Research Article

Nonfragile Robust H_∞ Synchronization Approach for Drive-Response Complex Dynamical Networks with Randomly Occurring Controller Gain Fluctuations and Uncertainties

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This paper investigates the nonfragile H_∞ synchronization problem of complex dynamical networks with randomly occurring controller gain fluctuations and uncertainties. These randomly occurring phenomena are described by independent stochastic variables satisfying Bernoulli distributions, which are adopted to model more realistic dynamical behaviors of the complex networks. By applying the Lyapunov-Krasovskii method, delay-dependent criteria are established to ensure that the synchronization can be achieved with the prescribed H_∞ disturbance attenuation. Moreover, the obtained results do not rely on the derivatives of time-varying delays. A set of nonfragile controllers are further designed in terms of linear matrix inequality (LMI) approach. Finally, a numerical example is given to illustrate the effectiveness of our theoretical results.

1. Introduction

In the past decade, there have been intensive researches on dynamical properties of complex networks. This increasing interest in complex dynamical networks is mainly due to their wide ranging implications and applications in the real world, such as computer networks, communication networks, biological networks, and social networks [1–8]. Roughly speaking, complex dynamical networks consisted of a large set of interconnected nodes displaying collective behaviors. Specially, the synchronization problem of complex dynamical networks is becoming a hot research area in recent years, which is a universal phenomenon in nature. As a result, many synchronization methods have been developed for complex dynamical networks [9–11].

It is worth mentioning that there are inevitable disturbances in complex dynamical networks caused by environmental circumstances, which gives rise to the H_∞ synchronization problem. Following this research line, several H_∞ synchronization results have been reported in the literature; see, for example, [12–15]. In addition, it is generally known that time delays exist ubiquitously in almost all practical

applications, which may lead to system performance degradation [16–19]. Due to their impact, there have been some efficient synchronization results of complex dynamical networks with various time delays [20, 21]. Another important issue in the complex dynamical networks is the uncertainty. Since explicit descriptions of the network parameters are always hardly obtained, robust synchronization approaches against uncertainties are proposed. Some initial attention has been focused on this problem [22].

On the other hand, there is a class of complex dynamical networks that cannot achieve synchronization by themselves, such that synchronization controllers should be designed. It should be pointed out that, due to A/D conversion, D/A conversion, and finite word length in computations, it is always very expensive or even impossible to implement the designed controller gains precisely [23–28]. Thus, nonfragile controllers are needed to tolerate the uncertainty in practical situations. Very recently, the issue of randomly occurring uncertainties has been introduced for neural networks and chaotic systems. However, to the best of the authors' knowledge, the nonfragile H_∞ synchronization problem for complex dynamical networks with randomly occurring controller

gain fluctuations and uncertainties has never been tackled in the previous literatures and remains challenging, which motivates us to conduct this study.

In this paper, by introducing the complex dynamical networks subject to time-varying delays, external disturbances, and randomly occurring uncertainties, the H_∞ synchronization problem is investigated. In particular, nonfragile controllers are designed, in which the randomly occurring controller gain fluctuations are taken into consideration. The parameter uncertainties and coupling delays are assumed to be time-varying and norm bounded. Compared with the existing results, the main contributions of this paper can be summarized as threefold. Firstly, the complex dynamical networks model adopted in this paper is more general, which can better simulate the networks in the applications. Both self-time-varying delays and coupling time-varying delays are considered in the model. Secondly, it is the first time to establish the nonfragile H_∞ synchronization for complex dynamical networks with randomly occurring controller gain fluctuations and uncertainties. By utilizing the Lyapunov-Krasovskii method, novel H_∞ synchronization criteria are derived in terms of LMIs. Finally, the obtained delay-dependent results impose no constraints on the derivative of time-varying delays, which are more applicable in the real world.

The remainder of this paper is organized as follows. Section 2 introduces the complex dynamical networks model and presents several assumptions and lemmas. The main results based on the LMI approach are established in Section 3. Section 4 gives a numerical example to demonstrate the effectiveness of our derived criteria. Finally, the paper is concluded in Section 5.

Notation. The notation in the paper is standard. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m \times n}$ represents the set of all $m \times n$ real matrices. I and 0 represent identity matrix and zero matrix with appropriate dimensions, respectively. The notation $P > 0$ means P is real symmetric and positive definite, and the superscript “ T ” denotes matrix transposition. $\Pr\{\alpha\}$ means the occurrence probability of the event α . $\mathbb{E}\{x\}$ stands for the mathematical expectation of the stochastic variable x . $A \otimes B$ denotes the Kronecker product of matrices A and B . Moreover, in symmetric block matrices, $*$ is used as an ellipsis for the terms that are introduced by symmetry and $\text{diag}\{\dots\}$ denotes a block-diagonal matrix. Finally, if not explicitly stated, all matrices are assumed to have compatible dimensions.

2. Problem Formulation and Preliminaries

Consider the following model of complex dynamical networks with time-varying delays:

$$\begin{aligned} \dot{x}_i(t) = & (A_i + \alpha(t) \Delta A_i(t)) x_i(t) \\ & + (A_{id} + \alpha(t) \Delta A_{id}(t)) x_i(t - \tau(t)) \\ & + (B_i + \alpha(t) \Delta B_i(t)) f(t, x_i(t)) \\ & + \sum_{j=1}^N c_{ij} \Gamma x_j(t - \sigma(t)), \quad i = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ represents the state vector of the i th node, A_i , A_{id} , and B_i are constant matrices, $f: \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth nonlinear function, $\tau(t)$ represents the self-time-varying delays, $\sigma(t)$ represents the coupling time-varying delays, $\Gamma \in \mathbb{R}^{n \times n}$ is the inner coupling matrix, and $C = (c_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$ is the outer coupling matrix describing the network topology. If there is a connection from node i to node j ($i \neq j$), then the coupling $c_{ij} \neq 0$; otherwise, $c_{ij} = 0$. Moreover, the diagonal elements c_{ii} are defined as $c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}$, $i = 1, 2, \dots, N$. $\Delta A_i(t)$, $\Delta A_{id}(t)$, and $\Delta B_i(t)$ are real valued matrices representing the time-varying parameter uncertainties and are assumed to be of the following form:

$$[\Delta A_i(t) \quad \Delta A_{id}(t) \quad \Delta B_i(t)] = D_i \Psi_i(t) [E_{i1} \quad E_{i2} \quad E_{i3}], \quad (2)$$

where D_i , E_{i1} , E_{i2} , and E_{i3} are known real constant matrices and $\Psi_i(t)$ is an unknown time-varying matrix satisfying

$$\Psi_i^T(t) \Psi_i(t) \leq I. \quad (3)$$

The initial conditions are $x_i(t) = \phi_i(t)$, $t \in [-\max(\bar{\tau}, \bar{\sigma}), 0]$, with some given continuous functions $\phi_i: [-\max(\bar{\tau}, \bar{\sigma}), 0] \rightarrow \mathbb{R}^n$. Without loss of generality, the initial conditions are chosen as constant functions on $[-\max(\bar{\tau}, \bar{\sigma}), 0]$. The stochastic variable $\alpha(t) \in \mathbb{R}$ is a Bernoulli distributed sequence defined by

$$\alpha(t) = \begin{cases} 1, & \text{uncertainties happen,} \\ 0, & \text{uncertainties do not happen,} \end{cases} \quad (4)$$

with

$$\begin{aligned} \Pr\{\alpha(t) = 1\} &= \alpha, \\ \Pr\{\alpha(t) = 0\} &= 1 - \alpha, \end{aligned} \quad (5)$$

where $\alpha \in [0, 1]$ is a known constant.

Assumption 1. The time-varying delays satisfy $0 \leq \tau(t) \leq \bar{\tau}$ and $0 \leq \sigma(t) \leq \bar{\sigma}$, $\bar{\tau}$ and $\bar{\sigma}$ are positive constants.

Assumption 2. The nonlinear function satisfies

$$\begin{aligned} & [f(t, x_i(t)) - f(t, y_i(t)) - F_1(x_i - y_i)]^T \\ & \cdot [f(t, x_i(t)) - f(t, y_i(t)) - F_2(x_i - y_i)] \leq 0, \end{aligned} \quad (6)$$

$\forall x_i, y_i \in \mathbb{R}^n$, where F_1 and F_2 are constant matrices of appropriate dimensions with $F_2 - F_1 \geq 0$.

By the drive-response synchronization concept, model (1) is set as the drive complex dynamical networks.

Correspondingly, the response complex dynamical networks can be given as

$$\begin{aligned} \dot{y}_i(t) = & (A_i + \alpha(t) \Delta A_i(t)) y_i(t) \\ & + (A_{id} + \alpha(t) \Delta A_{id}(t)) y_i(t - \tau(t)) \\ & + (B_i + \alpha(t) \Delta B_i(t)) f(t, y_i(t)) \\ & + \sum_{j=1}^N c_{ij} \Gamma y_j(t - \sigma(t)) + \bar{\omega}_i(t) + u_i(t), \end{aligned} \quad (7)$$

$$i = 1, 2, \dots, N,$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \dots, y_{in}(t))^T \in \mathbb{R}^n$ represents the response state vector of the i th node, $u_i \in \mathbb{R}^n$ denotes the control input to be designed for the i th node, and $\bar{\omega}_i(t)$ is the disturbance on the i th node.

Remark 3. Compared with the existing results, our proposed drive-response complex dynamical networks model is more general. When $N = 1$, the model can be utilized to describe synchronization problem of chaotic systems or neural networks. Moreover, the random appearance or disappearance of the parameter uncertainties is introduced in the model to reflect more complex behaviors of the real networks.

Define the synchronization errors as follows:

$$e_i(t) = y_i(t) - x_i(t), \quad i = 1, 2, \dots, N, \quad (8)$$

such that the following synchronization error dynamics can be obtained:

$$\begin{aligned} \dot{e}_i(t) = & (A_i + \alpha(t) \Delta A_i(t)) e_i(t) \\ & + (A_{id} + \alpha(t) \Delta A_{id}(t)) e_i(t - \tau(t)) \\ & + (B_i + \alpha(t) \Delta B_i(t)) \tilde{f}(t, e_i(t)) \\ & + \sum_{j=1}^N c_{ij} \Gamma e_j(t - \sigma(t)) + \bar{\omega}_i(t) + u_i(t), \end{aligned} \quad (9)$$

$$i = 1, 2, \dots, N,$$

where $\tilde{f}(t, e_i(t)) := f(t, y_i(t)) - f(t, x_i(t))$. Then, in order to deal with the real world controller implementation, the following nonfragile state feedback controllers with controller gain fluctuations are designed

$$u_i(t) = (K_i + \beta(t) \Delta K_i(t)) e_i(t), \quad i = 1, 2, \dots, N, \quad (10)$$

where $K_i \in \mathbb{R}^{n \times n}$ is the controller gain to be determined and the real valued matrix $\Delta K_i(t)$ denotes the controller gain fluctuation. Moreover, ΔK_i is assumed to have the following structure:

$$\Delta K_i(t) = H_i \Upsilon_i(t) G_i, \quad (11)$$

where $\Upsilon_i(t) \in \mathbb{R}^{k \times l}$ and $i = 1, 2, \dots, N$ is an unknown time-varying matrix satisfying

$$\Upsilon_i^T(t) \Upsilon_i(t) \leq I, \quad (12)$$

and H_i, G_i are known constant matrices. The stochastic variable $\beta(t) \in \mathbb{R}$ is a Bernoulli distributed sequence, which describes the phenomena of randomly occurring controller gain fluctuation. $\beta(t)$ is defined by

$$\begin{aligned} \beta(t) &= \begin{cases} 1, & \text{controller gain fluctuations happen,} \\ 0, & \text{controller gain fluctuations do not happen,} \end{cases} \end{aligned} \quad (13)$$

with

$$\begin{aligned} \Pr \{\beta(t) = 1\} &= \beta, \\ \Pr \{\beta(t) = 0\} &= 1 - \beta, \end{aligned} \quad (14)$$

where $\beta \in [0, 1]$ is a known constant. Moreover, it is assumed that the stochastic variables $\alpha(t)$ and $\beta(t)$ are mutually independent.

Consequently, the closed-loop synchronization error dynamics can be obtained as

$$\begin{aligned} \dot{e}_i(t) = & (A_i + \alpha(t) \Delta A_i(t) + K_i + \beta(t) \Delta K_i(t)) e_i(t) \\ & + (A_{id} + \alpha(t) \Delta A_{id}(t)) e_i(t - \tau(t)) \\ & + (B_i + \alpha(t) \Delta B_i(t)) \tilde{f}(t, e_i(t)) \\ & + \sum_{j=1}^N c_{ij} \Gamma e_j(t - \sigma(t)) + \bar{\omega}_i(t), \end{aligned} \quad (15)$$

$$i = 1, 2, \dots, N,$$

which can be further rewritten as

$$\begin{aligned} \dot{e}(t) = & (\bar{A} + \alpha(t) \Delta \bar{A}(t) + K + \beta(t) \Delta K(t)) e(t) \\ & + (\bar{A}_d + \alpha(t) \Delta \bar{A}_d(t)) e(t - \tau(t)) \\ & + (\bar{B} + \alpha(t) \Delta \bar{B}(t)) \bar{F}(t, e(t)) \\ & + (C \otimes \Gamma) e(t - \sigma(t)) + \bar{\omega}(t), \end{aligned} \quad (16)$$

$$i = 1, 2, \dots, N,$$

where

$$\begin{aligned} \bar{A} &:= \text{diag} \{A_1, A_2, \dots, A_N\}, \\ \Delta \bar{A}(t) &:= \text{diag} \{\Delta A_1(t), \Delta A_2(t), \dots, \Delta A_N(t)\}, \\ \bar{A}_d &:= \text{diag} \{A_{1d}, A_{2d}, \dots, A_{Nd}\}, \\ \Delta \bar{A}_d(t) &:= \text{diag} \{\Delta A_{1d}, \Delta A_{2d}, \dots, \Delta A_{Nd}\}, \\ \bar{B} &:= \text{diag} \{B_1, B_2, \dots, B_N\}, \\ \Delta \bar{B}(t) &:= \text{diag} \{\Delta B_1, \Delta B_2, \dots, \Delta B_N\}, \\ D &:= \text{diag} \{D_1, D_2, \dots, D_N\}, \end{aligned}$$

$$\Psi(t) := \text{diag}\{\Psi_1(t), \Psi_2(t), \dots, \Psi_N(t)\},$$

$$E_i := \text{diag}\{E_{1i}, E_{2i}, \dots, E_{Ni}\}, \quad i = 1, 2, 3,$$

$$K := \text{diag}\{K_1, K_2, \dots, K_N\},$$

$$H := \text{diag}\{H_1, H_2, \dots, H_N\},$$

$$\Upsilon(t) := \text{diag}\{\Upsilon_1(t), \Upsilon_2(t), \dots, \Upsilon_N(t)\},$$

$$G := \text{diag}\{G_1, G_2, \dots, G_N\},$$

$$e(t) := [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T,$$

$$\bar{F}(t, e(t)) := [f^T(t, y_1(t))$$

$$- f^T(t, x_1(t)), f^T(t, y_2(t))$$

$$- f^T(t, x_2(t)), \dots, f^T(t, y_N(t))$$

$$- f^T(t, x_N(t))]^T.$$

(17)

Before proceeding, the following definitions and lemmas are given for subsequent analysis.

Definition 4. The H_∞ synchronization is said to be reached if there exists a constant $\gamma > 0$ such that

$$\mathbb{E} \left\{ \int_0^\infty [e^T(t) e(t) - \gamma^2 \bar{\omega}^T(t) \bar{\omega}(t)] dt \right\} < 0, \quad (18)$$

for any nonzero $\bar{\omega}(t) \in \mathbb{L}_2$.

Lemma 5 (see [29]). For any positive symmetric constant matrix $Q \in \mathbb{R}^{n \times n}$, scalars h_1, h_2 satisfying $h_1 < h_2$, a vector function $\phi : [h_1, h_2] \rightarrow \mathbb{R}^n$, such that the integrations concerned are well defined; then

$$\begin{aligned} & \left(\int_{h_1}^{h_2} \phi(s) ds \right)^T Q \left(\int_{h_1}^{h_2} \phi(s) ds \right) \\ & \leq (h_2 - h_1) \left(\int_{h_1}^{h_2} \phi^T(s) Q \phi(s) ds \right). \end{aligned} \quad (19)$$

Lemma 6 (see [30]). Let $L^T = L, H$, and E be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t)F(t) \leq I$. Then, $L + HFE + E^T F^T H^T < 0$, if and only if there exists a scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1} H H^T + \varepsilon E^T E < 0$, or equivalently

$$\begin{bmatrix} L & H & \varepsilon E^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \quad (20)$$

The main purpose of the paper is to design a set of nonfragile controllers (10) for the response complex dynamical networks such that, for admissible external bounded disturbances, the H_∞ synchronization is achieved with a prescribed attenuation level γ .

3. Main Results

In this section, delay-dependent criteria will be established to solve the H_∞ synchronization problem of complex dynamical networks in the presence of randomly occurring controller gain fluctuations and uncertainties. Then, a design procedure is proposed for the nonfragile controllers in terms of LMIs.

Theorem 7. For given scalars $\alpha, \beta \in [0, 1]$, and γ and the upper bounds of time-varying delays $\bar{\tau}$ and $\bar{\sigma}$, the H_∞ synchronization of the complex dynamical networks can be achieved with the given controller gains $K_i, i = 1, 2, \dots, N$, if there exist symmetric positive definite matrices $P, Q, R \in \mathbb{R}^{nN \times nN}$, and constants $\varepsilon_1, \varepsilon_2 > 0$ such that $\Pi < 0$, where

$$\Pi := \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix}, \quad (21)$$

with

$$\Pi_1 := \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{13} \end{bmatrix},$$

$$\Pi_2 := \begin{bmatrix} \Pi_{21} & \Pi_{22} \\ \Pi_{23} & \Pi_{24} \end{bmatrix},$$

Π_{11}

$$:= \begin{bmatrix} P\bar{A} + \bar{A}^T P - \hat{F}_1 + I & -P\bar{A}_d & -P(C \otimes \Gamma) & P\bar{B} - \hat{F}_2 & P \\ * & -\frac{Q}{\bar{\tau}} & 0 & 0 & 0 \\ * & * & -\frac{R}{\bar{\sigma}} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Pi_{12} := \begin{bmatrix} \bar{A}^T P & 0 & 0 & 0 & 0 \\ -\bar{A}_d^T P & 0 & 0 & 0 & 0 \\ -(C \otimes \Gamma)^T P & 0 & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{13} := \text{diag} \left\{ \frac{Q - 2P}{\bar{\tau}} + \frac{R - 2P}{\bar{\sigma}}, \frac{Q - 2P}{\bar{\tau}} + \frac{R - 2P}{\bar{\sigma}}, \frac{Q - 2P}{\bar{\tau}} + \frac{R - 2P}{\bar{\sigma}}, \frac{Q - 2P}{\bar{\tau}} + \frac{R - 2P}{\bar{\sigma}}, \frac{Q - 2P}{\bar{\tau}} + \frac{R - 2P}{\bar{\sigma}} \right\},$$

$$\Pi_{21} := \begin{bmatrix} \beta PH & \varepsilon_1 G^T & \alpha PD & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta PH & 0 & \alpha PD & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\Pi_{22} &:= \begin{bmatrix} 0 & \varepsilon_2 E_1^T + \varepsilon_2 E_2^T & 0 & \varepsilon_2 E_1^T + \varepsilon_2 E_2^T & 0 \\ 0 & -\varepsilon_2 E_2^T & \varepsilon_2 E_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_2 E_3^T & 0 & 0 & \varepsilon_2 E_3^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Pi_{23} &:= \begin{bmatrix} 0 & 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD \\ \sqrt{\beta(1-\beta)}PH & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Pi_{24} &:= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha(1-\alpha)}PD & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Pi_3 &:= \text{diag} \{-\varepsilon_1 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, \\
&\quad -\varepsilon_2 I, -\varepsilon_2 I\}, \\
\tilde{A} &:= \bar{A} + K + \bar{A}_d + C \otimes \Gamma, \\
\hat{F}_1 &:= \frac{F_{1\Lambda}^T F_{2\Lambda} + F_{2\Lambda}^T F_{1\Lambda}}{2}, \\
\hat{F}_2 &:= -\frac{F_{1\Lambda}^T + F_{2\Lambda}^T}{2}, \\
F_{1\Lambda} &:= \text{diag} \left\{ \underbrace{F_1, F_1, \dots, F_1}_N \right\}, \\
F_{2\Lambda} &:= \text{diag} \left\{ \underbrace{F_2, F_2, \dots, F_2}_N \right\}.
\end{aligned} \tag{22}$$

Proof. Choose the Lyapunov-Krasovskii function candidate as follows:

$$\begin{aligned}
V(t) &= e^T(t) P e(t) + \int_{-\bar{\tau}}^0 \int_{t+\varphi}^t \dot{e}^T(\eta) Q \dot{e}(\eta) d\eta d\varphi \\
&\quad + \int_{-\bar{\sigma}}^0 \int_{t+\varphi}^t \dot{e}^T(\eta) R \dot{e}(\eta) d\eta d\varphi.
\end{aligned} \tag{23}$$

Note that (16) can be rewritten as

$$\begin{aligned}
&\dot{e}(t) \\
&= (\tilde{A} + \alpha(t) \Delta \bar{A}(t) + \beta(t) \Delta K(t) + \alpha(t) \Delta \bar{A}_d(t)) \\
&\quad \cdot e(t) - (\bar{A}_d + \alpha(t) \Delta \bar{A}_d(t)) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi
\end{aligned}$$

$$\begin{aligned}
&+ (\bar{B} + \alpha(t) \Delta \bar{B}(t)) \bar{F}(t, e(t)) - (C \otimes \Gamma) \\
&\quad \cdot \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + \omega(t),
\end{aligned} \tag{24}$$

where $\tilde{A}$ is defined in (21).

The infinitesimal operator $\mathcal{L}$ of $V(t)$ is defined as follows:

$$\mathcal{L}V(t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \{ \mathbb{E} \{ V(t + \Delta) \mid t \} - V(t) \}. \tag{25}$$

Taking the derivative of (23) along the solution of system (24) yields

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V(t) \} &= \mathbb{E} \left\{ e^T(t) P \dot{e}(t) + \dot{e}^T(t) P e(t) \right. \\
&\quad + \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) - \int_{t-\bar{\tau}}^t \dot{e}^T(\varphi) Q \dot{e}(\varphi) d\varphi \\
&\quad \left. + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) - \int_{t-\bar{\sigma}}^t \dot{e}^T(\varphi) R \dot{e}(\varphi) d\varphi \right\}.
\end{aligned} \tag{26}$$

By Lemma 5, it holds that

$$\begin{aligned}
& - \int_{t-\bar{\tau}}^t \dot{e}^T(\varphi) Q \dot{e}(\varphi) d\varphi \\
& \leq -\frac{1}{\bar{\tau}} \int_{t-\bar{\tau}}^t \dot{e}^T(\varphi) d\varphi Q \int_{t-\bar{\tau}}^t \dot{e}(\varphi) d\varphi \\
& \leq -\frac{1}{\bar{\tau}} \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi Q \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi, \\
& - \int_{t-\bar{\sigma}}^t \dot{e}^T(\varphi) R \dot{e}(\varphi) d\varphi \\
& \leq -\frac{1}{\bar{\sigma}} \int_{t-\bar{\sigma}}^t \dot{e}^T(\varphi) d\varphi R \int_{t-\bar{\sigma}}^t \dot{e}(\varphi) d\varphi \\
& \leq -\frac{1}{\bar{\sigma}} \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi R \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi.
\end{aligned} \tag{27}$$

Moreover, it can be obtained by Assumption 1 that

$$- \begin{bmatrix} e(t) \\ \bar{F}(t, e(t)) \end{bmatrix}^T \begin{bmatrix} \hat{F}_1 & \hat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} e(t) \\ \bar{F}(t, e(t)) \end{bmatrix} \geq 0, \tag{28}$$

where $\hat{F}_1$ and $\hat{F}_2$ are defined in (21).

Combining inequalities (27)-(28), one has

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V(t) \} &\leq \mathbb{E} \left\{ 2e^T(t) P \left[(\tilde{A} + \alpha(t) \Delta \bar{A}(t) \right. \right. \\
&\quad + \beta(t) \Delta K(t) + \alpha(t) \Delta \bar{A}_d(t)) e(t) - (\bar{A}_d \\
&\quad + \alpha(t) \Delta \bar{A}_d(t)) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi + (\bar{B} \\
&\quad \left. \left. + \alpha(t) \Delta \bar{B}(t)) \bar{F}(t, e(t)) - (C \otimes \Gamma) \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \bar{\omega}(t) \Big] + \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) - \frac{1}{\bar{\tau}} \\
& \cdot \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi Q \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi - \frac{1}{\bar{\sigma}} \\
& \cdot \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi R \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi \Big\} \\
& \leq \mathbb{E} \left\{ \left[2e^T(t) \right. \right. \\
& \cdot P \left[\left(\bar{A} + \alpha(\Delta \bar{A}(t) + \Delta \bar{A}_d(t)) + \beta \Delta K(t) \right) e(t) \right. \\
& - \left(\bar{A}_d + \alpha \Delta \bar{A}_d(t) \right) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi \\
& + \left(\bar{B} + \alpha \Delta \bar{B}(t) \right) \bar{F}(t, e(t)) \\
& - (C \otimes \Gamma) \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + \bar{\omega}(t) \Big] + \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) \\
& + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) - \frac{1}{\bar{\tau}} \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi Q \\
& \cdot \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi - \frac{1}{\bar{\sigma}} \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi R \\
& \cdot \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + 2(\alpha(t) - \alpha) e^T(t) P (\Delta \bar{A}(t) \\
& + \Delta \bar{A}_d(t)) e(t) + 2(\beta(t) - \beta) e^T(t) P \Delta K(t) e(t) \\
& - 2(\alpha(t) - \alpha) e^T(t) P \Delta \bar{A}_d(t) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi \\
& + 2(\alpha(t) - \alpha) e^T(t) P \Delta \bar{B}(t) \bar{F}(t, e(t)) \\
& \left. - \left[\begin{array}{c} e(t) \\ \bar{F}(t, e(t)) \end{array} \right]^T \left[\begin{array}{cc} \hat{F}_1 & \hat{F}_2 \\ * & I \end{array} \right] \left[\begin{array}{c} e(t) \\ \bar{F}(t, e(t)) \end{array} \right] \right\} \\
& = \mathbb{E} \left\{ \left[2e^T(t) \right. \right. \\
& \cdot P \left[\left(\bar{A} + \alpha(\Delta \bar{A}(t) + \Delta \bar{A}_d(t)) + \beta \Delta K(t) \right) e(t) \right. \\
& - \left(\bar{A}_d + \alpha \Delta \bar{A}_d(t) \right) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi \\
& + \left(\bar{B} + \alpha \Delta \bar{B}(t) \right) \bar{F}(t, e(t))
\end{aligned}$$

$$\begin{aligned}
& - (C \otimes \Gamma) \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + \bar{\omega}(t) \Big] + \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) \\
& + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) - \frac{1}{\bar{\tau}} \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi Q \\
& \cdot \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi - \frac{1}{\bar{\sigma}} \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi R \\
& \cdot \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi - \left[\begin{array}{c} e(t) \\ \bar{F}(t, e(t)) \end{array} \right]^T \\
& \cdot \left[\begin{array}{cc} \hat{F}_1 & \hat{F}_2 \\ * & I \end{array} \right] \left[\begin{array}{c} e(t) \\ \bar{F}(t, e(t)) \end{array} \right] \Big\}. \tag{29}
\end{aligned}$$

In order to show the H_∞ performance γ , the following function is defined:

$$J_P := \mathbb{E} \{ \mathcal{L}V(t) + e^T(t) e(t) - \gamma^2 \bar{\omega}^T(t) \bar{\omega}(t) \}. \tag{30}$$

Then, it can be obtained from (27)-(28) that $J_P < 0$, if $\mathbb{E} \{ \zeta^T(t) \bar{\Pi} \zeta(t) + \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) \} < 0$ holds, where

$$\begin{aligned}
\bar{\Pi} &:= \begin{bmatrix} \bar{\Pi}_1 & \bar{\Pi}_2 \\ * & \bar{\Pi}_3 \end{bmatrix}, \\
\bar{\Pi}_1 &:= P \bar{A} + \bar{A}^T P + \alpha (P \Delta \bar{A}(t) + P \Delta \bar{A}_d(t)) \\
&+ \alpha (\Delta \bar{A}^T(t) P + \Delta \bar{A}_d^T(t) P) + \beta P \Delta K(t) \\
&+ \beta \Delta K^T(t) P - \hat{F}_1 + I, \\
\bar{\Pi}_2 &:= [-P (\bar{A}_d + \alpha \Delta \bar{A}_d(t)), -P (C \otimes \Gamma), P \bar{B} \\
&+ \alpha P \Delta \bar{B}(t) - \hat{F}_2, P], \\
\bar{\Pi}_3 &:= \text{diag} \left\{ -\frac{Q}{\bar{\tau}}, -\frac{R}{\bar{\sigma}}, -I, -\gamma^2 I \right\}, \\
\zeta(t) &:= \left[e^T(t), \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi, \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi, \right. \\
&\left. \bar{F}^T(t), \bar{\omega}^T(t) \right]^T. \tag{31}
\end{aligned}$$

Furthermore, it can be derived that

$$\begin{aligned}
\mathbb{E} \{ \bar{\tau} \dot{e}^T(t) Q \dot{e}(t) + \bar{\sigma} \dot{e}^T(t) R \dot{e}(t) \} &= \mathbb{E} \{ \dot{e}^T(t) (\bar{\tau} Q + \bar{\sigma} R) \dot{e}(t) \} = \mathbb{E} \left\{ \left[e^T(t) \left(\bar{A}^T + \alpha \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) + \beta \Delta K^T(t) \right) \right. \right. \\
&\quad - \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi \left(\bar{A}_d^T + \alpha \Delta \bar{A}_d^T(t) \right) + \bar{F}^T(t, e(t)) \left(\bar{B}^T + \alpha \Delta \bar{B}^T(t) \right) - \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi (C \otimes \Gamma)^T + \bar{\omega}^T(t) + (\alpha(t) - \alpha) \\
&\quad \cdot e^T(t) \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) + (\beta(t) - \beta) e^T(t) \Delta K^T(t) - (\alpha(t) - \alpha) \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi \Delta \bar{A}_d^T(t) + (\alpha(t) - \alpha) \bar{F}^T(t, e(t)) \\
&\quad \cdot \Delta \bar{B}^T(t) \left. \right] (\bar{\tau} Q + \bar{\sigma} R) \left[\left(\bar{A} + \alpha \left(\Delta \bar{A}(t) + \Delta \bar{A}_d(t) \right) + \beta \Delta K(t) \right) e(t) - \left(\bar{A}_d + \alpha \Delta \bar{A}_d(t) \right) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi + \left(\bar{B} \right. \right. \\
&\quad + \alpha \Delta \bar{B}(t) \left. \right) \bar{F}(t, e(t)) - (C \otimes \Gamma) \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + \bar{\omega}(t) + (\alpha(t) - \alpha) \left(\Delta \bar{A}(t) + \Delta \bar{A}_d(t) \right) e(t) + (\beta(t) - \beta) \Delta K(t) e(t) \\
&\quad - (\alpha(t) - \alpha) \Delta \bar{A}_d(t) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi + (\alpha(t) - \alpha) \Delta \bar{B}(t) \bar{F}(t, e(t)) \left. \right] \Big\} = \mathbb{E} \left\{ \left[e^T(t) \left(\bar{A}^T + \alpha \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) \right. \right. \right. \\
&\quad + \beta \Delta K^T(t) \left. \right) - \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi \left(\bar{A}_d^T + \alpha \Delta \bar{A}_d^T(t) \right) + \bar{F}^T(t, e(t)) \left(\bar{B}^T + \alpha \Delta \bar{B}^T(t) \right) - \int_{t-\sigma(t)}^t \dot{e}^T(\varphi) d\varphi (C \otimes \Gamma)^T \\
&\quad + \bar{\omega}^T(t) \left. \right] (\bar{\tau} Q + \bar{\sigma} R) \left[\left(\bar{A} + \alpha \left(\Delta \bar{A}(t) + \Delta \bar{A}_d(t) \right) + \beta \Delta K(t) \right) e(t) - \left(\bar{A}_d + \alpha \Delta \bar{A}_d(t) \right) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi + \left(\bar{B} \right. \right. \\
&\quad + \alpha \Delta \bar{B}(t) \left. \right) \bar{F}(t, e(t)) - (C \otimes \Gamma) \int_{t-\sigma(t)}^t \dot{e}(\varphi) d\varphi + \bar{\omega}(t) \left. \right] + \alpha(1 - \alpha) e^T(t) \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) (\bar{\tau} Q + \bar{\sigma} R) \left(\Delta \bar{A}(t) \right. \\
&\quad + \Delta \bar{A}_d(t) \left. \right) e(t) + \beta(1 - \beta) e^T(t) \Delta K^T(t) (\bar{\tau} Q + \bar{\sigma} R) \Delta K(t) e(t) + \alpha(1 - \alpha) \\
&\quad \cdot \int_{t-\tau(t)}^t \dot{e}^T(\varphi) d\varphi \Delta \bar{A}_d^T(t) (\bar{\tau} Q + \bar{\sigma} R) \Delta \bar{A}_d(t) \int_{t-\tau(t)}^t \dot{e}(\varphi) d\varphi + \alpha(1 - \alpha) \bar{F}^T(t, e(t)) \\
&\quad \cdot \Delta \bar{B}^T(t) (\bar{\tau} Q + \bar{\sigma} R) \Delta \bar{B}(t) \bar{F}(t, e(t)) \Big\}.
\end{aligned} \tag{32}$$

Then, according to Schur complement lemma [31], it can be verified that $J_P < 0$ if $\bar{\Pi} < 0$ holds, where

$$\bar{\Pi} := \begin{bmatrix} \bar{\Pi} & \bar{\Pi}_1 \\ * & \bar{\Pi}_2 \end{bmatrix}, \tag{33}$$

$$\bar{\Pi}_1 := \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} \\ \bar{\Pi}_{13} & \bar{\Pi}_{14} \end{bmatrix},$$

$$\bar{\Pi}_{11} \tag{34}$$

$$:= \begin{bmatrix} \bar{\Pi}_{111} & \sqrt{\alpha(1-\alpha)} \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) \\ -\bar{A}_d^T - \alpha \Delta \bar{A}_d^T(t) & 0 \\ -(C \otimes \Gamma)^T & 0 \end{bmatrix},$$

$$\bar{\Pi}_{111} := \bar{A}^T + \alpha \left(\Delta \bar{A}^T(t) + \Delta \bar{A}_d^T(t) \right) + \beta \Delta K^T(t), \tag{35}$$

$$\begin{aligned}
\bar{\Pi}_{12} &:= \text{diag} \left\{ \sqrt{\beta(1-\beta)} \Delta K^T(t), \sqrt{\alpha(1-\alpha)} \Delta \bar{A}_d^T(t), 0 \right\}, \\
\bar{\Pi}_{13} &:= \left[\bar{B} + \alpha \Delta \bar{B}(t), I \right]^T,
\end{aligned} \tag{36}$$

$$\bar{\Pi}_{14} := \begin{bmatrix} 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)} \Delta \bar{B}^T(t) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\bar{\Pi}_2 &:= \text{diag} \left\{ -\frac{Q^{-1}}{\bar{\tau}} - \frac{R^{-1}}{\bar{\sigma}}, -\frac{Q^{-1}}{\bar{\tau}} - \frac{R^{-1}}{\bar{\sigma}}, -\frac{Q^{-1}}{\bar{\tau}} - \frac{R^{-1}}{\bar{\sigma}}, \right. \\
&\quad \left. -\frac{Q^{-1}}{\bar{\tau}} - \frac{R^{-1}}{\bar{\sigma}}, -\frac{Q^{-1}}{\bar{\tau}} - \frac{R^{-1}}{\bar{\sigma}} \right\}.
\end{aligned} \tag{37}$$

Note that $-PQ^{-1}P \leq Q - 2P$ holds. By substituting (2) and (11) into (33) and performing a congruence transformation of $\text{diag}\{I, I, I, I, P, P, P, P\}$ to (33), it follows that $\bar{\Pi} < 0$ if the following inequality holds:

$$\begin{aligned}
& \widehat{\Pi} + \begin{bmatrix} \alpha PD & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha PD & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\alpha(1-\alpha)}PD & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{\alpha(1-\alpha)}PD & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD \end{bmatrix} \begin{bmatrix} \Psi(t) & 0 & 0 & 0 \\ 0 & \Psi(t) & 0 & 0 \\ 0 & 0 & \Psi(t) & 0 \\ 0 & 0 & 0 & \Psi(t) \end{bmatrix} \begin{bmatrix} E_1+E_2 & -E_2 & 0 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ E_1+E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
& + \left(\begin{bmatrix} \alpha PD & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha PD & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\alpha(1-\alpha)}PD & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{\alpha(1-\alpha)}PD & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD \end{bmatrix} \begin{bmatrix} \Psi(t) & 0 & 0 & 0 \\ 0 & \Psi(t) & 0 & 0 \\ 0 & 0 & \Psi(t) & 0 \\ 0 & 0 & 0 & \Psi(t) \end{bmatrix} \begin{bmatrix} E_1+E_2 & -E_2 & 0 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ E_1+E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right)^T \quad (38) \\
& + \begin{bmatrix} \beta PH \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta PH \\ 0 \\ \sqrt{\beta(1-\beta)}PH \\ 0 \\ 0 \end{bmatrix} \Upsilon(t) [G \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] + \begin{bmatrix} \beta PH \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta PH \\ 0 \\ \sqrt{\beta(1-\beta)}PH \\ 0 \\ 0 \end{bmatrix} \Upsilon(t) [G \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \quad < 0,
\end{aligned}$$

where

$$\begin{aligned}\widehat{\Pi} &:= \begin{bmatrix} \widehat{\Pi}_1 & \widehat{\Pi}_2 \\ * & \widehat{\Pi}_3 \end{bmatrix}, \\ \widehat{\Pi}_1 &:= \begin{bmatrix} P\bar{A} + \bar{A}^T P - \widehat{F}_1 + I & -P\bar{A}_d & -P(C \otimes \Gamma) & P\bar{B} - \widehat{F}_2 & P \\ * & -\frac{Q}{\bar{\tau}} & 0 & 0 & 0 \\ * & * & -\frac{R}{\bar{\sigma}} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}, \\ \widehat{\Pi}_2 &:= \begin{bmatrix} \bar{A}^T P & 0 & 0 & 0 & 0 \\ -\bar{A}_d^T P & 0 & 0 & 0 & 0 \\ -(C \otimes \Gamma)^T P & 0 & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \widehat{\Pi}_3 &:= \text{diag} \left\{ \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}} \right\}. \end{aligned} \quad (39)$$

Therefore, it can be obtained by Lemma 6 that if $\Pi < 0$ holds, then $J_p < 0$. When $\varpi(t) \neq 0$, under zero initial conditions, it can be verified that the condition $\Pi < 0$ can guarantee that $\mathbb{E}\{\int_0^\infty [e^T(t)e(t) - \gamma^2 \varpi^T(t)\varpi(t)]dt\} < 0$, which implies that the H_∞ synchronization is achieved in the sense of Definition 4 and completes the proof. $\square$

Remark 8. The two stochastic variables satisfying Bernoulli random binary distributions are adopted to character the randomly occurring phenomena in the synchronization problem for complex dynamical networks. It can be observed that when $\beta = 1$, the controllers (11) are degenerated to the general nonfragile controllers. By utilizing the stochastic information, a better synchronization performance can be obtained with less conservatism.

Remark 9. It is worth mentioning that it is a very expensive or even impossible task to obtain the real values or the boundedness of the derivatives of time delays in many applications. Regarding this aspect, our established synchronization conditions impose no constraint on the derivatives of the time-varying delays $\tau(t)$ and $\sigma(t)$, which is applicable to more general case in the real world. Moreover, the derived results in this paper can be directly applied to deal with the cases of constant time delays.

Based on Theorem 7, the following theorem is presented to solve the H_∞ synchronization controller design problem in terms of LMIs.

Theorem 10. For given scalars $\alpha, \beta \in [0, 1]$, and γ and the upper bounds of time-varying delays $\bar{\tau}$ and $\bar{\sigma}$, the H_∞ synchronization of the complex dynamical networks can be achieved, if there exist symmetric positive definite matrices $P = \text{diag}\{P_1, P_2, \dots, P_N\}$, $Q = \text{diag}\{Q_1, Q_2, \dots, Q_N\}$, and $R = \text{diag}\{R_1, R_2, \dots, R_N\} \in \mathbb{R}^{nN \times nN}$, matrix $V = \text{diag}\{V_1, V_2, \dots, V_N\} \in \mathbb{R}^{nN \times nN}$, and constants $\varepsilon_1, \varepsilon_2 > 0$ such that $\Xi < 0$, where

$$\Xi := \begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix}, \quad (40)$$

with

$$\Xi_1 := \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{13} \end{bmatrix},$$

$$\Xi_2 := \begin{bmatrix} \Xi_{21} & \Xi_{22} \\ \Xi_{23} & \Xi_{24} \end{bmatrix},$$

$$\Xi_{11} := \begin{bmatrix} \Xi_{111} & \Xi_{112} \\ * & \Xi_{113} \end{bmatrix},$$

$$\Xi_{111} := P\bar{A} + \bar{A}^T P + V + V^T - \widehat{F}_1 + I,$$

$$\Xi_{112} := [-P\bar{A}_d, -P(C \otimes \Gamma), P\bar{B} - \widehat{F}_2, P],$$

$$\Xi_{113} := \text{diag} \left\{ -\frac{Q}{\bar{\tau}}, -\frac{R}{\bar{\sigma}}, -I, -\gamma^2 I \right\},$$

$$\Xi_{12} := \begin{bmatrix} \bar{A}^T P + V^T & 0 & 0 & 0 & 0 \\ -\bar{A}_d^T P & 0 & 0 & 0 & 0 \\ -(C \otimes \Gamma)^T P & 0 & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}\Xi_{13} &:= \text{diag} \left\{ \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}} \right\}, \\ &+ \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}}, \frac{Q-2P}{\bar{\tau}} + \frac{R-2P}{\bar{\sigma}} \Big\},\end{aligned}$$

$$\Xi_{21} := \begin{bmatrix} \beta PH & \varepsilon_1 G^T & \alpha PD & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta PH & 0 & \alpha PD & 0 & 0 \end{bmatrix},$$

$$\Xi_{22} := \begin{bmatrix} 0 & \varepsilon_2 E_1^T + \varepsilon_2 E_2^T & 0 & \varepsilon_2 E_1^T + \varepsilon_2 E_2^T & 0 \\ 0 & -\varepsilon_2 E_2^T & \varepsilon_2 E_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_2 E_3^T & 0 & 0 & \varepsilon_2 E_3^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{23} := \begin{bmatrix} 0 & 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD \\ \sqrt{\beta(1-\beta)}PH & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha(1-\alpha)}PD & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{24} := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha(1-\alpha)}PD & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_3 := \text{diag}\{-\varepsilon_1 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I\},$$

$$\widehat{A} := \overline{A} + \overline{A}_d + C \otimes \Gamma,$$

(41)

$\widehat{F}_1, \widehat{F}_2$ are defined in (21), and the controller gains can be obtained as $K_i = P_i^{-1}V_i, i = 1, 2, \dots, N$.

Proof. The proof follows directly from Theorem 7 by letting $V := PK$. $\square$

4. Numerical Example

In this section, an illustrative example is presented to show the effectiveness and usefulness of our proposed control strategy.

Consider two complex dynamical networks (1) and (7) with three nodes and the state vector of each node being two-dimensional, such that $N = 3$ and $n = 2$. The network matrices are given as

$$A_i = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.2 \end{bmatrix},$$

$$A_{id} = \begin{bmatrix} 0.5 & 0.4 \\ 0.3 & 0.1 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 1.2 & 0.1 \\ 0.2 & 1 \end{bmatrix},$$

$$i = 1, 2, 3.$$

(42)

The inner coupling matrix is assumed to be $\Gamma = \text{diag}\{1, 1\}$ and the outer coupling matrix C is given as

$$C = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}. \quad (43)$$

The time-varying delays are chosen as $\tau(t) = 0.2 + 0.05 \sin t$ and $\sigma(t) = 0.4 + 0.1 \cos t$. Accordingly, one has $\bar{\tau} = 0.25$ and $\bar{\sigma} = 0.5$. The nonlinear function $f(t, x_i(t))$ is taken as

$$f(t, x_i(t)) = \begin{bmatrix} 0.2x_{i1}(t) - \tanh(0.1x_{i1}(t)) \\ 0.1x_{i2}(t) \end{bmatrix}, \quad (44)$$

such that $f(t, x_i(t))$ can satisfy Assumption 1 with

$$F_1 = \text{diag}\{0.1, 0.1\},$$

$$F_2 = \text{diag}\{0.2, 0.1\}. \quad (45)$$

The parameter uncertainties satisfy condition (11) with

$$D_1 = \text{diag}\{0.3, 0.4\},$$

$$D_2 = \text{diag}\{0.5, 0.6\},$$

$$D_3 = \text{diag}\{0.7, 0.8\}, \quad (46)$$

$$\Psi_i(t) = \text{diag}\{\cos t, \sin t\}, \quad i = 1, 2, 3,$$

$$E_{ij} = \text{diag}\{0.5, 0.5\}, \quad i, j = 1, 2, 3,$$

and the controller gain fluctuations are assumed to be

$$H_1 = \text{diag}\{0.4, 0.4\},$$

$$H_2 = \text{diag}\{0.5, 0.5\},$$

$$H_3 = \text{diag}\{0.6, 0.6\},$$

$$\Upsilon_i(t) = \text{diag}\{\sin t, \cos t\}, \quad i = 1, 2, 3, \quad (47)$$

$$G_1 = \text{diag}\{0.3, 0.3\},$$

$$G_2 = \text{diag}\{0.2, 0.2\},$$

$$G_3 = \text{diag}\{0.1, 0.1\}.$$

The external disturbances are assumed to be $\omega_i(t) = 0.2 \sin t$, $i = 1, 2, 3$. The parameters γ , α , and β are set as $\gamma = 0.2$, $\alpha = 0.5$, and $\beta = 0.5$, respectively.

By solving the LMIs (40) in Theorem 10, the desired controller gains can be obtained as follows:

$$K_1 = \begin{bmatrix} -8.9075 & -0.9760 \\ -0.7894 & -7.8987 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -9.0663 & -0.9777 \\ -0.7892 & -8.0776 \end{bmatrix}, \quad (48)$$

$$K_3 = \begin{bmatrix} -9.1508 & -0.9740 \\ -0.7841 & -8.1728 \end{bmatrix}.$$

FIGURE 1: Synchronization errors without controllers.

FIGURE 2: Synchronization errors with designed controllers.

Set the initial states of the two complex dynamical networks with random values. Figure 1 shows the state trajectories of the synchronization errors without the controllers and the state trajectories of the controlled synchronization errors are shown in Figure 2. It can be clearly seen that the two complex dynamical networks can achieve the H_∞ synchronization with the designed controllers, which demonstrates our proposed method.

5. Conclusion

This paper is concerned with the problem of nonfragile H_∞ synchronization of complex dynamical networks with randomly occurring controller gain fluctuations and uncertainties. In particular, two independent stochastic variables satisfying Bernoulli distributions are adopted to model these randomly varying phenomena. By constructing appropriate

Lyapunov-Krasovskii functionals and using stochastic analysis, delay-dependent sufficient conditions are established to guarantee the H_∞ synchronization of the drive-response complex dynamical networks. Moreover, the nonfragile controllers are further developed based on the obtained results in terms of LMIs. In the end, the effectiveness of our proposed strategy is demonstrated through a simulation example. Our further research would be to extend the current results to the cases with time-varying topologies and pinning control strategy, which are more comprehensive in the real world.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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