Research Article

Contract Coordination in Dual Sourcing Supply Chain under Supply Disruption Risk

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This paper explores a coordination model for a three-echelon supply chain including two different manufacturers, one distributor and one retailer via the combined option and back contracts. And one manufacturer provides the high wholesale price with low supply disruption risk and the other is completely the opposite. This differs from the previous supply chain coordination model. Firstly, supply disruption is added to the three-echelon supply chain. Secondly, considering the coordination of the supply chain, we deploy the combined option and back contracts which are seldom used in the previous study. Furthermore, it is interesting that supply disruption risk and buyback factor do not affect the distributor's order quantity from the manufacturer who has low product price and unreliable operating ability, while the order quantity increases with the rise of option premium and option strike price. The distributor's order quantity from the manufacturer, which has high product price and reliable operating ability, increases with the rise of supply disruption risk but decreases when the buyback factor, option premium, and option strike price decrease.

1. Introduction

With the growing popularity of the online shopping, logistics industry in China has shown significant development recently. Nevertheless, many unexpected changes may hinder the normal operation of the supply chain, for instance, the insufficient supply of spare parts in Toyota in 1997, the shortage of chips in Apple in 1999, the fire at a supplier of Ericsson in 2000, and the sea earthquake in Miyagi in Japan resulting in disruption of supply of car spare parts on 11 March, 2011. All of the accidents discussed above have led to a sense of danger among the people or a great loss to both the local economy and people's lives. Supply chain can be disrupted by many events, such as natural disasters, bankruptcy, strikes by workers, terrorist attacks, and policy failures. Supply chain enterprises have to face diverse external risks and the internal risks of supply chains are ubiquitous. Supply chain enterprises are independent economic entities in market, pursuing the maximum individual profits, and

potential conflicts of interest exist as stance, rationality, knowledge background, and mindsets of enterprises vary and engender variations in understanding. All the players of a supply chain are greatly affected by such demand disruptions, which can also affect the performance of a supply chain significantly and cause irreversible losses to the supply chain. This poses a challenge to managers regarding what can be done to maintain coordination of the supply chain and reduce the damage. The issue of how to tackle the uncertain disruptions efficiently and effectively has become increasingly significant to the managers nowadays. To the best of our knowledge, little attention has been paid to such problems in current research and how to coordinate the supply chain with interruption risk is another problem we intend to tackle.

With regard to the problem of coordinating the supply chain, option contract has been widely used. And the option contract means that the player orders some product before the selling season with certain wholesale price per unit and purchases the product with another strike price per unit. Its

advantage is that the option contract is beneficial to solve the supply chain in an unstable environment, which had been investigated in the previous study. Ritchken and Tapiero [1] argued that option contracts could hedge the risk caused by product prices and quantity fluctuations, but Barnes-Schuster et al. [2] proposed a two-stage model in an option contract where the use of options could help sellers cope with market changes and improve flexibility. Burnetas and Ritchken [3] noted that the introduction of options could lead to a rise in wholesale prices, and retail prices tended to be stable at the same time, and therefore manufacturers had to take the relevant conditions into consideration when they select options. Wang and Liu [4] applied Stackelberg's game to analyzing and determining the conditions to achieve supply chain coordination where retailers with option contracts play a major role. Ning and Dai [5] developed a one-to-many model deploying options to coordinate supply chain enterprises to improve the capacity to cope with market changes. Shang et al. [6] discussed the three-stage processing and ordering strategies in option contracts and showed that, at the moment, manufacturers did not have the motivation to speculate and the overall efficiency of the supply chain system and its members had increased, but the extent of increase depended on the negotiating capacity of supply chain enterprises in relation to option purchasing prices. Lin and Fan [7] coordinated two-echelon supply chain with uncertainty via option contract. Du et al. [8] integrated the contract of wholesale price discounts with option contracts and identified the contract parameters which increase operational efficiency of supply chains and help supply chain members share profit increases equally. The literature has thus far shown that the option contract is based on the fixed ordering contract by individual suppliers in the two-stage model. Tian et al. [9] developed emergency supplies purchasing model based on capacity option contract with dual purchasing sources. Y. Luo and Y. J. Luo [10] studied the agricultural produce supply chains and obtained the optimal order and supply strategy with circulation loss and option contract. Liu et al. [11] applied option contract to tackling overloading problems in the delivery service supply chain. Luo et al. [12] studied the coordination of a supply chain with dual procurement sources via real-option contract. Ma and Zeng [13] explored order strategy of retailers with stochastic demand based on payment in advance and option contract.

In addition to the option contract, growing importance has also attached to the buyback contract because of its advantage. For example, it is easy to calculate the parameter of buyback or it is equal to the revenue sharing contract [14], which has received more attention. Pasternack [15] proposed that buyback contracts can be used to achieve supply chain coordination. Cachon [14] argued that recycling and compensation could help coordinate supply chain to some degree. Yu et al. [16] maintained that supply chains were very robust under buyback contracts and buyback contracts could be reset to achieve supply chain coordination after any disruptions or accidents. Jia et al. [17] testified that, in the two-echelon supply chain including suppliers and retailers, if the inventory cost of retailers is nonlinear, the type of buyback contract in supply chains depended on retail prices

of suppliers. Hu and Wang [18] discussed the coordinating function of buyback contracts in three-echelon supply chains in the event of an accident, on the basis of random demand. Some studies integrate buyback contracts with other types of contracts. For instance, Hou and Qiu [19] incorporated buyback contracts into revenue-sharing contracts, while a few studies have combined buyback contracts with option contracts. Xu et al. [20] noted that, in restricted buyback contracts, if the quantity of products eligible for buyback was limited, retailers had to choose the optimal retail prices and ordering quantities, and suppliers' profits increased as the buyback prices rise, which was vice versa for the retailers. Chen [21] analyzed the impact of sales return on ordering quantity and wholesale prices in buyback contracts by applying the Stackelberg game and developed a motivation mechanism for sharing information among supply chain members. Guang et al. [22] investigated a buyback contract in a two-echelon supply chain consisting of a risk-neutral supplier and a risk-averse retailer, and if retailers are risk-averse, supply chain can also achieve coordination. There is an enormous amount of research on buyback contracts, which extends into the three-echelon supply chains, whereas there are only a few models of network supply chains.

Most of the research discussed above is concerned with one single option contract or buyback contract based on two-echelon supply chains, whose external environment is relatively stable. It is seldom to see the use of combined contracts to tackle the supply chain with disruption risk under unstable environment, which is not inevitable in practice. The buyback contract has been extensively studied in the previous research, which has received considerable attention in practice. Certainly, the option contract has its own advantage, and thus we choose the combined contract. Therefore, this paper attempts to address these problems. Firstly, integrating the advantages of the option contract and the buyback contract, this paper applies the two contracts together to coordinating the supply chain. Secondly, to be closer to the real-life environment, we consider the three-echelon supply chain model with two different suppliers (called dual sourcing purchase), one distributor and one retailer. In addition, considering the recent situation, we cannot neglect the disruption risk factors in the supply chain. Consequently, supply disruption risk factors are considered in this paper. And the buyback contract is deployed to stimulate and lead the retailers to increase ordering quantity; the distributor shares the partial risk engendered by demand uncertainty; a balance can be struck between marginal revenue and marginal cost of distributors and retailers. The disruption risk can be hedged by distributors who select options.

2. Model Description

2.1. Notations and Assumptions. Here, the single three-echelon supply chain includes two manufacturers, one distributor and one retailer, among which Manufacturer 1 can reduce supplies of low-price products and supply disruption is likely to occur; Manufacturer 2's products have a relatively higher price, but they are stable and reliable. The upstream

FIGURE 1: Research network chart.

enterprises provide single products for downstream enterprises that are mutually independent, without cross-echelon relations. Before sales, the manufacturer and the distributor offer contracts to the downstream enterprises; retailers determine the ordering quantity in terms of the market demand and the contract provided by the distributor. At the same time, the distributor determines their ordering quantity according to the ordering quantity of retailers and the contract of the manufacturers (see Figure 1). Manufacturer 1 provides products for the distributor according to the wholesale prices w_m ; as the supply of Manufacturer 1 is likely to be disrupted, the distributor will determine the ordering quantity based on the option contract offered by Manufacturer 2 in order to ensure more stable sourcing of their products. Before selling seasons, the distributors reserve q_1 units of products from Manufacturer 1 at the wholesale price; they reserve units of option purchasing quantity from Manufacturer 2. At the initial stage of the selling season, the distributor buys products within the option purchasing quantity q_2 at a certain price from Manufacturer 2 on the basis of the disrupted information obtained from Manufacturer 1; in order to stimulate the product ordering from the retailer, the distributor is able to provide a buyback contract (β, w_d) for the retailer. The distributor sells products at price w_d per unit and after the selling season, unsold products are bought back at β -fold of the distributor's price.

At the same time, the symbols used in the modes are shown in Notations. The superscript $*$ denotes the optimum ordering value of retailers in allied contracts.

The following are the hypotheses used for building and testing the models.

Assumption 1. Participants in supply chains are completely rational, and they are risk-neutral.

Assumption 2. All distribution functions are two-echelon and differential, and there are strict single inverse functions.

Assumption 3. With $w_d > w_m > c > v$, the limited profits of Manufacturer 1 are ensured and profits of distributors are also guaranteed.

Assumption 4. With $w_d > e + h > w_m$, $e + v < w_m$, validity of the option contract is guaranteed.

Assumption 5. With $r > w_d$, $\beta w_d > v$, the retailer's profits are ensured, and validity of buyback contract is guaranteed.

Assumption 6. Manufacturers play leading roles and the distributor acts as the follower.

2.2. Optimum Ordering Strategies of Centralized Supply Chain. In centralized supply chains, the manufacturers, the distributor, and the retailer are considered as a whole, and the objective is to maximize the overall profits of the supply chain, regardless of the internal transference of payments between member enterprises. There is no "dual-marginalized effect" in supply chains, and it is a typical newsvendor model.

Below is the overall profit of supply chains when disruptions occur to Manufacturer 1:

$$\Pi = r \min(q_2, x) + v \max(q_2 - x, 0) - cq_2 - g \max(x - q_2, 0). \quad (1)$$

Below is the overall profit of supply chain when disruptions do not happen to Manufacturer 1:

$$\Pi = r \min(q_1 + q_2, x) + v \max(q_1 + q_2 - x, 0) - c(q_1 + q_2) - g \max(x - q_1 - q_2, 0). \quad (2)$$

Now, below is the overall profit expected of supply chain:

$$\begin{aligned} E(\Pi) = & p [r \min(q_2, x) + v \max(q_2 - x, 0) - cq_2 \\ & - g \max(x - q_2, 0)] + (1 - p) [r \min(q_1 + q_2, x) \\ & + v \max(q_1 + q_2 - x, 0) - c(q_1 + q_2) \\ & - g \max(x - q_1 - q_2, 0)]. \end{aligned} \quad (3)$$

With $\text{Max}(E(\Pi))$ and $q_1, q_2 \geq 0$, the expected profit is the concave function of q_1 and q_2 , making $\partial E(\Pi)/\partial q_1 = 0$ and $\partial E(\Pi)/\partial q_2 = 0$.

The optimum ordering quantity can be derived based on the above equations.

Theorem 1. The optimum ordering quantity is

$$Q_c^* = q_1^* + q_2^* = F^{-1} \left(\frac{r + g - c}{r + g - v} \right). \quad (4)$$

Proof. Equation (3) can be written as

$$\begin{aligned}
E(\Pi) &= p \left[r \left(\mu + \int_{q_2}^{+\infty} (q_2 - x) f(x) dx \right) + v \left(q_2 - \mu - \int_{q_2}^{+\infty} (q_2 - x) f(x) dx \right) - cq_2 \right. \\
&\quad \left. - g \left(q_2 - \mu - \int_{q_2}^{+\infty} (q_2 - x) f(x) dx \right) \right] + (1-p) \left[r \left(\mu + \int_{q_1+q_2}^{+\infty} (q_1 + q_2 - x) f(x) dx \right) \right. \\
&\quad \left. + v \left(q_1 + q_2 - \mu - \int_{q_1+q_2}^{+\infty} (q_1 + q_2 - x) f(x) dx \right) - c(q_1 + q_2) - g \left(q_1 + q_2 - \mu - \int_{q_1+q_2}^{+\infty} (q_1 + q_2 - x) f(x) dx \right) \right], \\
&\begin{cases} \frac{\partial E(\Pi)}{\partial q_1} = 0, \\ \frac{\partial E(\Pi)}{\partial q_2} = 0, \end{cases} \\
&\Rightarrow \begin{cases} (1-p) \left((r-w+g) \int_{q_1+q_2}^{+\infty} f(x) dx \right) + (w-c) = 0, \\ p \left((r-w+g) \int_{q_2}^{+\infty} f(x) dx + (w-c) \right) + (1-p) \left((r-w+g) \int_{q_1+q_2}^{+\infty} f(x) dx + (w-c) \right) = 0, \end{cases} \\
&\Rightarrow \begin{cases} (r-w+g) F(q_2) + (w-c) = 0, \\ (r-w+g) F(q_1 + q_2) + (w-c) = 0, \end{cases} \\
&\Rightarrow F(q_2) = F(q_1 + q_2) = \frac{r+g-c}{r+g-v}, \\
&\quad q_2 = F^{-1} \left(\frac{r+g-c}{r+g-v} \right), \quad q_1 = 0.
\end{aligned} \tag{5}$$

Then we can get Theorem 1. This is the end of the proof. $\square$

3. Coordination of the Decentralized Supply Chain via Allied Contracts

Manufacturers play a leading role in supply chains. The optimal wholesale price and option contracts can be determined by the possible responses of the distributors, and distributors follow manufacturers. The distributor's optimal ordering quantity is determined by the manufacturer's information. Then, the distributor plays a leading role and the Stackelberg game comes into play between the distributor and the retailer. The decision is made through Stackelberg reverse induction. The sequence of steps in the option order is as follows.

- At the beginning, the distributor reserves the future q_2 from Manufacturer 2.
- The distributor is informed about the disruption from Manufacturer 1.
- The distributor obtains the ordering information from the retailer.
- The distributor invokes some option contracts from Manufacturer 2.
- Manufacturer 2 satisfies the distributor's demands.

3.1. Distributors' Decision-Making Process. Calculating the first-order value of the distributor's profit, we can get the

optimal order quantity from the manufacturer. Below is the distributors' profit when disruptions occur, in which the notations p , e , and f denote the probability of supply disruptions, the order price per unit, and the strike price per unit in the option contract:

$$\begin{aligned}
\Pi_d &= (w_d - h) \min(q_2, x) - g \max(x - q_2, 0) \\
&\quad - (\beta w_d - v) \max(q_2 - x, 0) - eq_2.
\end{aligned} \tag{6}$$

Below is the distributor's profit without disruptions:

$$\begin{aligned}
\Pi_d &= w_d \min(q_1, x) \\
&\quad + (w_d - h) \max\{\min(x - q_1, q_2), 0\} \\
&\quad + (v - \beta w_d) \max(q_1 + q_2 - x, 0) \\
&\quad - g \max(x - q_1 - q_2, 0) - w_m q_1 - eq_2.
\end{aligned} \tag{7}$$

Now, below is the expected profit of the distributor:

$$\begin{aligned}
E(\Pi_d) &= p [(w_d - h) \min(q_2, x) - g \max(x - q_2, 0) \\
&\quad - (\beta w_d - v) \max(q_2 - x, 0) - eq_2] + (1-p) \\
&\quad \cdot [w_d \min(q_1, x)
\end{aligned}$$

$$\begin{aligned}
& + (w_d - h) \max \{ \min (x - q_1, q_2), 0 \} \\
& + (v - \beta w_d) \max (q_1 + q_2 - x, 0) \\
& - g \max (x - q_1 - q_2, 0) - w_m q_1 - e q_2 \}.
\end{aligned} \quad (8)$$

With $p < 1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d)$, the optimum ordering quantity of the supply chain system is

$$Q_d^* = q_1^* + q_2^* = F^{-1} \left(\frac{w_d + g - w_m}{w_d + g - v + \beta w_d} \right). \quad (9)$$

The proof of Q_d^* , q_1^* , and q_2^* is similar to that of (4), to which detailer process has been added. For brevity, it is not necessary to recalculate it.

With $p > 1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d)$, the optimum ordering quantity of the supply chain system is to satisfy the solution of $\partial[\Pi_d]/\partial q_1 = 0$, $\partial[\Pi_d]/\partial q_2 = 0$, $Q_d^* > F^{-1}((w_d - w_m + g - h)/(w_d - v + g - h + \beta w_d))$, and $Q_d^* > F^{-1}((w_d - e + g - h - p(w_d - v + g - h + \beta w_d))/(w_d - v + g - h + \beta w_d)(1 - p))$.

The optimal conditions of KKT (the Karush-Kuhn-Tucker) are based on the ideas proposed by Karush [23] and Kuhn and Tucker [24], which indicates that a linear programming problem is able to have the necessary and sufficient conditions for the best solution, equivalent to a Lagrange multiplication in a broad sense. One of the conditions of KKT (the Karush-Kuhn-Tucker) is that the optimum must be a possible solution and satisfy the restrictive conditions of inequality and equation.

Testifying. The target function is the strict concave function and its restriction is linearity and there is only one optimal solution, which is achieved through the condition of KKT:

$$\begin{aligned}
\frac{\partial \Pi}{\partial q_1} + \lambda_1 &= (1 - p) [w_d + g - w_m - hF(q_1) \\
& - (w_d - h - v + \beta w_d + g)F(q_1 + q_2)] + \lambda_1 = 0,
\end{aligned} \quad (10)$$

$$\begin{aligned}
\frac{\partial \Pi}{\partial q_2} + \lambda_2 &= w_d + g - e - h - p(w_d - v + \beta w_d - h \\
& + g)F(q_2) - (1 - p)(w_d - v + \beta w_d - h + g)F(q_1 \\
& + q_2) + \lambda_2 = 0,
\end{aligned} \quad (11)$$

$$\lambda_1 q_1 = 0, \quad (12)$$

$$\lambda_2 q_2 = 0, \quad (13)$$

$$\lambda_1, \lambda_2, q_1, q_2 \geq 0. \quad (14)$$

The four possibilities to be analyzed are (a) $\lambda_1, \lambda_2 > 0$; (b) $\lambda_1 > 0, \lambda_2 = 0$; (c) $\lambda_1 = 0, \lambda_2 > 0$; and (d) $\lambda_1 = 0, \lambda_2 = 0$.

Case (a). With $\lambda_1, \lambda_2 > 0$, from (11) and (12), $q_1, q_2 = 0$ can be derived; now, from (10), $\lambda_1 = -(1 - p)(w_d + g - w_m) < 0$

is known, which is in conflict with the hypothesis, and thus it is not the optimum.

Case (b). With $\lambda_1 > 0$ and $\lambda_2 = 0$, $q_1 = 0$ is derived from (11), and now $F(q_2) = (r + g - e - h)/(r + g - h)$ is derived from (11). When it is incorporated into (10), $\lambda_1 = -(1 - p)[(e + h) - w_m] < 0$ is derived, which is inconsistent with the hypothesis. Thus, the solution is not the optimum.

Case (c). With $\lambda_1 = 0$ and $\lambda_2 > 0$, $q_2 = 0$ is derived from (12), and now $F(q_1) = (w_d + g - w_m)/(w_d + g - v + \beta w_d)$ is derived from (10). It is incorporated into (11) and $\lambda_2 = (1 - p)(w_d + g - h - v + \beta w_d)((w_d + g - w_m)/(w_d + g - v + \beta w_d)) - (w_d + g - h - e)$. When $p < 1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d)$ is satisfied, $\lambda_2 > 0$ is satisfied, and now $q_1^* = F^{-1}((w_d + g - w_m)/(w_d + g - v + \beta w_d))$ and $q_2^* = 0$ is the optimal solution, and all the conditions of KKT are satisfied.

Case (d). As the only optimal solution is obtained, the corollary is that $p > 1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d)$ is the optimal solution, with $\lambda_1 = 0$ and $\lambda_2 = 0$, and the conditions of KKT are satisfied. The optimal solution has satisfied $\partial[\Pi_d]/\partial q_1 = 0$ and $\partial[\Pi_d]/\partial q_2 = 0$, with $q_1^* > 0$ and $q_2^* > 0$.

3.2. The Retailer's Decision-Making Process. Calculating the first derivation value of the retailer's profit, we can get the optimal order quantity from the distributor and the optimal wholesale price given by the distributor to the retailer. Below is the retailer's profit with disruptions:

$$\begin{aligned}
\Pi_r &= r \min (q_2, x) + \beta w_d \max (q_2 - x, 0) \\
& - g \max (x - q_2, 0) - w_d q_2.
\end{aligned} \quad (15)$$

Below is the retailer's profit without disruptions:

$$\begin{aligned}
\Pi_r &= r \min (q_1 + q_2, x) + \beta w_d \max (q_1 + q_2 - x, 0) \\
& - g \max (x - q_1 - q_2, 0) - w_d (q_1 + q_2).
\end{aligned} \quad (16)$$

Now, below is the expected retailer's profit:

$$\begin{aligned}
E(\Pi_r) &= p [r \min (q_2, x) + \beta w_d \max (q_2 - x, 0) - g \\
& \cdot \max (x - q_2, 0) - w_d q_2] + (1 - p) [r \\
& \cdot \min (q_1 + q_2, x) + \beta w_d \max (q_1 + q_2 - x, 0) \\
& - g \max (x - q_1 - q_2, 0) - w_d (q_1 + q_2)].
\end{aligned} \quad (17)$$

When the above equation is calculated for q_1 and q_2 , $F(q_2^*) = (r + g - w_d)/(r + g - \beta w_d)$, $q_1^* = 0$ are derived, and below is the optimum ordering quantity of the retailer:

$$Q_r^* = F^{-1} \left(\frac{r + g - w_d}{r + g - \beta w_d} \right). \quad (18)$$

3.3. Coordination of the Decentralized Supply Chain. Here, the option contract is combined with buyback contract, and

the three-echelon supply chain is coordinated and optimized through designing the corresponding parameters. In the allied contracts, the following conditions must be satisfied to achieve complete coordination of the supply chain.

Proposition 2. *In the allied contract consisting of the option contract and the buyback contract, if complete coordination is to be achieved in the supply chain, then the contract parameters must satisfy the following cases.*

Case 1. Consider the following:

$$p < 1 - \frac{(w_d + g - e - h)(w_d + g - v + \beta w_d)}{(w_d + g - w_m)(w_d + g - h - v + \beta w_d)}, \quad (19)$$

$$\frac{r + g - w_d}{r + g - \beta w_d} = \frac{r + g - c}{r + g - v}, \quad (20)$$

$$\frac{w_d + g - w_m}{w_d + g - v - \beta w_d} \geq \frac{r + g - w_d}{r + g - \beta w_d}. \quad (21)$$

Case 2. Consider the following:

$$p > 1 - \frac{(w_d + g - e - h)(w_d + g - v + \beta w_d)}{(w_d + g - w_m)(w_d + g - h - v + \beta w_d)}, \quad (22)$$

$$\frac{r + g - w_d}{r + g - \beta w_d} = \frac{r + g - c}{r + g - v}, \quad (23)$$

$$\max \left(\frac{w_d - w_m + g - h}{w_d - v + g - h + \beta w_d}, \frac{w_d - e + g - h - p(w_d - v + g - h + \beta w_d)}{(w_d - v + g - h + \beta w_d)(1 - p)} \right) \geq \frac{r + g - w_d}{r + g - \beta w_d}. \quad (24)$$

Equations (19) and (22) are two different sets of circumstances where disruptions are likely to occur; in (20) and (23), the validity of contract coordination under different risks is ensured; in (21) and (24), the effective circulation of supply chain products under different risks is ensured. In order to guarantee the validity and the continuity of supply chain operations, the ordering quantity purchased by distributors from manufacturers has to be more than or equal to that purchased by the retailer from the distributor. Only if the ordering quantity of the retailer is consistent with that of the centralized decision can coordination be achieved.

Proposition 3. *When the three-stage supply chain is coordinated, the buyback factor β must satisfy the following equation: $\beta = ((r + g)(w_d + v - c) - vw_d)/(w_d(r + g - c))$.*

Corollary 4. *When $(w_d + g + \beta w_d - v)(e + h - w_m) < h(w_d + g - w_m)$, the reliable Manufacturer 2 is always deployed.*

Corollary 5. *The increase of the ordering quantity from Manufacturer 1 does not follow the rise of the disruption risks, and the ordering quantity from Manufacturer 2 does not decline with the rise of the disruption risks.*

Corollary 6. *The larger the buyback factor β is, the larger the ordering quantity from retailer is. The ordering quantity by distributors from Manufacturer 1 does not increase with the rise of β , and the ordering quantity from Manufacturer 2 does not decline with the rise of β .*

Corollary 7. *The ordering quantity by distributors from Manufacturer 1 does not decline with the rise of option purchasing prices and option strike prices, and the ordering quantity by the distributor does not increase with the rise of option premium and the option strike prices.*

Testifying. It can be deduced from (20) and (23).

Proof of Corollary 4. When $(w_d + g + \beta w_d - v)(e + h - w_m) < h(w_d + g - w_m)$, then $1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d) = ((w_d + g + \beta w_d - v)(e + h - w_m) - h(w_d + g - w_m))/(w_d + g - w_m)(w_d + g - h - v + \beta w_d) < 0$; from $p \geq 0$, $p > 1 - (w_d + g - e - h)(w_d + g - v + \beta w_d)/(w_d + g - w_m)(w_d + g - h - v + \beta w_d)$ is derived; from Case (d) in Section 3.1, $q_2^* > 0$ is derived, and the test is completed. $\square$

Proof of Corollary 5. With $q_1' = dq_1^*(p)/dp$ and $q_2' = dq_2^*(p)/dp$, from $\partial \Pi_d / \partial q_1 = 0$, the following can be derived:

$$w_d + g - w_m - hF(q_1) - (w_d - h - v + \beta w_d + g)F(q_1 + q_2) = 0. \quad (25)$$

As the first derivation of p is carried out with $\partial \Pi_d / \partial q_1 = 0$, $hq_1'f(q_1) + (w_d - h + \beta w_d - v + g)(q_1' + q_2')f(q_1 + q_2) = 0$ is achieved. Thus, the plus and minus signs of q_1' and q_2' are the opposite. As the first derivation of p is conducted by substituting (25) into $\partial \Pi_d / \partial q_2 = 0$, $(w_d - h + \beta w_d - v + g)[-F(q_2) + pq_2'f(q_2) + F(q_1 + q_2)] + (1 - p)hq_1'f(q_1) = 0$ is achieved. To satisfy the equation, with $q_1' \leq 0$, $q_2' \geq 0$, and the test is completed. $\square$

Proof of Corollary 6. With $Q_r^* = F^{-1}((r + g - w_d)/(r + g - \beta w_d))$, the more the buyback factor β is, the greater $(r + g - w_d)/(r + g - \beta w_d)$ is. Also, since $F(x)$ is continuous and differentiable within intervals and it is strictly increasing, the ordering quantity of the retailer Q_r^* increases with the rise of the buyback factor β . With $q_1' = dq_1^*(p)/d\beta$ and $q_2' = dq_2^*(p)/d\beta$, the first derivation of β is carried out for $\partial \Pi_d / \partial q_1 = 0$, and $hq_1'f(q_1) + (w_d - h + \beta w_d - v + g)(q_1' + q_2')f(q_1 + q_2) + w_dF(q_1 + q_2) = 0$ is achieved. As such, the sign of q_1' is not positive. Substituting (25) into $\partial \Pi_d / \partial q_2 = 0$ for the first derivation of β , $pw_dF(q_2) + p(w_d - h + \beta w_d - v + g)q_2'f(q_2) - (1 - p)hq_1'f(q_1) = 0$ is achieved. To satisfy the equation, the sign of q_1' is not positive and $q_2' < 0$ is known, and hence $q_1' \leq 0$, $q_2' < 0$, and the test is completed. $\square$

Proof of Corollary 7. With $q_1' = dq_1^*(p)/de$ and $q_2' = dq_2^*(p)/de$, (25) is obtained from $\partial \Pi_d / \partial q_1 = 0$.

The first derivation e is carried out for $\partial \Pi_d / \partial q_1 = 0$, and $hq_1'f(q_1) + (w_d - h + \beta w_d - v + g)(q_1' + q_2')f(q_1 + q_2) = 0$ is achieved. As such, the plus and minus signs of q_1' , q_2' are

TABLE 1

r	c	e	h	w_m	w_d	v	g	p	β	μ	σ
150	25	5	45	40	80	5	10	0.1	0.1	1350	380

TABLE 2: The optimal ordering quantity under decentralization and each contract.

	Decentralized	Buyback contract	Option contract
Economic ordering quantity	1365.367393	1375.084489	1849.519365

the opposite. Substituting (25) into $\partial \Pi_d / \partial q_2 = 0$ for the first derivation of e , $-pq_2' f(q_2)(w_d - h + \beta w_d - v + g) + (1 - p)hq_1' f(q_1) = 1$ is achieved. To satisfy the equation, $q_1' \geq 0$, $q_2' \leq 0$, is known, and the test is completed.

Likewise, with $q_1' = dq_1^*(p)/\partial h$ and $q_2' = dq_2^*(p)/\partial h$, from $\partial \Pi_d / \partial q_1 = 0$, (25) is derived.

The first derivation of h is carried out for $\partial \Pi_d / \partial q_1 = 0$, and $F(q_1) + hq_1' f(q_1) + (w_d - h + \beta w_d - v + g)(q_1' + q_2') f(q_1 + q_2) = 0$ is achieved. As such, the plus and minus signs of q_1' , q_2' are the opposite. Substituting (25) into $\partial \Pi_d / \partial q_2 = 0$ for the first derivation of h , $-pq_2' f(q_2)(w_d - h + \beta w_d - v + g) + pF(q_2) + (1 - p)[F(q_1) + hq_1' f(q_1)] = 1$ is achieved. To satisfy the equation, $q_1' \geq 0$, $q_2' \leq 0$ is known, and the test is completed. $\square$

4. Numerical Analysis

We deploy MATLAB to do the simulation. Similar to Li et al. [25] and Tian et al. [9], it is suitable to suppose that the market demand is subject to the normal distribution: $N(1350, 380^2)$, and the relevant parameters are as in Table 1.

The optimal ordering quantity under decentralization and each contract is shown in Table 2.

The optimal profit under decentralization and contract coordination is shown in Table 3.

Tables 2 and 3 show that the ordering quantity and profits in the centralized mode are smaller than those of buyback contracts and option contracts, among which the ordering quantity of distributors in the option contracts is more than those of distributors in the buyback contracts. The reason might be that the option purchases from the manufacturer cannot be exercised according to the real-life circumstances, and the coordinated contracts play a role in optimization.

If other parameters are definite, the possibility of occurrence of different disruption risks, buyback factors, option strike prices, and changes of option premium will have an impact on ordering quantity (Figures 2 to 5).

Figure 2 shows that the ordering quantity of distributors from Manufacturer 1 does not decline with the rise of disruption risks, because the cost of products offered by Manufacturer 1 is always lower than that of Manufacturer 2. When the disruptions do not occur in practice, distributors earn relatively higher profits from the low-cost products by Manufacturer 1, which can mitigate the possible loss caused by the disruption of Manufacturer 1.

FIGURE 2: The impact of different disruption risks on ordering quantity.

FIGURE 3: The impact of different buyback factors β on ordering quantity.

It can be seen from Figure 3 that the ordering quantity of the retailer's increases with increase of the buyback factor. Distributors share the partial risk of surplus inventory with the retailer, which stimulates the ordering from the retailer to some extent. The ordering quantity of distributors from Manufacturer 1 does not change with the variation of buyback factor, and the ordering quantity of Manufacturer 2 does not decline with the rise of the buyback factor. The larger the buyback factor is, the higher the buyback cost of the distributor is. The profit margin tends to be smaller, and the order of products with higher prices will decline. When the buyback price is beyond a certain degree, stockout is preferred.

Figures 4 and 5 show that the ordering quantity of the distributor from Manufacturer 1 increases when option strike prices and purchasing prices rise, whereas the ordering quantity by the distributor from Manufacturer 2 decreases with the rise of option strike prices and option premium. With the rise of the option premium, distributors tend to reduce the ordering quantity from Manufacturer 2, and the ordering from Manufacturer 1 increases to obtain the revenue when there is no risk in ordering from Manufacturer 1.

TABLE 3: The optimal profits with decentralized model and contract coordination.

Supply chain type	Retailer's profits	Distributor's profits	Overall profits of manufacturers	Total profits of supply chain
Decentralized model	77745.8466	40961.0218	34134.18483	152841.0533
Contract coordination	78259.9824	45268.9764	37137.92283	160666.8817

FIGURE 4: The impact of different option strike prices e on ordering quantity. FIGURE 5: The impact of different option premium e on ordering quantity.

5. Conclusions

The main findings of this paper are as follows. Firstly, integrating the advantages of the option contract and the buyback contract, this paper applies these two contracts together to coordinating the supply chain. Secondly, to be closer to the practical environment, we consider the three-echelon supply chain model with two different suppliers (called dual sourcing purchase): one distributor and one retailer. In addition, considering the practical situation in recent years, we cannot ignore the disruption risk factors in the supply chain. Consequently, supply disruption risk factors

are considered in this paper. And the buyback contract is used to stimulate and lead the retailer to increase ordering quantity; the distributor shares the partial risk engendered by demand uncertainty; a balance can be struck between marginal revenue and marginal cost of the distributor and the retailer. The disruption risk can be hedged by the distributor who selects options.

This study has investigated the three-echelon supply chain mode with random demand, where distributors are able to choose one of two manufacturers; one has lower-price products but disruptions are more likely to occur, and the other has stable supply but its price of products is relatively higher. The proposed model incorporates an option contract and a buyback contract, whose coordination can help achieve the optimal ordering strategies. It is revealed that increased ordering from a stable source can mitigate the disruption risks in supply chains, suggesting greater adaptability and robustness in optimization of the operation of supply chains. In dual sourcing purchasing, disruption risks do not affect the ordering quantity from enterprises with lower-price products and unstable operations, whereas the ordering quantity of option contracts from enterprises with high-price products and stable supply will increase accordingly. In specific circumstances, it is likely to choose suppliers with relatively stable operations but relatively higher prices. The buyback factor, option premium, and option strike prices influence the ordering decisions by the distributor. When the buyback factor, option purchasing prices, and option strike prices are greater, the distributor reduces the ordering quantity of products which are stable but costly and tends to buy unstable and cheaper products which tend to increase with the rise of option premium and option strike prices but will not be affected by the buyback efficiency. The buyback factor can stimulate the order from the retailer. These conclusions might provide important references for supply chain members when they make decisions.

Here, it is assumed that the risks of supply chain enterprises are neutral, and it is worth investigating the circumstances with different risk preferences. In addition, there are many measures to mitigate the supply chain disruptions, but we only consider the dual sourcing purchasing strategy, whereas other different strategies can also be addressed in future research to develop the many-to-many models. The disruption risks and demand uncertainties can be estimated, and further research can consider the impact engendered by estimation errors of measurement or different strategies in misjudgment on profits of supply chain enterprises.

Notations

q_1 : The ordering quantity of distributors from Manufacturer 1

- q_2 : The option purchasing quantity of distributors from Manufacturer 2
 e : Option premium
 h : Option strike prices
 p : The probability of disruptions for Manufacturer 1 ($0 < p < 1$)
 c : Production cost of the two manufacturers
 w_m : Wholesale prices provided by Manufacturer 1 for distributors
 w_d : Wholesale prices provided by distributors 1 for the retailer
 r : Sale prices of the retailer
 β : Buyback price factor of distributors ($0 < \beta < 1$)
 g, v : Shortage cost of the retailer and commodity salvage, respectively
 x : The random demand of the retailer's market; the random variable is continuous
 $f(x)$: The random demand probability density function of the retailer's market; random demand cumulative distribution function of the retailer's market
 $F(x)$: The random demand probability distribution function of the retailer's market; $F(x)$ is continuous and differentiable within intervals, and it is strictly increasing as $F(0) = 0$
 Π : The overall profit of centralized supply chain without contract
 $E(\Pi)$: The expected overall profit of centralized supply chains without contracts
 Q : Ordering quantity of the retailer
 Q_c^* : The optimum ordering quantity of the centralized supply chain system without contract
 Q_D^* : The optimum ordering quantity of the decentralized supply chain system without contracts
 Π_r : Profits of the retailer
 Π_d : Profits of the distributor
 $E(\Pi_r)$: Expected profits of the retailer
 $E(\Pi_d)$: Expected profits of the distributor.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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