Research Article

Neural Network-Based Adaptive Backstepping Control for Hypersonic Flight Vehicles with Prescribed Tracking Performance

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1. Introduction

During the past decades, hypersonic flight vehicles (HFVs) have received a great deal of attention. They may represent more cost-efficient and reliable access to space routine and are especially suitable for prompt global response, as well as offering worldwide air superiority because of the high speed and endurance [1–5]. In this paper a nonlinear generic model of HFVs is adopted, which has been widely used by various researchers [6–8]. The dynamics of HFVs are highly nonlinear with strong couplings between the propulsive and aerodynamic effects. The requirements of flight stability and high speed response make the onboard flight control of HFVs quite difficult [9, 10]. Besides, modeling inaccuracy can result in strong adverse effects on the performance of HFVs control systems. Thus, the controller design for HFVs is challenging and must guarantee closed-loop stability and desired performance [11].

Recently, feedback control strategy based on nonlinear control theory has been used for HFVs, such as sliding mode control [3], minimax linear quadratic regulator control [12, 13], genetic algorithm [14], and sequential loop closure controller design [15]. In [16], the adaptive backstepping method was used to design controller for the HFVs model, while fuzzy logic and neural networks were used to approximate the unknown system dynamics in [17–19]. Adaptive dynamic surface control schemes were proposed by [20, 21] to avoid the derivatives of nonlinear functions. The nonlinear dynamic inversion method was used to design a robust controller. In [3, 14], feedback linearization techniques were applied to design nonlinear controllers for the longitudinal motion of a hypersonic aircraft containing aerodynamic uncertain parameters. This approach leads to a complicated high-order Lie derivatives and is hard to perform a robustness analysis when considering uncertainties. In [22], a neural network controller for a nonlinear flight dynamic system was designed by using the adaptation mechanism to deal with the effects of aerodynamic modeling errors.

In the control design for HFVs, an important issue is tracking performance. Traditionally, the controller for HFVs guarantees the tracking error convergence to a residual set. Moreover, the transient behavior such as overshoot, undershoot, and convergence rate are difficult to be established analytically. In [23–25], a prescribed performance scheme is proposed for one-class nonlinear systems; this approach is to construct a prescribed performance function that converts
the tracking error into a new variable. Therefore the tracking performance can be characterized by a prescribed constraint function. Besides, the prescribed performance approach with new definition is applied in a class of uncertain strict-feedback systems \[26\], strict-feedback time-delay systems \[27\], and MIMO systems \[28\], respectively.

A drawback of adaptive NNs \[22\] or FLSs \[29, 30\] schemes is that the number of adaptation laws generally depends on the neural network nodes or the fuzzy rules. That is, with an increase of the nodes or the rules, the parameters to be estimated may be greatly increased. To solve this problem, we propose a new method by estimating the norm of the NNs weights rather than estimating every item of the weight vector \[31–33\].

In this paper, we separate the longitudinal model of HFVs into two parts: the velocity subsystem and the altitude subsystem. Velocity and altitude controllers are designed separately. For the velocity subsystem, a dynamic inversion controller with radial basis function neural networks (RBF NNs) is proposed to track a desired velocity trajectory. The altitude subsystem is transformed into a strict-feedback form. Then an adaptive backstepping controller is designed to track a desired altitude trajectory. The main contribution of this paper is described as follows:

1. We introduce a performance function, and a new error constraint variable is used as a virtual tracking error variable to ensure the prescribed transient performance. By extending the prescribed tracking performance technique proposed in \[23, 24\] to HFVs, it is shown that the tracking errors can converge to predefined arbitrarily small residue sets with prescribed convergence rate and maximum overshoot.

2. RBF NNs are employed to compensate for complex and uncertain terms to solve the problem of controller complexity. By using the minimal learning technique \[31–33\], only one parameter needs to be updated online at each design step regardless of the NNs input-output dimension and the number of NNs nodes. As a result, the number of adaptation laws, which generally depends on the neural network nodes, and the computational burden are greatly reduced.

3. With the bounded of the virtual control gain \( g_i(\cdot) \), the singularity problem by the estimation of \( g_i(\cdot) \) is avoided without any effort, and both low and up bounded will not appear in the control law and will be used only for analysis; they can be unknown.

The rest of this paper is organized as follows. In Section 2, the nonlinear longitudinal dynamic model of HFVs is presented. The controllers design and the stability analysis are given in Section 3. The simulation results are illustrated in Section 4, followed by conclusions of this paper in Section 5.

2. Problem Formulation and Preliminaries

2.1. Longitudinal Model of HFVs. The model considered in this paper is taken from the NASA Langley Research Center [2, 3]. Cruising at a Mach number of 15 and at an altitude of 110000 ft, the longitudinal hypersonic flight model is given by

\[
\dot{V} = \frac{T(V, \beta) \cos \alpha - D(V, \alpha)}{m} - \frac{\mu \sin \gamma}{r^2},
\]

\[
\dot{h} = V \sin \gamma,
\]

\[
\dot{\gamma} = \frac{L(V, \alpha) + T(V, \beta) \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{Vp^2},
\]

\[
\dot{\alpha} = q - \dot{\gamma},
\]

\[
\dot{q} = \frac{M_{yy}(V, \alpha, q, \delta_E)}{I_{yy}},
\]

where \( V \) is the velocity, \( \gamma \) the flight path angle, \( h \) the altitude, \( \alpha \) the attack angle, \( q \) the pitch rate, \( \delta_E \) the elevator deflection, and \( \beta \) the throttle setting. \( T(V, \beta) \), \( D(V, \alpha) \), \( L(V, \alpha) \), and \( M_{yy}(V, \alpha, q, \delta_E) \) represent the thrust, drag, lift-force, and pitching moment, respectively, which can be expressed as

\[
T(V, \beta) = \frac{\rho V^2 SC_{r}}{2},
\]

\[
L(V, \alpha) = \frac{\rho V^2 SC_{l}}{2},
\]

\[
D(V, \alpha) = \frac{\rho V^2 SC_{D}}{2},
\]

\[
M_{yy} = \frac{1}{2} \rho V^2 S \left[ C_M(\alpha) + C_M(\delta_E) + C_M(q) \right],
\]

with

\[
C_L = 0.6203\alpha,
\]

\[
C_D = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772,
\]

\[
C_T = \begin{cases} 0.02576\beta & \text{if } \beta < 1 \\ 0.0224 + 0.00336\beta & \text{if } \beta > 1, \end{cases}
\]

\[
r = h + R_E,
\]

\[
C_M(\alpha) = -0.035\alpha^2 + 0.036617(1 + \Delta C_{Mx})\alpha + 5.326 \times 10^{-6},
\]

\[
C_M(q) = \left( \frac{c}{2V} \right)q \left( -6.796\alpha^2 + 0.3015\alpha - 0.2289 \right),
\]

\[
C_M(\delta_E) = c_s(\delta_E - \alpha).
\]

The nominal values of inertial and aerodynamic parameters are given in Table 1. Besides, at trimmed cruise condition, \( V = 15060 \text{ ft/s}, \ h = 11000 \text{ ft}, \ \gamma = 0 \text{ rad}, \ \alpha = 0.0315, \ \text{and} \ q = 0 \text{ rad/s}. \)

The engine dynamics can be modeled by a second-order system:

\[
\ddot{\beta} = - 2\xi_\omega \dot{\beta} - \omega_n^2 \beta + \omega_p^2 \beta_c.
\]
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Table 1: Parameters of the HFV.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Mean aerodynamic chord</td>
<td>80 ft</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>Moment of inertia</td>
<td>$7 \times 10^6$ slug ft$^2$</td>
</tr>
<tr>
<td>$S$</td>
<td>Reference area</td>
<td>3603 ft$^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
<td>0.24325 $\times 10^{-4}$ slugs ft$^{-3}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of aircraft</td>
<td>9375 slug</td>
</tr>
<tr>
<td>$R_E$</td>
<td>Radius of the earth</td>
<td>20903500 ft</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gravitational constant</td>
<td>$1.39 \times 10^{-6}$ ft/s$^2$</td>
</tr>
</tbody>
</table>


Therefore, by selecting the commanded value $\beta_4$ as the new control input, the HFV is composed of five state variables $X = \{V, h, \alpha, q\}$ and two control inputs $U = \{\beta_c, \delta_e\}$, while the outputs to be controlled are selected as $Y = \{V, h\}^T$. The design objective is that the outputs track the desired altitude and velocity commands $Y_d = \{V_d, h_d\}^T$ with prescribed tracking performance.

From (1), it can be inferred that the main contribution in the change of flight vehicle velocity is from the throttle setting $\beta_c$. The altitude change is related mainly to the elevator deflection $\delta_e$. Thus, it is reasonable to divide the system into two loops: the velocity loop and the altitude loop.

Note that the thrust term $T \sin \alpha$ is generally much smaller than the lift $L$, velocity $V$ is high, and the flight path angle $\gamma$ is typically very small during the trimmed cruise condition, which justify the following approximation.

Assumption 1 (see [7, 19]). The thrust term $T \sin \alpha = 0$, and the term $V \sin \gamma = V \gamma$.

Defining that $\theta$ denotes the pitch angle, we have $\theta = \alpha + \gamma$.

Then, we define state variables as $x = \{x_1, x_2, x_3, x_4\}$, with $x_1 = h$, $x_2 = \gamma$, $x_3 = \theta$, and $x_4 = q$. For simplicity, let $\xi = \{x_1, \ldots, x_4\}$, so the altitude subsystem can be written as

\[\begin{align*}
\dot{x}_1 &= f_1(\xi) + g_1(\xi)x_2, \\
\dot{x}_2 &= f_2(\xi) + g_2(\xi)x_2, \\
\dot{x}_3 &= f_3(\xi) + g_3(\xi)x_4, \\
\dot{x}_4 &= f_4(\xi) + g_4(\xi)\delta_e, \\
y &= x_1,
\end{align*}\]

where

\[\begin{align*}
f_1(\xi) &= 0, \\
f_2(\xi) &= 0, \\
f_3(\xi) &= \frac{\rho V^2S}{2mV}0.6203x_2 - \frac{(\mu - V^2r)}{Vr^2} \cos x_2, \\
f_4(\xi) &= \frac{e\rho V^2S}{2I_{yy}} (C_M (x_3 - x_2) + C_M (x_4) - 0.0292(x_3 - x_2)), \\
g_1(\xi) &= V,
\end{align*}\]

\[
g_2(\xi) = \frac{\rho V^2S}{2mV}0.6203, \\
g_3(\xi) = 1, \\
g_4(\xi) = \frac{e^2\rho V^2S}{2I_{yy}}.
\]

Since the values of the inertial and the aerodynamic parameters are uncertain, the aforementioned $f_i(\xi)$ and $g_i(\xi)$, $i = 1, 2, 3, 4$, are unknown smooth functions. Moreover, it is easy to check that $g_i(\xi)$ are always strictly positive. With these observations in mind, we have the following assumption.

Assumption 2. There exist positive constants $b_i$ and $d_i$ such that $0 < b_i \leq g_i(\xi) \leq d_i$.

Remark 3. It is worth noting that, in the proposed scheme, both $h_l$ and $d_l$ will not appear in the control law and will be used only for analysis; they can be unknown.

Assumption 4. $V_d$ and its first derivative are known and bounded, while $h_l$ and its first four derivatives are continuous and bounded.

2.2. Description of RBF NN. In this paper, RBF NNs will be employed to approximate unknown functions. Mathematically, an RBF NN can be expressed as

\[F(\xi) = W^T \psi(\xi),\]

where $F \in \mathbb{R}$ and $\xi \in \mathbb{R}^n$ are the NN outputs and input, $W \in \mathbb{R}^n$ is the weight vector, and $\psi(\xi) = [\psi_1(\xi), \ldots, \psi_N(\xi)]^T$ is the basis function vector with $\psi_i(\xi)$ commonly chosen as the Gaussian functions:

\[\psi_i(\xi) = \frac{1}{\sqrt{2\pi}\phi} \exp\left(-\frac{\|\xi - \xi_i\|^2}{2\phi^2}\right),\]

\[
\phi > 0, \quad i = 0, \ldots, N,
\]

where $\xi_i \in \mathbb{R}^n$ and $\phi \in \mathbb{R}$ are constants called the center and width of the basis function, respectively.

Lemma 5 (see [17]). Given any continuous function $F(\xi) : \Omega \to \mathbb{R}$ with $\Omega \subset \mathbb{R}^n$ a compact set and any constant $\epsilon > 0$, by appropriately choosing $\phi$ and $\xi_i$, $i = 1, \ldots, N$, for some sufficiently large integer $N$, there exists an RBF NN $W^*^T \psi(\xi)$ such that

\[F(\xi) = W^*^T \psi(\xi) + \Delta(\xi), \quad |\Delta(\xi)| \leq \epsilon, \quad \forall \xi \in \Omega,\]

where $W^*$ is the optimal weight vector defined as

\[W^* = \arg \min_{W \in \mathbb{R}^n} \left\{ \sup_{\xi \in \Omega} |\psi(\xi) - W^T \psi(\xi)| \right\},\]

and $\Delta(\xi)$ denotes the approximation error.
3. Adaptive Neural Controller Design

3.1. Performance and Error Transformation Functions. Let the tracking error be defined as

\[ e = y - y_d, \]

where \( y_d \) is the desired trajectory. Similar to [23, 24], the mathematical expression of the prescribed tracking performance is given by

\[ -\kappa \varepsilon(t) < e(t) < \tau \varepsilon(t), \]

where \( \kappa \) and \( \tau \) are given positive constants and the smooth function is given by

\[ \varepsilon(t) = (\varepsilon_0 - \varepsilon_{\text{so}}) \exp(-lt) + \varepsilon_{\text{co}}, \]

in which \( \varepsilon_0 \) is the initial value of \( \varepsilon(t) \), \( \varepsilon_{\text{so}} \) represents the value of \( \varepsilon(t) \) at the steady state, and \( l \) is the decreasing rate of \( \varepsilon(t) \). Then, introduce the following error transformation:

\[ S(z) = \frac{e(t)}{\varepsilon(t)} \]

where \( z \) is the transformed error and \( S(z) \) is a smooth, strictly increasing, and thus invertible function possessing the following properties:

\[ \lim_{z \to -\infty} S(z) = -\kappa, \]

\[ \lim_{z \to +\infty} S(z) = \tau. \]

Note that if \( z \) is kept bounded, we have \(-\kappa < S(z) < \tau\), and thus (12) holds. The inverse transformation of \( S(z) \) can be written as

\[ z = S^{-1}\left(\frac{e(t)}{\varepsilon(t)}\right) = \Theta\left(\frac{e(t)}{\varepsilon(t)}\right). \]

Differentiating (17) yields

\[ \dot{z} = \eta \dot{y} - \eta \nu, \]

where \( \eta = (\partial \Theta / \partial (e/\varepsilon))(1/\varepsilon) \) and \( \nu = \dot{y}_d + \dot{e}/\varepsilon \). From the properties of the transformation, it is clear that \( \eta \) and \( \nu \) are bounded and \( 0 < \eta_0 \leq \eta \).

Remark 6. From (12) and (13), one can see that \( \tau \varepsilon(0) \) and \(-\kappa \varepsilon(0)\) serve as the upper bound of the overshoot and the lower bound of the undershoot of \( \varepsilon(t) \), respectively, the decreasing rate of \( \varepsilon(t) \) introduces a lower bound of the convergence rate of \( \varepsilon(t) \), and \( \max(\kappa \varepsilon_{\text{so}}, \tau \varepsilon_{\text{co}}) \) represents the maximum allowable size of the steady-state value of \( \varepsilon(t) \). Note that \( \varepsilon(0), \tau \), and \( \kappa \) should be properly chosen such that \(-\kappa \varepsilon(0) < e(0) < \tau \varepsilon(0)\).

3.2. Attitude Controller Design via Backstepping. After the error transformation (18), the altitude subsystem (5) is equivalent to

\[ \begin{align*}
\dot{z}_1 &= \eta_1 g_1 (x_1) x_2 - \eta_1 \nu_1, \\
\dot{x}_2 &= f_2 (x_2) + \eta_2 g_2 (x_2) x_3, \\
\dot{x}_3 &= \eta_3 g_3 (x_3) x_4, \\
\dot{x}_4 &= \eta_4 g_4 (x_4) \delta_E + f_4 (x_4),
\end{align*} \]

where \( 0 < \eta_{10} \leq \eta_1 \) and \( \eta_k = \eta_{k0} = 1, k = 2, 3, 4 \). The stabilization of the transformed system (19) is sufficient to guarantee the prescribed tracking performance of system (5).

Based on the backstepping approach, a trajectory tracking controller is designed for the dynamics model given in (19). The design procedure contains 4 steps, and the actual control law will be deduced at the last step. For convenience, let \( F_i(\xi_i) \) and \( \Omega_i \) denote the unknown function to be estimated by RBF NNs and the corresponding compact set in the \( i \)th step, respectively. Then by using Lemma 5, we have

\[ F_i(\xi_i) = W_i^* T(\xi_i + \Delta_i(\xi_i)) \]

where \( \psi_i(\xi_i) \) and \( \xi_i \) denote the vector valued function and the RBF NN input in the step with proper dimensions that are given below.

Step 1. Let \( z_1 \) given by (19) be the first error variable. Define \( z_2 = x_2 - u_1 \), where \( u_1 \) is the first virtual control signal. Then the derivative of \( z_1 \) can be expressed as

\[ \begin{align*}
\dot{z}_1 &= \eta_1 g_1 (x_1) z_2 + u_1 - \frac{1}{2} z_2 + F_1(\xi_1),
\end{align*} \]

where \( F_1(\xi_1) = (1/2)z_1 - \eta_1 \nu_1 \) and \( \xi_1 := [x_1, h_1, h_4, e_1]^T \subset \mathbb{R}^4 \). Since \( F_1(\xi_1) \) is unknown, we employ an RBF NN to approximate it on a compact set \( \Omega_{\xi_1} \subset \mathbb{R}^4 \). By properly choosing the basis function vectors we have

\[ F_1(\xi_1) = W_{i1}^* \psi_1(\xi_1) + \Delta_1(\xi_1), \]

where \( \epsilon_1 \) is a positive constant. With respect to the unknown optimal weight vector in (22), define

\[ \theta_1 = \frac{\|W_{i1}^*\|^2}{\eta_1 \delta_1}. \]

Besides, let \( \bar{\theta}_1 \) be the estimation of \( \theta_1 \) and \( \bar{\theta}_1 := \bar{\theta}_1 - \theta_1 \).

Consider the first Lyapunov function

\[ L_1 = \frac{1}{2} z_2^2 + \frac{\eta_{10} \theta_1}{2\lambda_1} \bar{\theta}_1^2. \]

Taking the time derivation of (24) yields

\[ \begin{align*}
\dot{L}_1 &= z_1 \eta_1 g_1 (x_1) z_2 + z_1 \eta_1 g_1 (x_1) u_1 - \frac{1}{2} z_2^2 \\
&+ z_2 W_{i1}^* \psi_1(\xi_1) + z_1 \Delta_1(\xi_1) + \frac{\eta_{10}}{\lambda_1} \bar{\theta}_1 \dot{\bar{\theta}}_1.
\end{align*} \]
Using Young's inequality and (23), it can be verified that
\[
z_i W_i^T \psi_i (\xi_i) \leq \frac{1}{2} z_i^2 \| W_i \| \| \psi_i (\xi_i) \| + \frac{1}{2} \lambda_i \psi_i (\xi_i) + \frac{1}{2} z_i^2 + \frac{1}{2} \lambda_i \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i). \tag{26}
\]
(26)

\[
z_i \Delta_i (\xi_i) \leq \frac{1}{2} z_i^2 + \frac{1}{2} \lambda_i^2.
\]
Thus, (25) can be rewritten as
\[
\dot{L}_1 \leq \eta \eta_\lambda g_1 (\xi_1) z_2 + z_i \eta_i g_i (\xi_i) u_i + \frac{1}{2} \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i), \tag{27}
\]
which suggests that we choose the first virtual control signal \( u_1 \) as
\[
u_1 = -c_1 z_1 - \frac{1}{2} \delta_1 z_1^2 \psi_i (\xi_i). \tag{28}
\]
Let
\[
\delta_1 = \frac{1}{2} \psi_i (\xi_i) - \lambda_i \sigma_i \delta_i, \quad \delta_1 (0) \geq 0, \tag{29}
\]
where \( c_1, \lambda_1, \) and \( \sigma_1 \) are positive design parameters. Then substituting (28) and (29) into (27), we get
\[
\dot{L}_1 \leq - \frac{1}{2} \eta \eta_\lambda g_1 (\xi_1) z_2 + z_i \eta_i g_i (\xi_i) u_i + \frac{1}{2} \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i), \tag{30}
\]
where \( F_i (\xi_i) = f_i (\xi_i) + \eta_\lambda g_1 (\xi_1) z_2 + (1/2) z_i - u_i \) and \( \xi_i = [\xi_i, \eta_\lambda, \ldots, \eta_\lambda, \delta_1, \delta_2, \delta_3, \delta_4, \xi_i] \in \Omega_i. \) Consider the virtual Lyapunov function
\[
L_i = L_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i), \tag{32}
\]
where \( \lambda_i \) is a positive design parameter, \( \delta_i = \delta_i - \delta_i \) with \( \delta_i = \| W_i \| \| \psi_i \| / \eta_\lambda \eta_i. \) By taking the time derivative of (32), we have
\[
\dot{L}_i = \dot{L}_{i-1} - \eta_\lambda g_1 (\xi_1) z_i - z_i + z_i \eta_i g_i (\xi_i) z_i + z_i \eta_i g_i (\xi_i) u_i - \frac{1}{2} z_i^2 + z_i F_i (\xi_i) + \frac{1}{2} \eta \eta_\lambda \psi_i (\xi_i) \tag{33}
\]
Similar to (26), we have
\[
\dot{L}_i \leq \dot{L}_{i-1} - \eta_\lambda g_1 (\xi_1) z_i - z_i + z_i \eta_i g_i (\xi_i) z_i + z_i \eta_i g_i (\xi_i) u_i + \frac{1}{2} \eta \eta_\lambda \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i) \tag{34}
\]
Choose the control signal
\[
u_i = -c_1 z_1 - \frac{1}{2} \delta_1 z_1^2 \psi_i (\xi_i). \tag{35}
\]
where \( \delta_i \) is updated by
\[
\delta_i = \frac{1}{2} \psi_i (\xi_i) - \lambda_i \sigma_i \delta_i, \quad \delta_i (0) \geq 0, \tag{36}
\]
with \( c_i, \lambda_i, \) and \( \sigma_i \) being positive design parameters. Substituting (35), (36), and (30) into (34), we get
\[
\dot{L}_i \leq - \frac{1}{2} \eta \eta_\lambda c_i z_i^2 + z_i \eta_i g_i (\xi_1) z_i + \sum_{i=1}^{n} \psi_i (\xi_i) + \psi_i (\xi_i), \tag{37}
\]
Step 4. The time derivative of \( z_i \) is
\[
\dot{z}_i = - \eta_3 g_3 (\xi_i) z_i + \eta_4 g_4 (\xi_i), \tag{38}
\]
where \( F_4 (\xi_i) = f_4 (\xi_i) + \eta_3 g_3 (\xi_i) z_i + (1/2) z_i - u_i \) and \( \xi_i = [\xi_i, h_1, \ldots, h_2, \delta_1, \delta_2, \delta_3, \delta_4] \in \Omega_i. \) Consider the virtual Lyapunov function
\[
L_i = L_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \psi_i (\xi_i) + \frac{1}{2} \psi_i (\xi_i) \tag{39}
\]
where \( \lambda_i \) is a positive design parameter and \( \delta_i = \delta_i - \delta_i \) with \( \delta_i = \| W_i \| \| \psi_i \| / \eta_\lambda \eta_i. \) Differentiating (39) we have
\[
\dot{L}_i = \dot{L}_{i-1} - \eta_3 g_3 (\xi_i) z_i + \eta_4 g_4 (\xi_i), \tag{40}
\]
Similarly to (26), (40) can be rewritten as
\[
\dot{L}_i \leq \dot{L}_{i-1} - \eta_3 g_3 (\xi_i) z_i + \eta_4 g_4 (\xi_i), \tag{41}
\]
Choose the control signal
\[
u = -c_3 z_3 - \frac{1}{2} \delta_3 z_3^2 \psi_i (\xi_i). \tag{42}
\]
where $\hat{\theta}_4$ is updated by
\[
\dot{\hat{\theta}}_4 = \frac{\lambda_4}{2} z_4^2 \psi_4^T (\xi_4) \psi_4 (\xi_4) - \lambda_4 \sigma_4 \hat{\theta}_4, \quad \hat{\theta}_4 (0) \geq 0, \tag{43}
\]
with $c_4$, $\lambda_4$, and $\sigma_4$ being positive design parameters. Substituting (42), (43), and (37) into (41), we arrive at
\[
\dot{L}_4 \leq - \sum_{k=1}^{4} c_k \eta_k b_k z_k^2 - \sum_{k=1}^{4} \eta_k b_k \sigma_k \vec{b}_k \vec{b}_k^T + \frac{1}{2} \sum_{k=1}^{4} \left( \frac{1}{2} + \frac{1}{2} \epsilon_k^2 \right). \tag{44}
\]

Remark 7. The RBF NNs are used to compensate for the complex and uncertain terms to solve the problem of controller complexity, and the repeated derivation of virtual control signal $u_t$ can be avoided. Compared with the neural based control [16, 21], in each design step, by using the estimation of the norm of the NNs weights, only one parameter needs to be updated online; therefore the design procedure can be greatly simplified and the computational burden is greatly reduced. Moreover, the lower bound of the virtual control coefficient $\eta_i g_i$ is used to avoid the singularity problem without any additional effort.

Remark 8. Since the approximation ability of RBF NNs is on a compact set, we can only guarantee the semiglobal stability of the control scheme.

Theorem 9. Consider system (5) under the Assumptions 2 and 4, with the error transformation (17), the virtual control signals (28) and (35), the control law (42), and the adaptive laws (29), (36), and (43). Then all closed-loop signals are uniformly bounded and the prescribed tracking performance (12) can be guaranteed.

Proof. Using the following facts:
\[
- \vec{b}_i \vec{b}_i^T \leq \frac{1}{2} \vec{b}_i^2 + \frac{1}{2} \vec{b}_i^2, \quad i = 1, \ldots, 4, \tag{45}
\]
we rewrite (44) as
\[
\dot{L}_4 \leq - \sum_{k=1}^{4} c_k \eta_k b_k z_k^2 - \sum_{k=1}^{4} \eta_k b_k \sigma_k \vec{b}_k \vec{b}_k^T + \zeta, \tag{46}
\]
where
\[
\zeta = \sum_{k=1}^{4} \frac{1}{2} \eta_k b_k \sigma_k \vec{b}_k \vec{b}_k^T + \frac{1}{2} \sum_{k=1}^{4} \left( \frac{1}{2} + \frac{1}{2} \epsilon_k^2 \right). \tag{47}
\]
Let
\[
h = \min \left\{ 2 \eta_k b_k \epsilon_k, \lambda_i \sigma_i, \quad i = 1, \ldots, 4 \right\}. \tag{48}
\]
Then we have
\[
\dot{L}_4 \leq - h \dot{L}_4 + \zeta. \tag{49}
\]
Solving (49) gives
\[
0 \leq L_4 (t) \leq \frac{\zeta}{h} + \left( L_4 (0) - \frac{\zeta}{h} \right) e^{-ht}, \quad t \geq 0, \tag{50}
\]
which implies that $L_4$, $z_t$, $\hat{\theta}_4$, and $\hat{\theta}_4$ are bounded. Since $z_t$ is bounded, according to the error transformation of (15), to (17) we can obtain that $- \kappa = S (z) < \tau$; as a result, we have $- \kappa \varepsilon (t) < \varepsilon (t) < \tau (t)$; that is, the prescribed tracking performance is guaranteed. This completes the proof. \hfill $\square$

3.3. Velocity Controller Design via Dynamic Inversion. The velocity subsystem of (1) can be rewritten as follows:
\[
\dot{V} = f_V (\vec{x}_4, V) + g_V (\vec{x}_4, V) \beta_c, \tag{51}
\]
where $f_V (\vec{x}_4, V)$ and $g_V (\vec{x}_4, V)$ are unknown nonlinear functions $g_V (\vec{x}_4, V) \geq b_c > 0$. Then define the velocity tracking error as $e_V = V - V_d$. According to (17) and (18) we obtain
\[
\dot{\varepsilon}_V = - \eta V_u V + \eta V f_V (\vec{x}_4, V) + \eta V g_V (\vec{x}_4, V) \beta_c. \tag{52}
\]
We assume that $0 < \eta V \leq \eta V$. The transformed system dynamics of (52) can be rewritten as
\[
\dot{\varepsilon}_V = \eta V g_V (\vec{x}_4, V) \beta_c - \frac{1}{2} z_2 + f_V (\vec{x}_4), \tag{53}
\]
where $f_V (\vec{x}_4) = (1/2) z_2 - \eta V u_V + \eta V f_V (\vec{x}_4, V)$ and $\xi_V := [\vec{x}_4, V, V_d, e_V]^T \in \Omega_c$. Since $F_V (\xi_V)$ is an unknown nonlinear function, we use an RBF NN to approximate it:
\[
F_V (\xi_V) = W_{iV}^T \psi_V (\xi_V) + \Delta V (\xi_V), \quad | \Delta V (\xi_V) | \leq \epsilon_V. \tag{54}
\]
The control law and the adaptive update law are designed as follows:
\[
\beta_c = - \epsilon V z_V - \frac{1}{2} z_V \dot{V} \psi_V^T (\xi_V) \psi_V (\xi_V) \tag{55}
\]
\[
\dot{\delta}_V = \frac{\lambda V}{2} z_V \dot{V} \psi_V^T (\xi_V) \psi_V (\xi_V) - \lambda V \sigma V \dot{\delta}_V, \tag{56}
\]
where $\delta_V$ is the estimate of $\delta_V = \| W_{iV} \|^2 / \eta V b_i V$, and $\epsilon_V$, $\eta V$, and $\sigma V$ are positive design parameters. Consider the Lyapunov function
\[
L_V = \frac{1}{2} z_V^2 + \frac{\eta V b_i V \delta_V^2}{2 \lambda V}. \tag{57}
\]
Differentiating $L_V$ we have
\[
\dot{L}_V \leq z_V \eta V b_i V \beta_c + \frac{\eta V b_i V \delta_V^2}{2} z_V \dot{V} \psi_V^T (\xi_V) \psi_V (\xi_V)
\]
\[
+ \lambda V \delta_V \left( \dot{\delta}_V - \frac{\lambda V}{2} z_V \dot{V} \psi_V^T (\xi_V) \psi_V (\xi_V) \right) + \frac{1}{2}
\]
\[
+ \frac{1}{2} \epsilon_V^2. \tag{58}
\]
Substituting (55) into (57) then
\[
\dot{L}_V \leq - \epsilon V \eta V b_i V \delta_V + \eta V b_i V \sigma V \delta_V \delta_V + \frac{1}{2} + \frac{1}{2} \epsilon_V^2, \tag{59}
\]
which implies that $L_4$, $z_t$, $\hat{\theta}_4$, and $\hat{\theta}_4$ are bounded. Since $z_t$ is bounded, according to the error transformation of (15), to (17) we can obtain that $- \kappa = S (z) < \tau$; as a result, we have $- \kappa \varepsilon (t) < \varepsilon (t) < \tau (t)$; that is, the prescribed tracking performance is guaranteed. This completes the proof. \hfill $\square$
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with inequation
\[ -\overline{\gamma}_V \overline{\gamma}_V \leq -\frac{1}{2} \overline{\gamma}_V^2 + \frac{1}{2} \overline{\gamma}_V^2, \tag{59} \]
and then
\[ L_V \leq -c_V \eta_V \lambda_V z_V^2 + \frac{1}{2} \eta_V \lambda_V \sigma_V \overline{\gamma}_V^2 + \frac{1}{2} \eta_V \lambda_V \sigma_V \overline{\gamma}_V^2 + \frac{1}{2} \epsilon_V \overline{\gamma}_V^2 \leq -\kappa_V L_V + \mu_V, \tag{60} \]
where
\[ \kappa_V = \min (2c_V \eta_V \lambda_V, \lambda_V \sigma_V), \]
\[ \mu_V = \frac{1}{2} \eta_V \lambda_V \sigma_V \overline{\gamma}_V^2 + \frac{1}{2} \epsilon_V \overline{\gamma}_V^2. \]

Solving (60) gives
\[ 0 \leq L_V (t) \leq \frac{\mu_V}{\kappa_V} + \left( L_V (0) - \frac{\mu_V}{\kappa_V} \right) e^{-\kappa_V t}, \quad t \geq 0. \tag{62} \]

It is clear that \( z_V, \overline{\gamma}_V, \) and \( \overline{\gamma}_V \) are bounded. Owning that \( z_V \) is bounded, together with the error transformation of (11) into (17), implies that the prescribed tracking performance is guaranteed.

4. Simulation Results

In this section, the numerical simulation results are presented to show the performance of the control scheme. Simulation of the HFV model is conducted for trimmed cruise conditions of 110000 ft and Mach 15. The parameters of simulation model are taken from [16, 21]. The control objective is to track the step change of 100 ft/s in airspeed and 2000 ft in altitude. Linear command filters are used to generate the differentiable commands:

\[ h_d = \frac{a_{n1}^2 \omega_n^2}{(p + \omega_n)^2 (p^2 + 2\omega_n p + \omega_n^2)}, \]
\[ V_d = \frac{a_{n1}^2}{p^2 + 2\omega_n p + \omega_n^2}. \tag{63} \]

where \( p \) is Laplace operator, \( \omega_{n1} = 0.5, \omega_{n2} = 0.3, \) and \( \xi_1 = 0.95. h_c \) and \( V_c \) are step commands. The inputs of the RBF NNs are \( \xi_1, \xi_2, \xi_3, \) and \( \xi_4, \) and the nonlinear functions are approximated with width \( \phi = 1. \) The initial values of the adaptive laws are \( \overline{\gamma}_1(0) = \overline{\gamma}_2(0) = \overline{\gamma}_3(0) = \overline{\gamma}_4(0) = 0. \) In addition we choose the design parameters \( c_1 = 0.04, c_2 = 150, c_3 = 120, c_4 = 100, c_5 = 0.5, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.01, \lambda_4 = 0.01, \lambda_5 = 0.01, \sigma_1 = 0.1, \sigma_2 = 0.1, \sigma_3 = 0.1, \sigma_4 = 0.1, \) and \( \sigma_5 = 0.1. \) The parameters of performance functions are given by \( \epsilon_{h0} = 30, \epsilon_{\nu \nu} = 5, h_0 = 0.2, \epsilon_{V0} = 30, \epsilon_{V \nu} = 2, \nu_0 = 0.5, \) and \( \kappa = \tau = 1. \) The initial states are chosen as \( V = 15060 \) ft/s, \( h = 110000 \) ft, \( \nu = 0 \) rad, \( \theta = 0.0315 \) rad, and \( q = 0 \) rad/s.

The simulation results are presented in Figures 1–10. The responses to 100-ft/s step-velocity and 2000-ft step-altitude command in trimmed condition are depicted in Figures 1–3 and Figures 4–6, respectively. From Figures 2 and 5, we see that the tracking errors performance are guaranteed. Figures 7–10 show the simulation results of altitude tracking with square wave trajectory. From the results of simulations, the maximum value of terms \( Tsina/(mV) \) is \( 3 \times 10^{-5}; \) it is much smaller than the term \( L/(mV) \) whose minimum value is \( 1 \times 10^{-3} \) and the maximum value of flight path angle \( \gamma \) is less than \( 0.012 \) rad. Therefore, Assumption 1 is reasonable.

5. Conclusion

An adaptive neural control scheme has been proposed for a class of longitudinal dynamics of a generic hypersonic
Figure 3: Throttle setting $\beta$.

Figure 4: The altitude tracking.

Figure 5: Altitude tracking error $e_h$ and the prescribed error bounds.

Figure 6: Elevator deflection $\delta_E$.

Figure 7: Altitude tracking.

Figure 8: Altitude tracking error $e_h$ and the prescribed error bounds.
flight vehicle. We have shown that, by using a new constraint variable, the prescribed tracking performance can be achieved. The unknown nonlinear functions associated with each recursive step of backstepping control were approximated by using RBF NNs. For each design step, only one parameter needs to be updated online. Thus the explosion of the complex problem in backstepping control scheme and the computational burden can be greatly reduced. Numerical simulations revealed that the tracking error clearly satisfies the prescribed performance specification and verified the proposed design scheme. Currently, we assume that all of the system states are available and the controller is based on state feedback. However, some states cannot be obtained in some circumstances, especially when the sensor fault occurs. As a result, future work will be focused on output feedback control law design.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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