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Research Article

Maintaining Track Continuity for Extended Targets Using Gaussian-Mixture Probability Hypothesis Density Filter

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A multiextended-target tracker based on the extended target Gaussian-mixture probability hypothesis density (ET-GMPHD) filter, which can provide the tracks of the extended targets, is proposed to maintain the track continuity for the extended targets. To identify the extended targets, each individual Gaussian term of the mixture representing the posterior intensity function will be assigned a label, which is evolved through time. Then a track management scheme, including track initiation, track confirmation, track propagation, and termination, is developed to form the tracks for the extended targets. Furthermore, to improve the performance of the extended target tracker we also propose a mixture partitioning algorithm for resolving the identities of the extended targets in close proximity. The simulation results show that our proposed tracker achieves the less error of the position estimates and decreases the probability of incorrect label assignments from 0.6 to 0.25.

1. Introduction

In general target tracking applications, it is assumed that each target produces at most one measurement per time step. This is true when their extension is assumed to be negligible in comparison with sensor resolution. However, with the increasing sensor resolution capability this assumption is no longer valid. For instance, in short-range and maritime surveillance applications, different scattering centers, which vary from scan to scan both in its number and the relative origin location, of the objects may give rise to several distinct detections.

Extended target tracking has attracted much attention in the last decade. Gilholm and Salmond in [1] presented an approach for tracking extended targets under the assumption that the number of received target measurements in each time step is Poisson distributed. An inhomogeneous Poisson point process measurement model was developed in [2]. This measurement model could imply that the extended target is sufficiently far away from the sensor for its measurements to resemble a cluster of points, rather than a geometrically structured ensemble. A similar approach was proposed in [3] where track-before-detect theory was applied to track a 1D extent target. Baum et al. presented a random hypersurface model, which was used to track elliptic targets in [4, 5] and more general shapes in [6]. Another method to elliptic target model is an approximate Bayesian approach based on the random matrix by Koch in [7]. The target kinematical states are modeled by a Gaussian distribution, and the ellipsoidal target extension is modeled by a random matrix which follows the inverse Wishart distribution. In [8], Koch and Feldmann applied the filter based on the random matrix to track group targets under kinematical constraints. Modifications and improvements to the random matrix model of [7] were developed in [9]. A comparison of the random matrix model and the random hypersurface model was discussed in [10]. Other methods to obtain the target extension information were given in [11–14]. However, almost all those methods mentioned above are for single extended target.

Wieneke and Koch in [15] integrated random matrix, and Baum et al. in [16] integrated Mixture RHM into a probabilistic multihypothesis tracking (PMHT) framework to track multiply extended objects. However the complexity grows dramatically with the number of extended targets and measurements increasing. Another way to track multiple extended targets is based on the random finite set (RFS).

In [17], Mahler presented an extension of the probability hypothesis density (PHD) filter [18] to handle extended targets of the type presented in [2]. Orguner et al. proposed a cardinalized probability hypothesis density (CPHD) filter for extended targets [19]. Vo and Ma presented an extension of the Gaussian-mixture PHD filter [20] for extended targets, called the extended target GMPHD filter (ET-GMPHD) [21], and described much more details and extensive investigations of the methodology [22]. Random matrix framework was adapted into the ET-GMPHD framework by Granström and Orguner in [23], resulting in the Gaussian inverse Wishart PHD (GIW-PHD) filter. For the sake of convenience, the ET-GMPHD filter [21, 22] is referred to as the original ET-GMPHD filter from here onwards. However, the object identities were not involved in the implementations of the PHD filter such as the particle PHD filter and the GMPHD filter.

For postprocessing such as the behavior of objects and activity recognition, the track continuity of objects needs to be obtained. There are some studies on the track continuity in implementations of the PHD filter. The multiple hypothesis tracker and assignment algorithms are applied to the particle PHD filter to form the tracks of targets in [24] and [25, 26], respectively. Moreover, there are some methods which analyze the propagation of particles to maintain the track continuity [27, 28]. Due to the unreliability of the clustering methods in the particle PHD filter, the performance of these approaches may be affected. Recently, Clark et al. introduced a technique to identify the state estimates of objects in the GMPHD filter [29]. This method was successfully applied in sonar image tracking [30]. However, the temporal information which adversely affects the performance was not included. Pham et al. [31] proposed a method for maintaining the continuity of state estimates of objects in the GMPHD filter. The set of labels from Gaussian components was used to create hypotheses for label association process and the Hungarian algorithm was applied to search for the best hypothesis association. Panta et al. [32] proposed a GMPHD filter-based multitarget tracker, which provided track labels and the association amongst state estimates of targets over time. Various issues regarding initiating, propagating, and terminating tracks were discussed. However, to the best of our knowledge, no work was carried out on the track continuity in the ET-GMPHD filter.

In the works based on RFS discussed above, theoretically all the possible partitions of the measurements should be considered. However, the number of all the possible partitions grows dramatically with the increase in the number of measurements. Distance partitioning (DP) and distance partitioning with subpartitioning (DP-SP) were proposed to obtain a reasonable subset of all the possible partitions in [22]. In [33], to further reduce the computation time, Zhang and Ji proposed a ART (adaptive resonance theory) partitioning algorithm based on the fuzzy ART. Moreover, subpartitioning was applied to handle spatially close targets. Scheel proposed a mixture clustering algorithm, which could decompose the GIW-PHD filtering procedure into independent problems and thus reduce the combinatorial and computational complexity significantly [34]. By using the mixture cluster method, the GIW-PHD filter could be applied to the real-time tracking applications. In [33, 34], the problem with underestimation of the target number when there are extended targets in close proximity was not discussed further.

In this paper, we propose a multiextended-target tracker based on the original ET-GMPHD filter [21, 22], which provides not only the state estimates of targets at each time step but also the association of state estimates to targets over time. Three main contributions have been made to achieve this purpose, just as follows.

(i) To obtain the temporal association for the state estimates of individual extended targets, we assign the labels to individual Gaussian terms and develop a method of the label evolution through time. State trajectories of the individual extended targets can be obtained directly from the evolution of the Gaussian mixture.

(ii) We propose a track management scheme of initiating, confirming, propagating, and terminating tracks to construct the tracks of the extended targets.

(iii) To reduce the label assignment error when there are spatially close extended targets, mixture partitioning algorithm is proposed.

The rest of the paper is organized as follows. We briefly describe the extended target tracking problem in Section 2. Section 3 provides a summary of the original ET-GMPHD filter [22]. Section 4 presents the ET-GMPHD tracker proposed in this paper. In Section 5, simulation results are given to demonstrate the performance of the proposed ET-GMPHD tracker. Section 6 draws the conclusion and outlines future research directions.

2. Problem Formulation

The aim of the ET-GMPHD filter is to estimate the set of the extended target states $X_k = \{\mathbf{x}_k^{(i)}\}_{i=1}^{N_{x,k}}$, given the sets of measurements $\mathbf{Z}_k = \{\mathbf{z}_k^{(i)}\}_{i=1}^{N_{z,k}}$, for discrete time instants $k = 1, \ldots, K$, where $N_{x,k}$ is the unknown number of targets and $N_{z,k}$ is the number of measurements. The purpose of the ET-GMPHD tracker is to provide the track set $\mathcal{T}_k = \{T_k^{(i)}\}_{i=1}^{N_k^T}$, where N_k^T is the number of tracks in track set $\mathcal{T}_k$. Here, each track $T_k^{(i)}$ contains the set of estimated target states $\mathbf{X}_k^{(i)}$, corresponding time steps set $t_k^{(i)}$ and its label $l_k^{(i)}$. $\mathbf{X}_k^{(i)}$ contains all the estimated states of the extended target, whose label equals $l_k^{(i)}$, and from the time step it appears to the time step k.

The dynamic evolution of each target state $\mathbf{x}_k^{(i)}$ is assumed to be modeled by a linear Gaussian dynamic model

$$\mathbf{x}_{k+1}^{(i)} = \mathbf{F}_k \mathbf{x}_k^{(i)} + \mathbf{G}_k \mathbf{w}_k^{(i)} \tag{1}$$

for $i = 1, \ldots, N_{x,k}$, where $\mathbf{w}_k^{(i)}$ is Gaussian white noise with the covariance $\mathbf{Q}_k^{(i)}$. It is assumed that each target state evolves according to the same dynamic model independent of the other targets.

The measurements originating from the ith target are assumed to be related to the target state according to a linear Gaussian model

$$\mathbf{z}_k^{(j)} = \mathbf{H}_k \mathbf{x}_k^{(i)} + \mathbf{G}_k \mathbf{v}_k^{(j)}, \tag{2}$$

where $\mathbf{v}_k^{(j)}$ is white Gaussian noise with covariance $\mathbf{R}_k$. Each target is assumed to give rise to measurements independently of the other targets. We emphasize here that in an RFS framework both the set of measurements $\mathbf{Z}_k$ and the set of target states X_k are unlabeled, and hence no assumptions are made regarding which target gives rise to which measurement. The number of measurements generated by the ith target at each time step, denoted by $N_{m,k}^{(i)}$, is a Poisson distributed random variable with rate $\gamma(\mathbf{x}_k^{(i)})$ measurements per scan.

At each time step, clutter measurements are also generated. The number of clutter measurements N_k^c is a Poisson distributed random variable with rate $\beta_{FA,k}$ clutter measurements per surveillance volume per scan. The clutter measurements uniform over the surveillance volume.

3. Review of the Original ET-GMPHD Filter

The original ET-GMPHD filter [21, 22] is reviewed in Section 3.1. Section 3.2 describes two methods of partitioning the measurement set.

3.1. The Original ET-GMPHD Filter. Since the Gaussian mixture prediction equations of the ET-GMPHD filter are the same as those of the standard GMPHD filter [20], only measurement update formulas of the ET-GMPHD filter are introduced below. Here, six assumptions which are made in [20] hold here.

In the standard GMPHD-filter measurement update, each measurement is used to update each Gaussian component. In the ET-GMPHD filter, each cell of each partition is applied to update each Gaussian component. The corrected PHD-intensity, which is derived in [17], is the multiplication of the predicted PHD and the measurement pseudolikelihood function L_{Z_k},

$$\upsilon_{k|k}(\mathbf{x} \mid Z) = L_{Z_k}(\mathbf{x})\, \upsilon_{k|k-1}(\mathbf{x} \mid Z). \tag{3}$$

The measurement pseudolikelihood function L_{Z_k} is defined as

$$\begin{aligned} L_{Z_k}(\mathbf{x}) &= 1 - \left(1 - e^{-\gamma(\mathbf{x})}\right) p_D(\mathbf{x}) \\ &\quad + e^{-\gamma(\mathbf{x})} p_D(\mathbf{x}) \sum_{p \angle Z_k} \omega_p \sum_{W \in p} \frac{\gamma(\mathbf{x})^{|W|}}{d_W} \prod_{z \in W} \frac{\phi_{\mathbf{z}}(\mathbf{x})}{\lambda_k c_k(\mathbf{z})}, \end{aligned} \tag{4}$$

where $p_D(\mathbf{x})$ is the detected probability of the extended target; ω_p and d_W are nonnegative coefficients defined for each partition and cell, respectively; $\phi_{\mathbf{z}}(\mathbf{x})$ is the likelihood function for a single target-generated measurement, which is a Gaussian density under the measurement model; λ_k is the mean number of clutter measurements per scan; $c_k(\mathbf{z})$ is the spatial distribution of the clutter over the surveillance region; the notation $p \angle Z_k$ means that p partitions the measurement set Z_k into nonempty cells W. The first summation is taken over all partitions p of the measurement set Z_k. The second summation is taken over all cells W in the current partition p, and the product is over all measurements in the cell W. For each partition, the measurements in cells containing more than one measurement can be interpreted as from the same target. Measurements in cells with just one measurement can be either from clutter or from target.

The first part of (4), $1 - (1 - e^{-\gamma(\mathbf{x})})\, p_D(\mathbf{x})$, handles the targets for which there are no detections. The second part handles targets for which there is at least one detection.

Assuming that the predicted intensity has the Gaussian-mixture form

$$\upsilon_{k|k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k|k-1}} \omega_{k|k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\right), \tag{5}$$

where $\mathcal{N}(\cdot; m, P)$ denotes a Gaussian density with mean m and covariance P and ω is nonnegative weight; the posterior intensity at time k is also a Gaussian mixture, as shown as follows:

$$\upsilon_{k|k}(\mathbf{x}) = \upsilon_{k|k}^{ND}(\mathbf{x}) + \sum_{p \angle Z_k} \sum_{W \in p} \upsilon_{k|k}^{D}(\mathbf{x}, W). \tag{6}$$

The Gaussian-mixture $\upsilon_{k|k}^{ND}(\cdot)$, handling the no detections case, is given by

$$\begin{aligned} \upsilon_{k|k}^{ND}(\mathbf{x}) &= \sum_{j=1}^{J_{k|k-1}} \omega_{k|k}^{(j)} \mathcal{N}\left(\mathbf{x}; m_{k|k}^{(j)}, P_{k|k}^{(j)}\right), \\ \omega_{k|k}^{(j)} &= \left(1 - \left(1 - e^{-\gamma^{(j)}}\right) p_D^{(j)}\right) \omega_{k|k-1}^{(j)}, \\ m_{k|k}^{(j)} &= m_{k|k-1}^{(j)}, \\ P_{k|k}^{(j)} &= P_{k|k-1}^{(j)}, \end{aligned} \tag{7}$$

where $\gamma^{(j)}$ and $p_D^{(j)}$ are short for $\gamma(m_{k|k-1}^{(j)})$ and $p_D(m_{k|k}^{(j)})$, respectively.

The Gaussian-mixture $\upsilon_{k|k}^{D}(\mathbf{x}, W)$, handling detected targets, is given by

$$\begin{aligned} \upsilon_{k|k}^{D}(\mathbf{x}, W) &= \sum_{j=1}^{J_{k|k-1}} \omega_{k|k}^{(j)} \mathcal{N}\left(\mathbf{x}; m_{k|k}^{(j)}, P_{k|k}^{(j)}\right), \\ \omega_{k|k}^{(j)} &= \omega_p \frac{\Gamma^{(j)} p_D^{(j)}}{d_W} \Phi_W^{(j)} \omega_{k|k-1}^{(j)}, \\ \Gamma^{(j)} &= e^{-\gamma^{(j)}} \left(\gamma^{(j)}\right)^{|W|}, \\ \Phi_W^{(j)} &= \prod_{\mathbf{z} \in W} \frac{\phi_{\mathbf{z}}\left(m_{k|k}^{(j)}\right)}{\lambda_k c_k(\mathbf{z})} \end{aligned} \tag{8}$$

and the likelihood function of one measurement is

$$\phi_{\mathbf{z}}\left(m_{k|k}^{(j)}\right) = \mathcal{N}\left(\mathbf{z} \mid \mathbf{H}_k m_{k|k-1}^{(j)}, \mathbf{R}_k + \mathbf{H}_k P_{k|k-1}^{(j)} \mathbf{H}_k^T\right). \tag{9}$$

The partition weights ω_p can be considered as the probability of the partition p being true and can be written as

$$\omega_p = \frac{\prod_{W\in p} d_W}{\sum_{p'\angle Z_k} \prod_{W'\in p'} d_{W'}},$$
$$d_W = \delta_{|W|,1} + \sum_{l=1}^{J_{k|k-1}} \Gamma^{(l)} p_D^{(l)} \Phi_W^{(l)} \omega_{k|k-1}^{(l)}, \tag{10}$$

where $\delta_{i,j}$ is the Kronecker delta. The mean and covariance of the Gaussian components are updated by using the standard Kalman measurement update,

$$m_{k|k}^{(j)} = m_{k|k-1}^{(j)} + \mathbf{K}_k^{(j)} \left(\mathbf{z}_W - \mathbf{H}_W m_{k|k-1}^{(j)}\right),$$
$$P_{k|k}^{(j)} = \left(I - \mathbf{K}_k^{(j)} \mathbf{H}_W\right) P_{k|k-1}^{(j)}, \tag{11}$$
$$\mathbf{K}_k^{(j)} = P_{k|k}^{(j)} \mathbf{H}_W^T \left(\mathbf{H}_W P_{k|k}^{(j)} \mathbf{H}_W^T + \mathbf{R}_W\right)^{-1},$$

where $\mathbf{z}_W$, $\mathbf{H}_W$, and $\mathbf{R}_W$ are defined as

$$\mathbf{z}_W \triangleq \bigoplus_{\mathbf{z}_k\in W} \mathbf{z}_k, \qquad \mathbf{H}_W = \left[\underbrace{\mathbf{H}_k^T, \mathbf{H}_k^T, \ldots, \mathbf{H}_k^T}_{|W| \text{ times}}\right]^T,$$
$$\mathbf{R}_W = \text{blkdiag}\left(\underbrace{\mathbf{R}_k, \mathbf{R}_k, \ldots, \mathbf{R}_k}_{|W| \text{ times}}\right). \tag{12}$$

The operation $\oplus$ is vertical vectorial concatenation. The number of Gaussian components increases rapidly as the time progresses. To keep the number of Gaussian components at a computationally tractable level, pruning and merging are applied as in [20].

3.2. Partitioning the Measurement Set. In (4), all the possible partitions of the measurement set are considered in an ideal situation. However, the number of all the possible partitions would grow dramatically as the size of the measurement set increases. Thus choosing a subset of all the possible partitions is necessary to achieve the acceptable computational complexity. This section describes distance partitioning and subpartitioning proposed in [22].

3.2.1. Distance Partitioning (DP). Given a set of measurements $Z = \{\mathbf{z}^{(i)}\}_{i=1}^{N_z}$ and a distance measure $\mathbf{d}(\cdot,\cdot)$, the distances between each pair of measurements can be calculated as

$$\Delta_{ij} \triangleq \mathbf{d}\left(\mathbf{z}^{(i)}, \mathbf{z}^{(j)}\right), \quad \text{for } 1 \le i \ne j \le N_z. \tag{13}$$

It is proved in [22] that there is a unique partition that leaves all pairs (i, j) of measurements satisfying $\Delta_{ij} \le d_l$ in the same cell. N_d alternative partitions of the measurement set Z are generated by selecting N_d different thresholds

$$\{d_l\}_{l=1}^{N_d}, \quad d_l < d_{l+1}, \quad \text{for } l = 1, \ldots, N_d - 1. \tag{14}$$

For each d_l, one partition is obtained where the cells constitute sets of measurements that are no more than d_l apart from their closest cell neighbor.

The thresholds $\{d_l\}_{l=1}^{N_d}$ are selected from the set

$$\mathscr{D} \triangleq \{0\} \cup \left\{\Delta_{ij} \mid 1 \le i < j \le N_z\right\}. \tag{15}$$

If all of the elements in $\mathscr{D}$ are used to form alternative partitions, $|\mathscr{D}| = N_z(N_z - 1)/2 + 1$ partitions are achieved. To further reduce the computational load, only a subset of thresholds in the set $\mathscr{D}$ are applied to generate partitions.

The Mahalanobis distance is selected as the distance measure $\mathbf{d}(\cdot,\cdot)$. For two measurements $\mathbf{z}^{(i)}$ and $\mathbf{z}^{(j)}$ belonging to the same target, $\mathbf{d}_M(\mathbf{z}^{(i)}, \mathbf{z}^{(j)})$ is χ^2 distributed with degrees of freedom equal to the dimension of the measurement vector. A unitless distance threshold, denoted by δ_{P_G}, can be calculated as

$$\delta_{P_G} = \text{invchi2}\left(P_G\right) \tag{16}$$

for a given probability P_G, where invchi2$(\cdot)$ is the inverse cumulative χ^2 distribution function. In [21], it is illustrated that good target tracking results could be obtained in the situation that the subset of distance thresholds in $\mathscr{D}$ satisfies the condition $\delta_{P_L} < d_l < \delta_{P_U}$ with lower probabilities $P_L \le 0.3$ and upper probabilities $P_U \ge 0.8$.

3.2.2. Subpartitioning (SP). The results given by the ET-GMPHD filter with DP show the problem with underestimation of target set cardinality in situations where two or more extended targets are spatially close [21]. When targets are spatially close, so are their measurements. In this case, measurements from more than one extended target would be included in the same cell W in all partitions obtained by DP, and subsequently the ET-GMPHD filter interprets measurements from multiple targets as originating from the same target. SP was proposed in [22] to form additional partitions after performing DP.

Suppose that a set of partitions using DP have been obtained. Then, for each partition p_i, the estimates $\widehat{N}_x^j$ of the number of targets for each cell W_j^i are calculated as

$$\widehat{N}_x^j = \arg\max_n p\left(\left|W_j^i\right| \mid N_x^j = n\right). \tag{17}$$

If $\widehat{N}_x^j$ is larger than one, split the cell W_j^i into $\widehat{N}_x^j$ smaller cells, denoted by $\{W_s^+\}_{s=1}^{\widehat{N}_x^j}$ (Granström et al. [22] use K-means++ clustering to split the measurements in the cell). Then add a new partition, consisting of the new cells along with the other cells in p_i, to the list of partitions obtained by DP. For simplicity, DP-SP is short for the partition method whose partitions are obtained by the distance partitioning with subpartitioning.

4. The Proposed ET-GMPHD Tracker

The trajectories of extended targets were not provided in the original ET-GMPHD filter [21, 22]. This section describes

the proposed ET-GMPHD tracker which can provide the trajectories of individual extended targets according the state estimates of extended targets and their labels. It assigns the labels to the Gaussian terms of the mixture representing the posterior intensity function and evolves these labels through time without affecting the ET-GMPHD tracker recursion. This idea is inspired by the GMPHD tracker proposed in [32] which only adapts to point targets. Here, we extend it to the extended targets and achieve the ET-GMPHD tracker, in which the update step is different from that of the GMPHD tracker. Moreover, the method of the label processing when Gaussian terms of the mixture are merged was not provided. It will be also discussed in this section.

4.1. Label Evolvement for the ET-GMPHD Tracker. At time step $k = 0$, a unique label is assigned to each Gaussian term of the intensity function v_0

$$v_0(\mathbf{x}) = \sum_{i=1}^{J_0} \omega_0^{(i)} \mathcal{N}\left(\mathbf{x}; m_0^{(i)}, P_0^{(i)}\right) \tag{18}$$

to form the set

$$\mathcal{L}_0 = \left\{l_0^{(1)}, \ldots, l_0^{(J_0)}\right\}, \tag{19}$$

where $l_0^{(j)}$ denotes the label of jth Gaussian term with mean $m_0^{(j)}$ and covariance $P_0^{(j)}$.

The structure of propagating the Gaussian term and its label evolvement is shown as in Figure 1.

Given the posterior intensity $v_{k-1}(\mathbf{x})$ at time step $k-1$

$$v_{k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k-1}} \omega_{k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; m_{k-1}^{(i)}, P_{k-1}^{(i)}\right), \tag{20}$$

the predicted intensity at time step k is also a Gaussian mixture and can be expressed as

$$v_{k|k-1}(\mathbf{x}) = v_{S,k|k-1}(\mathbf{x}) + v_{\beta,k|k-1}(\mathbf{x}) + \eta_k(\mathbf{x}), \tag{21}$$

where

$$\begin{aligned}
v_{S,k|k-1}(\mathbf{x}) &= p_{S,k} \sum_{i=1}^{J_{k-1}} \omega_{k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; m_{S,k|k-1}^{(i)}, P_{S,k|k-1}^{(i)}\right), \\
m_{S,k|k-1}^{(i)} &= \mathbf{F}_{k-1} m_{k-1}^{(i)}, \\
P_{S,k|k-1}^{(i)} &= \mathbf{Q}_{k-1} + \mathbf{F}_{k-1} P_{k-1}^{(i)} \left(\mathbf{F}_{k-1}\right)^T, \\
v_{\beta,k|k-1}(\mathbf{x}) &= \sum_{i=1}^{J_{k-1}} \sum_{j=1}^{J_{\beta,k}} \omega_{k-1}^{(i)} \omega_{\beta,k}^{(j)} \mathcal{N}\left(\mathbf{x}; m_{\beta,k|k-1}^{(i,j)}, P_{\beta,k|k-1}^{(i,j)}\right), \\
m_{\beta,k|k-1}^{(i,j)} &= \mathbf{F}_{\beta,k-1}^{(j)} m_{k-1}^{(i)}, \\
P_{\beta,k|k-1}^{(i,j)} &= \mathbf{Q}_{\beta,k-1}^{(j)} + \mathbf{F}_{\beta,k-1}^{(j)} P_{k-1}^{(i)} \left(\mathbf{F}_{\beta,k-1}^{(j)}\right)^T, \\
\eta_k(\mathbf{x}) &= \sum_{i=1}^{J_{\eta,k}} \omega_{\eta,k}^{(i)} \mathcal{N}\left(\mathbf{x}; m_{\eta,k}^{(i)}, P_{\eta,k}^{(i)}\right).
\end{aligned} \tag{22}$$

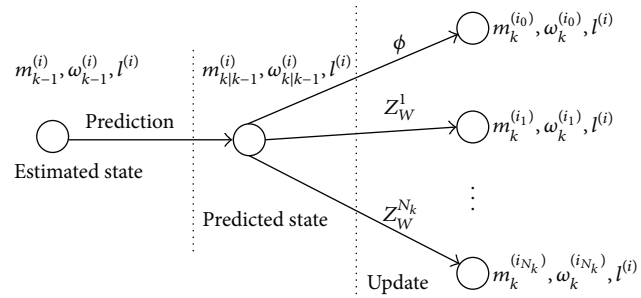

Figure 1: Tree structure of propagating ith Gaussian term and its label evolvement.

We construct the set of labels as follows:

$$\mathcal{L}_{k|k-1} = \mathcal{L}_{k-1} \cup \left\{l_{\eta,k}^{(1)}, \ldots, l_{\eta,k}^{(J_{\eta,k})}\right\} \cup \left\{l_{\beta,k}^{(1,1)}, \ldots, l_{\beta,k}^{(J_{k-1},J_{\beta,k})}\right\}, \tag{23}$$

where $\mathcal{N}(\mathbf{x}; m_{S,k|k-1}^{(i)}, P_{S,k|k-1}^{(i)})$ retains the label of its prior $\mathcal{N}(\mathbf{x}; m_{k-1}^{(i)}, P_{k-1}^{(i)})$, $l_{\eta,k}^{(i)}$ is the new label associated with ith Gaussian term introduced by the birth process, and $l_{\beta,k}^{(i,j)}$ is the new label of jth Gaussian term spawned by ith Gaussian term of the mixture.

The predicted intensity is updated according to (6). Each term in the predicted Gaussian mixture gives rise to $(1 + N_k)$ terms in the updated mixture. N_k is the number of the cells in all the partitions of the measurement set Z_k, as shown as follows:

$$N_k = \sum_{i=1}^{N_p} N_W^{(i)}, \tag{24}$$

where N_P is the number of the partitions of the measurement set Z_k and $N_W^{(i)}$ is the number of the cells in ith partition. We assign the same label to each of the updated Gaussian terms as its associated predicted term. As shown in Figure 1, the Gaussian term $\mathcal{N}(\mathbf{x}; m_k^{(i_j)}, P_k^{(i_j)})$ $(0 \le j \le N_k)$ gets the same label as that of $\mathcal{N}(\mathbf{x}; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)})$. As a result, we can obtain a number of updated Gaussian terms with different weights for every predicted Gaussian term.

As shown in Figure 1, each tree has its unique label that is the same as the label of the Gaussian term at its root. Each branch of a tree is a possible track of a target. The likelihood of each track is given by its weight. As time goes on, the number of Gaussian components increases sharply. Thus it is necessary to take measures to keep the number of Gaussian components at a computationally tractable level. After discarding those Gaussian components with weights below a preset truncation threshold T, three steps need to be carried out for each tree (Figure 1). First, the branch with the largest weight, B_L, is found. Second, we need to find those branches, represented as B_N, whose Gaussian components are so close to the branch B_L that they could be approximated by a single Gaussian. Finally, we will merge the branch B_L and the branches B_N into one branch and discard other branches of the tree. After the above three steps, only one branch

Given $\{\omega_k^{(i)}, m_k^{(i)}, P_k^{(i)}, l_k^{(i)}\}_{i=1}^{J_k}$, a truncation threshold T, a merging threshold U and a maximum allowable number of Gaussian terms $J_{\max}$.
Set $n = 0$, and $I = \{i = 1, \ldots, J_k \mid \omega_k^{(i)} > T\}$
Repeat:

$$j := \arg\max_{i \in I} \omega_k^{(i)}$$
$$M := \left\{i \in I \mid l_k^{(i)} == l_k^{(j)}\right\}$$
$$n := n + 1$$
$$\widetilde{l}_k^{(n)} = l_k^{(j)}$$
$$N := \left\{i \in M \mid \left(m_k^{(i)} - m_k^{(j)}\right)^T \left(P_k^{(i)}\right)^{-1} \left(m_k^{(i)} - m_k^{(j)}\right) \le U\right\}$$
$$\widetilde{\omega}_k^{(n)} = \sum_{i \in N} \omega_k^{(i)}$$
$$\widetilde{m}_k^{(n)} = \frac{1}{\widetilde{\omega}_k^{(n)}} \sum_{i \in N} \omega_k^{(i)} m_k^{(i)}$$
$$\widetilde{P}_k^{(n)} = \frac{1}{\widetilde{\omega}_k^{(n)}} \sum_{i \in N} \omega_k^{(i)} \left(P_k^{(i)} + \left(\widetilde{m}_k^{(n)} - \widetilde{m}_k^{(i)}\right)\left(\widetilde{m}_k^{(n)} - \widetilde{m}_k^{(i)}\right)^T\right)$$
$$I := I \setminus M$$

Until $I = \emptyset$
If $n > J_{\max}$, replace $\{\widetilde{\omega}_k^{(i)}, \widetilde{m}_k^{(i)}, \widetilde{P}_k^{(i)}, \widetilde{l}_k^{(i)}\}_{i=1}^{n}$ by those the $J_{\max}$ Gaussian terms with largest weights.
Output $\{\widetilde{\omega}_k^{(i)}, \widetilde{m}_k^{(i)}, \widetilde{P}_k^{(i)}, \widetilde{l}_k^{(i)}\}_{i=1}^{\widetilde{J}_k}$, where $\widetilde{J}_k = \min(J_{\max}, n)$

Algorithm 1: Pruning and merging algorithm for the ET-GMPHD tracker.

of each tree is obtained, which contains the estimated state and corresponding label. The proposed pruning and merging algorithm are summarized in Algorithm 1.

Let $\{\widetilde{\omega}_k^{(i)}, \widetilde{m}_k^{(i)}, \widetilde{P}_k^{(i)}, \widetilde{l}_k^{(i)}\}_{i=1}^{\widetilde{J}_k}$ denote the remaining Gaussian components after pruning and merging, and the intensity function can be expressed as

$$\widetilde{v}_k(\mathbf{x}) = \sum_{i=1}^{\widetilde{J}_k} \widetilde{\omega}_k^{(i)} \mathcal{N}\left(\mathbf{x}; \widetilde{m}_k^{(i)}, \widetilde{P}_k^{(i)}\right). \tag{25}$$

At time step k, state estimates of individual extended targets and their labels are extracted by picking the means of the Gaussian terms whose weights are greater than a chosen threshold, as shown as follows:

$$\begin{aligned} \widehat{X}_k &= \left\{m_k^{(i)} : \omega_k^{(i)} > \omega_{\mathrm{Th}},\ i = 1, \ldots, \widetilde{J}_k\right\}, \\ \widehat{L}_k &= \left\{l_k^{(i)} : \omega_k^{(i)} > \omega_{\mathrm{Th}},\ i = 1, \ldots, \widetilde{J}_k\right\}. \end{aligned} \tag{26}$$

Thus the trajectories of the targets can be determined directly by the evolution of the Gaussian mixture.

4.2. Track Management Scheme for the ET-GMPHD Tracker. For the ET-GMPHD tracker, to form the tracks of the extended targets, a scheme of initiating, confirming, propagating, and terminating tracks is described below.

4.2.1. Track Initiation. At $k = 0$, initialize a tree with $m_0^{(i)}$ as its root and $l_0^{(i)}$ as its label for $i = 1, \ldots, J_0$. At time step $k > 0$, we initialize a tree for every Gaussian term introduced by new birth process. For the tree, $m_{\eta,k}^{(i)}$ and $l_{\eta,k}^{(i)}$ can be regarded as its root and label, respectively.

4.2.2. Track Confirmation. As mentioned in the preceding section, after pruning and merging algorithm each tree has only one branch. We classify a tree as a confirmed tree, if its merged branch weight satisfies $\omega_k^{(i)} > \omega_{\mathrm{Th}}$ in the past three time steps. A confirmed tree provides one confirmed track whose label is the same as that of the tree it belongs to. All the confirmed tracks form a track set $\mathcal{T}_k = \{T_k^{(i)}\}_{i=1}^{N_k^T}$, where N_k^T is the number of tracks in track set $\mathcal{T}_k$. Each track $T_k^{(i)}$ contains the set of estimated target states $\mathbf{X}_k^{(i)}$, corresponding time steps set $t_k^{(i)}$ and its label $l_k^{(i)}$. $\mathbf{X}_k^{(i)}$ contains all the estimated states of the extended target, whose label is equal to $l_k^{(i)}$, from the time step it appears to the time step k.

4.2.3. Track Propagation and Termination. In order to achieve a good performance of the ET-GMPHD tracker in the presence of the detection uncertainty, an undefined tree set $\mathcal{U}_k = \{U_k^{(i)}\}_{i=1}^{N_k^U}$ is constructed, where N_k^U is the number of undefined trees. If a tree has been confirmed before the time step k and the branch weight $\omega_k^{(i)} \le \omega_{\mathrm{Th}}$ at current time step k, we consider the tree from k as an undefined tree. If the undefined tree lasts three time steps, its corresponding track is terminated. Otherwise, we combine the undefined tree with its corresponding confirmed tree, and then its corresponding track is propagated. The management scheme of the track

Given:
Output of Algorithm 1, $\{\widetilde{\omega}_k^{(i)}, \widetilde{m}_k^{(i)}, \widetilde{P}_k^{(i)}, \widetilde{l}_k^{(i)}\}_{i=1}^{\widetilde{J}_k}$; Extended targets state estimates $\widehat{X}_k = \{m_k^{(i)}\}_{i=1}^{N_k^{\widehat{X}}}$, and their labels $\widehat{L}_k = \{l_k^{(i)}\}_{i=1}^{N_k^{\widehat{X}}}$;
Track set $\mathscr{T}_{k-1}$, Undefined tree set $\mathscr{U}_{k-1}$,
Candidate terminated track set $\mathscr{C} = \{C^{(i)}\}_{i=1}^{N_C}$.
Propagation and Termination:
$j = 0$, $\mathscr{U}_k = \mathscr{U}_{k-1}$
For $i = 1 : N_k^{\widehat{X}}$
  If $l_k^{(i)} \in \mathscr{L}_{k-1}$ $(l_k^{(i)} == l_{k-1}^{(T,n)})$
    $\{\mathscr{L}_{k-1} = \{l_{k-1}^{(T,n)}\}_{n=1}^{N_{k-1}^T}$: set of the labels of track set $\mathscr{T}_{k-1}\}$
    $j = j + 1$,
    $\mathbf{X}_k^{(j)} = \mathbf{X}_{k-1}^{(n)} \cup m_k^{(i)}$, $t_k^{(j)} = t_{k-1}^{(i)} \cup k$,
    $T_k^{(j)} = \{\mathbf{X}_k^{(j)}, t_k^{(j)}, l_k^{(i)}\}$.
  Else If $l_k^{(i)} \in \mathscr{L}_k^U$ $(l_k^{(i)} == l_k^{(U,n)})$
    $\mathscr{L}_k^U = \{l_k^{(U,n)}\}_{n=1}^{N_k^U}$: set of the labels of $\mathscr{U}_k$
    $\mathscr{U}_k = \mathscr{U}_k \setminus U_k^{(n)}$,
    $j = j + 1$,
    $T_k^{(j)} = C^{(n)} \cup U_k^{(n)} \cup \{m_k^{(i)}, k\}$ $(l^{(C,n)} == l_k^{(i)})$
    $\{\mathscr{L}^C = \{l^{(C,n)}\}_{n=1}^{N_C}$: set of the labels of $\mathscr{C}\}$,
    $\mathscr{C} = \mathscr{C} \setminus C^{(n)}$.
  Else
    Track Initiation and Confirmation Steps.
  End If
End For
If $N_k^U \neq 0$
  $A = \emptyset$,
  For $i = 1 : N_k^U$
    If $N_T^{U_k^{(i)}} == 2$ (i.e. $U_k^{(i)}$ lasts three time steps)
    $\{N_T^{U_k^{(i)}}$ is the time steps that undefined tree $U_k^{(i)}$ lasts$\}$
      $A = A \cup U_k^{(i)}$,
      Terminate the track in $\mathscr{C}$ whose label equals to $l_k^{(U,i)}$.
    Else
      $\mathbf{X}_k^{(U,i)} = \mathbf{X}_k^{(U,i)} \cup \widetilde{m}_k^{(n)}$ $\{l_k^{(U,i)} == \widetilde{l}_k^{(n)}\}$,
      $t_k^{(U,i)} = t_k^{(U,i)} \cup k$, $U_k^{(i)} = \{\mathbf{X}_k^{(U,i)}, t_k^{(U,i)}, l_k^{(U,i)}\}$.
    End If
  End For
  $\mathscr{U}_k = \mathscr{U}_k \setminus A$,
  Update the number of undefined trees N_k^U.
End If
$B = \mathscr{L}_{k-1} \setminus \mathscr{L}_k$,
If $B \neq \emptyset$
  $\mathscr{C} = \mathscr{C} \cup \{$tracks in $\mathscr{T}_{k-1}$ whose labels are the same as $B\}$,
  $j = N_k^U$,
  For $i = 1 : N_B$ $\{N_B$ is the cardinality of set $B\}$
    $j = j + 1$,
    $\mathbf{X}_k^{(U,j)} = \widetilde{m}_k^{(n)}$ $\{\widetilde{l}_k^{(n)} == l_B^i\}$,
    $t_k^{(U,j)} = k$,

Algorithm 2: Continued.

$U_k^{(j)} = \{\mathbf{X}_k^{(U,j)}, t_k^{(U,j)}, l_B^i\}$.
End for
End If

Algorithm 2: Scheme of the track propagation and termination.

Require: Measurement set Z_k
Step 1. Obtain the partitions set $Z^p = \{p_1, \ldots, p_{N_p}\}$ using distance partitioning, where N_p is the number of partitions.
Step 2.
for $i = 1, \ldots, N_p$ do
  for $j = 1, \ldots, |p_i|$ do
    $\widehat{N}_x^j = \arg\max_n p\left(\left|W_j^i\right| \mid N_x^j = n\right)$
    if $\widehat{N}_x^j > 1$, then
      $p_i = p_i \setminus W_j^i$
      $\{W_k^+\}_{k=1}^{\widehat{N}_x^j} = \text{split}\left(\widehat{N}_x^j, W_j^i\right)$
      $p_i = p_i \cup \{W_k^+\}_{k=1}^{\widehat{N}_x^j}$
    end if
  end for
end for

Algorithm 3: Mixture partitioning algorithm.

propagation and termination for the ET-GMPHD tracker is summarized in Algorithm 2.

4.3. Mixture Partitioning. It is obvious that the performance of the ET-GMPHD tracker largely depends on the performance of the update step which depends, to a great extent, on the partitioning algorithm of measurements. When targets are spatially close, so are their position measurements. If the region occupied by measurements from an extended target overlaps with that of the other extended target to a certain degree, the cluster algorithm could not reasonably split the cell which contains measurements from the two extended targets. The weight of the partition, in which the cell containing measurements from the spatially close extended targets is split into the right number cells by SP, may be smaller than that of the partition, in which measurements from spatially close extended targets are put in one cell. Therefore, the partition obtained by SP does not take effect in the original ET-GMPHD filter algorithm. In the case discussed above, the original ET-GMPHD filter with DP-SP still has the problem of underestimation of target set cardinality. If two or more extended targets are considered as one, it would be assigned a new label or one of their labels. When they separate from each other again, they would be also labeled as new.

Since DP [21] works well when extended targets are away from each other, in this case we still employ DP. In the situation that two or more extended targets are spatially close, the main error results from the cardinality error of the target set and the error of the target state estimates is relatively small. The mixture partitioning algorithm is proposed to reduce the cardinality error of the target set. It can be described as follows. After partitioning the measurement set by DP, the number of targets for each cell W_j^i is estimated using (17) for each partition p_i, denoted by $\widehat{N}_x^j$. If $\widehat{N}_x^j > 1$, we split the cell W_j^i into $\widehat{N}_x^j$ small cells by Kernel Fuzzy c-means (KFCM) cluster. Kernel functions in KFCM cluster can map the data in the original space to a high-dimensional feature space, in which we can perform clustering more efficiently than c-means cluster and Fuzzy c-means cluster. The Gaussian function is chosen as the kernel function in this papers. The mixture partitioning algorithm is shown in Algorithm 3.

We will describe how to choose the partitioning method as follows. In practical applications, the extended target state always contains position and velocity $\mathbf{x}_k = [x_k, y_k, v_k^x, v_k^y]^T$. The measurement is mainly for position component, sometimes velocity component included. Here two situations are discussed. One is that the measurement only contains the position component, and the other is that the measurement contains not only the position component but also the velocity component.

According to the estimated extended target state set $\widehat{X}_{k-1}$ at time step $k-1$, we calculate the predicted target state set $\widehat{X}_{k|k-1} = \{\mathbf{x}_{k|k-1}^{(i)}\}_{i=1}^{N_{x,k|k-1}}$ by prediction and then obtain the corresponding estimated measurement set $\widehat{\mathbf{Z}}_k = \{\widehat{\mathbf{z}}^{(i)}\}_{i=1}^{N_{x,k|k-1}}$ for the centers of extended targets.

If the measurement only contains the position component, $\widehat{\mathbf{Z}}_k = \{\widehat{\mathbf{z}}^{(i)}\}_{i=1}^{N_{x,k|k-1}}$ is the set of the estimated position measurements of the centers of extended targets. To check

whether there exist extended targets which are spatially close to others, firstly we calculate the Mahalanobis distance between each pair of the estimated position measurements. Then since the smaller the Mahalanobis distance between two estimated position measurements is, the closer the two extended targets are, the following equation

$$\mathbf{d}_M\left(\hat{\mathbf{z}}^{(i)}, \hat{\mathbf{z}}^{(j)}\right) \le \delta_{P_D}, \quad \text{if } \exists i \ne j,\ 1 \le i, j \le N_{x,k|k-1} \tag{27}$$

is applied. If $\hat{\mathbf{Z}}_k$ satisfies (27), there exists spatially close extended targets, and the mixture partitioning is applied to remedy the problem with underestimation of the target number.

As mentioned in Section 3.2, for $\hat{\mathbf{z}}^{(i)}$ and $\hat{\mathbf{z}}^{(j)}$ belonging to the same target, the Mahalanobis distance $\mathbf{d}_M(\cdot,\cdot)$ is χ^2 distributed with degrees of freedom equal to the measurement vector dimension. Using the inverse cumulative χ^2 distribution function, a unitless distance threshold

$$\delta_{P_D} = \text{invchi2}\left(P_D\right) \tag{28}$$

can be computed for a given probability P_D. Simulations illustrate that the good target tracking results are achieved when P_D satisfies the condition $P_D \in [0.8\ \ 0.82]$.

If the measurement contains the velocity component, that is, $\mathbf{Z}_k = \{\mathbf{z}_k^{(i)}\}_{i=1}^{N_{z,k}}$, where $\mathbf{z}_k^{(i)} = [(\mathbf{p}_k^{(i)})^T, (\mathbf{v}_k^{(i)})^T]^T$, to improve the performance of the ET-GMPHD tracker, two parameters ρ_1, ρ_2 are introduced to adjust the weight of the position distance and velocity distance in DP. We adopt the following equation as the distance measure $\mathbf{d}(\cdot,\cdot)$:

$$\mathbf{d}\left(\mathbf{z}^{(i)}, \mathbf{z}^{(j)}\right) = \rho_1^{(i,j)}\mathbf{d}_M\left(\mathbf{p}^{(i)}, \mathbf{p}^{(j)}\right) + \rho_2^{(i,j)}\mathbf{d}_M\left(\mathbf{v}^{(i)}, \mathbf{v}^{(j)}\right), \tag{29}$$

where $\mathbf{d}_M$ is Mahalanobis distance which is unitless and $\rho_1^{(i,j)}$, $\rho_2^{(i,j)}$ are defined as follows:

$$\begin{aligned} \rho_1^{(i,j)} &= \frac{\chi^2\left(\mathbf{d}_M\left(\mathbf{p}^{(i)}, \mathbf{p}^{(j)}\right)\right)}{\chi^2\left(\mathbf{d}_M\left(\mathbf{p}^{(i)}, \mathbf{p}^{(j)}\right)\right) + \chi^2\left(\mathbf{d}_M\left(\mathbf{v}^{(i)}, \mathbf{v}^{(j)}\right)\right)}, \\ \rho_2^{(i,j)} &= \frac{\chi^2\left(\mathbf{d}_M\left(\mathbf{v}^{(i)}, \mathbf{v}^{(j)}\right)\right)}{\chi^2\left(\mathbf{d}_M\left(\mathbf{p}^{(i)}, \mathbf{p}^{(j)}\right)\right) + \chi^2\left(\mathbf{d}_M\left(\mathbf{v}^{(i)}, \mathbf{v}^{(j)}\right)\right)}. \end{aligned} \tag{30}$$

In (29), the distance measure $\mathbf{d}(\cdot,\cdot)$ becomes unitless by means of the Mahalanobis distance. $\rho_1^{(i,j)}$, $\rho_2^{(i,j)}$ regulate the ratio of the position distance and velocity distance in the distance measure. When the position distance and velocity distance of two targets are relatively large, their values of the χ^2 distribution function is nearly to 1, and ρ_1 is roughly equal to ρ_2. When the position distance is small and the velocity distance is relatively large, that is, the two targets are spatially close and their velocities are very different, we mainly use the velocity components to partition the measurement set and vice versa. If there exists the extended targets which are spatially close and whose velocity is similar, that is, the following two equations hold

$$\begin{aligned} &\mathbf{d}_M\left(\mathbf{p}^{(i)}, \mathbf{p}^{(j)}\right) \le \delta_{P_{D1}}, \quad \text{if } \exists i \ne j,\ 1 \le i, j \le N_{x,k|k-1}, \\ &\mathbf{d}_M\left(\mathbf{v}^{(i)}, \mathbf{v}^{(j)}\right) \le \delta_{P_{D2}}, \quad \text{if } \exists i \ne j,\ 1 \le i, j \le N_{x,k|k-1}, \end{aligned} \tag{31}$$

we employ the mixture partitioning algorithm.

5. Simulation Results

Section 5.1 presents the simulation setup. A comparison of the ET-GMPHD filter with DP and DP-SP is presented in Section 5.2, and the label assignment results based on the original ET-GMPHD filter are shown. In Section 5.3 a comparison between the ET-GMPHD filter with DP-SP and that with mixture partitioning algorithm is described. Finally, the simulation results of the proposed ET-GMPHD tracker are illustrated in Section 5.4.

5.1. Simulation Setup

5.1.1. Target Tracking Setup. We consider a two-dimensional scenario over the surveillance region $[-1000, 1000] \times [-1000, 1000]$ (in m). The state $\mathbf{x}_k = [x_k, y_k, v_k^x, v_k^y]^T$ of each extended target consists of its position (x_k, y_k) and velocity (v_k^x, v_k^y). $[\cdot]^T$ denotes a transpose of a matrix $[\cdot]$. The sensor measurements are given in batches of Cartesian x and y coordinates as shown as follows:

$$\mathbf{z}_k^j = \left[x_k^j, y_k^j\right]^T. \tag{32}$$

Reference back to (1) and (2) here. The parameters in the dynamic and measurement models are shown as follows:

$$\mathbf{F}_k = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{G}_k = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix},$$
$$\mathbf{H}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad \mathbf{Q}_k = \sigma_{\mathbf{w}}^2\mathbf{I}_2, \qquad \mathbf{R}_k = \sigma_{\mathbf{v}}^2\mathbf{I}_2 \tag{33}$$

with sampling time $T = 1$ s. Here, $\sigma_{\mathbf{w}} = 2$ m/s and $\sigma_{\mathbf{v}} = 20$ m.

The probability of survival is set to $p_S = 0.99$, and the probability of detection is $p_D = 0.99$. In each simulation, clutters are generated with a Poisson rate of 10 clutter measurements per scan, and each target generates measurements with a Poisson rate of 10 measurements per scan. The birth intensity in the simulations is

$$\upsilon_b(\mathbf{x}) = 0.1\mathcal{N}\left(\mathbf{x}; m_b^1, P_b\right) + 0.1\mathcal{N}\left(\mathbf{x}; m_b^2, P_b\right), \tag{34}$$

where

$$\begin{aligned} m_b^1 &= [250, 250, 0, 0]^T, \\ m_b^2 &= [-250, -250, 0, 0]^T, \\ P_b &= \text{diag}([100, 100, 25, 25]). \end{aligned} \tag{35}$$

The spawn intensity is

$$\upsilon_\beta(\mathbf{x}) = 0.05\mathcal{N}\left(\mathbf{x}; \xi, P_\beta\right), \tag{36}$$

where $P_\beta = \text{diag}([100, 100, 400, 400])$ and ξ is the state of the target from which the new target is spawned.

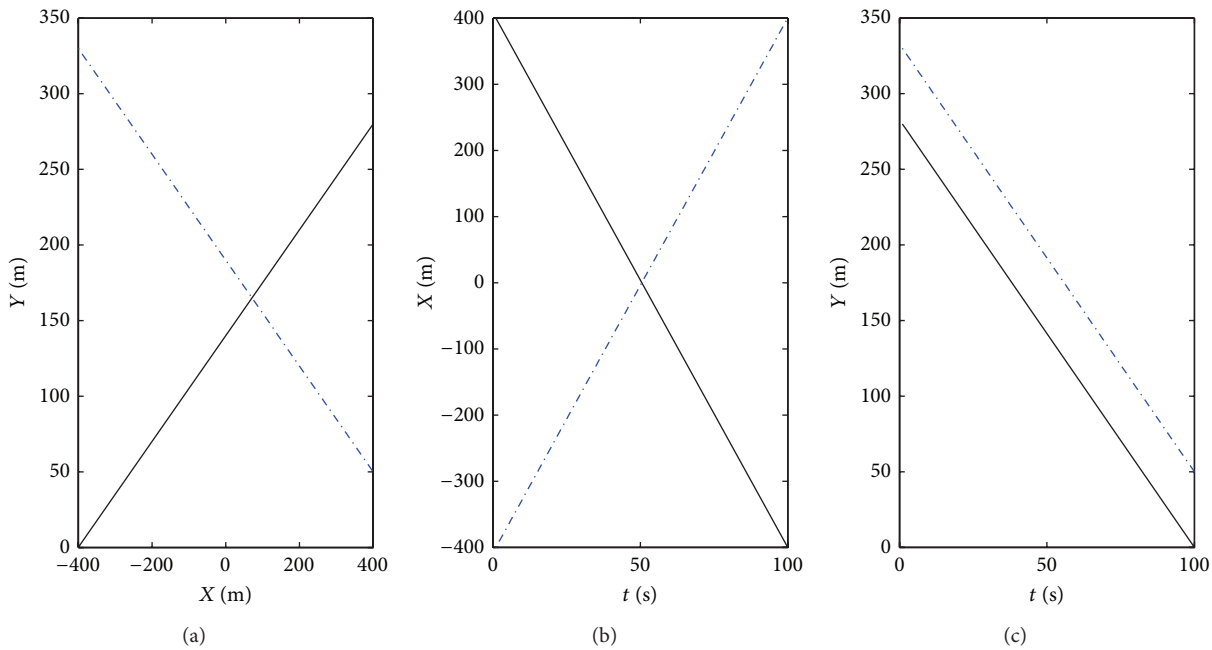

Figure 2: (a) Trajectories of extended targets for the first scenario. (b) The x positions changing with time. (c) The y positions changing with time.

In many practical applications (e.g., radar, laser, and stereo vision), the sensor provides range r and azimuth angle φ given by

$$\mathbf{z}_k^j = \left[r_k^j, \varphi_k^j\right]^T. \tag{37}$$

The work here could be applied to such case by using the appropriate polar to Cartesian conversion equations or by employing the unscented Kalman filter in the update step of the ET-GMPHD tracker.

5.1.2. True Target Track. Three different scenarios are employed in several simulations. The first two of them have two targets. The true x, y positions and their corresponding positions changing with time in the first scenario are shown in Figure 2. The trajectories of two targets in the second scenario are very similar to that in the first scenario: (1) the position of one target is the same with that of the target shown by black solid line in Figure 2; (2) the x position of the other is the same as that of the target shown by blue dotted line in Figure 2(b), and at each time step the y position is 10 m less than that of the target shown by blue dotted line in Figure 2(c). As shown in Figure 2(c), the distance of the two targets in the two scenarios does not change all the time in y direction. The distance of the two targets in y direction is 50 m in the first scenario and 40 m in the second scenario. In the third scenario there are five targets in total. The true x, y positions and their corresponding positions changing with time are shown in Figure 3.

In those simulations, targets generate measurements with standard deviation 20 m in both x and y directions. Thus, a measure of target extent can be considered as the two standard deviation measurement covariance circles with radius 40 m. In all scenarios these circles partly overlap in the case that the targets are close to each other.

5.2. ET-GMPHD Label Assignment. As mentioned in Section 4, the original ET-GMPHD [22] still has the problem that it could not distinguish those extended targets which are spatially close. In this subsection, the first two scenarios described in Section 5.1 are considered to illustrate the problem. The average number of extended targets estimated by the ET-GMPHD algorithm by using DP and DP-SP for two scenarios is given in Figures 4(a) and 4(b), which are obtained by averaging over 100 Monte Carlo runs, respectively.

As shown in Figure 4, though using SP would slightly increase the performance of ET-GMPHD filter, the original ET-GMPHD filter still has the problem of the underestimation of target set cardinality when there are spatially close extended targets. The problem would cause the error label assignment. To illustrate this, target tracks given by the tracker, which employs label assignment and track management scheme discussed in Section 4, based on the original ET-GMPHD filter by using DP-SP for the second scenario are shown in Figure 5. The results indicate that the identities of the extended targets change at time steps 45–55. This is because the ET-GMPHD tracker considers two extended target as one at time steps 47–55. There are four estimated tracks with two true tracks and the labels of estimated tracks are not correct from time step 47 on. For instance, the tracker assigns new labels from time step 56 and takes them as two new extended targets.

5.3. Mixture Partitioning. In this section, we present results that compare the performance of the ET-GMPHD filter

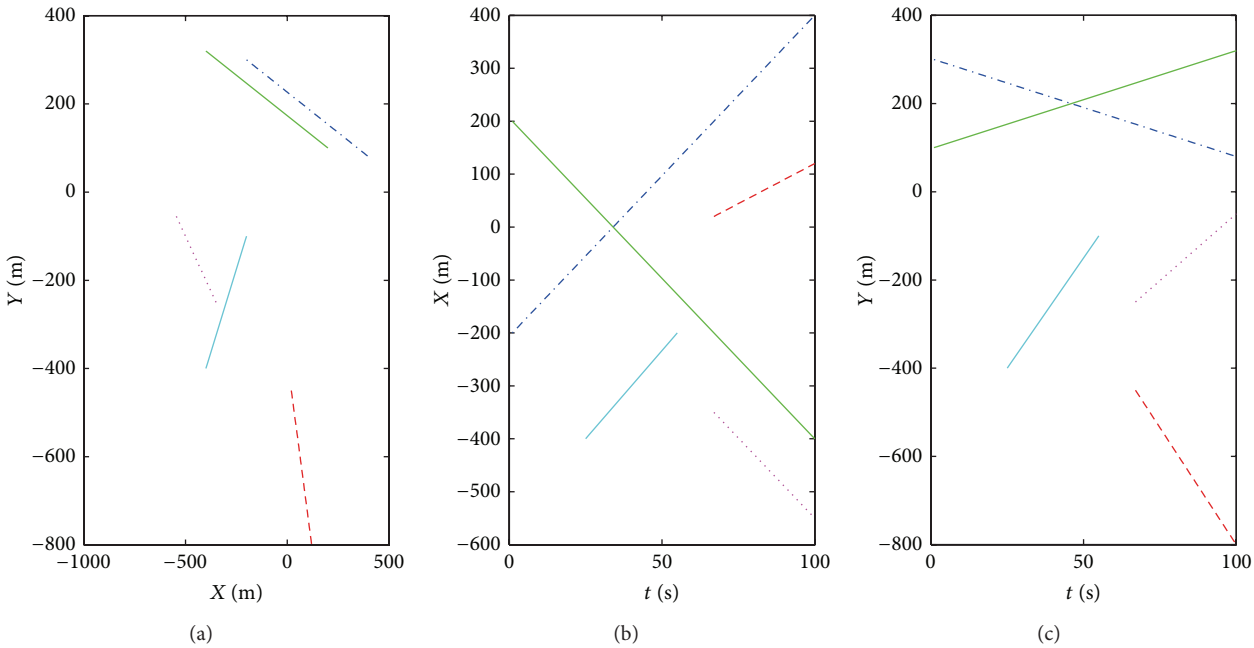

Figure 3: (a) Trajectories of extended targets for the third scenario. (b) The x positions changing with time. (c) The y positions changing with time.

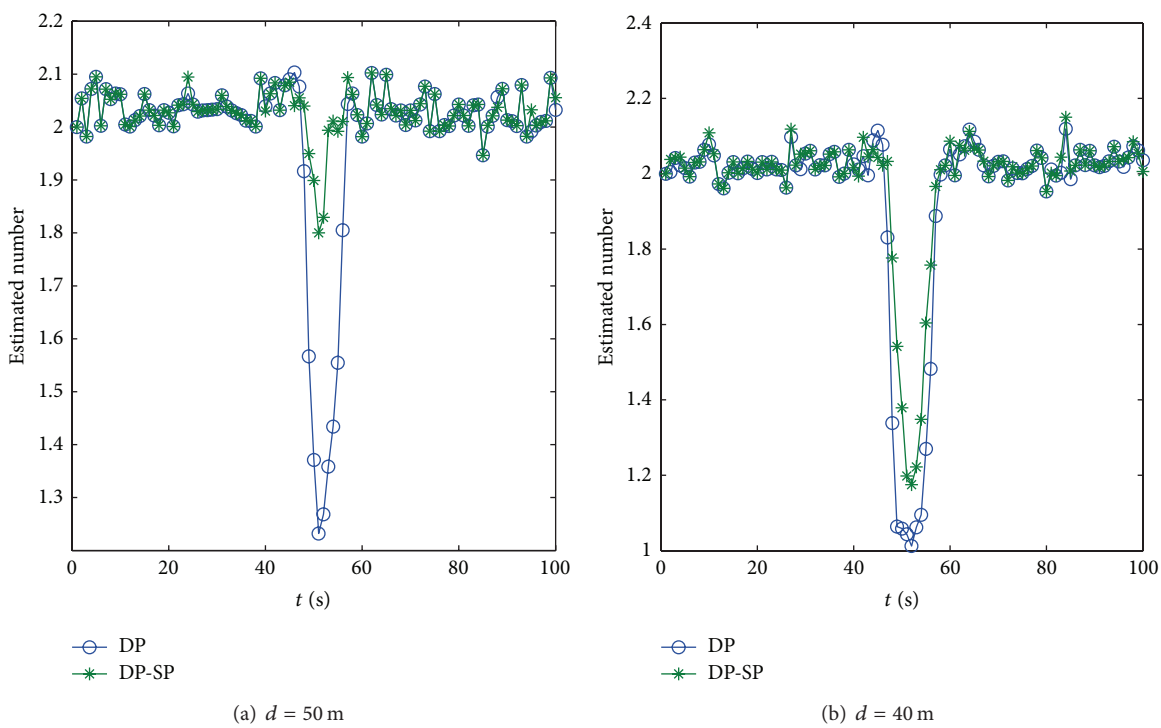

Figure 4: Estimated number of extended targets for two scenarios, compared with different partitioning methods (blue circle is only DP; green star shows DP-SP).

by using mixture partitioning and DP-SP. The estimated number of extended targets for the second scenario in Section 5.1 is obtained by averaging over 100 Monte Carlo runs and illustrated in Figure 6. As shown in Figure 6, the average estimated number of extended targets estimated by ET-GMPHD filter by using mixture partitioning is closer to the true number of targets.

5.4. Evaluation of the Proposed ET-GMPHD Tracker. In this section, simulation results are given to show the performance

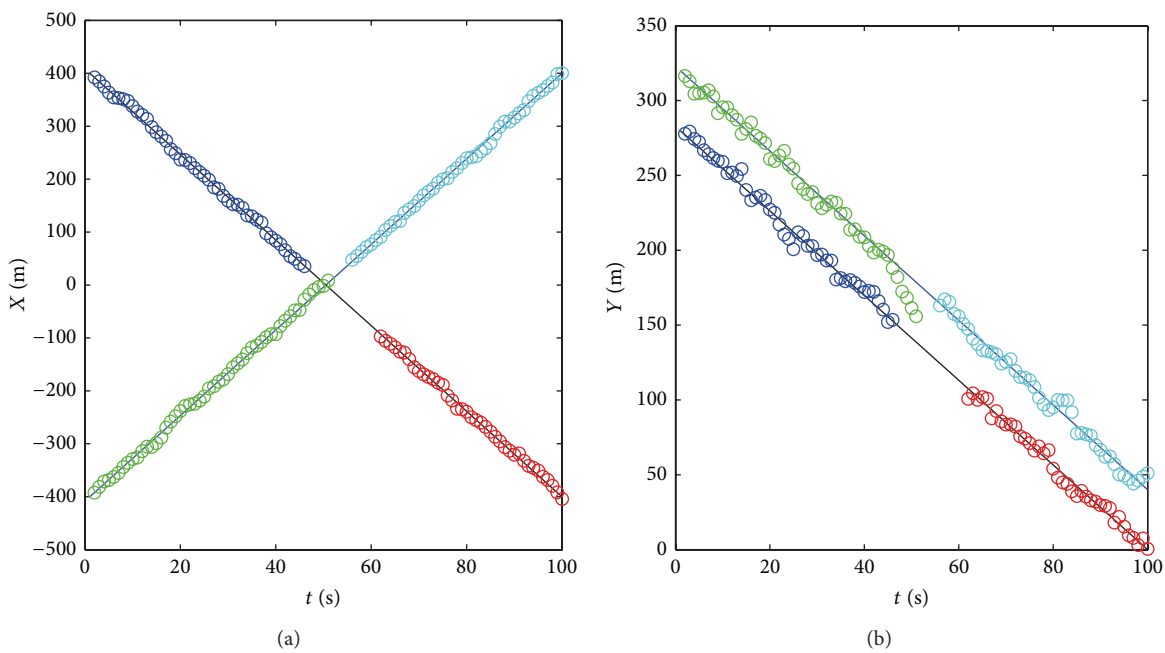

Figure 5: Tracks given by the tracker based on the original ET-GMPHD filter for the second scenario. (a) The x positions changing with time. (b) The y positions changing with time (lines are the true positions and circles are the estimated positions. Different color denotes different track).

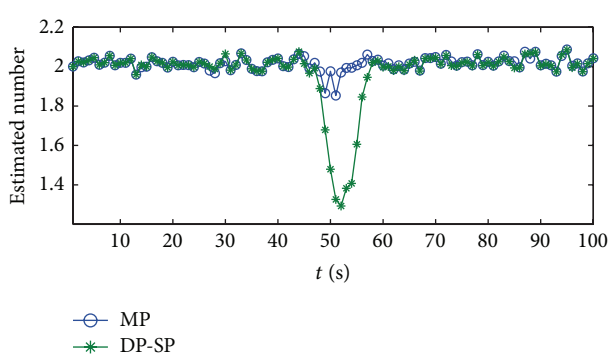

Figure 6: Estimated number of extended targets for the second scenario, compared with different partitioning methods (blue circle is mixture partitioning (MP); green star shows DP-SP).

of the proposed ET-GMPHD tracker and illustrate the effectiveness of the label evolution algorithm and the track management scheme. The last two scenarios in Section 5.1 are considered. The results of the proposed ET-GMPHD tracker for two scenarios are shown in Figures 7 and 8, which show that the proposed ET-GMPHD tracker could give good estimates of the true target trajectories and the estimates are almost free of false tracks.

The results in Figures 7 and 8 are acquired from only one simulation. To evaluate the proposed ET-GMPHD tracker, we choose the optimal subpattern assignment (OSPA) metric as the metric in this paper. Schuhmacher and Xia proposed firstly the OSPA metric in the point process [35] and demonstrated the OSPA metric in performance evaluation of multitarget filtering algorithms [36]. The OSPA metric was used in the multitarget tracking application by Ristic et al. [37]. It is an appropriate metric for multitarget tracking application: it incorporates both the cardinality error and the spatial distance of points and combines various aspects of tracking performance into a single metric. The brief introduction is given in the following.

Two track sets at t_k are represented by

$$\begin{aligned} \mathscr{A}_k &= \{(\ell_1, \mathbf{x}_{k,1}), \ldots, (\ell_m, \mathbf{x}_{k,m})\}, \\ \mathscr{B}_k &= \{(s_1, \mathbf{y}_{k,1}), \ldots, (s_n, \mathbf{y}_{k,n})\}, \end{aligned} \tag{38}$$

where m, n are the cardinalities of sets $\mathscr{A}_k$ and $\mathscr{B}_k$, respectively. For $m \le n$, the OSPA distance between $\mathscr{A}_k$ and $\mathscr{B}_k$ is defined as

$$D_{p,c}(\mathscr{A}_k, \mathscr{B}_k) = \left[\frac{1}{n}\left(\min_{\pi \in \Pi_n} \sum_{i=1}^{m} \left(d_c\left(\tilde{x}_{k,i}, \tilde{y}_{k,\pi(i)}\right)\right)^p + (n-m)\cdot c^p\right)\right]^{1/p}, \tag{39}$$

where $\tilde{x}_{k,i} = (\ell_i, \mathbf{x}_{k,i})$, $\tilde{y}_{k,\pi(i)} = (s_\pi(i), \mathbf{y}_{k,\pi(i)})$; $d_c(\tilde{x}, \tilde{y}) = \min(c, d(\tilde{x}, \tilde{y}))$ is the cutoff distance between two tracks at t_k, with $c > 0$ being the cutoff parameter; $d(\tilde{x}, \tilde{y})$ is the base distance between two tracks at t_k; Π_n represents the set of permutations of length m with elements taken from $\{1, 2, \ldots, n\}$; $1 \le p < \infty$ is the OSPA metric order parameter.

For the case $m > n$, the definition is $D_{p,c}(\mathscr{A}_k, \mathscr{B}_k) = D_{p,c}(\mathscr{B}_k, \mathscr{A}_k)$.

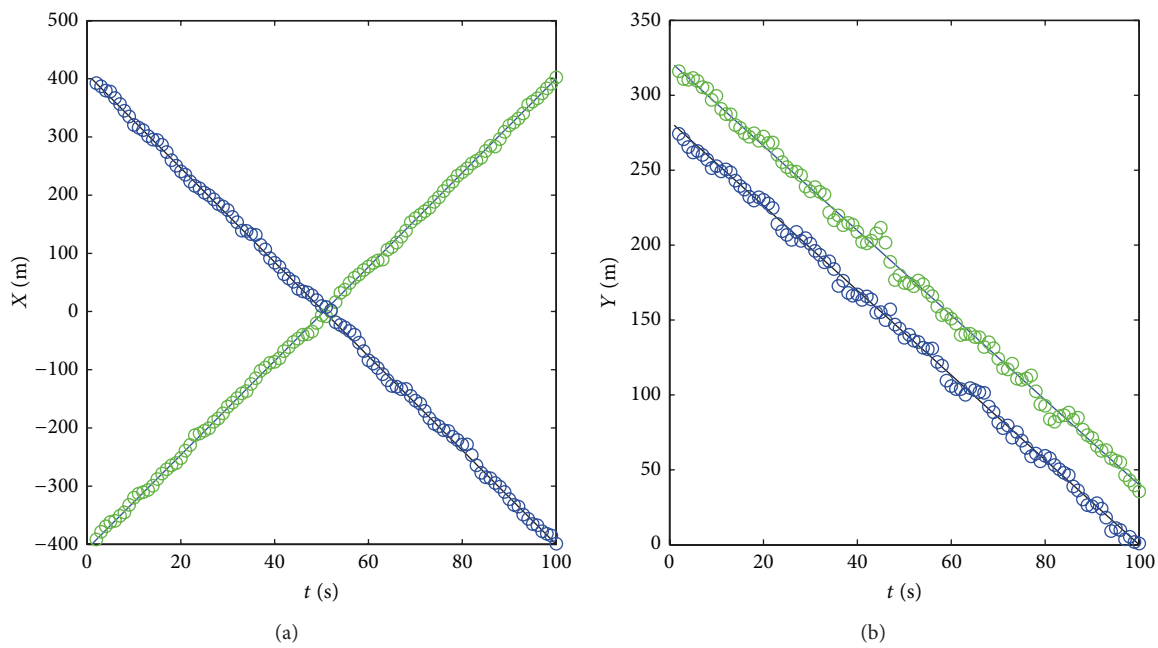

Figure 7: Tracks given by the proposed ET-GMPHD tracker for the second scenario. (a) The x positions changing with time. (b) The y positions changing with time (lines are the true positions and circles are the estimated positions. Different color denotes different track).

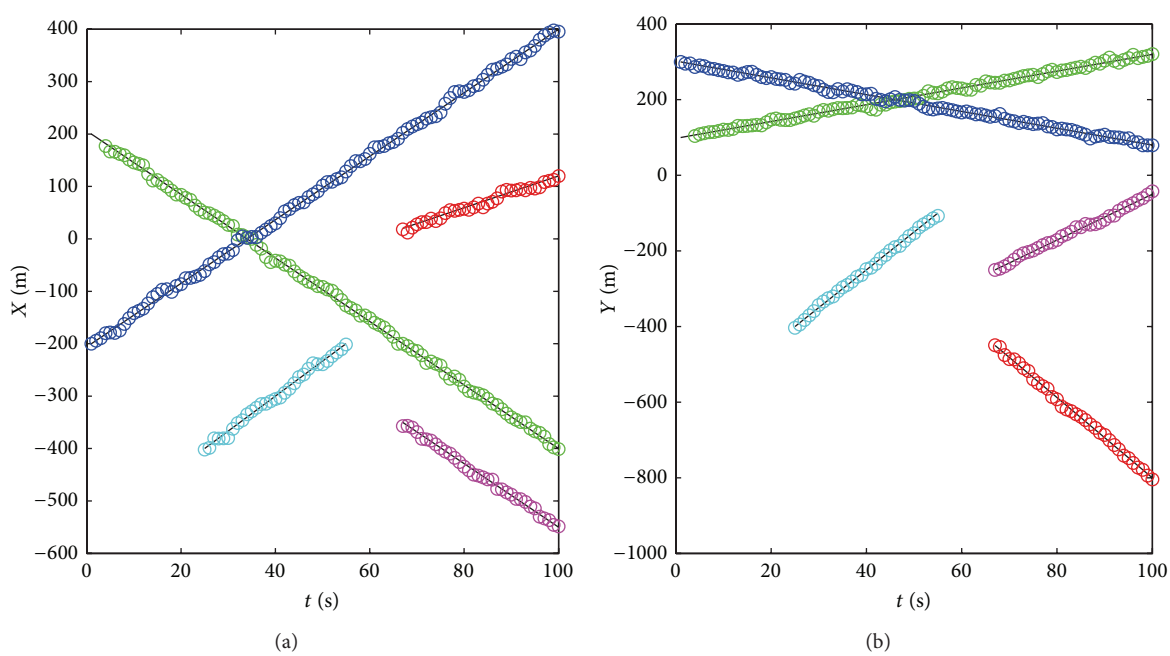

Figure 8: Tracks given by the proposed ET-GMPHD tracker for the third scenario. (a) The x positions changing with time. (b) The y positions changing with time (lines are the true positions and circles are the estimated positions. Different color denotes different track).

The base distance $d(\tilde{x}, \tilde{y})$ between tracks $\tilde{x} = (\ell, \mathbf{x})$ and $\tilde{y} = (s, \mathbf{y})$ is defined as

$$d(\tilde{x}, \tilde{y}) = \left(d(\mathbf{x}, \mathbf{y})^{p'} + d(\ell, s)^{p'}\right)^{1/p'}, \tag{40}$$

where $d(\mathbf{x}, \mathbf{y}) = \|x - y\|_{p'}$ is the localization base distance and $d(\ell, s) = \alpha\bar{\delta}(\ell, s)$ is the labeling error distance. $\bar{\delta}(i, j)$ is the complement of the Kronecker delta; that is, $\bar{\delta}(i, j) = 0$, if $i = j$ and $\bar{\delta}(i, j) = 1$, otherwise. Parameter $\alpha \in [0, c]$ controls the penalty assigned to the labeling error $d(\ell, s)$ interpreted relative to the localization distance $d(\mathbf{x}, \mathbf{y})$. The case $\alpha = 0$ assigns no penalty and $\alpha = c$ assigns the maximum penalty.

More details about the OSPA metric can be obtained in [37]. The choice of parameters c and p follows the guidelines

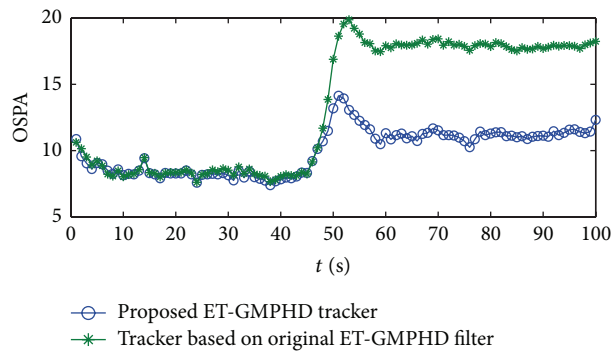

Figure 9: OSPA metric (blue circle is the proposed ET-GMPHD tracker; green star shows the tracker based on the original ET-GMPHD filter by using DP-SP).

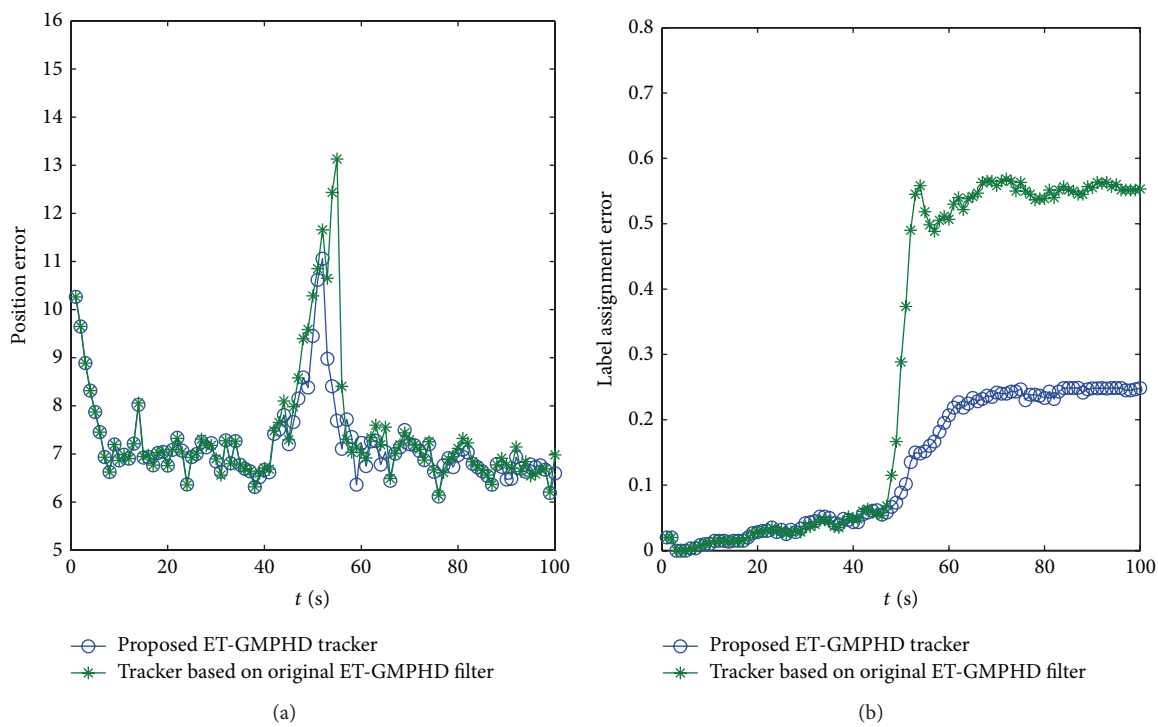

Figure 10: Position and label assignment error (blue circle is the proposed ET-GMPHD tracker; green star shows the tracker based on the original ET-GMPHD filter by using DP-SP).

in [36]. In this paper we set the parameters of the OSPA metric as follows: $c = 25\,\mathrm{m}$, $p = p' = 1$, and $\alpha = 25\,\mathrm{m}$. In Figures 9 and 10, the curves are obtained by averaging over 100 Monte Carlo runs.

To illustrate the performance of proposed ET-GMPHD tracker more intuitively, for $m \leq n$ we define the position error and label assignment error as follows:

$$e_p = \left[\frac{1}{m}\left(\min_{\pi\in\Pi_n}\sum_{i=1}^{m}\left(\min\left(d\left(x_{k,i}, y_{k,\pi(i)}\right), c\right)\right)^{p'}\right)\right]^{1/p'},$$
$$e_l = \frac{1}{n}\left(\min_{\pi\in\Pi_n}\sum_{i=1}^{m}\overline{\delta}\left(\ell_i, s_{\pi(i)}\right)\right), \tag{41}$$

where the distance $d(x, y)$ is the same with that in (40). The case $m > n$ is a trivial modification of (41). The OSPA, position error, and label assignment error curves for the first scenario are shown in Figures 9 and 10.

Figures 9 and 10 show that the proposed ET-GMPHD tracker significantly outperforms the tracker based on the original ET-GMPHD filter, and the proposed label processing and management scheme for tracker are reasonable. The position error of the proposed ET-GMPHD tracker is less than that of the tracker based on the original ET-GMPHD filter by using DP-SP in the situations, where the two extended targets are spatially close to each other. The main difference between two methods is the probability of error label assignment. With the target extension being a circle of 40 m radius and the measurements overlapping significantly at 20 m distance, the

probability of the incorrect label assignment of the proposed method is below 0.25, and the other is beyond 0.6.

6. Conclusion

This paper presents an ET-GMPHD filter-based multiextended-target tracker, which can give the tracks of extended targets according to their labels and the estimation of their states. The proposed ET-GMPHD tracker assigns labels to Gaussian terms and propagates these labels through time without affecting the ET-GMPHD recursion. In addition, a track management scheme for track initiation, track confirmation, track propagation, and track termination is proposed to obtain the tracks of individual extended targets. This paper also presents mixture partitioning algorithm to maintain separate track identities of the extended targets in close proximity and enhance the performance of the ET-GMPHD tracker.

In our future research, we will consider the shape of the extended target and the solution to track the nonlinear models and high maneuvering extended targets.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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