Research Article

Lot-Order Assignment Applying Priority Rules for the Single-Machine Total Tardiness Scheduling with Nonnegative Time-Dependent Processing Times

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Lot-order assignment is to assign items in lots being processed to orders to fulfill the orders. It is usually performed periodically for meeting the due dates of orders especially in a manufacturing industry with a long production cycle time such as the semiconductor manufacturing industry. In this paper, we consider the lot-order assignment problem (LOAP) with the objective of minimizing the total tardiness of the orders with distinct due dates. We show that we can solve the LOAP optimally by finding an optimal sequence for the single-machine total tardiness scheduling problem with nonnegative time-dependent processing times (SMTTSP-NNTDPT). Also, we address how the priority rules for the SMTTSP can be modified to those for the SMTTSP-NNTDPT to solve the LOAP. In computational experiments, we discuss the performances of the suggested priority rules and show the result of the proposed approach outperforms that of the commercial optimization software package.

1. Introduction

Lot-order assignment is to assign items in lots being processed to orders to fulfill the orders in make-to-order production systems. Typical lot-order assignment examples include assigning wafers in lots to orders in a semiconductor wafer fabrication facility and assigning panels in lots to orders in a color-filter fabrication facility of the thin film transistor liquid crystal display industry. In general, an order is satisfied with several lots because the order size is typically larger than the lot size; however, a lot may be used to satisfy several orders also. Additional lots should be released into the production facility if all the orders cannot be satisfied with lots currently in the production facility. Lot-order assignment is as important as production planning and scheduling for meeting the due dates of orders (especially in a manufacturing industry with a long production cycle time such as the semiconductor manufacturing industry). Lot-order assignment is usually

performed periodically with a one-day or one-shift cycle before production begins.

Compared to the production planning and scheduling problem, there have only been a few studies on lot-order assignment problems (LOAPs), most of which have involved the semiconductor manufacturing industry. Knutson et al. [1], Fowler et al. [2], Carlyle et al. [3], Boushell et al. [4], and Ng et al. [5] studied lot-order assignment problems in semiconductor assembly and test facilities, whereas Wu [6], Bang et al. [7], and Kim and Lim [8] studied these problems in wafer fabrication facilities. Kim and Lim [8] provided insight into the characteristics of the LOAP. They suggested the compact pegging method and showed that the LOAP becomes equivalent to an order sequence problem with this compact pegging method. On the other hand, Lim et al. [9] developed a Lagrangian heuristic for simultaneous order-lot pegging and release planning in semiconductor wafer fabrication facilities.

The existing algorithms for the LOAP are not practical since they are not easy to implement and require long computation time. In this study, we show that the LOAP can solve optimally by finding an optimal sequence for the single-machine total tardiness scheduling problem with nonnegative time-dependent processing times (SMTTSP-NNTDPT). Also, we address how the priority rules for the SMTTSP can be modified to those for the SMTTSP-NNTDPT to solve the LOAP.

2. Mathematical Formulation

The LOAP considered in this paper is to determine the assignment of items in lots to orders in a production facility, as well as a plan to release additional lots into the production facility (when needed) with the objective of minimizing the total tardiness of the orders over a finite time horizon. To simplify the problem, it is assumed that items in any lot can be assigned to any order. This assumption is realistic in many cases where lots and orders can be classified into different groups according to their product types. Thus, lots and orders of the same product type can be dealt with separately for lot-order assignments. In addition, we assume that new lots are released into the production facility on a daily basis (when needed) and lot-order assignment is performed with a one-day cycle before production begins. Further, it is assumed that the current day is represented as “day 0” and the current day’s production begins at time zero without loss of generality. We use the following notation for modeling the problem mathematically:

- N : number of orders,
- L : number of lots in process,
- T : length of the time horizon (day),
- i : index for order,
- l : index for lot,
- t : index for time period,
- q_i : quantity of order i ,
- w_l : number of items in lot l ,
- Q_t : limit on the total number of items that can be released into the production facility at day t ,
- r_l : remaining processing time of lot l (hour),
- d_i : due date of order i (hour),
- CT: production cycle time (hour),
- C_i : completion time of order i (hour),
- τ_i : tardiness of order i (hour),
- x_{il} : number of items in lot l that are assigned to order i ,
- y_{it} : number of items to be newly released at day t for order i ,
- z_{il} : equal to 1 if lot l is used to satisfy order i , 0 otherwise,
- u_{it} : equal to 1 if new lots are released at day t for order i , 0 otherwise.

Now, we present a mixed integer linear programming formulation of the LOAP:

$$[\text{LOAP}] \text{ Minimize } \sum_{i=1}^N \tau_i \quad (1)$$

$$\text{subject to } \sum_{l=1}^L x_{il} + \sum_{t=1}^T y_{it} = q_i \quad \forall i \quad (2)$$

$$\sum_{i=1}^N x_{il} \leq w_l \quad \forall l \quad (3)$$

$$\sum_{i=1}^N y_{it} \leq Q_t \quad \forall t \quad (4)$$

$$x_{il} \leq w_l z_{il} \quad \forall i, l \quad (5)$$

$$y_{it} \leq Q_t u_{it} \quad \forall i, t \quad (6)$$

$$C_i \geq r_l z_{il} \quad \forall i, l \quad (7)$$

$$C_i \geq (24t + \text{CT}) u_{it} \quad \forall i, t \quad (8)$$

$$\tau_i \geq C_i - d_i \quad \forall i \quad (9)$$

$$\tau_i \geq 0 \quad \forall i \quad (10)$$

$$x_{il} \geq 0 \quad \forall i, l \quad (11)$$

$$y_{it} \geq 0 \quad \forall i, t \quad (12)$$

$$z_{il} \in \{0, 1\} \quad \forall i, l \quad (13)$$

$$u_{it} \in \{0, 1\} \quad \forall i, t. \quad (14)$$

The objective function (1) to be minimized denotes the total tardiness of all orders. Constraint (2) ensures that order quantity should be satisfied with existing lots and/or newly released ones. Constraint (3) limits the maximum number of items in a lot that can be assigned to orders. Constraint (4) limits the maximum number of items that can be released at each period. Constraint (5) specifies the number of items in each lot which are assigned to orders. Constraint (6) specifies the number of items which are newly released in each time period for orders. Note that the number of lots newly released in each period can be automatically determined given the number of items newly released in each period and the lot sizes, although this is not included in the model. Constraints (7) and (8) specify the completion time of each order. Constraint (9) specifies the tardiness of each order. Constraints (10) to (14) represent decision variables.

Figure 1 shows a lot-order assignment for a small example with four orders and four lots. In the figure, $x_{11} = 20$, $x_{12} = 10$, $x_{22} = 10$, $x_{23} = 5$, $x_{33} = 10$, $x_{34} = 20$, $y_{40} = 25$, and the total tardiness is nine. Note that 25 new items should be released at day 0 (current day) to satisfy order 4 in this case.

FIGURE 1: Illustration of a lot-order assignment.

FIGURE 2: Lot-order assignments using the compact pegging method for two order sequences.

3. Relationship between the LOAP and the SMTTSP-NNTDPT

Kim and Lim [8] proved that the LOAP is strongly NP-hard and proposed a lot-order assignment method called the compact pegging method which generates the best lot-order assignment with the minimum total tardiness for a given order sequence for assigning items in lots to orders. In the compact pegging method, items (in lots) with less remaining time are assigned to orders in earlier positions in the order sequence. Figure 2 shows two lot-order assignments for the example, which were obtained by applying the compact pegging method to two order sequences. Lots are lined up in increasing order of the remaining time, and they are assigned to orders one by one according to the order sequence. In the

figure, the order sequence 2, 1, 3, 4 (Figure 2(b)) yields a better result than the order sequence 1, 2, 3, 4 (Figure 2(a)) in terms of total tardiness (7 versus 9).

Kim and Lim [8] proved that there exists an (optimal) order sequence from which an optimal solution (lot-order assignment) for the LOAP can be obtained using the compact pegging method. Thus, the LOAP becomes equivalent to the problem of finding an optimal order sequence for assigning items in lots to orders. In this section, we show that we can obtain an optimal order sequence for the LOAP by solving the SMTTSP with nonnegative time-dependent processing times. As a result, we can solve the LOAP optimally by finding an optimal sequence for the SMTTSP with nonnegative time-dependent processing times.

Scheduling with time-dependent processing times means that the job processing times can be changed by the time when the job is started. Many researches, including Cheng et al. ([10], survey paper), Cheng et al. [11], Yin et al. [12], and Yin and Xu [13], consider the cases in which processing times are changed by deterioration or learning effect. In real production systems, processing time including setup times can be changed with sequence (position) of jobs. There are a number of studies on scheduling problems with time-dependent processing time based on sequence-dependent setup times, as surveyed in Allahverdi et al. [14]. Recently, Hung et al. [15] consider scheduling problems with semiconductor multi-head testers in which the processing time is related to the product mix. However, there are few researches which deal with the scheduling problems with *nonnegative* time-dependent processing times. In this section, we suggest some properties of the LOAP and prove that the optimal solution can be obtained by finding the optimal sequence for SMTTSP-NNTDPT.

Let C_i^π be the completion time of order i in order sequence π when the compact pegging method is applied to order sequence π , and let $C(\cdot)$ be the total completion time for orders in order set $\cdot$, that is, the required time to complete all orders in order set $\cdot$.

Property 1. For any order set Ω , $C(\Omega)$ is constant regardless of the order sequence.

Proof. $C(\Omega)$ is equal to the completion time of the lot with the longest remaining time among the lots assigned to the orders in Ω . In the compact pegging method, items in lots are assigned to orders in an increasing order of the remaining processing times of the lots until the total demand of the orders is satisfied. Because the total demand of the orders is constant regardless of the order sequence, the last lot assigned to an order is not changed. Thus, the total completion time of the orders is constant regardless of their sequence (e.g., the total completion time is 20 for any order sequence in Figure 2). $\square$

Property 2. For any two sets of orders Ω_1 and Ω_2 , the total completion times for Ω_1 and Ω_2 have the following relationships:

- (1) if $\sum_{i \in \Omega_1} q_i = \sum_{i \in \Omega_2} q_i$, then $C(\Omega_1) = C(\Omega_2)$,
- (2) if $\sum_{i \in \Omega_1} q_i < \sum_{i \in \Omega_2} q_i$, then $C(\Omega_1) \leq C(\Omega_2)$.

Proof. The relationship (1) holds because the last lot assigned to the two sets of orders, Ω_1 and Ω_2 , is the same when the two sets of orders have the same total number of items. The relationship (2) also holds because it may be necessary to assign additional lots with longer remaining times to the orders in Ω_2 compared to ones in Ω_1 . $\square$

Property 3. If order i precedes order j in any order sequence π , then $C_i^\pi \leq C_j^\pi$.

Proof. In the lot-order assignment method, lots with less remaining times are assigned to orders at the earlier positions

in the order sequence. Therefore, the lots assigned to order j have the same or longer completion times than those assigned to order i . $\square$

We define the processing time of order i in order sequence π as the time duration to complete order i after completing the preceding order in π and denote it by ρ_i^π . Let U_i^π be a set of orders which precede order i in π , and let $[i]$ be an index of the i th order in an order sequence. Then, $\rho_{[i]}^\pi$ is given as below:

$$\rho_{[i]}^\pi = C_{[i]}^\pi - C_{[i-1]}^\pi \quad \forall i. \quad (15)$$

Property 4. Consider $C_i^\pi = C(U_i^\pi \cup \{i\})$.

Proof. By definition of $C(\cdot)$, $C(U_i^\pi \cup \{i\}) = \max_{j \in U_i^\pi \cup \{i\}} C_j^\pi$. By Property 3, $\max_{j \in U_i^\pi} C_j^\pi \leq C_i^\pi$. It follows that $C(U_i^\pi \cup \{i\}) = C_i^\pi$. $\square$

By Property 4, (15) can be rewritten as

$$\rho_i^\pi = C(U_i^\pi \cup \{i\}) - C(U_i^\pi) \quad \forall i. \quad (16)$$

Property 5. If $q_i \geq q_j$ and $U_i^\pi = U_j^{\pi'}$, $\rho_i^\pi \geq \rho_j^{\pi'}$.

Proof. It is obvious by Properties 2 and 4. $\square$

Property 6. The completion time of the last order in any order sequence is equal to the total completion time of the orders; that is, $C_{[N]}^\pi = C(\Omega)$ for any π .

Proof. It is obvious by Properties 1 and 4. $\square$

Property 7. The sum of the processing times of all orders for any order sequence π is equal to the total completion time of the orders; that is, $\sum_{i \in \Omega} \rho_i^\pi = C(\Omega)$.

Proof. Consider $\sum_{i \in \Omega} \rho_i^\pi = C_{[1]}^\pi + (C_{[2]}^\pi - C_{[1]}^\pi) + (C_{[3]}^\pi - C_{[2]}^\pi) + \dots + (C_{[N]}^\pi - C_{[N-1]}^\pi) = C_{[N]}^\pi = C(\Omega)$. $\square$

Property 8. The processing times of orders are time-dependent.

Proof. The completion times of orders are determined by both the order quantities and the lots assigned to the orders. The lots assigned to each order are determined by the sequence of the orders. Therefore, the processing times of orders are time-dependent. $\square$

Property 9. The processing times of orders are nonnegative.

Proof. By Property 3 and (15), the completion time of order $[i]$ is greater than that of order $[i-1]$; that is, $C_{[i]}^\pi \geq C_{[i-1]}^\pi$, so that, $\rho_{[i]}^\pi = C_{[i]}^\pi - C_{[i-1]}^\pi \geq 0$. $\square$

Properties 8 and 9 can be easily verified in the example shown in Figure 2. In Figure 2, $\rho_1^\pi = 8$, $\rho_2^\pi = 2$, $\rho_3^\pi = 0$, $\rho_4^\pi = 10$, and $\rho_1^{\pi'} = 3$, $\rho_2^{\pi'} = 7$, $\rho_3^{\pi'} = 0$, $\rho_4^{\pi'} = 10$, where $\pi = (1, 2, 3, 4)$ and $\pi' = (2, 1, 3, 4)$. Note that the *processing time of order 3 becomes zero* as shown in Figure 2, although the processing time is strictly greater than zero

FIGURE 3: The effect of interchanging two orders in an order sequence in Property 10.

in a typical SMTTSP. This is a distinct characteristic not considered in previous research on the scheduling problem with time-dependent processing times.

Theorem 1. *An optimal solution for the LOAP can be obtained by finding an optimal sequence for the SMTTSP with nonnegative time-dependent processing times (SMTTSP-NNTDPT).*

Proof. By applying the compact pegging method to any order sequence π , we can obtain ρ_i^π and C_i^π for all i . According to Properties 8 and 9, ρ_i^π is time-dependent and nonnegative. If we consider ρ_i^π and C_i^π as the processing time and the completion time of job i in the SMTTSP-NNTDPT, respectively, finding an optimal order sequence for the LOAP becomes equivalent to the problem of finding an optimal job sequence for the SMTTSP-NNTDPT. $\square$

4. Priority Rules for SMTTSP-NNTDPT

Because the SMTTSP-NNTDPT is NP-hard, it is considerably difficult to obtain an optimal solution when the problem size is large, similar to the SMTTSP. In this paper, we suggest several priority rules for the SMTTSP-NNTDPT, which can be easily applied to practice. Priority rules are used to determine the order sequence to which the compact pegging method is applied to obtain lot-order assignments for the LOAP. Various priority rules exist for the SMTTSP, such as SPT, EDD, MDD, SLACK, COVERT, and ATC [16, 17]. In this section, we address how the priority rules for the SMTTSP can be modified to those for the SMTTSP-NNTDPT. Among the many priority rules for the SMTTSP, six priority rules (EDD, SPT, MDD, SLACK, COVERT, and ATC) are most widely used in practice and are known to perform better than others [18–21]. Therefore, these six priority rules are modified for the SMTTSP-NNTDPT in this paper.

4.1. EDD Rule. In the EDD rule, orders with earlier due dates have higher priorities and are considered before those with later due dates in an order sequence. Ties are broken by selecting the order with the minimum order quantity. The EDD rule can be simply represented by EDD: $\min(d_i)$. We derive the optimality property of the EDD rule for the SMTTSP-NNTDPT [22].

Property 10. If order quantities and due dates are agreeable for any pair of orders, total tardiness is minimized by EDD sequencing.

Proof. Consider order sequence π in which order k precedes order j with $q_j \leq q_k$ and $d_j \leq d_k$. Let T_s and T_e be the times at which order k starts and order j ends, respectively (see Figure 3). Let π' be a sequence which is obtained by interchanging orders j and k in π . Because $q_j \leq q_k$, $\rho_k^\pi \geq \rho_j^{\pi'}$ and $\rho_j^\pi \leq \rho_k^{\pi'}$ according to Property 2. Note that there exists no (in)equality relationship between ρ_j^π and $\rho_j^{\pi'}$ (or ρ_k^π and $\rho_k^{\pi'}$). We will show that interchanging the two orders must decrease, or possibly leave unchanged, the total tardiness. Clearly, all orders which precede order k or follow order j in the original sequence are unaffected when the two orders are interchanged. All orders between order k and j are advanced in time, which decreases or leaves unchanged their tardiness. The changes in tardiness of orders j and k are investigated by considering following three cases.

- (i) If $d_j \leq d_k < T_e$, as illustrated in Figure 3, then the decrease of tardiness of order j is $\Delta\tau_j = T_e - \max(T_s + \rho_j^{\pi'}, d_j)$, the increase of tardiness of order k is $\Delta\tau_k = T_e - \max(T_s + \rho_k^\pi, d_k)$, and the net decrease due to the two changes is $\Delta\tau_j - \Delta\tau_k = \max(T_s + \rho_k^\pi, d_k) - \max(T_s + \rho_j^{\pi'}, d_j)$, which is nonnegative because $d_k \geq d_j$ and $\rho_k^\pi \geq \rho_j^{\pi'}$.
- (ii) If $d_j \leq T_e \leq d_k$, $\Delta\tau_j - \Delta\tau_k = \Delta\tau_j \geq 0$.
- (iii) If $T_e < d_j \leq d_k$, $\Delta\tau_j = \Delta\tau_k = 0$. Thus in all cases the total decrease of tardiness is positive, or at worst zero, so the change should be made. $\square$

4.2. MOQ Rule. The MOQ rule corresponds to the SPT rule for the SMTTSP; that is, the order with the shortest processing time has the highest priority. Although processing times of the SMTTSP-NNTDPT are nonnegative and time-dependent, they can be replaced with the order quantities according to Property 5 when using the MOQ rule. Thus, the order with the minimum order quantity has the highest priority in the MOQ rule. Ties are broken by selecting an

FIGURE 4: The effect of interchanging two orders in an order sequence in Property 11.

order with the earliest due date. The MOQ rule can be simply represented by MOQ: $\min(q_i)$.

4.3. MDD Rule. The MDD rule was originally suggested by Baker and Bertrand [23] for the SMTTSP. The MDD rule for the SMTTSP-NNTDPT can be modified based on Property 11 below.

Property 11. If order k comes before order j in order sequence π and $\max(d_k, C_k^\pi) > \max(d_j, C_j^{\pi'})$, π' gives less total tardiness than π , where π' is an order sequence in which positions of orders j and k are interchanged in π .

Proof. Assume that order k starts at time T_s and order j is completed at time T_e in π as shown in Figure 4 and $\max(d_k, C_k^\pi) > \max(d_j, C_j^{\pi'})$. We will show that interchanging the two orders must decrease, or possibly leave unchanged, the total tardiness. Let τ'_j and τ'_k be tardiness of order j and order k in π' , respectively. We can consider following four cases.

- (i) Consider $\max(d_j, C_j^{\pi'}) = d_j$, $\max(d_k, C_k^\pi) = d_k$, and $d_j < d_k$ as illustrated in Figure 4. Then $\tau_k = \max(C_k^\pi - d_k, 0) = 0$, $\tau_j = \max(T_e - d_j, 0)$, $\tau'_k = \max(T_e - d_k, 0)$, and $\tau'_j = \max(C_j^{\pi'} - d_j, 0) = 0$. The decrease of tardiness of order j is $\Delta\tau_j = \max(T_e - d_j, 0)$ and the increase of that of order k is $\Delta\tau_k = \max(T_e - d_k, 0)$. Because $d_j < d_k$, $\Delta\tau_j \geq \Delta\tau_k$.
- (ii) Consider $\max(d_j, C_j^{\pi'}) = C_j^{\pi'}$, $\max(d_k, C_k^\pi) = d_k$, and $C_j^{\pi'} < d_k$. Then $\tau_k = \max(C_k^\pi - d_k, 0) = 0$, $\tau_j = \max(T_e - d_j, 0) = T_e - d_j$, $\tau'_k = \max(T_e - d_k, 0)$, and $\tau'_j = \max(C_j^{\pi'} - d_j, 0) = C_j^{\pi'} - d_j$. The decrease of tardiness of order j is $\Delta\tau_j = T_e - C_j^{\pi'}$ and the increase of that of order k is $\Delta\tau_k = \max(T_e - d_k, 0)$. Because $C_j^{\pi'} < d_k$, $\Delta\tau_j > \Delta\tau_k$.
- (iii) Consider $\max(d_j, C_j^{\pi'}) = d_j$, $\max(d_k, C_k^\pi) = C_k^\pi$, and $d_j < C_k^\pi$. Then $\tau_k = \max(C_k^\pi - d_k, 0) = C_k^\pi - d_k$, $\tau_j = \max(T_e - d_j, 0) = T_e - d_j$, $\tau'_k = \max(T_e - d_k, 0) = T_e - d_k$, and $\tau'_j = \max(C_j^{\pi'} - d_j, 0) = 0$. The decrease of

tardiness of order j is $\Delta\tau_j = T_e - d_j$ and the increase of that of order k is $\Delta\tau_k = T_e - C_k^\pi$. Because $d_j < C_k^\pi$, $\Delta\tau_j > \Delta\tau_k$.

- (iv) Consider $\max(d_j, C_j^{\pi'}) = C_j^{\pi'}$, $\max(d_k, C_k^\pi) = C_k^\pi$, and $C_k^\pi < C_j^{\pi'}$. Then $\tau_k = \max(C_k^\pi - d_k, 0) = C_k^\pi - d_k$, $\tau_j = \max(T_e - d_j, 0) = T_e - d_j$, $\tau'_k = \max(T_e - d_k, 0) = T_e - d_k$, and $\tau'_j = \max(C_j^{\pi'} - d_j, 0) = C_j^{\pi'} - d_j$. The decrease of tardiness of order j is $\Delta\tau_j = T_e - C_j^{\pi'}$ and the increase of that of order k is $\Delta\tau_k = T_e - C_k^\pi$. Because $C_k^\pi < C_j^{\pi'}$, $\Delta\tau_j > \Delta\tau_k$.

□

We define the modified due date for an order as the larger value between the due date of the order and the earliest possible completion time of the order, which can be obtained by temporarily assigning lots with the least remaining time to the order until the order demand is satisfied. According to Property 11, orders with less modified due dates should be considered before ones with later modified due dates to minimize total tardiness. In the MDD rule, orders are selected in succession, and the i th selected order is placed at the i th position in the order sequence. When selecting an order, the order with the least modified due date among the unselected orders is selected. The modified due dates of orders should be updated each time a new order is selected because the earliest possible completion times of the unselected orders depend on those of selected ones. In this regard, the MDD rule is called the dynamic priority rule. MDD can be simply represented by MDD: $\min\{\max(d_i, t + \rho_i)\}$, where t is the total completion time of the orders selected until now, and ρ_i is the processing time of order i in the SMTTSP-NNTDPT. Note that $t + \rho_i$ is the earliest possible completion time of order i with the selected orders given. Because order sequence π is not given but needs to be determined, we use notation ρ_i instead of ρ_i^π for convenience. Although ρ_i is nonnegative and time-dependent, it can be calculated once the predecessors of order i are given. Let Ψ be the set of selected orders. Then $t = C(\Psi)$ and $\rho_i = C(\Psi \cup \{i\}) - C(\Psi)$. Procedure 1 shows the steps for selecting orders and assigning lots to these orders in the MDD rule.

TABLE 1: Summary of the priority rules.

Rule	Definition	Rank and priority index
EDD	Earliest due date	$\min(d_i)$
MOQ	Minimum order quantity	$\min(q_i)$
MDD	Modified due date	$\min\{\max(d_i, t + \rho_i)\}$
SLACK	Slack	$\min(d_i - t - \rho_i)$
COVERT	Cost over time	$\max\left\{\frac{1}{\rho_i} \max\left[0, 1 - \frac{\max(0, d_i - t - \rho_i)}{k\rho_i}\right]\right\}$
ATC	Apparent tardiness cost	$\max\left\{\frac{1}{\rho_i} \exp\left(\frac{-\max(0, d_i - t - \rho_i)}{k\bar{\rho}}\right)\right\}$

Procedure 1 (MDD).

Step 0. Let $p = 1$, $\Psi = \emptyset$, and $t = 0$.

Step 1. For order $i \notin \Psi$, obtain ρ_i by $\rho_i = C(\Psi \cup \{i\}) - C(\Psi)$ and obtain the modified due date of the order, that is, $\max(d_i, t + \rho_i)$.

Step 2. Select $i^* = \arg \min_{i \notin \Psi} \{\max(d_i, t + \rho_i)\}$ and place order i^* at the p th position in an order sequence. Let $\Psi \leftarrow \Psi \cup \{i^*\}$, $t \leftarrow t + C(\Psi \cup \{i^*\})$, and $p \leftarrow p + 1$. If $p \leq N$, go to step 1.

Step 3. Assign lots to order i^* using the compact pegging method.

4.4. SLACK Rule. In the SLACK rule, the order with the least slack time has the highest priority. The slack time of an order is defined as the maximum available time to delay the completion of the order without violating its due date. The order slack time is computed as the difference between the order due date and the earliest possible completion time for the order. As with the MDD rule, the SLACK rule is also a dynamic priority rule because the order slack time is not static but should be updated each time a new order is selected and lots are assigned to the order. The SLACK rule can be simply represented by SLACK: $\min(\text{SLACK}_i)$, where $\text{SLACK}_i = d_i - t - \rho_i$.

4.5. COVERT Rule. The original COVERT priority index represents the expected tardiness cost per unit of imminent processing time, or cost over time [17]. The COVERT rule is a ratio-based priority rule which combines the SLACK rule and the SPT rule. It puts the order with the largest COVERT ratio in the first position. The COVERT ratio of an order is computed by dividing a derived urgency ratio of the order by the processing time of the order. The COVERT rule can be simply represented by COVERT: $\max\{(1/\rho_i) \max[0, 1 - \max(0, d_i - t - \rho_i)/k\rho_i]\}$, where k is a parameter. The value of k is usually determined through experimental analysis. The COVERT rule is also a dynamic priority rule. It is known that it performs well on the due date-based objectives, especially on the total tardiness for the SMTTSP [24].

4.6. ATC Rule. The ATC rule was developed based on the COVERT rule for the SMTTSP [17]. The basic concept of

the ATC rule is the same with the COVERT rule with two main differences. First, the ATC rule uses an exponential function rather than linear one to emphasize the part of slack. Second, the ratio is calculated by dividing the order slack by the average processing time, instead of the order processing time. The ATC rule can be simply represented by ATC: $\max\{(1/\rho_i) \exp(-\max(0, d_i - t - \rho_i)/k\bar{\rho})\}$, where k is a parameter and $\bar{\rho} = \sum_{i \notin \Psi} \rho_i$.

Table 1 summarizes the six priority rules for the SMTTSP-NNTDPT. The first two rules, EDD and MOQ, are static rules, and the rest rules on the other hand are dynamic ones. In the static rules, the order priority index values do not change over time, whereas they might change over time in the dynamic rules.

5. Computational Experiments

To test the performances of the suggested priority rules, we randomly generated 540 problem instances of the LOAP while varying the number of orders, number of lots, and due-date tightness. We considered 27 combinations: three levels for the number of orders (50, 100, and 200), three levels for the number of processing lots (sufficient, limited, and short), and three levels for the due-date tightness (loose, normal, and tight). We then generated 20 replication problem instances for each combination to obtain 540 problem instances. In each problem instance, the relevant data were generated as follows. Here, $DU(a, b)$ and $U(a, b)$ denote random numbers generated from the discrete and continuous distributions with a range $[a, b]$, respectively.

- (1) The order size, N , was set to $DU(50, 300)$ for each order.
- (2) The production cycle time, CT , was set to $DU(150, 200)$ hours.
- (3) The remaining time of lot l , r_l , was set to $CT \cdot U(0, 1)$ hours.
- (4) The length of the time horizon was set to 4 weeks.
- (5) The lot sizes were set to $DU(10, 20)$ for each lot.
- (6) Q_i was set to 1000 units for all i .
- (7) Due dates of orders were set to $\alpha \cdot CT$ for each order, where α is a due-date factor which is set to 1.7, 1.3, and

TABLE 2: Results with CPLEX.

Number of orders	Due date tightness	Lots	Number of optimal solutions found	Number of feasible solutions found
50	Loose	Sufficient	20	20
		Limited	4	20
		Short	0	20
	Normal	Sufficient	15	20
		Limited	0	20
		Short	0	20
	Tight	Sufficient	0	20
		Limited	0	20
		Short	0	20
100	Loose	Sufficient	16	20
		Limited	3	20
		Short	0	20
	Normal	Sufficient	12	20
		Limited	0	15
		Short	0	17
	Tight	Sufficient	0	0
		Limited	0	19
		Short	0	16
200	Loose	Sufficient	16	16
		Limited	0	0
		Short	0	1
	Normal	Sufficient	12	12
		Limited	0	0
		Short	0	0
	Tight	Sufficient	0	0
		Limited	0	0
		Short	0	0

0.9 for generating loose, normal, and tight due dates, respectively.

- (8) The number of processing lots was set to $\lceil \beta \cdot (\text{the sum of order sizes/average lot size}) \rceil$, where β is a number of lots factor which is set to 1.0, 0.7 and 0.5 for generating sufficient, limited, and short numbers of lots, respectively, and $\lceil \cdot \rceil$ is the closest integer to $\cdot$.

Using the data sets, we compared the priority rules to each other and to CPLEX 12.1, which is one of the best commercial optimization software packages. All of the tests were performed on a personal computer with an Intel Core i5 processor operating at a 3.20 GHz clock speed with 4 GB of RAM. First, the problem instances were solved using CPLEX with a 1 h time limit. Table 2 lists the test results with CPLEX. CPLEX found optimal and feasible solutions for 98 and 356 problem instances out of the 540, respectively. As the number of orders increased, the problem size increased, and CPLEX hardly found optimal solutions as a result. In addition, as the due dates became tighter and the number of lots decreased,

the problems became more difficult to solve, and it became more difficult to find optimal solutions using CPLEX. When the due date was tight or the lots were short, CPLEX could not find optimal solutions for all the instances. For the 200-order problem instances, CPLEX failed to find even feasible solutions for 84%, which means CPLEX is not a practical tool for solving real-world lot-order assignment problems.

Table 3 lists the priority-rule test results for 50-order problem instances. In the table, rule performance is represented by two measures: the number of best solutions found by each rule (NB) out of the 20 replication problem instances and the average relative deviation index (ARDI). A relative deviation index (RDI) value for rule A represents how relatively good a solution obtained by rule A is compared to those obtained by other rules, and it is calculated as $(T_{\text{worst}} - T_A)/(T_{\text{worst}} - T_{\text{best}})$, where T_A , T_{best} , and T_{worst} are the total tardiness by rule A, along with the best (smallest) and worst (largest) total tardiness values among all the rules, respectively. The RDI value is between 0 and 1, where 1 means the rule found the best solution among all the rules, and 0 means the opposite. In the table, MDD performed the best in terms of both NB and ARDI. Further, COVERT and ATC found favorable solutions that were considerably close to the best solutions. MOQ showed the worst performance because it does not consider the due date.

Table 4 lists the test results for the 100-order problem instances. These results are similar to those in Table 2. MDD found the greatest number of best solutions, and MDD, COVERT, and ATC showed the best performances in terms of solution quality.

Table 5 lists the test results for the 200-order problem instances. In this test, COVERT and ATC beat MDD in both of the performance measures. Although MDD worked best when the number of lots was sufficiently large to be assigned to orders, its performance deteriorated significantly when there were insufficient lots, and therefore, new lots needed to be released into the production facility. Summarizing the test results in Tables 2–5, MDD, COVERT, and ATC outperformed the other rules. The MDD performance seemed to be affected by the problem characteristics such as the problem size, due-date tightness, and work-in-process level, whereas COVERT and ATC seemed to be robust rules, which guaranteed favorable performances in any case.

To validate the absolute performances of the suggested rules, we compared the suggested rules with CPLEX. For the sake of convenience, we only compared MDD with CPLEX because MDD is one of the best rules. Table 6 lists the results of the comparison between MDD and CPLEX in terms of two measures: average optimality gap (AOG) and average CPLEX gap (ACG). The AOG denotes a percentage of optimal solutions found by MDD for the 20 replication instances in each problem group for which CPLEX also found optimal solutions, and the ACG denotes an average total tardiness gap percentage between the CPLEX and MDD solutions in each problem group. Note that “N.A.” in the AOG (ACG) column in the table means that CPLEX failed to find optimal (feasible) solutions for all the problem instances in the problem group, and no optimal (feasible) solutions are available. In the table, the AOG values are close to 100%,

TABLE 3: Test results of priority rules for 50-order problems.

Due date tightness	Number of lots	EDD		MOQ		MDD		SLACK		COVERT		ATC	
		NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]
Loose	Sufficient	17	0.99	0	0.00	20	1.00	15	0.99	10	0.99	17	0.99
	Limited	3	0.97	0	0.00	20	1.00	1	0.96	0	0.94	1	0.98
	Short	0	0.76	0	0.00	13	0.99	0	0.70	1	0.95	6	0.99
Normal	Sufficient	13	0.99	0	0.00	19	1.00	10	0.99	6	0.99	14	0.99
	Limited	0	0.80	0	0.00	16	0.99	0	0.73	1	0.97	4	0.98
	Short	0	0.23	0	0.11	4	0.97	0	0.11	9	0.98	9	0.99
Tight	Sufficient	0	0.79	0	0.00	15	0.99	0	0.75	5	0.98	1	0.98
	Limited	0	0.12	0	0.26	11	0.99	0	0.00	6	0.99	6	0.98
	Short	0	0.11	0	0.68	6	0.98	0	0.00	11	0.99	7	0.98
Sum/average		33	0.64	0	0.12	124	0.99	26	0.58	49	0.98	65	0.99

[†]Number of the best solutions found.[‡]Average relative deviation index.

TABLE 4: Test results of priority rules for 100 order problems.

Due date tightness	Number of lots	EDD		MOQ		MDD		SLACK		COVERT		ATC	
		NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]
Loose	Sufficient	15	0.99	0	0.00	20	1.00	12	0.99	7	0.99	8	0.99
	Limited	4	0.98	0	0.00	19	1.00	1	0.96	0	0.91	0	0.96
	Short	0	0.53	0	0.00	3	0.96	0	0.45	4	0.97	13	0.99
Normal	Sufficient	10	0.99	0	0.00	19	1.00	9	0.99	3	0.99	7	0.99
	Limited	0	0.67	0	0.00	5	0.97	0	0.60	9	0.99	6	0.98
	Short	0	0.09	0	0.38	4	0.97	0	0.00	12	0.98	4	0.97
Tight	Sufficient	0	0.80	0	0.00	12	0.99	0	0.77	8	0.99	0	0.96
	Limited	0	0.07	0	0.41	7	0.98	0	0.00	8	0.99	5	0.98
	Short	0	0.06	0	0.77	8	0.98	0	0.00	9	0.99	3	0.98
Sum/average		29	0.58	0	0.17	97	0.98	22	0.53	60	0.98	46	0.98

[†]Number of the best solutions found.[‡]Average relative deviation index.

TABLE 5: Test results of priority rules for 200 order problems.

Due date tightness	Number of lots	EDD		MOQ		MDD		SLACK		COVERT		ATC	
		NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]	NB [†]	ARDI [‡]
Loose	Sufficient	17	0.99	0	0.00	20	1.00	14	0.99	5	0.99	9	0.99
	Limited	1	0.92	0	0.00	9	0.98	0	0.89	0	0.89	10	0.99
	Short	0	0.11	0	0.34	1	0.82	0	0.01	3	0.90	16	0.99
Normal	Sufficient	12	0.99	0	0.00	20	1.00	9	0.99	5	0.99	2	0.99
	Limited	0	0.46	0	0.00	2	0.93	0	0.37	7	0.97	11	0.99
	Short	0	0.06	0	0.81	0	0.93	0	0.00	8	0.99	12	0.99
Tight	Sufficient	0	0.81	0	0.00	13	0.99	0	0.79	7	0.99	0	0.94
	Limited	0	0.06	0	0.60	1	0.95	0	0.00	19	1.00	0	0.95
	Short	0	0.05	0	0.93	2	0.95	0	0.00	15	0.99	3	0.97
Sum/average		30	0.50	0	0.30	68	0.95	23	0.45	69	0.97	63	0.98

[†]Number of the best solutions found.[‡]Average relative deviation index.

TABLE 6: Comparison between MDD and CPLEX.

Due date tightness	Number of orders Number of lots	50		100		200	
		AOG [†] (%)	ACG [‡] (%)	AOG (%)	ACG (%)	AOG (%)	ACG (%)
Loose	Sufficient	100.0	0.00	100.0	−11.1	100.0	0.00
	Limited	100.0	−38.5	33.3	−56.1	N.A.	N.A.
	Short	N.A.	−40.7	N.A.	−29.9	N.A.	−20.11
Normal	Sufficient	93.3	−11.9	91.7	−35.9	100.0	−7.4
	Limited	N.A.	−44.0	N.A.	−60.5	N.A.	N.A.
	Short	N.A.	−45.6	N.A.	−44.4	N.A.	N.A.
Tight	Sufficient	N.A.	−69.3	N.A.	N.A.	N.A.	N.A.
	Limited	N.A.	−59.5	N.A.	−48.2	N.A.	N.A.
	Short	N.A.	−62.8	N.A.	−56.3	N.A.	N.A.

[†] Average optimality gap.

[‡] Average CPLEX gap, where CPLEX gap = (total tardiness of MDD − total tardiness of CPLEX)/(total tardiness of CPLEX) × 100.

which means the MDD found optimal solutions for most problem instances for which CPLEX found optimal solutions. All ACG values are zero or negative and most of them are less than −30%, which means the MDD solutions were much better than those obtained by CPLEX. Furthermore, all suggested priority rules required very short CPU times (less than 2 s even for the 200-order problem instances). Thus, we can argue that the suggested priority rules are considerably more effective than commercial optimization software packages for lot-order assignment.

6. Conclusions

In this paper, we showed that we can solve the LOAP by solving the SMTTSP with nonnegative time-dependent processing times and suggested six priority rules for the SMTTSP-NNTDPT. The results of computational experiments showed that the priority rules are considerably more effective than CPLEX, with three of the rules (MDD, COVERT, and ATC) being the best. The suggested rules are very simple and can be easily implemented in a manufacturing information system to generate candidate lot-order assignment plans. A production manager can start with the candidate plans to draft a final plan using his or her expertise and experience considering various production conditions. In addition, the suggested rules can be implemented in an order management system, and a sales manager can predict the tardiness of a new order when he or she is attempting to assign a due date to the order after receiving it. In the further research, we will develop more sophisticated scheduling algorithms by using the feature of the nonnegative processing time in the SMTTSP-NNTDPT.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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