Research Article

An Analytical Approach for Deformation Shapes of a Cylindrical Shell with Internal Medium Subjected to Lateral Contact Explosive Loading

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Received 27 February 2015; Revised 18 June 2015; Accepted 24 June 2015

Academic Editor: Yuri Petryna

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An experimental investigation on deformation shape of a cylindrical shell with internal medium subjected to lateral contact explosion was carried out briefly. Deformation shapes at different covered width of lateral explosive were recovered experimentally. Based on the experimental results, a corresponding analytical approach has been undertaken with rigid plastic hinge theory. In the analytical model, the cylindrical shell is divided into end-to-end rigid square bars. Deformation process of the cylindrical shell is described by using the translations and rotations of all rigid square bars. Expressions of the spring force, buckling moment, and deflection angle between adjacent rigid square bars are conducted theoretically. Given the structure parameters of the cylinder and the type of the lateral explosive charge, deformation processes and shapes are reported and discussed using the analytical approach. A good agreement has been obtained between calculated and experimental results, and thus the analytical approach can be considered as a valuable tool in understanding the deformation mechanism and predicting the deformation shapes of the cylindrical shell with internal medium subjected to lateral contact explosion. Finally, parametric studies are carried out to analyze the effects of deformation shape, including the covered width of the lateral explosive, explosive charge material, and distribution of initial velocity.

1. Introduction

Cylindrical shells are used in a wide variety of engineering applications, from the containment pressure vessels of nuclear reactors to the bracing elements of aerospace structures. Such structures may be subjected to a wide variety of short duration transient loads throughout the course of their working life, such as air blasts, underwater explosions, and high velocity impact. Accurate prediction of the dynamic plastic deformation and rupture of the cylindrical shell subjected to high intensity transient loading is of great importance in many industrial applications.

Early researches on the dynamic buckling and failure of cylinders were restricted to axisymmetric external radial pulse loading [1] and axial impact loading. However, the corresponding analysis, though presented in an elegant analytical form, is of limited applicability because axisymmetric dynamic loading seldom occurs in practice. In real world situations loading is usually applied to one side of the cylinder and is characterized by various degrees of locality. It may consist of a projectile, missile or mass impact, standoff explosion described by a pressure pulse, or contact explosion often approximated as an ideal impulsive loading.

Depending on the load intensity and the special distribution of contact pressures, various forms of damage may result ranging from large amplitude lateral deflections to punch-through penetration, fracture initiation at the base plate, progression of tearing fracture, and finally massive structural damage. Yakupov [2, 3] studied the dynamic response of the cylindrical shell subjected to a planar plastic shock wave with rigid plastic hinge theory and presented the residual deformation of the cylindrical shell as a function of a planar wave pressure. Gefken et al. [4] extended the earlier analysis by Lindberg and Florence to one-side inward radial pressure that varied as the cosine of the angular position around the shell and was uniform along length, to identify the structural
response modes of thin cylindrical shell, with and without internal pressure, subjected to external radial impulsive loads. For unpressurized shells the response modes consisted of dynamic pulse buckling followed by large inward deflections of the loaded surface. In shell with high internal pressure, these response modes were followed by an outward motion driven by the internal pressure. Fatt and Wierzbicki [5] and Wierzbicki and Fatt [6] investigated the large amplitude transient response of plastic cylindrical shells using a string-on-foundation model, and the model incorporated two main load-resistance mechanisms in the shell: stretching in the longitudinal direction and bending in the circumferential direction. Jiang and Olson [7] presented a numerical model for large deflection, elastic-plastic analysis of the cylindrical shell structures under air blast loading condition based on a modal analysis approach. Given the structural parameters of the ring and the type of the explosive charge, deformation processes and shapes are reported using the analytical approach. A good agreement has been obtained between calculated and experimental results. Finally, parametric studies are carried out to analyze the effects of deformation shapes, depending on the covered width of the lateral explosive, explosive charge material, and distribution of initial velocity.

2. Experimental Procedure and Results

2.1. Experimental Procedure. In order to investigate the dynamic response of the cylindrical shell with internal medium loaded by lateral contact explosion loading, several experiments have been conducted, and the experimental results are presented and discussed in detail in this paper. Based on the experiments, a corresponding analytical approach was conducted with rigid plastic hinge theory. Deformation processes of cylindrical shells are described using the translations and rotations of all rigid square bars. In this study, an infinitely long cylindrical shell filled with medium subject to lateral contact explosion is performed. Because the cylindrical shell has the unique deformation shape in the symmetric axis direction, we take a ring representing the cylinder. Due to solving the inertia moment of the ring, a unit height ring represents deformation of the infinite cylinder. Because the ring has the same value of thickness and height in the radial and axial direction, the cross section of the ring in the circumferential direction is square, so we call it “square bar.”

Given the structural parameters of the ring and the type of the explosive charge, deformation processes and shapes are reported using the analytical approach. A good agreement has been obtained between calculated and experimental results. Finally, parametric studies are carried out to analyze the effects of deformation shapes, depending on the covered width of the lateral explosive, explosive charge material, and distribution of initial velocity.
Table 1: Material properties of 1020 Steel, LY12 Aluminium, and sand.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Yield stress (MPa)</th>
<th>Young's modulus (GPa)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 Steel</td>
<td>7.85</td>
<td>275</td>
<td>210</td>
<td>0.29</td>
</tr>
<tr>
<td>LY12 Aluminium</td>
<td>2.78</td>
<td>230</td>
<td>70</td>
<td>0.29</td>
</tr>
<tr>
<td>Sand</td>
<td>1.75</td>
<td>4.23</td>
<td>0.01</td>
<td>0.26</td>
</tr>
</tbody>
</table>

2.2. Experimental Results. Deformation shapes of the cylindrical shell with internal medium at three different covered widths of the lateral explosive charge were recovered. Deformation shapes after tests are shown in Figure 3. From the deformation shapes recovered, the angle of the lateral charge has a significant effect on the deformation shape, and the deformation shapes are concave, linear, and convex when the covered widths of lateral charges are 45°, 90°, and 135°, respectively.

3. Analytical Model

3.1. Basic Assumptions. Deformation processes of the cylindrical shell with internal medium subjected to lateral contact explosion are a high nonlinear problem. Due to complexities introduced by unsymmetric loading, large displacements, and rotations of the cylinder amplified by material non-linearities, the problem does not lend itself easily to an analytical treatment. However, by introducing a suitable set of assumptions, a simple and realistic model can be established to describe the deformation processes of the cylindrical shell with internal medium.

Basic assumptions are as follows: (1) the cylindrical shell is infinitely long, and the axial thickness selected is equal to the thickness of cylinder; (2) the cylinder is divided into end-to-end rigid square bars along the circumferential direction; (3) the square bars close to the lateral explosive charges have an instantaneous velocity pointing to the centre of the cylinder.

During deformation processes of the cylindrical shell, some parameters of square bars may vary at different moments, such as translational displacements, translational velocities, rotational displacements, rotational angles, and area surrounded by square bars, which affect the value of the spring force, the bending moment, and the deflection angle intensively.

3.2. Analytical Approach. Based on the above-mentioned assumptions, the analytical model is established, shown in Figure 4, where the origins for x and y are given, and the range x is from 0 to 2r_i, and the range y is from -r_i to r_i. The cylindrical shell is divided into end-to-end rigid square bars along the circumferential direction. The arc length l of each square bar is 2πr_i/N (N is the total number of square bars). The relationships between two adjacent square bars are established by using a spring force, a bending moment, and a deflection angle.

The spring force between adjacent square bars assumed as the perfect elastic-plastic is described by a changeable spring force (Figure 5). The spring force between adjacent square bars is expressed by the equation

\[ F = \begin{cases} E \cdot s \cdot t^2 & s \leq s_0 \\ \sigma_y \cdot t^2 & s > s_0 \end{cases} \]  

where \( E, \sigma_y, \) and \( s_0 \) are Young’s modulus, the yield stress, and the elastic limit displacement of bar, respectively. \( F \) is the spring force between adjacent square bars, and \( s \) is the relative displacement between the end of the current bar and the head of the next bar.

The relative displacements between the end of the current bar and the head of the next bar are obtained by using the end displacement of the current square bar subtracting the head displacement of the next square bar. The relative displacements of adjacent square bars are described by the equation

\[ s_x(i) = \left[ dt \cdot v_x(i) - \frac{1}{2} \cdot \theta(i) \cdot \sin \alpha(i) \right] - \left[ dt \cdot v_x(i + 1) - \frac{1}{2} \cdot \theta(i + 1) \cdot \sin \alpha(i + 1) \right], \]

\[ s_y(i) = \left[ dt \cdot v_y(i) + \frac{1}{2} \cdot \theta(i) \cdot \cos \alpha(i) \right] - \left[ dt \cdot v_y(i + 1) + \frac{1}{2} \cdot \theta(i + 1) \cdot \cos \alpha(i + 1) \right], \]

where \( s_x(i) \) and \( s_y(i) \) are relative displacements between two adjacent square bars in the x-axis and y-axis directions,
respectively. $dt$ is a time step. $v_x(i)$ and $v_y(i)$ are translational velocities of the current square at the $x$-axis and $y$-axis directions, respectively. $v_x(i+1)$ and $v_y(i+1)$ are translational velocities of the next square at the $x$-axis and $y$-axis directions, respectively. $\theta(i)$ is the relative deflection angle between two adjacent square bars. The spring force between two adjacent square bars is calculated by the following equation when $s_x(i)$ and $s_y(i)$ are less than $s_0$:

$$F_x(i) = E \cdot t^2 \cdot s_x(i),$$

$$F_y(i) = E \cdot t^2 \cdot s_y(i),$$

where $F_x(i)$ and $F_y(i)$ are spring forces between two adjacent square bars in the $x$-axis and $y$-axis directions, respectively.

Bending moment and corresponding deflection angle between two adjacent square bars are shown in Figure 6. Based on the rigid plastic hinge theory, the plastic hinges are achieved when the bending moment of the square bar reaches the plastic ultimate bending moment. During the deformation processes of the cylindrical shell, the relationships between the bending moment $M$ and the deflection angle $\theta$ are assumed to be linear when the bending moment is less than the plastic ultimate bending moment $M_p$. Meanwhile, the deflection $\theta$ is set to the plastic limit deflection angle $\theta_p$ when the bending moment is greater than or equal to the plastic ultimate bending moment.

Figure 2: Assembly schematics of experimental setups: (a) section drawing and (b) profile drawing.

Figure 3: Experimental result of deformation shapes after tests at three different covered widths of the lateral explosive charge.
The internal medium is compressed because of the shock pressure and volume is assumed to be linear:

\[ p = \frac{\eta (dV - j_0 \cdot V_0)}{(1 - j_0) \cdot V_0}, \]  

where \( p \) is the shock pressure; \( \eta \) is a scale factor; \( V_0 \) and \( dV \) are the initial volume and the compression volume of internal medium, respectively; \( j_0 \) is the compression ratio of medium.

By calculating the spring forces between adjacent square bars and the resistance forces generated by the internal medium, translational accelerations of all square bars are obtained at corresponding time. By calculating the bending moments of all square bars, the rotational accelerations are obtained at corresponding time. The translational and rotational accelerations of square bars are described by

\[ \ddot{\mathbf{a}} (i) = \frac{\ddot{\mathbf{F}} (i) + \ddot{\mathbf{F}} (i - 1) + \ddot{\mathbf{F}} (i)}{m}, \]

where \( \ddot{\mathbf{a}}(i) \) and \( \ddot{\mathbf{a}}(i) \) are the translational and rotational accelerations of the current square bar, respectively.

Utilizing (8), the translational and rotational accelerations are calculated, which are the initial conditions at the next time step.

3.3. Initial Conditions. Initial translational acceleration, rotational acceleration, and velocities of all square bars are set to 0, and the distributions of initial velocities are as follows:

\[ \mathbf{v} = \begin{cases} \mathbf{v}_0 \cos \alpha, \mathbf{v}_0 \sin \alpha & -\frac{\varphi}{2} \leq \alpha \leq \frac{\varphi}{2} \\ 0 & \alpha < -\frac{\varphi}{2}, \alpha > \frac{\varphi}{2} \end{cases}, \]

where \( \varphi \) is the half angle covered by the lateral explosive charge, \( \alpha \) is the angle between the current square bar and \( x \)-axis, and \( \mathbf{v}_0 \) is the initial velocity of cylindrical shell close to the lateral explosive charge.

According to the Gurney equations on contact explosion, the ring velocity can be obtained by using the equation [17]

\[ v_0 = \sqrt{2E_\gamma \left( \frac{3}{1 + 5M_f/C_e + 4 (M_f/C_e)^2} \right)}, \]

where \( v_0 \) is the ring velocity (m/s); \( \sqrt{2E_\gamma} \) is the Gurney energy unit mass (m/s); \( C_e \) is the mass of lateral explosive charge (kg); \( M_f \) is the mass of the cylindrical shell close to the lateral charge (kg). The Gurney equation is a classical method to solve the metal velocity in recent decades, the error between the experimental and calculated results is about 5%, and its accuracy is acceptable for our work.
4. Calculated Results

According to specific structure parameters and the type of the explosive charge, the velocity of the ring can be obtained with (8). It is assumed that the ring, made of 1020 Steel, has a thickness of 2 mm, and the explosive, made of DL103-80, has the thickness of 5 mm. Based on detonation parameters of DL103-80, substituting these parameters into the Gurney equation, the velocity is approximately 200 m/s. If the explosive DL103-80 is replaced with RDX or HMX, the velocity is greater, and the different initial velocity distribution can be obtained by varying the explosive charge material and the explosive mass.

Based on the analytical approach, calculated results are reported and discussed. Deformation processes of the cylindrical shell consist of two stages: stage I, velocities of the cylindrical shell obtained by the lateral contact explosion loading, and stage II, interactions between the cylindrical shell and internal medium. Figure 8 illustrates distribution of positions and velocities of the cylindrical shell at five different moments under the covered width 45° and the initial velocity of the cylindrical shell 200 m/s, respectively. Figures 9 and 10 show distributions of positions and velocities of the cylindrical shell at five classical times under the covered widths 90° and 135°, respectively. The ultimate translational velocities of the cylindrical shell are 6.1 m/s, 7.3 m/s, and 10.9 m/s with the covered widths 45°, 90°, and 135°, respectively.

From the results of deformation shapes, a good agreement has been obtained between calculated and experimental results, and thus the analytical approach can be considered as...
5. Parametric Studies

5.1. Effect of Covered Width of Lateral Explosive. From the calculated and experimental results, it is obvious that the covered width of the lateral explosive charge is a key factor to the deformation shapes. Deformation shapes of various covered widths of the lateral charge are shown in Figure 11, where initial velocity of the ring equals 200 m/s. Various deformation shapes can be achieved by changing the width of lateral charge.

5.2. Effect of Lateral Explosive Materials. In order to investigate the effect of lateral charge, a series of calculated results are obtained by adjusting various initial velocities

a valuable tool in understanding the deformation mechanism and predicting the deformation shape of the cylindrical shell under lateral contact explosion loading.

5. Parametric Studies

Deformation shapes of the ring have a significant relationship with the covered width of lateral explosive, explosive materials, and initial velocities distribution. In order to better understand the deformation mechanism, parametric studies are carried out for the deformation shapes and corresponding results were discussed.
of the cylindrical shell, because higher initial velocities of cylindrical shell represent greater power of charge.

Figure 12 shows the deformation shapes of the cylindrical shell at various initial velocities with the covered width 60°, in which velocities are 50 m/s, 100 m/s, 150 m/s, 200 m/s, 250 m/s, and 300 m/s, respectively. The compression capacity of the sand medium increases with initial velocities; while initial velocities reach a certain extent, there is little change in deformation shapes.

5.3. Effect of Initial Velocities Distribution. In general, the lateral explosive charge has a uniform thickness in the circumferential direction within the central angle, and the ring close to the lateral explosive has the same velocity value. The velocity values varies with thickness of the circumferential explosive. The simplest assumption is that the thickness of the lateral explosive is a linear distribution from one side to
The deformation shapes of the cylindrical shell with internal medium subjected to lateral contact explosion. Finally, a parametric study is carried out to analyze the effects of deformation shapes, depending on the covered width of the lateral explosive, explosive materials, and distribution of initial velocities. Therefore, an optimal deformation shape can be achieved by adjusting the covered width of lateral and initial velocities distribution.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment
The authors wish to acknowledge, with thanks, the financial support from the China National Natural Science Funding under Grants nos. 11202237 and 11132012.

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