}^{1/2}, \tilde{A})$ detectable.

Lemma 6. The pair $(\tilde{A}, \tilde{B})$ is stabilizable if and only if (A, B) is stabilizable and the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.

Lemma 7. The pair $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable if and only if (C, A) is detectable.

Lemmas 6 and 7 are both the results of [22].

Note that we can put assumption 3 another way: suppose the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank, the pair (A, B) is stabilizable, and (C, A) is detectable.

Note the following. As seen from Lemmas 6 and 7, when (A_0, B_1) or (A_1, B_1) is stabilizable, this cannot guarantee the stabilizability of $(\tilde{A}, \tilde{B})$; similarly, when (C, A_0) or (C, A_1) is detectable, we cannot guarantee the detectability of $(\tilde{Q}^{1/2}, \tilde{A})$. These are characteristics of time-delay systems.

In summary, we obtain the main theorem in this paper.

Theorem 8. Suppose the following conditions are satisfied:

- (1) A_0 is invertible (Assumption 1).
- (2) The reference signal $y_d(t)$ is a piecewise-continuously differentiable function (Assumption 2).
- (3) The matrix $\begin{bmatrix} A_0 + MA_1 & NB_1 \\ C & 0 \end{bmatrix}$ is of full row rank, the pair $(A_0 + MA_1, NB_1)$ is stabilizable, and the pair $(C, A_0 + MA_1)$ is detectable (Assumption 3).
- (4) Q_e and R are both positive definite matrices.

We then have the following closed-loop system:

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + A_1 x(t - \beta) + B_1 u(t - \tau), \\ y(t) &= Cx(t), \\ u(t) &= -K_e \int_0^t e(\sigma) d\sigma - K_x x(t) \\ &\quad - K_x \int_{t-\beta}^t e^{A(t-\beta-\theta)} A_1 x(\theta) d\theta \\ &\quad - K_x \int_{t-\tau}^t e^{A(t-\theta)} B u(\theta) d\theta,\end{aligned}\quad (34)$$

where K_e , K_x , and P are all defined in Theorem 5 and the output vector $y(t)$ of the closed-loop system can track the reference signal $y_d(t)$ asymptotically without static errors.

6. Numerical Simulation

Consider the system

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x(t - \beta) \\ &\quad + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t - \tau), \\ y(t) &= [1 \ 0] x(t).\end{aligned}\quad (35)$$

In comparison with system (1), the coefficient matrices are as follows:

$$\begin{aligned}A_0 &= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\ C &= [1 \ 0].\end{aligned}\quad (36)$$

Taking the sampling period $T = 0.01$ s and using the discretization technique for (35), we obtain the following discrete-time system:

$$\begin{aligned}x((k+1)T) &= (TA_0 + I)x(kT) + TA_1 x(kT - \beta) \\ &\quad + TB_1 u(kT - \tau), \\ y(kT) &= Cx(kT).\end{aligned}\quad (37)$$

According to Theorem 8, we get the closed-loop system:

$$\begin{aligned}x((k+1)T) &= (TA_0 + I)x(kT) + TA_1 x(kT - \beta) \\ &\quad + TB_1 u(kT - \tau), \\ y(kT) &= Cx(kT), \\ e(kT) &= y(kT) - y_d(kT), \\ u(kT) &= -K_e \sum_{i=0}^{k-1} T e(iT) - K_x x(kT) - K_x \\ &\quad \cdot \sum_{i=k-\beta/T}^{k-1} T e^{A(kT-\beta-iT)} A_1 x(iT) - K_x \\ &\quad \cdot \sum_{i=k-\tau/T}^{k-1} T e^{A(kT-iT)} B u(iT).\end{aligned}\quad (38)$$

Take the step signal below as the reference signal:

$$y_d(t) = \begin{cases} 0, & t \leq 3 \\ 1, & t > 3 \end{cases}\quad (39)$$

and the weight matrices $Q_e = 1$, $R = 50$. Now we do numerical simulations for several different time-delay combinations.

(1) The state delay is $\beta = 0.30$, and the input delays are $\tau = 0$, $\tau = 0.15$, and $\tau = 0.30$, respectively. Here, $\tau = 0$ means that the system has no input delay. In each case, A and B separately are

$$\beta = 0.30, \tau = 0:$$

$$\begin{aligned}A &= \begin{bmatrix} -0.43342315236725 & 1.62880145390177 \\ -0.43342315236725 & 1.62880145390177 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 \\ 2 \end{bmatrix},\end{aligned}\quad (40)$$

$$\beta = 0.30, \tau = 0.15:$$

$$\begin{aligned}A &= \begin{bmatrix} -0.43342315236725 & 1.62880145390177 \\ -0.43342315236725 & 1.62880145390177 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.52165561961054 \\ 1.52165561961054 \end{bmatrix},\end{aligned}\quad (41)$$

$\beta = 0.30, \tau = 0.30$:

$$A = \begin{bmatrix} -0.43342315236725 & 1.62880145390177 \\ -0.43342315236725 & 1.62880145390177 \end{bmatrix}, \quad (42)$$

$$B = \begin{bmatrix} 0.19537830153452 \\ 1.19537830153452 \end{bmatrix}.$$

It is verified that (A, B) is stabilizable and (C, A) is detectable in any case. Together with the reference signal $y_d(t)$, which is piecewise-continuously differentiable, the conditions of Theorem 8 are all satisfied.

In each case, we can get $\tilde{A}$, $\tilde{B}$, $\tilde{D}$, and $\tilde{C}$ (omitted here) and have the solutions of Riccati equations, K_e and K_x :

$\beta = 0.30, \tau = 0$:

$$P = \begin{bmatrix} 3.39789449121363 & 5.57360670710854 & 0.74873055237845 \\ 5.57360670710854 & 20.07087062826246 & -7.71030153174014 \\ 0.74873055237845 & -7.71030153174014 & 40.26348538482850 \end{bmatrix}, \quad (43)$$

$$K_e = 0.14142135623731,$$

$$K_x = [0.09300535129564 \quad 1.45633338475834],$$

$\beta = 0.30, \tau = 0.15$:

$$P = \begin{bmatrix} 3.65168053633590 & 5.97598941968832 & 2.59826158915244 \\ 5.97598941968832 & 21.42986989326477 & -7.64679690172430 \\ 2.59826158915244 & -7.64679690172430 & 67.51973067284588 \end{bmatrix}, \quad (44)$$

$$K_e = 0.14142135623731,$$

$$K_x = [-0.00913558840370 \quad 1.97505566074242],$$

$\beta = 0.30, \tau = 0.30$:

$$P = \begin{bmatrix} 3.839072604513 & 6.058678257824 & 4.925079815110 \\ 6.058678257824 & 22.054985350715 & -9.056007357195 \\ 4.925079815110 & -9.056007357195 & 107.307706538711 \end{bmatrix}, \quad (45)$$

$$K_e = 0.14142135623731,$$

$$K_x = [-0.13032578230273 \quad 2.53007913295354].$$

Take the allowed initial state to be $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and define $x(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $u(t) = 0$ when $t < 0$.

Figure 1 shows the situation in which the output response of the closed-loop system tracks the reference signal. As the input delay τ decreases, the effect of tracking improves when the state delay β is fixed. The control input curves of the closed-loop system are shown in Figure 2, in each case.

(2) The input delay is $\tau = 0.20$, and the state delays are $\beta = 0.01, \beta = 0.15$, and $\beta = 0.30$, respectively. Figure 3 shows

the situation in which the output response of the closed-loop system tracks the reference signal. As the state delay β decreases, the effect of tracking improves when the input delay τ is fixed. The control input curves of the closed-loop system are shown in Figure 4, in each case.

7. Conclusions

In this paper, we have studied the optimal tracking control for a class of linear continuous-time control systems with state and input delays. The main contribution of this paper is applying the methods of preview control to the optimal tracking control of linear continuous time-delay control systems. First, the time-delay system is converted to a nondelayed system in form, by a transformation. Second, utilizing the methods of preview control theory to derive an augmented system for the nondelayed system, we can design a controller with preview function, assuming that the reference signal is previewable. Finally, we propose an optimal control law of the augmented error system when the preview length of the reference signal goes to zero and obtain the optimal control law of the original time-delay system. Numerical simulations

FIGURE 1: Closed-loop responses for different input delays with a fixed state delay.

FIGURE 3: Closed-loop responses for different state delays with a fixed input delay.

FIGURE 2: The control inputs for different input delays with a fixed state delay.

FIGURE 4: The control inputs for different state delays with a fixed input delay.

show the effectiveness and validity of the conclusions in this paper.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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