Research Article

Finite-Time H_∞ Filtering for Singular Stochastic Markovian Jump Systems with Time-Varying Delays

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The issue of finite-time H_∞ filtering for singular stochastic Markovian jump systems with time-varying delays is concerned in this paper. H_∞ filtering is designed for underlying closed-loop singular Markovian jump system and system state does not exceed a given bound over some finite-time interval. Considering the full information of underlying Markov process, sufficient conditions are obtained to guarantee that the described system is finite-time stability and H_∞ filtering finite-time boundedness. By establishing the results of stochastic character and finite-time boundedness, the closed-loop singular Markovian jump system trajectory stays within the given bound. At last, a numerical example is supplied to show the efficiency of the proposed method.

1. Introduction

As the fundamental problem in the field of control application and output measurement is used, the H_∞ filtering problem has been attracting much attention [1, 2]. Among the existing robust control approaches, the traditional Kalman filtering is a well-known effective way to deal with the state estimation problems, which requires the exact knowledge of the statistics of system model and external noise signal. The disturbances of stationary Gaussian noises with known statistics and system model are exactly known under the consideration. Over the past two decades, because of their applications in a variety of areas, the H_∞ filtering problems have been widely studied in practical engineering. As mentioned above, H_∞ filtering technique is employed in most applications to describe the input or output behavior of the controlled system. It is important to investigate H_∞ filtering problem, and many results have been established [3–8].

On the other hand, as an important kind of hybrid systems, Markovian jump systems have a strong practical background in physical systems, which are subjected to abrupt variations in their structures. Moreover, Markovian

chain in a finite mode set always determines the transitions models, which can describe a set of linear Markovian jump systems. In the past two years, Markovian jump systems have received much attention because they continue being a hot research area of mathematical models to represent many physical systems. It should be noted that, the existence of abrupt changes on the modes of operation in many practical systems, and the class of systems known as Markov jump systems has been proved to be useful to model a great number of them. Recently, a large number of results on the stability, estimation, and control problems related to such systems have been reported [9–16].

In many engineering systems, singular systems have the extensive applications in power systems, electrical circuits, and other fields [17, 18]. Singular systems are famous for its better performance in physical systems when compared with state-space ones. Recently, the H_∞ filtering problems with theory of state-space ones have also found their wide application in the singular systems. Furthermore, as one special family of stochastic systems associated with time delays, the transition probabilities of Markovian jump systems are better to decide the singular system performance. Up to now, many

results on H_∞ filtering problem for singular Markovian jump systems with or without time delay have been reported [19–24], while there are some related issues that need to be solved.

It should be pointed out that the classical filtering problems concern the asymptotic performance over the infinite-time interval. However, the transient behavior of systems is always considered in practical problems and traditional asymptotical stability is not applicable; the finite-time stability was proposed [25]. In the finite-time interval, finite-time stability is investigated to address these transient behaviors of control systems when compared with classical asymptotical stability. Recently, the concept of finite-time stability has been revisited in the light of linear matrix inequalities (LMIs) and theory of Lyapunov function; many results are reported subject to finite-time stability and finite-time boundedness [25–33]. It should be noted that there are still some related issues to be solved; to the best of our knowledge, the finite-time H_∞ filtering problems for singular Markovian jump systems have not been fully studied. The topic remains interesting and challenging, which motivates the present study.

This paper is concerned with the finite-time boundedness of H_∞ filtering for singular Markovian jump systems. Finite-time stability can be affected by switching behavior significantly, which is an independent concept from traditional Lyapunov stability; thus, it deserves our investigation. In this paper, we choose the appropriate Lyapunov-Krasovskii functional; making full information of the underlying Markov process, the sufficient conditions are derived to guarantee the finite-time boundedness of the systems. The finite-time boundedness (FTB) criteria can be tackled in the form of LMIs and optimization algorithms. At last, an example is given to illustrate the effectiveness of the developed method.

2. Preliminaries

A probability space (M, F, P) is given, where M , F , and P , respectively, represent the sample space, the algebra of events, and the probability measure defined on F . Let the random form process $\{r_t, t \geq 0\}$ be the Markov stochastic process taking values on a finite set $\mathcal{N} = \{1, 2, \dots, N\}$ with transition rate matrix $\Omega = \{\mu_{ij}\}$, $i, j \in \mathcal{R}$. Define the following transition probability from mode i at time t to mode j at time $t + \Delta t$ as

$$\begin{aligned} P_{ij} &= P(r_{t+\Delta t} = j \mid r_t = i) \\ &= \begin{cases} \mu_{ij}\Delta t + o(\Delta t), & \text{if } i \neq j, \\ 1 + \mu_{ii}\Delta t + o(\Delta t), & \text{if } i = j, \end{cases} \end{aligned} \quad (1)$$

where $P_r\{r_0 = i\} > 0$, $\forall i \in \mathbb{R}$, $\Delta t > 0$, and $\lim_{\Delta t \rightarrow 0} (o(\Delta t)/\Delta t) \rightarrow 0$ with transition probability rates $\mu_{ij} \geq 0$ for $i, j \in \mathbb{R}$, $i \neq j$, and $\sum_{j=1}^N \mu_{j\neq i} = -\mu_{ii}$.

In this paper, we consider the singular systems as follows:

$$\begin{aligned} E\dot{x}(t) &= A_{r(t)}x(t) + A_{\tau r(t)}x(t - \tau(t)) + B_{r(t)}\omega(t), \\ y(t) &= C_{r(t)}x(t) + D_{r(t)}\omega(t), \\ z(t) &= L_{r(t)}x(t), \end{aligned} \quad (2)$$

where $x(t) \in \mathcal{R}^n$ is the state vector of the system, $y(t) \in \mathcal{R}^l$ is the measured output, $z(t) \in \mathcal{R}^m$ is the controlled output, and E is a singular matrix with $\text{rank}(E) = r < n$. A_{r_t} , B_{r_t} , C_{r_t} , D_{r_t} , and L_{r_t} are known mode-dependent constant matrices with appropriate dimension. $\omega(t) \in L_2^q[0, \infty]$ is the external disturbance vector which satisfies

$$\int_0^T \omega^\top(t) \omega(t) dt < d, \quad d \geq 0, \quad (3)$$

and time-varying delay $\tau(t)$ is the continuous function satisfying

$$0 \leq \tau_0 \leq \tau(t) \leq \tau_M < \infty, \quad (4)$$

$$0 \leq d_1 \leq \dot{\tau}(t) \leq d_2. \quad (5)$$

For system (2), we are interested in designing the filter described by

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{fr(t)}\hat{x}_f(t) + A_{f\tau r(t)}\hat{x}_f(t - \tau(t)) + B_{fr(t)}y(t), \\ z_f(t) &= C_{fr(t)}\hat{x}_f(t), \end{aligned} \quad (6)$$

where $\hat{x}_f(t) \in \mathcal{R}^n$ is the filter state, $z_f(t) \in \mathcal{R}^q$ is the estimation of $z(t)$ in system (2), and the matrices $A_{fr(t)}$, $B_{fr(t)}$, and $C_{fr(t)}$ are the unknown filter parameters to be designed.

Augmenting the model of (2) to include filter (6), we obtain the following filtering error system:

$$\begin{aligned} \bar{E}\dot{\eta}(t) &= \bar{A}_{r(t)}\eta(t) + \bar{A}_{\tau r(t)}\eta(t - \tau(t)) + \bar{B}_{r(t)}\omega(t), \\ e(t) &= \bar{C}_{r(t)}\eta(t), \end{aligned} \quad (7)$$

where the state estimate error $\eta(t) = [\bar{x}^\top(t), x_f^\top(t)]^\top$ and the output error $e(t) = z_f(t) - z(t)$ and

$$\begin{aligned} \bar{E} &= \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \bar{A}_{r(t)} = \begin{bmatrix} A_{r(t)} & 0 \\ B_{fr(t)}C_{r(t)} & A_{fr(t)} \end{bmatrix}, \\ \bar{A}_{\tau r(t)} &= \begin{bmatrix} A_{\tau r(t)} & 0 \\ 0 & A_{f\tau r(t)} \end{bmatrix}, \quad \bar{B}_{r(t)} = \begin{bmatrix} B_{r(t)} \\ B_{fr(t)}D_{r(t)} \end{bmatrix}, \\ \bar{C}_{r(t)} &= [-L_{r(t)}, C_{fr(t)}]. \end{aligned} \quad (8)$$

For brevity, in the sequel, $A_{r(t)} = A_i$, $A_{fr(t)} = A_{fi}$, and $A_{f\tau r(t)} = A_{f\tau i}$, for every $r(t) \in \mathcal{N}$, and the other symbols are similarly denoted.

Some definitions and lemmas should be introduced to facilitate the following discussion.

Definition 1 (see [21]). (i) The singular system (7) is said to be regular if $\text{rank}(sE - A_i)$ is not identically zero for every $r(t) = i \in \mathcal{R}$.

(ii) The singular system (7) is said to be impulse-free if $\deg(\det(sE_i - A_i)) = \text{rank}(E)$ for every $r(t) = i \in \mathcal{R}$.

Definition 2 (see [21]). The closed-loop continuous-time singular Markovian jump system (SMJS) (7) satisfying (3) is said to be singular stochastic finite-time boundedness (SSFTB) with respect to $(c_1, c_2, T, R, r(t), d)$ with $0 \leq c_1 < c_2$ and $R > 0$, if the stochastic system is regular and impulse-free in time interval $[0, T]$ and

$$\mathbb{E} \{x^\top(0) E^\top R E x(0)\} \leq c_1 \implies \mathbb{E} \{x^\top(t) E^\top R E x(t)\} < c_2, \quad \forall t \in [0, T]. \quad (9)$$

Definition 3 (see [21]). The closed-loop continuous-time singular Markovian jump system (SMJS) (7) satisfying (3) is said to be singular stochastic H_∞ finite-time boundedness with respect to $(c_1, c_2, T, R, \gamma, d)$, if the closed-loop continuous-time SMJS (7) is with respect to (c_1, c_2, T, R, d) and under the zero-initial condition the controlled output $z(t)$ satisfies

$$\mathbb{E} \left\{ \int_0^T e^\top(t) e(t) dt \right\} < \gamma^2 \mathbb{E} \left\{ \int_0^T \omega^\top(t) \omega(t) dt \right\} \quad (10)$$

for any nonzero $\omega(t)$ which satisfies (3), where γ is a prescribed positive scalar.

Definition 4 (see [16]). Consider $V(x_t, r_t, t > 0)$ as the stochastic Lyapunov function of the resulting system (7); its weak infinitesimal operator is defined as

$$\begin{aligned} \mathcal{L}V(x_t, r_t) &= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} [\mathbb{E} \{V(x(t + \Delta t), r_{t+\Delta t}) \mid x(t) = x_t, r_t = i\} \\ &\quad - V(x_t, i)] \\ &= \frac{\partial}{\partial t} V(x_t, i) + \frac{\partial}{\partial x} V(x_t, i) \dot{x}(t, i) + \sum_{j=1}^N \mu_{ij} V(x_t, j). \end{aligned} \quad (11)$$

Definition 5 (see [27]). The jump rates of the visited modes from a given mode i are assumed to satisfy

$$0 < \min \mu_i \leq \mu_{ij} \leq \max \mu_i, \quad \forall i, j \in \mathbb{R}, i \neq j, \quad (12)$$

where $\min \mu_i$ and $\max \mu_i$ are known parameters for a given mode i and represent the lower and upper bounds when all the jump rates are known; that is, $0 < \min \mu_i = \min\{\mu_{ij} \neq 0, i \neq j, j \in \mathbb{R}\}$ and $\min \mu_i \leq \max \mu_i$. Meanwhile, the number of the visited modes from a given mode i is denoted by N_i including the mode itself.

Lemma 6 (see [4]). For the positive matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \geq 0, \quad (13)$$

the following integral inequality holds:

$$\begin{aligned} & - \int_{t-\tau(t)}^t \dot{x}(s)^\top E^\top X_{33} E \dot{x}(s) ds \\ & \leq \int_{t-\tau(t)}^t \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ E \dot{x}(s) \end{bmatrix}^\top \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ E \dot{x}(s) \end{bmatrix} ds. \end{aligned} \quad (14)$$

3. Finite-Time H_∞ Performance Analysis

Theorem 7. Consider the closed-loop continuous-time SMJS (7); there exist positive scalars c_1 and c_2 with $0 \leq c_1 < c_2$, for all admissible $\omega(t)$ subject to condition (3), if there exist symmetric positive definite matrices $X = X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix}$, P_i , $i \in \mathcal{N}$, and $Q_{ji} > 0$, $j = 1, 2, 3$, such that, for all $i \in \mathcal{R}$, the following linear matrix inequalities

$$P_i E = E^\top P_i^\top \geq 0, \quad (15)$$

$$\begin{aligned} & \Xi_i \\ & = \begin{bmatrix} \Xi_{11i} & P_i \bar{A}_{\tau i} & 0 & 0 & P_i \bar{B}_i & e^{\alpha \tau_0} (\tau_M - \tau_0) A_i K_i \\ * & \Xi_{22i} & \Xi_{23i} & \Xi_{24i} & 0 & e^{\alpha \tau_0} (\tau_M - \tau_0) A_{\tau i} K_i \\ * & * & \Xi_{33i} & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 \\ * & * & * & * & -\alpha Q_i & e^{\alpha \tau_0} (\tau_M - \tau_0) B_i K_i \\ * & * & * & * & * & -K_i \end{bmatrix} \\ & < 0, \end{aligned} \quad (16)$$

$$P_i E = E^\top R^{1/2} S_{1i} R^{1/2} E,$$

$$Q_{ji} = R^{1/2} T_{ji} R^{1/2}, \quad (j = 1, 2, 3), \quad K_i = R^{1/2} U_i R^{1/2},$$

$$\begin{aligned} & c_1 [\lambda_2 + \tau_0 \alpha^{\tau_0} \lambda_3 + \tau_M \alpha^{\tau_M} (\lambda_4 + \lambda_5)] + \frac{1}{2} (\tau_M - \tau_0) e^{\alpha \tau_M} \lambda_6 \\ & + \alpha d \lambda_{\max}(Q_i) (1 - e^{-\alpha T}) < \lambda_1 c_2 e^{-\alpha T}, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \Xi_{11i} &= P_i \bar{A}_i + \bar{A}_i^\top P_i - [(1 - N_i) (\min \mu_i) + \alpha] P_i E \\ &+ (\max \mu_i) \sum_{j=1, j \neq i}^N P_j E + Q_{1i} + Q_{2i} + Q_{3i}, \\ \Xi_{22i} &= -e^{\alpha \tau_0} Q_{2i} + e^{\alpha \tau_M} Q_{3i} - 2e^{\alpha \tau_0} E^\top (K_i - X_{33i}) E \\ &+ e^{\alpha \tau_0} [(\tau_M - \tau_0) X_{22i} - X_{23i} E - E^\top X_{23i}^\top] \\ &+ e^{\alpha \tau_0} [(\tau_M - \tau_0) X_{11i} + X_{13i} E + E^\top X_{13i}^\top], \end{aligned}$$

$$\begin{aligned}
\Xi_{23i} &= -e^{\alpha\tau_0} E^\top Y_1^\top E \\
&\quad + e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{12i}^\top + E^\top X_{13i}^\top + X_{23i} E], \\
\Xi_{24i} &= -e^{\alpha\tau_0} E^\top Y_2^\top E \\
&\quad + e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{12i}^\top + E^\top X_{13i}^\top + X_{23i} E], \\
\Xi_{33i} &= -e^{\alpha\tau_0} (Q_{1i} - Q_{2i}) - e^{\alpha\tau_0} E^\top (K_i - X_{33i})^\top E \\
&\quad + e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{11i} + X_{13i} E + E^\top X_{13i}^\top], \\
\Xi_{44i} &= -e^{\alpha\tau_M} Q_{3i} - e^{\alpha\tau_0} E^\top K_i^\top E \\
&\quad + e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{22i} - X_{23i} E - E^\top X_{23i}^\top], \\
\lambda_1 &= \max_{i \in \mathcal{N}'} \lambda_{\min} (R^{-1/2} E^\top S_i R^{-1/2}), \\
\lambda_2 &= \max_{i \in \mathcal{N}'} \lambda_{\max} (R^{-1/2} E^\top S_i R^{-1/2}), \\
\lambda_3 &= \max_{i \in \mathcal{N}'} \lambda_{\max} (R^{-1/2} T_{1i} R^{-1/2}), \\
\lambda_4 &= \max_{i \in \mathcal{N}'} \lambda_{\max} (R^{-1/2} T_{2i} R^{-1/2}), \\
\lambda_5 &= \max_{i \in \mathcal{N}'} \lambda_{\max} (R^{-1/2} T_{3i} R^{-1/2}), \\
\lambda_6 &= \max_{i \in \mathcal{N}'} \lambda_{\max} (R^{-1/2} U_i R^{-1/2})
\end{aligned} \tag{18}$$

hold; the closed-loop SMJS (7) is SSFTB with respect to (c_1, c_2, T, R, d) .

Proof. At first, we will prove that the singular Markovian jump system (7) is regular and impulse-free.

Noting (16), we can have that

$$\begin{aligned}
&\bar{A}_i P_i^\top + P_i \bar{A}_i^\top + [(1 - N_i) (\min \mu_i) - \alpha] E P_i^\top \\
&\quad + W_i < -(\max \mu_i) \sum_{j=1, j \neq i}^N P_j^{-1} E \leq 0.
\end{aligned} \tag{19}$$

Since $0 < \text{rank } E_i = r_i < n$, there must exist two invertible matrices M_i and N_i such that

$$\tilde{E}_i = M_i E N_i = \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix}. \tag{20}$$

We denote

$$M_i \bar{A}_i N_i = \begin{bmatrix} \bar{A}_{11i} & \bar{A}_{12i} \\ \bar{A}_{21i} & \bar{A}_{22i} \end{bmatrix}, \quad M_i P_i N_i^{-\top} = \begin{bmatrix} \bar{P}_{11i} & \bar{P}_{12i} \\ \bar{P}_{21i} & \bar{P}_{22i} \end{bmatrix}. \tag{21}$$

According to (15), it can be obtained that

$$\bar{P}_{11i} = \bar{P}_{11i}^\top \geq 0, \quad \bar{P}_{12i} = 0. \tag{22}$$

Pre- and postmultiplying (22) by M and $M^\top$, we can easily obtain $\bar{A}_{22i} \bar{P}_{22i}^\top + \bar{P}_{22i} \bar{A}_{22i}^\top < 0$. Therefore, $\bar{A}_{22i}$ is nonsingular.

Otherwise, supposing $\bar{A}_{22i}$ is singular, there exists a nonzero vector $\varsigma_i \in \mathfrak{R}^{n-r_i}$ which ensures that $\bar{A}_{22i} \varsigma_i = 0$. And then we can conclude that $\varsigma_i^\top \bar{A}_{22i} \bar{P}_{22i}^\top + \bar{P}_{22i} \bar{A}_{22i}^\top \varsigma_i = 0$, and this contradicts $\bar{A}_{22i} \bar{P}_{22i}^\top + \bar{P}_{22i} \bar{A}_{22i}^\top < 0$.

Then, it can be shown that

$$\begin{aligned}
&\det(sE_i - A_i) \\
&= \det(M_i^{-1}) \det(-\bar{A}_{22i}) \\
&\quad \cdot \det(sI_{r_i} - (\bar{A}_{11i} - \bar{A}_{12i} \bar{A}_{22i}^{-1} \bar{A}_{21i})) \det(N_i^{-1})
\end{aligned} \tag{23}$$

which implies that $\det(sE_i - A_i)$ is not identically zero and $\deg(\det(sE_i - A_i)) = \text{rank}(E_i) = r_i$. Therefore, by Definition 1, we can obtain that the closed-loop SMJS (7) is regular and impulse-free in time interval $[0, T]$.

We consider the following Lyapunov-Krasovskii functional:

$$V(\eta_t, r(t)) = V_1(\eta_t, r(t)) + V_2(\eta_t, r(t)) + V_3(\eta_t, r(t)), \tag{24}$$

$$V_1(\eta_t, r(t)) = \eta^\top(t) P_{r(t)} E \eta(t),$$

$$\begin{aligned}
V_2(\eta_t, r(t)) &= \int_{t-\tau_0}^t \eta^\top(s) e^{\alpha(t-s)} Q_{1r_t} \eta(s) ds \\
&\quad + \int_{t-\tau(t)}^{t-\tau_0} \eta^\top(s) e^{\alpha(t-s)} Q_{2r_t} \eta(s) ds \\
&\quad + \int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) e^{\alpha(t-s)} Q_{3r_t} \eta(s) ds,
\end{aligned}$$

$$V_3(\eta(t), r(t)) = \int_{-\tau_M}^{-\tau_0} \int_{t+\theta}^t \dot{\eta}^\top(s) E^\top e^{\alpha(t-s)} K_{r_t} E \dot{\eta}(s) ds d\theta. \tag{25}$$

Taking the time derivative of $V(\eta_t, r(t))$ along the trajectory of system (7), one has

$$\begin{aligned}
&\mathcal{E}V_1(\eta_t, r(t)) \\
&= \eta^\top(t) (P_i \bar{A}_i + W_i) \eta(t) \\
&\quad + 2\eta^\top(t) P_i \bar{B}_i \omega(t) + \eta^\top(t) \sum_{j=1}^N \mu_{ij} P_j E \eta(t),
\end{aligned}$$

$$\begin{aligned}
&\mathcal{E}V_2(\eta_t, r(t)) - \alpha \mathcal{E}V_2(\eta_t, r(t)) \\
&= \eta^\top(t) (Q_{1i} + Q_{2i} + Q_{3i}) \eta(t) \\
&\quad - \eta^\top(t - \tau_0) e^{\alpha\tau_0} (Q_{1i} - Q_{2i}) \eta(t - \tau_0) \\
&\quad - \eta^\top(t - \tau(t)) (e^{\alpha\tau_0} Q_{2i} - e^{\alpha\tau_M} Q_{3i}) \\
&\quad \cdot \eta(t - \tau(t)) - \eta^\top(t - \tau_M) e^{\alpha\tau_M} Q_{3i} \eta(t - \tau_M),
\end{aligned}$$

$$\begin{aligned}
& \mathbb{E}V_3(\eta_t, r(t)) - \alpha \mathbb{E}V_3(\eta_t, r(t)) \\
&= e^{\alpha\tau_0} (\tau_M - \tau_0) \dot{\eta}^\top(t) E^\top K_i E \dot{\eta}(t) \\
&\quad - \int_{t-\tau_M}^{t-\tau_0} e^{\alpha(t-s)} \dot{\eta}^\top(s) E^\top K_i E \dot{\eta}(s) ds \\
&\leq e^{\alpha\tau_M} (\tau_M - \tau_0) \dot{\eta}^\top(t) E^\top K_i E \dot{\eta}(t) \\
&\quad - e^{\alpha\tau_0} \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}^\top(s) E^\top (K_i - X_{33i}) E \dot{\eta}(s) ds \\
&\quad - e^{\alpha\tau_0} \int_{t-\tau(t)}^{t-\tau_0} \dot{\eta}^\top(s) E^\top (K_i - X_{33i}) E \dot{\eta}(s) ds \\
&\quad - e^{\alpha\tau_0} \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}^\top(s) E^\top X_{33i} E \dot{\eta}(s) ds \\
&\quad - e^{\alpha\tau_0} \int_{t-\tau(t)}^{t-\tau_0} \dot{\eta}^\top(s) E^\top X_{33i} E \dot{\eta}(s) ds.
\end{aligned} \tag{26}$$

It should be noted that

$$\begin{aligned}
& -e^{\alpha\tau_0} \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}^\top(s) E^\top (K_i - X_{33i}) E \dot{\eta}(s) ds \\
& -e^{\alpha\tau_0} \int_{t-\tau(t)}^{t-\tau_0} \dot{\eta}^\top(s) E^\top (K_i - X_{33i}) E \dot{\eta}(s) ds \\
&= - \begin{bmatrix} \eta(t - \tau_0) \\ \eta(t - \tau(t)) \end{bmatrix}^\top \\
& \cdot \begin{bmatrix} e^{\alpha\tau_0} E^\top (K_i - X_{33i}) E & e^{\alpha\tau_0} E^\top Y_{1i} E \\ * & e^{\alpha\tau_0} E^\top (K_i - X_{33i}) E \end{bmatrix} \\
& \cdot \begin{bmatrix} \eta(t - \tau_0) \\ \eta(t - \tau(t)) \end{bmatrix} - \begin{bmatrix} \eta(t - \tau(t)) \\ \eta(t - \tau_M) \end{bmatrix}^\top \\
& \cdot \begin{bmatrix} e^{\alpha\tau_0} E^\top (K_i - X_{33i}) E & e^{\alpha\tau_0} E^\top Y_{2i} E \\ * & e^{\alpha\tau_0} E^\top (K_i - X_{33i}) E \end{bmatrix} \\
& \cdot \begin{bmatrix} \eta(t - \tau(t)) \\ \eta(t - \tau_M) \end{bmatrix}.
\end{aligned} \tag{27}$$

From Lemma 6, one has

$$\begin{aligned}
& -e^{\alpha\tau_0} \int_{t-\tau(t)}^{t-\tau_0} \dot{\eta}^\top(s) E^\top X_{33i} E \dot{\eta}(s) ds \\
&\leq \eta^\top(t - \tau_0) e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{11i} + X_{13i} E + E^\top X_{13i}^\top] \\
&\quad \cdot \eta(t - \tau_0) + 2\eta^\top(t - \tau_0) e^{\alpha\tau_0} \\
&\quad \cdot [(\tau_M - \tau_0) X_{12i} + X_{13i} E + E^\top X_{23i}^\top] \eta(t - \tau(t)) \\
&\quad + \eta^\top(t - \tau(t)) e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{22i} - X_{23i} E - E^\top X_{23i}^\top] \\
&\quad \cdot \eta(t - \tau(t)),
\end{aligned}$$

$$\begin{aligned}
& -e^{\alpha\tau_0} \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}^\top(s) E^\top X_{33i} E \dot{\eta}(s) ds \\
&\leq \eta^\top(t - \tau(t)) e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{11i} + X_{13i} E + E^\top X_{13i}^\top] \\
&\quad \cdot \eta(t - \tau(t)) + 2\eta^\top(t - \tau(t)) e^{\alpha\tau_0} \\
&\quad \cdot [(\tau_M - \tau_0) X_{12i} + X_{13i} E + E^\top X_{23i}^\top] \eta(t - \tau_M) \\
&\quad + \eta^\top(t - \tau_M) e^{\alpha\tau_0} [(\tau_M - \tau_0) X_{22i} - X_{23i} E - E^\top X_{23i}^\top] \\
&\quad \cdot \eta(t - \tau_M).
\end{aligned} \tag{28}$$

Moreover, we have

$$\begin{aligned}
& \sum_{j=1}^N \mu_{ij} P_j E = \mu_{ii} P_i E + \sum_{j=1, j \neq i}^N \mu_{ij} P_j E \\
&= - \sum_{j=1, j \neq i}^N \mu_{ij} P_i E + \sum_{j=1, j \neq i}^N \mu_{ij} P_j E \\
&\leq -(N_i - 1) (\min \mu_i) P_i + (\max \mu_i) \sum_{j=1, j \neq i}^N P_j E.
\end{aligned} \tag{29}$$

Then, from (24)–(29) and Schur complement, one can obtain

$$\begin{aligned}
& \mathbb{E}V(\eta_t, r(t)) - \alpha V(\eta_t, r(t)) \\
&\leq \chi^\top(t) \Xi_i \chi(t) + \alpha \omega^\top(t) Q_i \omega(t),
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
& \chi^\top(t) \\
&= [\eta^\top(t), \eta^\top(t - \tau(t)), \eta^\top(t - \tau_0), \eta^\top(t - \tau_M), \omega^\top(t)].
\end{aligned} \tag{31}$$

From condition (15), we have

$$\mathbb{E}V(\eta_t, r(t)) - \alpha V(\eta_t, r(t)) < \alpha \omega^\top(t) Q_i \omega(t). \tag{32}$$

Multiplying (32) by $e^{-\alpha t}$, it yields

$$\mathbb{E} \{ e^{-\alpha t} V(\eta_t, r(t)) \} < \alpha e^{-\alpha t} \omega^\top(t) Q_i \omega(t). \tag{33}$$

Integrating (33) from 0 to t , then we can obtain

$$\begin{aligned}
& e^{-\alpha t} \mathbb{E} \{ V(\eta_t, r(t)) \} \\
&< \{ V(x_0, r(0)) \} + \alpha \int_0^t e^{-\alpha s} \omega^\top(s) Q_i \omega(s) ds.
\end{aligned} \tag{34}$$

Noting that $\alpha \geq 0$, $t \in [0, T]$. Define $P_i E = E^T R^{1/2} S_{1i} R^{1/2} E$, $Q_{ji} = R^{1/2} T_{ji} R^{1/2}$, $j = 1, 2, 3$, and $K_i = R^{1/2} U_i R^{1/2}$; by Dynkin's formula, we have

$$\begin{aligned}
& \mathbb{E} \{V(\eta_t, r(t))\} \\
& < e^{\alpha t} \{V(x_0, r(0))\} + \alpha e^{\alpha t} \int_0^t e^{-\alpha s} \omega^T(s) Q_i \omega(s) ds \\
& \leq e^{\alpha T} \left\{ c_1 \left[\lambda_{\max}(S_{1i}) + \tau_0 \alpha^{\tau_0} \lambda_{\max}(T_{1i}) \right. \right. \\
& \quad \left. \left. + \tau_M \alpha^{\tau_M} (\lambda_{\max}(T_{2i}) + \lambda_{\max}(T_{3i})) \right. \right. \\
& \quad \left. \left. + \frac{1}{2} (\tau_M - \tau_0) e^{\alpha \tau_M} \lambda_{\max}(U_i) \right] \right. \\
& \quad \left. + \alpha d \lambda_{\max}(Q_i) \int_0^T e^{-\alpha s} ds \right\} \\
& \leq e^{\alpha T} \left\{ c_1 \left[\lambda_2 + \tau_0 \alpha^{\tau_0} \lambda_3 + \tau_M \alpha^{\tau_M} (\lambda_4 + \lambda_5) \right. \right. \\
& \quad \left. \left. + \frac{1}{2} (\tau_M - \tau_0) e^{\alpha \tau_M} \lambda_6 \right] \right. \\
& \quad \left. + \alpha d \lambda_{\max}(Q_i) (1 - e^{-\alpha T}) \right\}. \tag{35}
\end{aligned}$$

On the other hand, it follows from (24) that

$$\begin{aligned}
\mathbb{E} \{V(\eta_t, i)\} &= \mathbb{E} \{\eta^T(t) P_i E \eta(t)\} \\
&\geq \lambda_{\min}(S_{1i}) \mathbb{E} \{\eta^T(t) E^T R_i E \eta(t)\}. \tag{36}
\end{aligned}$$

Then, one has

$$\begin{aligned}
& \mathbb{E} \{\eta^T(t) E^T R_i E \eta(t)\} \\
& < e^{\alpha T} \cdot \left(\left(c_1 [\lambda_2 + \tau_0 \alpha^{\tau_0} \lambda_3 + \tau_M \alpha^{\tau_M} (\lambda_4 + \lambda_5)] \right. \right. \\
& \quad \left. \left. + \frac{1}{2} (\tau_M - \tau_0) e^{\alpha \tau_M} \lambda_6 \right. \right. \\
& \quad \left. \left. + \alpha d \lambda_{\max}(Q_i) (1 - e^{-\alpha T}) \right) \right. \\
& \quad \left. \cdot (\lambda_1)^{-1} \right). \tag{37}
\end{aligned}$$

Therefore, from (37) and by Definition 2, we conclude that $\mathbb{E} \{\eta^T(t) E^T R_i E \eta(t)\} < c_2$. This completes the proof. $\square$

Theorem 8. For a given constant $T > 0$ and $\alpha > 0$, system (4) is robust finite-time stability with respect to (c_1, c_2, N, R_r, d) , for all admissible $\omega(t)$ subject to condition (6), if there exist

symmetric positive definite matrices P_i, Q_{ji} , $j = 1, 2, 3$, $1 \leq i \leq N$, such that the following linear matrix inequalities hold:

$$\begin{aligned}
& \Xi_i \\
& = \begin{bmatrix} \Xi_{11i} & P_i \bar{A}_{\tau i} & 0 & 0 & P_i \bar{B}_i & e^{\alpha \tau_0} (\tau_M - \tau_0) A_i K_i & \bar{C}_i \\ * & \Xi_{22i} & \Xi_{22i} & \Xi_{24i} & 0 & e^{\alpha \tau_0} (\tau_M - \tau_0) A_{\tau i} K_i & 0 \\ * & * & \Xi_{33i} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 & 0 \\ * & * & * & * & -\alpha Q_i & e^{\alpha \tau_0} (\tau_M - \tau_0) B_i K_i & 0 \\ * & * & * & * & * & -K_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \\
& < 0. \tag{38}
\end{aligned}$$

Proof. We will show the H_∞ performance of system (7); from Theorem 7, we have

$$\begin{aligned}
& \mathbb{E} V(\eta_t, r(t)) - \alpha V(\eta_t, r(t)) + e^T(t) e(t) \\
& \quad - \gamma^2 \omega^T(t) \omega(t) \leq \chi^T(t) \Omega_i \chi(t). \tag{39}
\end{aligned}$$

Condition (38) yields

$$\mathbb{E} V(\eta_t, r_t) - \alpha V(\eta_t, r_t) + e^T(t) e(t) - \gamma^2 \omega^T(t) \omega(t) < 0. \tag{40}$$

Define

$$J(t) = e^T(t) e(t) - \gamma^2 \omega^T(t) \omega(t). \tag{41}$$

Multiplying (41) by $e^{-\alpha t}$, it follows that

$$\mathbb{E} e^{-\alpha t} V(\eta_t, r_t) < e^{-\alpha t} J(t). \tag{42}$$

According to condition (42), integrating this inequality during $[0, T]$ yields

$$0 \leq e^{-\alpha T} \mathbb{E} \{V(\eta_t, r_t)\} < \mathbb{E} \left\{ \int_0^T e^{-\alpha t} J(t) dt \right\}. \tag{43}$$

Then we have

$$\begin{aligned}
& e^{-\alpha T} \int_0^T e^T(t) e(t) dt < \int_0^T e^{-\alpha t} e^T(t) e(t) dt \\
& < \gamma^2 \int_0^T e^{-\alpha t} \omega^T(t) \omega(t) dt \tag{44} \\
& < \gamma^2 \int_0^T \omega^T(t) \omega(t) dt.
\end{aligned}$$

Thus it is concluded by Definition 3 that the closed-loop SMJS (7) is SSFTB with H_∞ performance γ . The proof is completed. $\square$

4. Finite-Time H_∞ Filter Design

Theorem 9. Consider the finite-time switched discrete-time system (2) and a given scalar $\gamma > 0$. Then there exists a switched filter in the form of (6) such that the filtering error system (7) is

finite-time boundedness with H_∞ performance level γ , if there exist real matrixes $P_i > 0$, $S_{ij} > 0$, $\bar{P}_i > 0$, $\bar{S}_{ij} > 0$, and $\bar{G}_i$, $\bar{A}_{fi}$, $\bar{B}_{fi}$, and $\bar{C}_{fi}$ ($i, j \in \mathcal{N}$) of appropriate dimensions and real matrices G_i ($i \in \mathcal{N}$), such that $\forall i \in \mathcal{N}$ and

$$\begin{bmatrix} \Sigma_{1i} & \Sigma_{2i} \\ * & \Sigma_{3i} \end{bmatrix} < 0, \quad (45)$$

where

$$\begin{aligned} \Sigma_{1i} &= \begin{bmatrix} Q_{1i} + Q_{2i} + Q_{3i} - G_i - G_i^\top & & 0 \\ * & -[(1 - N_i)(\min \mu_i) + \alpha] H_i E & X_{2i} \\ * & * & \Xi_{22i} \end{bmatrix}, \\ \Sigma_{2i} &= \begin{bmatrix} 0 & 0 & 0 & \begin{bmatrix} -L_i \\ C_{fi} \end{bmatrix} & \mathcal{G}_i & 0 \\ \Sigma_{23i} & \Sigma_{24i} & X_{3i} & 0 & 0 & 0 \end{bmatrix}, \\ \Sigma_{3i} &= \begin{bmatrix} \Xi_{33i} & 0 & 0 & 0 & 0 & 0 \\ * & \Xi_{44i} & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -K_i & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\mathcal{H}_i \end{bmatrix}, \end{aligned} \quad (46)$$

with

$$X_{1i} = \begin{bmatrix} G_{1i}A_i + V_iC_i & W_i \\ G_{3i}A_i + V_iC_i & W_i \end{bmatrix}, \quad X_{2i} = \begin{bmatrix} G_{1i}A_{\tau i} & U_i \\ G_{3i}A_{\tau i} & U_i \end{bmatrix},$$

$$X_{3i} = \begin{bmatrix} G_{1i}B_i + V_iD_i \\ G_{3i}B_i + V_iD_i \end{bmatrix},$$

$$\mathcal{G}_i = \begin{bmatrix} \sqrt{\max \mu_i G_i^\top} & \sqrt{\max \mu_i G_i^\top} & \dots & \sqrt{\max \mu_i G_i^\top} \end{bmatrix},$$

$$\mathcal{H}_i = \text{diag} \{EH_1, EH_2, \dots, EH_{i-1}, EH_{i+1}, \dots, EH_N\}, \quad (47)$$

hold; then the closed-loop SMJS (7) is SSFTB with respect to (c_1, c_2, T, R, d) with H_∞ performance γ .

In this case, desired H_∞ filter parameter matrices are given by

$$A_{fi} = G_{2i}^{-1}W_i, \quad A_{\tau fi} = G_{2i}^{-1}U_i, \quad A_{fi} = G_{2i}^{-1}V_i, \quad C_{fi} = C_{fi}. \quad (48)$$

Proof. The Lyapunov matrices P_i can be parameterized as follows:

$$P_i = G_i H_i^{-1} G_i, \quad i = 1, 2, \dots, n, \quad (49)$$

where H_i are symmetric positive definite mode-dependent matrices. We also parameterize G_i as

$$G_i = \begin{bmatrix} G_{1i} & G_{2i} \\ G_{3i} & G_{2i} \end{bmatrix}, \quad (50)$$

in which G_{2i} are nonsingular matrices. Note that P_i are $2n \times 2n$ matrices and G_{2i} belong to $\mathbb{R}^{n \times n}$. Also we will assume $\mu_{ii} \neq 0$ which excludes the existence of a permanent mode in the system.

First, we have

$$\sum_{j=1, j \neq i}^N P_j E = \mathcal{G}_i^\top \mathcal{H}_i^{-1} \mathcal{G}_i. \quad (51)$$

According to Schur's complement, (37) can be rewritten as

$$\begin{bmatrix} \Omega_i & \bar{\mathcal{G}}_i^\top \\ * & -\mathcal{H}_i \end{bmatrix} < 0, \quad (52)$$

where

$$\bar{\mathcal{G}}_i = \begin{bmatrix} \mathcal{G}_i & 0_{(2nN-2n) \times (n_{\Omega_i} - 2n)} \end{bmatrix}. \quad (53)$$

With matrices P_i , inequalities (51) could be decomposed into

$$\Sigma_i^\top \Pi_{ji} \Sigma_i < 0, \quad (54)$$

where

$$\Sigma_i = \begin{bmatrix} \tilde{\Sigma}_i & 0 \\ 0 & I_{3n \times 3n} \end{bmatrix}, \quad \tilde{\Sigma}_i = \begin{bmatrix} I_{2n} & 0 \\ H_i^{-1} G_i & 0 \end{bmatrix},$$

$$\Pi_i = \begin{bmatrix} \Pi_{1i} \begin{bmatrix} 0 \\ G_i \bar{A}_{\tau i} \end{bmatrix} & 0 & 0 & \begin{bmatrix} 0 \\ G_i \bar{B}_i \end{bmatrix} & e^{\alpha \tau_0} (\tau_M - \tau_0) \bar{A}_{\tau i} K_i & \begin{bmatrix} \bar{C}_i \\ 0 \end{bmatrix} & \begin{bmatrix} \mathcal{G}_i \\ 0 \end{bmatrix} \\ * & \Xi_{22i} & \Xi_{23i} & \Xi_{24i} & 0 & 0 & 0 \\ * & * & \Xi_{33i} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & e^{\alpha \tau_0} (\tau_M - \tau_0) B_i K_i & 0 \\ * & * & * & * & * & -K_i & 0 \\ * & * & * & * & * & * & -I \\ * & * & * & * & * & * & -\mathcal{H}_i \end{bmatrix} < 0, \quad (55)$$

$$\Pi_{1i} = \begin{bmatrix} Q_{1i} + Q_{2i} + Q_{3i} & \bar{A}_i^\top G_i^\top \\ * & -[(1 - N_i)(\min \mu_i) + \alpha] H_i E \end{bmatrix}.$$

Since the equality

$$\begin{bmatrix} -G_i & N_i & 0 \\ 0 & 0 & 0 \end{bmatrix} \Sigma_i = 0 \quad (56)$$

holds for every Σ_i , $\Sigma_i^\top \Pi_{ji} \Sigma_i < 0$ is equivalent to

$$\begin{aligned} \bar{\Pi}_i &= \Pi_i + \begin{bmatrix} I_{2n} & 0 \\ 0 & I_{2n} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -G_i & H_i & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} -G_i^\top & 0 \\ H_i & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_{2n} & 0 & 0 \\ 0 & I_{2n} & 0 \end{bmatrix} < 0. \end{aligned} \quad (57)$$

Also we have

$$\begin{aligned} G_i \bar{A}_i &= \begin{bmatrix} G_{1i} A_i + G_{2i} B_{fi} C_i & G_{2i} A_{fi} \\ G_{3i} A_i + G_{2i} B_{fi} C_i & G_{2i} A_{fi} \end{bmatrix}, \\ G_i \bar{A}_{\tau i} &= \begin{bmatrix} G_{1i} A_{\tau i} & G_{2i} A_{fi} \\ G_{3i} A_{\tau i} & G_{2i} A_{fi} \end{bmatrix}, \\ G_i \bar{B}_i &= \begin{bmatrix} G_{1i} B_i + G_{2i} B_{fi} D_i \\ G_{3i} B_i + G_{2i} B_{fi} D_i \end{bmatrix}. \end{aligned} \quad (58)$$

Taking $G_{2i} A_{fi} = W_i$, $G_{2i} B_{fi} = V_i$, and $G_{2i} A_{fi} = U_i$, we get

$$\begin{aligned} X_{1i} &= G_i \bar{A}_i = \begin{bmatrix} G_{1i} A_i + V_i C_i & W_i \\ G_{3i} A_i + V_i C_i & W_i \end{bmatrix}, \\ X_{2i} &= G_i \bar{A}_{\tau i} = \begin{bmatrix} G_{1i} A_{\tau i} & U_i \\ G_{3i} A_{\tau i} & U_i \end{bmatrix}, \end{aligned}$$

$$X_{3i} = G_i \bar{B}_i = \begin{bmatrix} G_{1i} B_i + V_i D_i \\ G_{3i} B_i + V_i D_i \end{bmatrix}. \quad (59)$$

Therefore, we can get (44) and the proof is complete. $\square$

5. Illustrative Example

Consider a finite-time stabilization of switched system as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.2597 & 0.0111 \\ 0.1699 & 0.5738 \end{bmatrix}, & A_2 &= \begin{bmatrix} -0.1332 & 0.3211 \\ 0.1904 & 0.6603 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -1.0599 & 1.1997 \\ 0.1183 & 0.1451 \end{bmatrix}, & B_2 &= \begin{bmatrix} 1.8472 & 0.9060 \\ 0.7990 & -0.6362 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.0540 & -0.4714 \\ -0.6740 & 0.1520 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.2616 & -0.6257 \\ -0.1612 & 0.4696 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} -0.5932 & 1.2077 \\ -0.4571 & -0.1404 \end{bmatrix}, & D_2 &= \begin{bmatrix} 1.2855 & 1.7521 \\ 1.0255 & 0.7475 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} 0.4992 & 0.1807 \\ -1.4114 & -0.4540 \end{bmatrix}, & L_2 &= \begin{bmatrix} 0.4028 & 1.4335 \\ 0.0917 & 0.2119 \end{bmatrix}, \\ E &= \begin{bmatrix} 0.025 & 0 \\ 0.020 & 0.030 \end{bmatrix}. \end{aligned} \quad (60)$$

Initial states satisfy $\|x(0)\| \leq 1$; the filtering objective is to find feedback filter parameter ensuring that system (5) is finite-time boundedness with minimum value of $\gamma_{\min}^2 + c_2 \min$. Choosing $c_1 = 4$, $d = 0.01$. According to Theorem 9, we can also obtain that feasible solution $c_2 = 97.08$, $\alpha = 0.420$, and performance level is $\gamma = 1.54$. Furthermore, Figures 1 and 2 are given to illustrate the optimal value with different value of

FIGURE 1: The local optimal bound of c_1 . FIGURE 2: The local optimal bound of c_2 .

α . By solving the matrix equalities in Theorem 9, we have the following filter parameters:

$$\begin{aligned}
 A_{f1} &= \begin{bmatrix} 1.4870 & -1.9748 \\ -0.9898 & 1.7607 \end{bmatrix}, \\
 A_{f2} &= \begin{bmatrix} -0.9214 & 1.1072 \\ -0.4963 & -0.6799 \end{bmatrix}, \\
 B_{f1} &= \begin{bmatrix} -1.4222 & 2.1525 \\ -2.1576 & 3.0650 \end{bmatrix}, \\
 B_{f2} &= \begin{bmatrix} -0.7484 & 1.6955 \\ -2.0858 & -1.2667 \end{bmatrix}, \\
 C_{f1} &= \begin{bmatrix} -0.0279 & -0.0098 \\ 0.0667 & 0.0214 \end{bmatrix}, \\
 C_{f2} &= \begin{bmatrix} -0.0183 & -0.0649 \\ -0.0032 & -0.0068 \end{bmatrix}.
 \end{aligned} \tag{61}$$

Two classes of filters are involved in this example; filter (4) serves in short-time singular Markovian jump systems to guarantee the error state bounded in the prescribed boundary.

6. Conclusion

In this paper, we have examined the problems of H_∞ filtering for short-time singular Markovian jump systems. Based on the analysis result, the static state feedback finite-time boundedness is given. Although the derived result is not in LMIs form, we can turn it into LMIs feasibility problem by fixing some parameters. At last, a numerical example is also given to illustrate the effectiveness of the proposed approach. Furthermore, the mode-dependent singular matrices E_{r_i} and uncertain matrices will be studied in the future for the recurrent singular Markovian jump system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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