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Research Article

Block Hybrid Collocation Method with Application to Fourth Order Differential Equations

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The block hybrid collocation method with three off-step points is proposed for the direct solution of fourth order ordinary differential equations. The interpolation and collocation techniques are applied on basic polynomial to generate the main and additional methods. These methods are implemented in block form to obtain the approximation at seven points simultaneously. Numerical experiments are conducted to illustrate the efficiency of the method. The method is also applied to solve the fourth order problem from ship dynamics.

1. Introduction

Fourth order ordinary differential equations (ODEs) arise in several fields such as fluid dynamics (see [1]), beam theory (see [2, 3]), electric circuits (see [4]), ship dynamics (see [5–7]), and neural networks (see [8]). Therefore, many theoretical and numerical studies dealing with such equations have appeared in the literature.

Here, we consider general fourth order ordinary differential equations:

$$y^{(4)} = f\left(t, y, y', y'', y'''\right) \tag{1}$$

with the initial conditions

$$\begin{gathered} y(a) = y_0, \qquad y'(a) = y_0', \\ y''(a) = y_0'', \qquad y'''(a) = y_0''', \qquad t \in [a, b]. \end{gathered} \tag{2}$$

Conventionally, fourth order problems (1) are reduced to system of first order ODEs and solved with the methods available in the literature. Many investigators [2, 9, 10] remarked the drawback of this approach as it requires heavier computational work and longer execution time. Thus, the direct approach on higher order ODEs has attracted considerable attention.

Recent developments have led to the implementation of collocation method for the direct solution of fourth order ODEs (1). Awoyemi [9] proposed a multiderivative collocation method to obtain the approximation of y at t_{n+4}. Moreover, Kayode [11, 12] developed collocation methods for the approximation of y at t_{n+5} with the predictor of orders five and six, respectively. These schemes [9, 11, 12] are implemented in predictor-corrector mode with the employment of Taylor series expansions for the computation of starting values. Jator [2] remarked that the implementation of these schemes is more costly since the subroutines for incorporating the starting values lead to lengthy computational time. Thus, some attempts have been made on the self-starting collocation method which eliminates the requirement of either predictors or starting values from other methods. Jator [2] derived a collocation multistep method and used it to generate a new self-starting finite difference method. On the other hand, Olabode and Alabi [13] developed a self-starting direct block method for the approximation of y at t_{n+j}, $j = 1, 2, 3, 4$.

Here, we are going to derive a block hybrid collocation method for the direct solution of general fourth order ODEs (1). The method is extended from the line proposed by Jator [14] and Yap et al. [15]. We apply the interpolation and

collocation technique on basic polynomials to derive the main and additional methods which are combined and used as block hybrid collocation method. This method generates the approximation of y at four main points and three off-step points concurrently.

2. Derivation of Block Hybrid Collocation Methods

The hybrid collocation method that generates the approximations to the general fourth order ODEs (1) is defined as follows:

$$\sum_{j=0}^{k}\alpha_j y_{n+j} + \sum_{j=1}^{3}\alpha_{\nu_j} y_{n+\nu_j} = h^4\left(\sum_{j=0}^{k}\beta_j f_{n+j} + \sum_{j=1}^{3}\beta_{\nu_j} f_{n+\nu_j}\right). \tag{3}$$

We approximate the solution by considering the interpolating function

$$Y(t) = \sum_{j=0}^{r+s-1}\phi_j t^j, \tag{4}$$

where $t \in [a,b]$, ϕ_j are unknown coefficients to be determined, r is the number of interpolations for $4 \le r \le k$, and s is the number of distinct collocation points with $s > 0$. The continuous approximation is constructed by imposing the conditions as follows:

$$Y\left(t_{n+j}\right) = y_{n+j}, \quad j = 0,1,2,\ldots,r-1, \tag{5}$$

$$Y^{(4)}\left(t_{n+\mu}\right) = f_{n+\mu}, \quad \mu = \{j,\nu_1,\nu_2,\nu_3\}, \quad j = 0,1,2,\ldots,k, \tag{6}$$

where ν_1, ν_2, and ν_3 are not integers. By considering $r = 4$ and $s = 8$, we interpolate (5) at the points t_n, t_{n+1}, t_{n+2}, and t_{n+3} and collocate (6) at the points $t_n, t_{n+1/2}, t_{n+1}, t_{n+3/2}, t_{n+2}, t_{n+5/2}$, t_{n+3}, and t_{n+4}. This leads to a system of twelve equations which is solved by Mathematica. The values of ϕ_j are substituted into (4) to develop the multistep method:

$$Y(t) = \sum_{j=0}^{k}\alpha_j y_{n+j} + h^4\left(\sum_{j=0}^{k}\beta_j f_{n+j} + \sum_{j=1}^{3}\beta_{\nu_j} f_{n+\nu_j}\right), \tag{7}$$

where α_j, β_j, and β_{ν_j} are constant coefficients. Hence, the block hybrid collocation method can be derived as follows.

Main Method. Consider the following

$$\begin{aligned} y_{n+4} = {} & 4y_{n+3} - 6y_{n+2} + 4y_{n+1} - y_n + \frac{h^4}{15120} \\ & \times \left(19f_n + 1804f_{n+1} + 2560f_{n+3/2} + 6354f_{n+2}\right. \\ & \left. + 2560f_{n+5/2} + 1804f_{n+3} + 19f_{n+4}\right). \end{aligned} \tag{8}$$

Additional Method. Consider the following

$$\begin{aligned} y_{n+1/2} = {} & \frac{1}{16}y_{n+3} - \frac{5}{16}y_{n+2} + \frac{15}{16}y_{n+1} + \frac{5}{16}y_n + \frac{h^4}{7741440} \\ & \times \left(75f_n - 25032f_{n+1/2} - 122530f_{n+1}\right. \\ & - 107760f_{n+3/2} - 43080f_{n+2} - 3880f_{n+5/2} \\ & \left. - 198f_{n+3} + 5f_{n+4}\right), \\ y_{n+3/2} = {} & -\frac{1}{16}y_{n+3} + \frac{9}{16}y_{n+2} + \frac{9}{16}y_{n+1} - \frac{1}{16}y_n + \frac{h^4}{430080} \\ & \times \left(f_n + 260f_{n+1/2} + 2311f_{n+1} + 4936f_{n+3/2}\right. \\ & \left. + 2311f_{n+2} + 260f_{n+5/2} + f_{n+3}\right), \\ y_{n+5/2} = {} & \frac{5}{16}y_{n+3} + \frac{15}{16}y_{n+2} - \frac{5}{16}y_{n+1} + \frac{1}{16}y_n - \frac{h^4}{7741440} \\ & \times \left(23f_n + 4680f_{n+1/2} + 41330f_{n+1}\right. \\ & + 110000f_{n+3/2} + 120780f_{n+2} + 25832f_{n+5/2} \\ & \left. - 250f_{n+3} + 5f_{n+4}\right). \end{aligned} \tag{9}$$

The general fourth order differential equations involve the first, second, and third derivatives. In order to generate the formula for the derivatives, the values of ϕ_j are substituted into

$$\begin{aligned} Y'(t) &= \sum_{j=1}^{r+s-1} j\phi_j t^{j-1}, \\ Y''(t) &= \sum_{j=2}^{r+s-1} j(j-1)\phi_j t^{j-2}, \\ Y'''(t) &= \sum_{j=3}^{r+s-1} j(j-1)(j-2)\phi_j t^{j-3}. \end{aligned} \tag{10}$$

This is obtained by imposing that

$$Y'(t) = \frac{1}{h}\left(\sum_{j=0}^{k}\alpha'_j y_{n+j} + h^4 \times \left(\sum_{j=0}^{k}\beta'_j f_{n+j} + \sum_{j=1}^{3}\beta'_{\nu_j} f_{n+\nu_j}\right)\right), \tag{11}$$

$$Y''(t) = \frac{1}{h^2}\left(\sum_{j=0}^{k}\alpha''_j y_{n+j} + h^4 \times \left(\sum_{j=0}^{k}\beta''_j f_{n+j} + \sum_{j=1}^{3}\beta''_{\nu_j} f_{n+\nu_j}\right)\right), \tag{12}$$

Table 1: Coefficients α_i' and β_i' for the method (11) evaluated at $t_{n+i/2}$ for $i = 0, 1, \ldots, 6$ and t_{n+4}.

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	$f_{n+3/2}$	f_{n+2}	$f_{n+5/2}$	f_{n+3}	f_{n+4}
t_n	$-\frac{11}{6}$	3	$-\frac{3}{2}$	$\frac{1}{3}$	$-\frac{3397}{3326400}$	$-\frac{146}{3465}$	$-\frac{16201}{166320}$	$-\frac{116}{1485}$	$-\frac{3089}{110880}$	$-\frac{26}{7425}$	$\frac{1}{23760}$	$-\frac{1}{665280}$
$t_{n+1/2}$	$-\frac{23}{24}$	$\frac{7}{8}$	$\frac{1}{8}$	$-\frac{1}{24}$	$\frac{7}{12165120}$	$\frac{114727}{13305600}$	$\frac{636767}{31933440}$	$\frac{2461}{295680}$	$\frac{28903}{6082560}$	$-\frac{127}{1995840}$	$\frac{179}{1900800}$	$-\frac{593}{255467520}$
t_{n+1}	$-\frac{1}{3}$	$-\frac{1}{2}$	1	$-\frac{1}{6}$	$\frac{127}{9979200}$	$\frac{113}{34650}$	$\frac{13613}{498960}$	$\frac{5827}{155925}$	$\frac{503}{36960}$	$\frac{569}{311850}$	$-\frac{71}{2494800}$	$\frac{1}{1425600}$
$t_{n+3/2}$	$\frac{1}{24}$	$-\frac{9}{8}$	$\frac{9}{8}$	$-\frac{1}{24}$	$-\frac{197}{17031168}$	$-\frac{13}{42240}$	$-\frac{4231}{1064448}$	$-\frac{409}{1900800}$	$\frac{4079}{946176}$	$\frac{41}{266112}$	$\frac{481}{10644480}$	$-\frac{409}{425779200}$
t_{n+2}	$\frac{1}{6}$	-1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{13}{3326400}$	$-\frac{89}{51975}$	$-\frac{6913}{498960}$	$-\frac{214}{5775}$	$-\frac{9157}{332640}$	$-\frac{491}{155925}$	$-\frac{31}{831600}$	$\frac{1}{1425600}$
$t_{n+5/2}$	$\frac{1}{24}$	$-\frac{1}{8}$	$-\frac{7}{8}$	$\frac{23}{24}$	$-\frac{2359}{182476800}$	$-\frac{13}{42240}$	$-\frac{17971}{4561920}$	$-\frac{74749}{7983360}$	$-\frac{271477}{14192640}$	$-\frac{179503}{19958400}$	$\frac{23}{285120}$	$-\frac{593}{255467520}$
t_{n+3}	$-\frac{1}{3}$	$\frac{3}{2}$	-3	$\frac{11}{6}$	$\frac{1}{95040}$	$\frac{113}{34650}$	$\frac{4721}{166320}$	$\frac{23}{297}$	$\frac{10859}{110880}$	$\frac{871}{20790}$	$\frac{893}{831600}$	$-\frac{1}{665280}$
t_{n+4}	$-\frac{11}{6}$	7	$-\frac{19}{2}$	$\frac{13}{3}$	$\frac{2159}{285120}$	$-\frac{146}{3465}$	$\frac{36541}{99792}$	$\frac{2956}{155925}$	$\frac{3697}{3168}$	$\frac{2918}{31185}$	$\frac{33047}{71280}$	$\frac{85741}{9979200}$

Table 2: Coefficients α_i'' and β_i'' for the method (12) evaluated at $t_{n+i/2}$ for $i = 0, 1, \ldots, 6$ and t_{n+4}.

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	$f_{n+3/2}$	f_{n+2}	$f_{n+5/2}$	f_{n+3}	f_{n+4}
t_n	2	-5	4	-1	$\frac{1597}{100800}$	$\frac{169}{675}$	$\frac{4439}{15120}$	$\frac{446}{1575}$	$\frac{1601}{30240}$	$\frac{37}{1575}$	$-\frac{71}{25200}$	$\frac{23}{302400}$
$t_{n+1/2}$	$\frac{3}{2}$	$-\frac{7}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	$-\frac{1973}{3225600}$	$\frac{107}{19200}$	$\frac{1933}{15120}$	$\frac{65911}{604800}$	$\frac{14927}{322560}$	$\frac{437}{134400}$	$\frac{439}{1209600}$	$-\frac{13}{1382400}$
t_{n+1}	1	-2	1	0	$\frac{1}{12096}$	$-\frac{107}{9450}$	$-\frac{185}{3024}$	$-\frac{7}{675}$	$-\frac{29}{30240}$	$\frac{1}{1890}$	$-\frac{1}{8400}$	$\frac{1}{302400}$
$t_{n+3/2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{59}{2419200}$	$-\frac{5753}{1209600}$	$-\frac{4253}{96768}$	$-\frac{13411}{120960}$	$-\frac{4253}{96768}$	$-\frac{5753}{1209600}$	$-\frac{59}{2419200}$	0
t_{n+2}	0	1	-2	1	$-\frac{1}{302400}$	0	$\frac{1}{5040}$	$-\frac{8}{675}$	$-\frac{121}{2016}$	$-\frac{8}{675}$	$\frac{1}{5040}$	$-\frac{1}{302400}$
$t_{n+5/2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{7}{2}$	$\frac{3}{2}$	$\frac{109}{3225600}$	$\frac{5753}{1209600}$	$\frac{10399}{241920}$	$\frac{68459}{604800}$	$\frac{120527}{967680}$	$\frac{1223}{172800}$	$-\frac{569}{604800}$	$\frac{13}{1382400}$
t_{n+3}	-1	4	-5	2	$-\frac{47}{302400}$	$\frac{107}{9450}$	$\frac{401}{5040}$	$\frac{1177}{4725}$	$\frac{9683}{30240}$	$\frac{2251}{9450}$	$\frac{1399}{75600}$	$-\frac{23}{302400}$
t_{n+4}	-2	7	-8	3	$\frac{10247}{302400}$	$-\frac{169}{675}$	$\frac{1121}{1008}$	$-\frac{146}{105}$	$\frac{18653}{6048}$	$-\frac{1781}{1575}$	$\frac{106483}{75600}$	$\frac{143}{2880}$

$$Y'''(t) = \frac{1}{h^3}\left(\sum_{j=0}^{k}\alpha_j''' y_{n+j} + h^4 \times\left(\sum_{j=0}^{k}\beta_j''' f_{n+j} + \sum_{j=1}^{3}\beta_{\nu_j}''' f_{n+\nu_j}\right)\right). \tag{13}$$

The formula for the first, second, and third derivatives is depicted in Tables 1, 2, and 3, respectively.

3. Order and Stability Properties

Following the idea of Henrici [16] and Jator [2, 14], the linear difference operator associated with (3) is defined as

$$L[y(t);h] = \sum_{j=0}^{k}\left[\alpha_j y(t+jh) - h^4\beta_j y^{(4)}(t+jh)\right] + \sum_{j=1}^{3}\left[\alpha_{\nu_j} y(t+\nu_j h) - h^4\beta_{\nu_j} y^{(4)}(t+\nu_j h)\right], \tag{14}$$

Table 3: Coefficients α_i''' and β_i''' for the method (13) evaluated at $t_{n+i/2}$ for $i = 0, 1, \ldots, 6$ and t_{n+4}.

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	$f_{n+3/2}$	f_{n+2}	$f_{n+5/2}$	f_{n+3}	f_{n+4}
t_n	-1	3	-3	1	$-\frac{33569}{226800}$	$-\frac{2357}{3150}$	$-\frac{275}{4536}$	$-\frac{8747}{14175}$	$\frac{61}{360}$	$-\frac{3317}{28350}$	$\frac{2507}{113400}$	$-\frac{19}{32400}$
$t_{n+1/2}$	-1	3	-3	1	$\frac{139673}{29030400}$	$-\frac{35453}{201600}$	$-\frac{789907}{1451520}$	$-\frac{134153}{907200}$	$-\frac{9473}{64512}$	$\frac{26767}{1814400}$	$-\frac{5101}{1036800}$	$\frac{3601}{29030400}$
t_{n+1}	-1	3	-3	1	$-\frac{239}{226800}$	$\frac{73}{3150}$	$-\frac{739}{5670}$	$-\frac{4577}{14175}$	$-\frac{61}{1260}$	$-\frac{647}{28350}$	$\frac{34}{14175}$	$-\frac{13}{226800}$
$t_{n+3/2}$	-1	3	-3	1	$\frac{14393}{29030400}$	$\frac{1027}{201600}$	$\frac{160589}{1451520}$	$\frac{1201}{129600}$	$-\frac{40357}{322560}$	$\frac{2767}{1814400}$	$-\frac{14107}{7257600}$	$\frac{1201}{29030400}$
t_{n+2}	-1	3	-3	1	$-\frac{89}{226800}$	$\frac{43}{3150}$	$\frac{1553}{22680}$	$\frac{4213}{14175}$	$\frac{379}{2520}$	$-\frac{131}{4050}$	$\frac{347}{113400}$	$-\frac{13}{226800}$
$t_{n+5/2}$	-1	3	-3	1	$\frac{2399}{4147200}$	$\frac{1027}{201600}$	$\frac{30025}{290304}$	$\frac{184567}{907200}$	$\frac{161531}{322560}$	$\frac{355087}{1814400}$	$-\frac{66427}{7257600}$	$\frac{3601}{29030400}$
t_{n+3}	-1	3	-3	1	$-\frac{359}{226800}$	$\frac{73}{3150}$	$\frac{29}{810}$	$\frac{5023}{14175}$	$\frac{67}{252}$	$\frac{18553}{28350}$	$\frac{2389}{14175}$	$-\frac{19}{32400}$
t_{n+4}	-1	3	-3	1	$\frac{21631}{226800}$	$-\frac{2357}{3150}$	$\frac{61601}{22680}$	$-\frac{70187}{14175}$	$\frac{17131}{2520}$	$-\frac{126197}{28350}$	$\frac{45341}{16200}$	$\frac{55067}{226800}$

where $y(t)$ is an arbitrary function that is sufficiently differentiable. Expanding the test functions $y(t + jh)$ and $y^{(4)}(t + jh)$ about t and collecting the terms we obtain

$$L[y(x); h] = C_0 y(x) + C_1 h y'(x) + \cdots + C_q h^q y^{(q)}(x) + \cdots \tag{15}$$

whose coefficients C_q for $q = 0, 1, \ldots$ are constants and given as

$$\begin{aligned} C_0 &= \sum_{j=0}^{k} \alpha_j + \sum_{j=1}^{3} \alpha_{\nu_j}, \\ C_1 &= \sum_{j=1}^{k} j\alpha_j + \sum_{j=1}^{3} \nu_j \alpha_{\nu_j}, \\ &\vdots \\ C_q &= \frac{1}{q!}\left[\sum_{j=1}^{k} j^q \alpha_j + \sum_{j=1}^{3} \nu_j^q \alpha_{\nu_j} - q(q-1)\right. \\ &\qquad \left.\times (q-2)(q-3)\left(\sum_{j=1}^{k} j^{q-4}\beta_j + \sum_{j=1}^{3} \nu_j^{q-4}\beta_{\nu_j}\right)\right]. \end{aligned} \tag{16}$$

According to Jator [2], the linear multistep method is said to be of order p if

$$C_0 = C_1 = \cdots = C_{p+3} = 0, \qquad C_{p+4} \neq 0. \tag{17}$$

The main method (8) and the additional methods (9) are the order eight methods with the error constants; C_{12} are $-1/207360$, $-13/679477248$, $-1/7927234560$, and $29/4756340736$, respectively. With the order $p > 1$, we stipulate the consistency of the method (see [2, 16]).

In the sense of Jator [2], the hybrid methods (8)-(9) are normalized in block form to analyze the zero stability. The first characteristic polynomial is defined as

$$\rho(z) = \operatorname{Det}\left[zA^{(0)} - A^{(1)}\right] = z^6(z-1) \tag{18}$$

with

$$A^{(0)} = 7 \times 7 \text{ Identity matrix},$$

$$A^{(1)} = \begin{bmatrix} 0 & -\frac{5}{4} & 0 & -\frac{5}{2} & 0 & -\frac{10}{3} & \frac{5}{16} \\ 0 & \frac{22}{3} & 0 & \frac{44}{3} & 0 & \frac{176}{9} & -\frac{11}{6} \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{2}{3} & -\frac{1}{16} \\ 0 & -8 & 0 & -16 & 0 & -\frac{64}{3} & 2 \\ 0 & -\frac{1}{4} & 0 & -\frac{1}{2} & 0 & -\frac{2}{3} & \frac{1}{16} \\ 0 & 4 & 0 & 8 & 0 & \frac{32}{3} & -1 \\ 0 & 4 & 0 & 8 & 0 & \frac{32}{3} & -1 \end{bmatrix}. \tag{19}$$

Since the roots of (18) satisfy $|z_i| \leq 1$ for $i = 1, 2, \ldots, 7$, the method is zero stable.

4. Numerical Experiment

The following problems are solved numerically to illustrate the efficiency of the block hybrid collocation method.

Table 4: Numerical results for Problem 1.

h	Method	Absolute error at $t = 2$
0.1	BHCM4	1.74 (−8)
	Adams	2.11 (−3)
	Jator	1.26 (−4)
0.05	BHCM4	8.45 (−11)
	Adams	5.37 (−4)
	Jator	1.91 (−6)
0.025	BHCM4	3.69 (−13)
	Adams	5.09 (−5)
	Jator	2.96 (−8)
0.02	BHCM4	7.11 (−14)
	Adams	2.25 (−5)
	Jator	8.65 (−9)

Problem 1. Consider the linear fourth order problem (see [2]):

$$\begin{aligned} y^{(4)} &= y''' + y'' + y' + 2y, \quad 0 \le t \le 2, \\ y(0) &= y'(0) = y''(0) = 0, \qquad y'''(0) = 30, \end{aligned} \tag{20}$$

and theoretical solution: $y(t) = 2e^{2t} - 5e^{-t} + 3\cos t - 9\sin t$.

Problem 2. Consider the nonlinear fourth order problem (see [9]):

$$\begin{aligned} y^{(4)} &= (y')^2 - yy''' - 4t^2 + e^t\left(1 + t^2 - 4t\right), \quad 0 \le t \le 1, \\ y(0) &= y'(0) = 1, \quad y''(0) = 3, \quad y'''(0) = 1, \end{aligned} \tag{21}$$

and theoretical solution: $y(t) = t^2 + e^t$.

The block hybrid collocation method is implemented together with the *Mathematica* built-in packages, namely, `Solve` and `FindRoot` for the solution of linear and nonlinear problems, respectively.

The performance comparison between block hybrid collocation method with the existing methods [2, 9] and the Adams Bashforth-Adams Moulton method is presented in Tables 4 and 5. The following notations are used in the tables:

h: step size;

BHCM4: block hybrid collocation method;

Adams: Adams Bashforth-Adams Moulton method;

Awoyemi: multiderivative collocation method in Awoyemi [9];

Jator: finite difference method in Jator [2].

Tables 4 and 5 show the superiority of BHCM4 in terms of accuracy over the existing Adams method, Jator finite difference method [2], and Awoyemi multiderivative collocation method [9].

Table 5: Numerical results for Problem 2.

h	Method	Absolute error at $t = 1$
0.2	BHCM4	2.38 (−12)
	Adams	5.01 (−7)
	Awoyemi	5.84 (−4)
0.1	BHCM4	1.95 (−14)
	Adams	2.44 (−6)
	Awoyemi	9.26 (−5)

Table 6: Performance comparison for Wu equation with $\epsilon = 0$.

h	Method	Absolute error at $t = 15$
0.25	BHCM4	5.2 (−7)
	Adams	4.9 (−3)
	Twizell	1.9 (−4)
0.1	BHCM4	2.8 (−10)
	Adams	8.4 (−5)
	Cortell	3.7 (−5)

5. Application to Problem from Ship Dynamics [5–7]

The proposed method is also applied to solve a physical problem from ship dynamics. As stated by Wu et al. [5], when a sinusoidal wave of frequency Ω passes along a ship or offshore structure, the resultant fluid actions vary with time t. In a particular case study by Wu et al. [5], the fourth order problem is defined as

$$y^{(4)} + 3y'' + y(2 + \epsilon\cos(\Omega t)) = 0, \quad t > 0, \tag{22}$$

which is subjected to the following initial conditions:

$$y(0) = 1, \qquad y'(0) = y''(0) = y'''(0) = 0, \tag{23}$$

where $\epsilon = 0$ for the existence of the theoretical solution, $y(t) = 2\cos t - \cos(t\sqrt{2})$. The theoretical solution is undefined when $\epsilon \neq 0$ (see [6]).

In the literature, some numerical methods for solving fourth order ODEs have been extended to solve the problem from ship dynamics. Numerical investigation was presented in Twizell [6] and Cortell [7] concerning the fourth order ODEs (22) for the cases $\epsilon = 0$ and $\epsilon = 1$ with $\Omega = 0.25(\sqrt{2}-1)$. Instead of solving the fourth order ODEs directly, Twizell [6] and Cortell [7] considered the conventional approach of reduction to system of first order ODEs. Twizell [6] developed a family of numerical methods with the global extrapolation to increase the order of the methods. On the other hand, Cortell [7] proposed the extension of the classical Runge-Kutta method.

Table 6 shows the comparison in terms of accuracy for y at the end point $t = 15$. BHCM4 manages to achieve better accuracy compared to Adams Bashforth-Adams Moulton method, Twizell [6], and Cortell [7] when $h = 0.25$ and $h = 0.1$, respectively.

Figure 1 depicts the numerical solution for Wu equation (22) with $\epsilon = 1$ and $\Omega = 0.25(\sqrt{2} - 1)$ in the interval

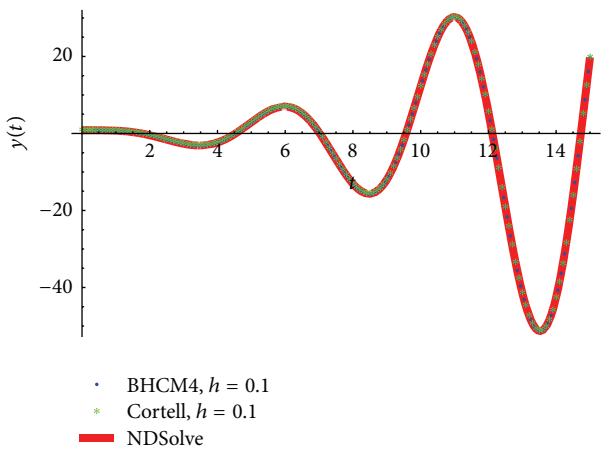

Figure 1: Response curve for Wu equation with $\epsilon = 1$, $\Omega = 0.25(\sqrt{2} - 1)$.

[0, 15]. The solutions obtained by BHCM4 are in agreement with the observation of Cortell [7] and *Mathematica* built-in package `NDSolve`.

6. Conclusion

As indicated in the numerical results, the block hybrid collocation method has significant improvement over the existing methods. Furthermore, it is applicable for the solution of physical problem from ship dynamics.

As a conclusion, the block hybrid collocation method is proposed for the direct solution of general fourth order ODEs whereby it is implemented as self-starting method that generates the solution of y at four main points and three off-step points concurrently.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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