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Research Article

The Effect of the Integrated Service Mode and Travel Time Uncertainty on Taxis Network Equilibrium

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This paper aims to discuss the trip mode choice problem by using cumulative prospect theory (CPT) rather than utility maximization from the network uncertainty perspective and evaluates the effect of the integrated service mode on taxi network equilibrium. The integrated service mode means taxis either are actively moving through traffic zones to pick up customers (cruising mode) or are queued at the center of a zone waiting for customers (dispatch mode). Based on this, CPT models are adopted to analyze the choice of customers' trip mode. The travel time uncertainty of the network and the applicability of CPT are considered first, and the Nested Logit model was used to complete the trip mode split problem. Further, several relevant relationships including supply-demand equilibrium, network conditions, taxi behavior, and customer behavior perspectives were analyzed with respect to the integrated mode. Moreover, a network equilibrium model was established and its algorithm was designed. Finally, this paper presented a numerical example and discussed the taxi network equilibrium's characteristic after introducing the integrated service mode.

1. Introduction

Public transport via taxi has increasingly become an important service to modernity because of its personalized and flexible service, comfort, and available speed among other things. Despite its growing popularity, problems have emerged in the practical operation of the taxi market. For instance, it is well known that automobiles used in the taxi service are responsible for consuming a considerable amount of energy. This energy consumption is magnified under the conditions where there is a higher vacancy ratio in the taxi market. The pollution produced as a by-product of this industry is therefore much worse and seriously threatens energy sustainability as well as posing a serious risk to the environment. Based on this, two types of measures are needed in order to alleviate the negative impacts of such an important transportation service. To address the environmental impact of taxi trip, both the alternatively fueled vehicles and cleaner fuels have been proposed to provide taxi service [1]. Management policies which effectively advance the operational efficiency needed to be explored have also been proposed as a means of curbing the concerns associated with the industry. This investigation seeks to elucidate ways to improve the level of service and the operational situation through improvement to the management policies.

Different taxi policies have been heavily researched since the 1970s with current policies in the taxi field primarily concentrated on market regulations (i.e., entry restrictions and price control). Douglas, who first studied the aggregated model as a precursor in the realm of taxi networks, introduced the realm to economists that have proceeded to investigate the characteristics of different taxi markets, enabling them to understand the equilibrium between supply and demand in regulated markets [2–7]. Despite all the research, it is known and accepted that the taxi market is not the idealized market for conventional economic analyses since the spatial features of the road network are not considered [8]. From this standpoint, Yang and Wong et al. devised a series of models considering fluctuations in supply and demand over variations in time and space of Hong Kong for 1998–2011.

In a previous attempt, Yang et al. sought to characterize taxi movements in a road network for a given origin-destination demand pattern, discussing the relationship between customer demand and taxi utilization by analyzing the impact of congestion, various markets, multiple user classes, stochastic travel time, and price formulation among other things [9–14]. Yang et al. [15] analyzed bilateral searching and meeting relationships between taxis and customers and studied fare control in competitive taxi markets. Further, by considering Pareto efficiency, Yang et al. [16] were able to evaluate quality of service and taxi utilization.

Despite all of the research over the years, models discussed typically account for the cruising service mode when determining regulations and the equilibrium problem; limited attention has been paid to other modes. In one of the first studies discussing multiple modes, Arnott [17] suggested that the dispatch mode was more effective in small cities and the cruising mode was more effective in large cities. von Massow and Canbolat [18] developed a model used to simulate the dispatch mode. One of the first to suggest this, Maria Salanora et al. [19], asserted that the integrated taxi service mode (e.g., cruising and dispatch modes) would be better in big cities.

Recently, in terms of prospect theory which assumes that lotteries are evaluated in a two-step process: an initial phase of editing and a subsequent phase of evaluation, a lot of achievements have been also published [20]. van de Kaa [21, 22] investigated the application of prospect theory and expected utility theory (EUT) where it is assumed that decision maker's attitude toward risk can be rationalized by an expected utility function in trip choice behavior and concluded that, by extending the prospect theory further, it would be suitable to describe traveler behavior. Supporting this assumption, Li and Hensher [23] demonstrated that prospect theory was more suitable for researching travelers decision by analyzing the application of prospect theory in physics, behavior economics, and transportation fields and finally summarized the application constraints of prospect theory in transportation field.

By constructing and analyzing a supply-demand model, the impact factor on the market is evaluated as well as the taxis. The aim was to explore the essential characteristics of the taxi market and provide effective, valuable management policies based on the findings.

In this model, the expected utility theory was replaced with cumulative prospect theory to describe the split in customer trip mode. Cumulative prospect theory is regarded as a bounded rationality model reflective of the uncertainty and complication inherent to real road networks, expressing a decision maker's attitude toward risk. Based on this, we consider the coexistence of cruising and dispatch modes to generate the model used for this paper. The equilibrium model describes the customer-search behavior of vacant-taxi drivers and the interactions between cruising and dispatch modes. Finally, parameters, such as customer-waiting time and taxi-searching time, were identified and evaluated based on variation in traffic zones and fleet sizes.

The rest of this paper is structured as follows: Section 2 is an analysis of the applicable characteristics of cumulative prospect theory and establishes the trip mode model for travelers based on CPT. Section 3 describes the relationship between cruising mode and dispatch modes presenting a supply-demand equilibrium relationship with customer demand variation in taxi market. Section 4 promotes a joint taxi network model of trip distribution and assignment. Section 5 explains the detailed steps of the algorithm developed to establish the supply-demand equilibrium mode. Section 6 describes the network conditions and parameters utilizing a numerical example and discusses the characteristic of equilibrium state with respect to the results. Finally, the last section makes a conclusion and proposes future extensions for the current taxi model.

2. The Trip Mode Choice Model Based on CPT

It was assumed that there are two modes of taxi service available (i.e., cruising and dispatch) as well as other transit modes (bus, rail system, etc.) for customers on a given network. In terms of cruising mode, it means that taxi cruises on roadway to pick up next customers. In terms of dispatch mode, its implementation effect is greatly dependent on the installation rate of in-vehicle navigation system. However, there is a smaller popularizing rate of intelligent system in most current taxi markets. Thus, this paper adopts a simple dispatch policy which means that taxis were required to queue at the center of zone or the gravity point of zone to search for next customers by means of the achievements of von Massow and Canbolat [18]. Based on this assumption, the trip mode choice process may be broken down as follows.

From Figure 1, it can be seen that subsets of alternatives whose properties are similar are grouped in hierarchies or nests. The multinominal logit model (MNL) is used to estimate the probability of trip mode choice for the alternatives of the lower nest, including both cruising and dispatching service modes. And furthermore, at the higher nest, the MNL model consisting of composite taxi mode and other transit modes can be also estimated. Thus, the Nested Logit model is suitable to be used to study the trip mode choice problem described by this paper.

2.1. The Descriptions of Travel Cost. The total travel time from zone i to zone j of customers involves the travel time on roadway and the waiting time which may be expressed as

$$\begin{aligned} t_c &= t_c^{ij} + W_i^{cc}, \\ t_d &= t_d^{ij} + W_i^{dc}, \\ t_T &= t_T^{ij} + W_i^{Tc}. \end{aligned} \tag{1}$$

In the preceding equations t_c is the total time of cruising mode, t_d is the total time of dispatch mode, and t_T is the total time of other transit modes, while t_c^{ij} and t_d^{ij} are the travel time of taxis and t_T^{ij} is the travel time of other forms of transit from zone i to zone j. W_i^{cc} is the waiting time of customers that choose cruising mode in zone i and W_i^{dc} is the waiting time of customers choosing cruising mode in zone i, and W_i^{Tc} refers to the waiting time of customers choosing other modes of transit in zone i.

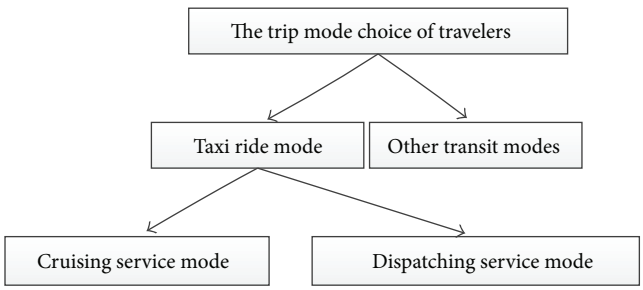


Figure 1: The trip mode choice process of travelers.

With U defined as trip cost the variable $U(t_l)$ indicates the cost brought by the perception time of mode l while $U(t_{\text{expected}})$ denotes the cost of the expected travel time relevant to a reference point and can be expressed as follows:

$$\begin{aligned} U(t_l) &= \theta_{\text{trip}} t_l, \\ U(t_{\text{expected}}) &= \theta_{\text{trip}} t_{\text{expected}}, \\ x &= U(t_{\text{expected}}) - U(t_l). \end{aligned} \tag{2}$$

In the model, x refers to the gain obtained by trip mode if $U(t_{\text{expected}}) \geq U(t_l)$, while x is the loss obtained by trip mode if $U(t_{\text{expected}}) < U(t_l)$.

A set reference point makes distinguishing between gain and loss of a trip with respect to CPT possible for prospective customers. Customers typically perceive gain or loss by comparing trip cost with the reference point cost. In this context, it was assumed that a reference point may be related to the actual travel time experienced, typically about 30 minutes, and was thus used for this study [20].

2.2. Cumulative Prospect Theory. CPT refers to the application of cumulative function to gains and losses. An uncertain prospect f is a function that assigns to each state of S a corresponding outcome. The prospect values are required to be arranged in increasing order to calculate the cumulative function. The prospect f can be regarded as a sequence of pairs (x_l, A_l), which yields x_l if A_l occurs, and A_l is a partition of S. The variable f^+ indicates the positive outcomes of f and f^- does the negative outcomes. For $f = (x_l, A_l)$ and $-m \leq l \leq n$, the relevant expressions are as follows:

$$V(f) = V(f^+) + V(f^-), \tag{3}$$

$$V(f^+) = \sum_{l=1}^{n} \pi_l^+(f^+)\, v(x_l), \tag{4}$$

$$V(f^-) = \sum_{l=-m}^{-1} \pi_l^-(f^-)\, v(x_l), \tag{5}$$

where $V(f^+)$ is the projected gains and $V(f^-)$ is the projected losses. Further, if A_l is given by a probability distribution $P(A_l) = P_l$, f can be regarded as an uncertain prospect (x_l, P_l). Here, $\pi_l^+(f^+)$ can be expressed as

$$\pi_n^+ = w^+(p_n) \quad l = n, \tag{6}$$

$$\pi_l^+ = w^+(p_l + \cdots + p_n) - w^+(p_{l+1} + \cdots + p_n) \quad 0 \leq l \leq n-1. \tag{7}$$

With the assumption of continuous traffic flow on a given roadway the distribution function can be introduced into this model as shown in

$$\begin{aligned} p_l + \cdots + p_n &= 1 - (p_{-m} + \cdots + p_{l-1}) = 1 - F(x_{l-1}), \\ p_{-m} + \cdots + p_l &= F(x_l). \end{aligned} \tag{8}$$

When combined with (8), (7) can be expressed as follows:

$$\begin{aligned} \pi_l^+ &= w^+(1 - F(x_{l-1})) - w^+(1 - F(x_l)) \\ &= -\frac{dw^+(1 - F(x))}{dx} dx. \end{aligned} \tag{9}$$

$\pi_l^-(f^-)$ can be also defined by

$$\pi_{-m}^- = w^-(p_{-m}) \quad l = -m, \tag{10}$$

$$\pi_l^- = w^-(p_{-m} + \cdots + p_l) - w^-(p_{-m} + \cdots + p_{l-1}) \quad 1 - m \leq l \leq 0. \tag{11}$$

From (8), (11) can also be expressed as

$$\pi_l^- = \frac{dw^-(F(x))}{dx} dx. \tag{12}$$

Thus cumulative prospect theory may be defined as follows:

$$\begin{aligned} V &= V^+ + V^- \\ &= \int_{x_0}^{\infty} -\frac{dw^+(1 - F(x))}{dx} v(x)\, dx \\ &\quad + \int_{-\infty}^{x_0} \frac{dw^-(F(x))}{dx} v(x)\, dx, \end{aligned} \tag{13}$$

where w^+ and w^- are the functions for gains and losses, respectively [24], and are defined by

$$\begin{aligned} w^+(p) &= \frac{p^r}{\left[p^r + (1-p)^r\right]^{1/r}}, \\ w^-(p) &= \frac{p^\delta}{\left[p^\delta + (1-p)^\delta\right]^{1/\delta}}. \end{aligned} \tag{14}$$

From (4) and (5), it is known that $v(x)$ is defined as the value function with the following formulation:

$$v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\beta & x < 0, \end{cases} \tag{15}$$

where x is the gain or loss; the variable x is defined as a gain if $x > 0$ and a loss if $x < 0$.

In these formulations, the parameters are $\alpha = \beta = 0.88$, $\lambda = 2.25$, $r = 0.61$, and $\delta = 0.69$ [24].

Based on this, the cumulative prospect theory of trip modes can be obtained so that a customer may choose the trip mode based on maximal cumulative prospect value.

2.3. The Relationship between Trip Mode Choices. Including cruising mode, dispatch mode, and other transit modes within the network the O-D pair (V_{ij}^{cc}, V_{ij}^{dc}, and V_{ij}^{T}) is used to indicate the cumulative prospect values, respectively.

By adopting the Nested Logit model, the probability P_{ij}^{cc} that a customer originating in zone i chooses cruising mode to travel to zone j is given as follows:

$$P_{ij}^{cc} = \frac{\exp\left\{-\ln\left[\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)\right]\right\}}{\exp\left(V_{ij}^{T}\right) + \exp\left\{-\ln\left[\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)\right]\right\}} \cdot \frac{\exp\left(\theta^f V_{ij}^{cc}\right)}{\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)}. \quad (16)$$

When the parameter θ^f is nonnegative, it may be used to reflect the degree of uncertainty in service from customers' perspective.

Meanwhile, the probability P_{ij}^{dc} of customers choosing dispatch mode is given as follows:

$$P_{ij}^{dc} = \frac{\exp\left\{-\ln\left[\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)\right]\right\}}{\exp\left(V_{ij}^{T}\right) + \exp\left\{-\ln\left[\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)\right]\right\}} \cdot \frac{\exp\left(\theta^f V_{ij}^{dc}\right)}{\exp\left(\theta^f V_{ij}^{cc}\right) + \exp\left(\theta^f V_{ij}^{dc}\right)}. \quad (17)$$

Therefore, the demand of customers choosing cruising service can be defined as follows:

$$Q_{ij}^{cc} = Q_{ij}^{c} P_{ij}^{cc}, \quad (18)$$

where Q_{ij}^{cc} is the customer demand for cruising mode and Q_{ij}^{c} is the total customer demand for taxis in a network from zone i to zone j.

Similarly, the customer demand for dispatch service may be defined as

$$Q_{ij}^{dc} = Q_{ij}^{c} P_{ij}^{dc}, \quad (19)$$

where Q_{ij}^{dc} is the customer demand for dispatch mode. Based on this, the total customer demand for taxi service is expressed by the following formula:

$$Q_{ij}^{\mathrm{tc}} = Q_{ij}^{cc} + Q_{ij}^{dc}, \quad (20)$$

where Q_{ij}^{tc} is the total customer demand for taxi service from zone i to zone j.

3. Characteristics of Taxi Movement in a Network

3.1. Basic Assumptions of the Model. The basic assumptions of the model established in this paper are described as follows:

(1) Assume that all occupied taxis follow the shortest path choice principle.

(2) Once a taxi is occupied in zone i, a driver will choose the shortest route to go to zone j.

(3) Given an O-D pair, (i, j), for example, after a taxi has delivered customers to zone j, the taxi driver either stays in the same zone or moves to other zones in search of the next customer in an attempt at minimizing search time.

(4) Based on the context, all taxis in a network are divided into one of three types: occupied taxis from zone i to zone j, vacant taxis moving from zone j to zone i, or taxis searching for customers in zone i.

(5) The vacant taxi driver in zone i can choose between cruising mode and dispatch mode to provide service for customers.

In other words, taxis on roadways can be divided into two types (occupied or vacant) and beginning to search for customers after arriving at zone i; those taxis can decide to make service mode choice.

3.2. Introduction of Network Model Variables. Suppose that a road network with a taxi market is in a state of supply-demand equilibrium, defined as $G(V, A)$, where V is the set of nodes and A is the set of links. I and J are the set of origination and destination zones, respectively. $t(v_a)$ is expressed as the travel time on link a ($a \in A$) and v_a is expressed as link flow including taxis and sources of traffic flow. Here, suppose that a taxi is only occupied by a customer.

Assume that t_{ij}^{k} is the travel time on route k between O-D pair (i, j) and is expressed as follows:

$$t_{ij}^{k} = \sum_{a \in A} t\left(v_a\right) \delta_{ij}^{ak}. \quad (21)$$

Further, $\delta_{ij}^{ak} = 1$ if route k between O-D pair (i, j) uses link a and 0 otherwise; k is included in the set R_{ij} which is the set of routes between O-D pair (i, j).

Based on this context, t_{ij} is considered as the travel time via the shortest route from zone i to zone j:

$$t_{ij} = \min\left(t_{ij}^{k}\right) \quad k \in R_{ij}. \quad (22)$$

3.3. The Choice Relationship of Taxi Service Modes. In accordance with basic assumptions above, the probability that

a vacant taxi arriving at zone i chooses cruising mode to search for the next customer is given as follows:

$$p^{ct} = \frac{\exp\left(-\theta^t w_i^{ct}\right)}{\exp\left(-\theta^t w_i^{ct}\right) + \exp\left(-\theta^t w_i^{dt}\right)}, \tag{23}$$

where p^{ct} is the probability of taxis choosing cruising mode, while θ^t is a nonnegative parameter, w_i^{ct} is the search time of taxi drivers that chose the cruising mode, and w_i^{dt} is the search time of taxi drivers that chose dispatch mode.

Similarly, the probability of vacant taxis choosing dispatch mode to search for the next customer is given as follows:

$$p^{dt} = \frac{\exp\left(-\theta^t w_i^{dt}\right)}{\exp\left(-\theta^t w_i^{ct}\right) + \exp\left(-\theta^t w_i^{dt}\right)}, \tag{24}$$

where p^{dt} is the probability of taxis choosing dispatch mode.

3.4. Taxi Service to Time Relationship. There is a given demand matrix OD and the 1 h interval is studied. This paper assumes that the total number of taxis in a network including occupied and vacant ones is Num and it is a constant.

The total number of occupied taxis is given as $\sum_{i\in I}\sum_{j\in J} T_{ij}^{\text{ot}} t_{ij}$, with T_{ij}^{ot} defined as the number of occupied taxis in the studied 1-h interval from zone i to zone j; "ot" refers to occupied taxis.

Similarly, the total number of vacant taxis can be expressed as follows:

$$\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} t_{ji} + \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{ct} w_i^{ct} + \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{dt} w_i^{dt}, \tag{25}$$

where T_{ji}^{vt} is the number of vacant taxis in the studied 1-h interval from zone j to zone i; "vt" refers to vacant taxis. In the above relationship, $\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} t_{ji}$ is the number of vacant taxis on roadway; $\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{ct} w_i^{ct}$ is the number of vacant taxis choosing cruising mode to search for customer in zone i; $\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{dt} w_i^{dt}$ is the number of vacant taxis choosing dispatch mode in zone i.

Therefore, the total number of taxis in a network can be expressed as

$$\begin{aligned}\sum_{i\in I}\sum_{j\in J} T_{ij}^{\text{ot}} t_{ij} + \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} t_{ji} + \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{ct} w_i^{ct} \\ + \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} p^{dt} w_i^{dt} = \text{Num}.\end{aligned} \tag{26}$$

3.5. Behavior Model of Taxi Drivers. The expected search time of drivers in one zone is assumed to be distributed in accordance with the Gumbel density function. Here w_i^t is the total searching time of vacant taxis in zone i and expressed as follows:

$$w_i^t = p^{ct} w_i^{ct} + p^{dt} w_i^{dt}. \tag{27}$$

The probability that a vacant taxi originating in zone j eventually meets a customer in zone i is given by

$$P = \frac{\exp\left\{-\theta\left(t_{ji} + p^{ct} w_i^{ct} + p^{dt} w_i^{dt}\right)\right\}}{\sum_{m\in I}\exp\left\{-\theta\left(t_{jm} + p^{ct} w_m^{ct} + p^{dt} w_m^{dt}\right)\right\}}, \quad i \in I,\ m \in I,\ j \in J, \tag{28}$$

where P is the probability that a taxi meets the next customer in zone i after having delivered a customer to zone j, θ is a nonnegative parameter and its explanation is referred to by Wong et al. [13].

3.6. The Relationship between Customer Wait Time and Taxi Search Time. According to Yang et al. [15], the function relationship between customer and taxi wait time can be described by applying the Cobb-Douglas model describing customer-taxi meeting function which is given by

$$W_i^{cc} = \frac{1}{0.01\sum_{j\in J} T_{ji}^{\text{vt}} p^{ct} w_i^{ct}}, \tag{29}$$

$$W_i^{dc} = \frac{1}{10^4\sum_{j\in J} T_{ji}^{\text{vt}} p^{dt} w_i^{dt}}, \tag{30}$$

where W_i^{cc} is the average waiting time of customers choosing cruising mode in zone i and W_i^{dc} is the average waiting time of customers choosing dispatch mode in zone i.

3.7. The Supply-Demand Equilibrium Relationship. When in an equilibrium state, the number of vacant taxis in a network should meet all customers' demands in their respective origination zones. That is to say, taxi service is available to every customer. Therefore, the supply-demand equilibrium relationship in a network is given as follows:

$$\begin{aligned}\sum_{i\in I} T_{ji}^{\text{vt}} = Q_j^{\text{tc}} = \sum_{i\in I} Q_{ij}^{\text{tc}}, \quad j \in J, \\ \sum_{j\in J} T_{ji}^{\text{vt}} = Q_i^{\text{tc}} = \sum_{j\in J} Q_{ij}^{\text{tc}}, \quad i \in I.\end{aligned} \tag{31}$$

In terms of cruising mode, its supply-demand equilibrium relationship is given by

$$\sum_{j\in J} Q_{ij}^{cc} = \sum_{j\in J} T_{ji}^{\text{vt}} p^{ct}. \tag{32}$$

In terms of dispatch mode, its supply-demand equilibrium relationship is expressed by

$$\sum_{j\in J} Q_{ij}^{dc} = \sum_{j\in J} T_{ji}^{\text{vt}} p^{dt}. \tag{33}$$

4. A Mathematical Programming Model

4.1. The Model Descriptions. A joint taxi network model of trip distribution and assignment can be given as follows:

$$\begin{aligned}
\text{minimize} \quad & Z \\
&= \sum_{a\in A}\int_0^{v_a} t_a(\omega)\,d\omega \\
&\quad + \frac{1}{\theta}\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}}\left(\ln T_{ji}^{\text{vt}} - 1\right) \qquad (34)\\
&\quad - \sum_{i\in I}\sum_{j\in J}\int_0^{Q_{ij}^{tc}} Q_{ij}^{-1}(\omega)\,d\omega
\end{aligned}$$

$$\text{subject to:} \quad \sum_{i\in I} T_{ji}^{\text{vt}} = Q_j^{tc} \qquad (34a)$$

$$\sum_{j\in J} T_{ji}^{\text{vt}} = Q_i^{tc} \qquad (34b)$$

$$\sum_{k\in R_{ij}} f_{ij}^k = T_{ij}^n + T_{ij}^{\text{ot}} + T_{ij}^{\text{vt}} \qquad (34c)$$

$$v_a = \sum_{i\in I}\sum_{j\in J}\sum_{k\in R_{ij}} \delta_{ij}^{ak} f_{ij}^k \qquad (34d)$$

$$f_{ij}^k \geq 0 \qquad (34e)$$

$$T_{ji}^{\text{vt}} > 0, \qquad (34f)$$

where T_{ij}^n is normal traffic flow in the studied 1-h interval from zone i to zone j and f_{ij}^k is the traffic flow on route k in the studied 1-h interval.

In terms of the established supply-demand equilibrium model, the following Lagrangian function can be formed:

$$\begin{aligned}
L = {} & \sum_{a\in A}\int_0^{v_a} t_a(\omega)\,d\omega + \frac{1}{\theta}\sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}}\left(\ln T_{ji}^{\text{vt}} - 1\right) \\
& - \sum_{i\in I}\sum_{j\in J}\int_0^{Q_{ij}^{tc}} Q_{ij}^{-1}(\omega)\,d\omega + \sum_{i\in I}\alpha_i\left(\sum_{j\in J} T_{ji}^{\text{vt}} - Q_i^{tc}\right) \\
& + \sum_{j\in J}\beta_j\left(\sum_{i\in I} T_{ji}^{\text{vt}} - Q_j^{tc}\right) \\
& + \sum_{i\in I}\sum_{j\in J}\mu_{ij}\left(T_{ij}^n + T_{ij}^{\text{ot}} + T_{ij}^{\text{vt}} - \sum_{k\in R_{ij}} f_{ij}^k\right).
\end{aligned} \qquad (35)$$

After differentiating the Lagrangian function and applying Kuhn-Tucker conditions, the following relationship can be obtained in an equilibrium state:

$$\begin{aligned}
T_{ji}^{\text{vt}} &= \exp\left(-\theta\left(t_{ji} + \alpha_i + \beta_j\right)\right) \\
&= \exp\left(-\theta t_{ji}\right)\exp\left(-\theta\alpha_i\right)\exp\left(-\theta\beta_j\right).
\end{aligned} \qquad (36)$$

4.2. The Deduction Process of the Model. Firstly, (36) can be rewritten as the following form of gravity model:

$$T_{ji}^{\text{vt}} = A_i B_j \exp\left(-\theta t_{ji}\right), \quad j\in J,\ i\in I, \qquad (37)$$

where $A_i = \exp(-\theta\alpha_i)$ and $B_j = \exp(-\theta\beta_j)$.

By using (34a) and (34b), it can be expressed as follows:

$$\begin{aligned}
A_i &= \frac{Q_i^{tc}}{\sum_{j\in J} B_j \exp\left(-\theta t_{ji}\right)}, \quad i\in I, \\
B_j &= \frac{Q_j^{tc}}{\sum_{i\in I} A_i \exp\left(-\theta t_{ji}\right)}, \quad j\in J,
\end{aligned} \qquad (38)$$

by first assuming $B_j = 1$ and then alternatively solving (38) until converged.

Secondly, according to (34a), (36) is also changed as follows:

$$\begin{aligned}
\sum_{i\in I} T_{ji}^{\text{vt}} &= \sum_{i\in I}\exp\left(-\theta\left(t_{ji} + \alpha_i + \beta_j\right)\right) \\
&= \sum_{i\in I}\exp\left(-\theta\left(t_{ji} + \alpha_i\right)\right)\exp\left(-\theta\beta_j\right) = Q_j^{tc}, \\
& \qquad j\in J.
\end{aligned} \qquad (39)$$

Then, by combining (36) with (39), the following relationship is established:

$$T_{ji}^{\text{vt}} = Q_j^{tc}\frac{\exp\left(-\theta\left(t_{ji} + \alpha_i\right)\right)}{\sum_{m\in I}\exp\left(-\theta\left(t_{jm} + \alpha_m\right)\right)}. \qquad (40)$$

By comparing (40) with (27), α_i can now be interpreted as the taxi searching time w_i^t in zone i. After making a set of deductions, w_i^t can be given as follows:

$$w_i^t = -\frac{\ln\left(\tau A_i\right)}{\theta}. \qquad (41)$$

Here, the variable τ can be calculated by using

$$\tau = \exp\left\{\frac{-\theta\left(\text{Num} - \sum_{i\in I}\sum_{j\in J} T_{ij}^{\text{ot}} t_{ij} - \sum_{j\in J}\sum_{i\in I} T_{ji}^{\text{vt}} t_{ji}\right) - \sum_{i\in I}\sum_{j\in J} Q_{ij}^{tc}\ln A_i}{\sum_{i\in I}\sum_{j\in J} Q_{ij}^{tc}}\right\}. \qquad (42)$$

The theoretical process of (41) and (42) can be derived in detail [13]. By substituting the solution to A_i in (42), the taxi searching time w_i^t in zone i can be then obtained.

Furthermore, by combining (28) with (41), the following form is also established as follows:

$$w_i^t = -\frac{\ln(\tau A_i)}{\theta} = p^{ct} w_i^{ct} + p^{dt} w_i^{dt}. \tag{43}$$

4.3. The Existence of Equilibrium. In the taxi supply relationships, by combining (42) and (43), it can be seen that the taxi-searching time, w_i^t, is a continuous function of customer demand vector Q^{tc}. Furthermore, based on the assumption of the meeting function, (29) and (30), it can be discerned that customer wait time, W_i, varies continuously with taxi search time, w_i^t, in zone i. Thus, it is concluded that the customer wait time W_i is a continuous function of Q^{tc}; continuous mapping relationship can be expressed as $W_i = W_i(Q^{tc})$. In the customer demand relationships, it can be seen that customer demand Q^{tc} is a continuous function of the cumulative prospect vector V. Additionally, based on the relationship between cumulative prospect V and trip cost U it may be deduced that cumulative prospect V varies continuously with customer wait time W_i. Such a continuous mapping relationship is also regarded as $Q^{tc} = Q^{tc}(W_i)$. Thus, the relationship is $Q^{tc} = \Gamma(Q^{tc})$. In addition, the solution set of equilibrium model variables is a compact and convex set Ω and the relationship $\Gamma(Q^{tc}) \in \Omega$ exists. By applying Brouwer's fixed-point theorem, it is easily concluded that Γ has at least one fixed point in Ω. Therefore the existence of the equilibrium solution is guaranteed [25].

5. The Solution Algorithm

The iterative algorithm of taxi equilibrium problem of a network considering the trip mode choice model based on CPT is as follows.

Step 1. Initialize customer waiting time as follows.

Set an initial set of customer waiting times $W_i^{cc(0)}$ and $W_i^{dc(0)}$, $i \in I$, then let $n = n + 1$.

Step 2. Updated customer demand value is as follows.

Substitute $W_i^{cc(n)}$ and $W_i^{dc(n)}$ into the cumulative prospect function and update $V_{ij}^{cc(n)}$, $V_{ij}^{dc(n)}$, and V_{ij}^T. Then $Q_{ij}^{cc(n)}$ and $Q_{ij}^{dc(n)}$ are updated in accordance with (16) and (17).

Step 3. Updated taxi search time is as follows.

(1) Utilize the equilibrium model for taxis in a network.

By applying the iterative balancing algorithm, A_i, B_j, and T_{ji}^{vt} can be obtained. Substituting a set of variable values involving A_i, B_j, T_{ji}^{vt}, $Q_{ij}^{cc(n)}$, and $Q_{ij}^{dc(n)}$, and so forth into (43) a formula involving $w_i^{ct(n)}$ and $w_i^{dt(n)}$ is established. The formula is defined by

$$p^{ct(n)} w_i^{ct(n)} + p^{dt(n)} w_i^{dt(n)} = -\frac{\ln(\tau A_i)}{\theta} \qquad i = 1, 2, \ldots, 6. \tag{44}$$

(2) Update the supply-demand equilibrium relationship.

Substitute T_{ji}^{vt} and $Q_{ij}^{cc(n)}$ into (32) or substitute T_{ji}^{vt} and $Q_{ij}^{dc(n)}$ into (33). Then update the appropriate variables. From this condition, another formulation is expressed as (45) and it includes $w_i^{ct(n)}$ and $w_i^{dt(n)}$.

$$\sum_{j \in J} T_{ji}^{\text{vt}} p^{ct(n)} = \sum_{j \in J} Q_{ij}^{cc(n)} \quad i = 1, 2, \ldots, 6. \tag{45}$$

(3) Combining (44) with (45) makes it possible to solve for the values of $w_i^{ct(n)}$ and $w_i^{dt(n)}$.

Step 4. Update customer wait time as follows.

Substituting $w_i^{ct(n)}$ and $w_i^{dt(n)}$ into the relationship between taxi search time and customer wait time, (29) and (30), respectively, makes it permissible to update $W_i^{cc(n)}$ and $W_i^{dc(n)}$.

Step 5. Check the following.

The standard to stop iteration is defined by

$$\begin{aligned} &\frac{\sqrt{\sum_i \left[W_i^{cc(n+1)} - W_i^{cc(n)}\right]^2}}{\sum_i W_i^{cc(n)}} \le \varepsilon, \\ &\frac{\sqrt{\sum_i \left[W_i^{dc(n+1)} - W_i^{dc(n)}\right]^2}}{\sum_i W_i^{dc(n)}} \le \varepsilon, \end{aligned} \tag{46}$$

where ε is a condition of iterative convergence.

If (46) is achieved, then stop. Otherwise, let $n = n + 1$ and go to Step 2.

6. A Numerical Example Analysis

A simple numerical example is given in order to check the convergence of the algorithm and present the resulting analysis of the equilibrium model.

6.1. Introduction of a Numerical Example. For the model a six-zone network is utilized with nodes representing traffic zones, arcs representing relationship of adjacent nodes, and arrows indicating road direction (Figure 2). Further, each node represents a potential origin or a potential destination for customers, as well as a searching location for vacant taxis.

The travel time function for the links is defined as follows [26]:

$$t_a = t_a^0 \left[1 + 0.15 \left(\frac{v_a}{C}\right)^4\right]. \tag{47}$$

Table 1: The parameters in the travel impedance functions.

Link	Start node	End node	t_a^0 (h)	C (veh/h)
1	1	2	0.25	250
2	1	4	0.20	200
3	2	1	0.25	300
4	2	3	0.15	200
5	2	5	0.30	250
6	4	1	0.25	200
7	4	5	0.15	150
8	5	4	0.20	200
9	5	6	0.25	250
10	5	2	0.30	300
11	3	2	0.15	150
12	3	6	0.15	150
13	6	5	0.20	200
14	6	3	0.20	200

Table 2: Matrix of the total customer demand of different zones.

Q_{ij}^c (person/h)	1	2	3	4	5	6	Total
1	0	50	20	20	10	15	115
2	40	0	15	25	20	10	110
3	20	10	0	50	10	25	115
4	10	20	30	0	20	15	95
5	25	20	15	20	0	20	100
6	15	30	25	25	20	0	115
Total	110	130	105	140	80	85	650

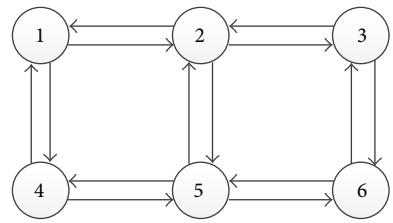


Figure 2: Network of the numerical example.

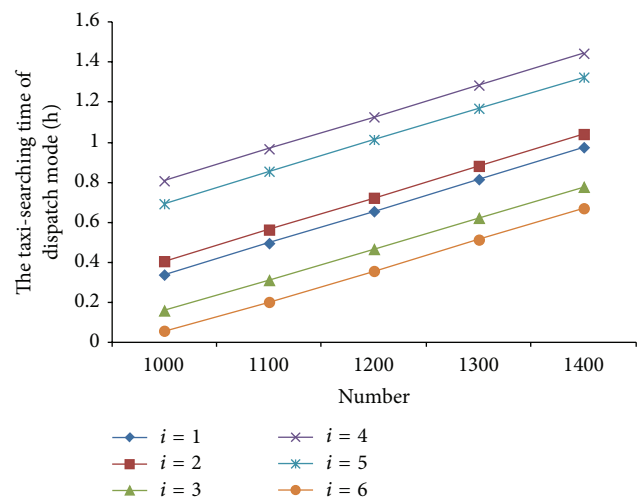


Figure 3: The taxi-searching time of dispatch mode versus taxi fleet size in six zones.

Values for the parameters are obtained from Table 1.

A matrix of the total customer demands for taxis and other transit modes from zone i to zone j is supposed in Table 2.

The parameters used in the example were $\theta_{\text{trip}} = 15$ yuan/h, $\theta^f = 0.3$, $\theta^t = 0.5$, $\theta = 0.3$, $\varepsilon = 0.03$, $W_i^{cc(0)} = W_i^{dc(0)} = 5$ min. Additionally, the 30-minute time used in this example was regarded as a reference point for different trip modes in the CPT function. When making the sensitivity analysis, this paper set 1000, 1100, 1200, 1300, and 1400 as the values of Num, respectively.

6.2. Result Analysis. Figure 3 shows the relationship of the taxi-searching times of dispatch mode versus the taxi fleet size in six zones. It can be seen that the taxi-searching times in zones are all positive and increase linearly with an increase in the taxi fleet size. This may be due to the fact that an increase in the taxi fleet size only enhances the availability of those taxis choosing dispatching service mode in taxi market but cannot decrease the searching time of dispatching service and achieve the improvement of dispatching service quality. That is to say, the changing trend of graphic in Figure 3 indicates that the demand for dispatch mode is less than its supply in taxi market. Specifically, taking a fixed taxi fleet size, for example, the taxi-searching times of dispatch mode in different zones are also different as shown in Figure 3. This is mainly because both the number of customer which chooses the dispatch mode and the one of taxi which provides the dispatching service are different in six zones. Figure 3 shows that, in terms of the taxi-searching time of dispatch mode, zone 4 is maximal and zone 6 is minimal. In addition, it can be seen from Figure 3 that the maximal gap of taxi-searching time in different zones reaches at least 0.7 h considering the taxi fleet size. It can be shown that the characteristic of a traffic zone should be discussed in terms of service and operation of taxi market.

Figure 4 presents the relationship of the taxi-searching times of cruising mode versus the taxi fleet size in six zones. These taxi-searching times in zones are all positive and increase nonlinearly with an increase in the taxi fleet size. It is attributed to the fact that an increase in the taxi fleet size leads to more greater supply of cruising mode and more worse service quality. Specifically, taking a fixed taxi fleet size, for example, the taxi-searching times of cruising mode in different zones are also different as depicted in Figure 4. It can be shown that, in terms of the taxi-searching time of cruising mode, zone 4 is maximal and zone 3 is minimal. The maximal gap of taxi-searching time in different zones reaches at least 0.6 h considering the taxi fleet size. In addition, combining Figure 3 with Figure 4, it can be seen that the taxi-searching times of cruising mode in zones are greater than the ones of dispatch mode, which reflects that the service quality of

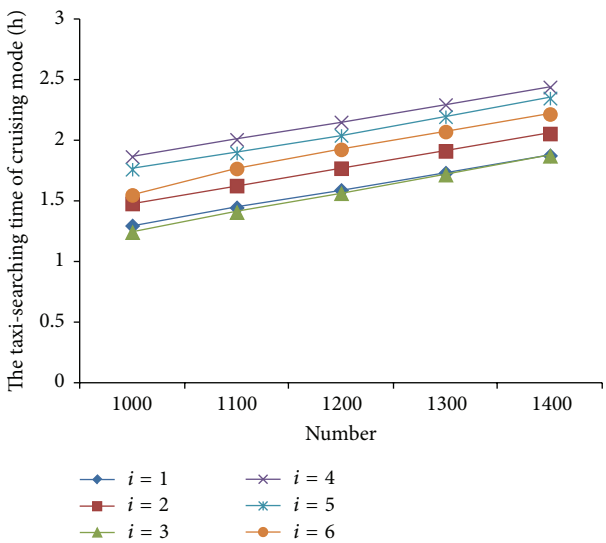


Figure 4: The taxi-searching time of cruising mode versus taxi fleet size in six zones.

dispatch mode is better than the one of cruising mode in taxi market.

Figure 5 depicts the relationship between the probability of different modes chosen and an increase in the taxi fleet size in six zones. Figure 5 shows that, in terms of these two service modes, their values have no great changes with an increase in the taxi fleet size. And furthermore, compared with the probabilities of dispatch mode chosen in six zones, the ones of cruising mode chosen are relatively less, which is mainly attributed to the policy of dispatch mode and its parameters adopted by this paper.

Figure 6 depicts the relationship of the total taxi-searching time versus the taxi fleet size in six zones. Figure 6 indicates that the total taxi-searching times in zones are all positive and increase linearly with an increase in the taxi fleet size. This is due to the fact that the increased taxis mainly search for customers instead of providing the occupied service. That is to say, an increase in the taxi fleet size can enhance taxi's availability in taxi market but cannot indicate the improvement of service quality. The complicated relationship between taxi supply and customer demand is totally determined by the market and network conditions. This conclusion is consistent with the achievement from Yang et al. [15]. And furthermore, taking a fixed taxi fleet size, for example, total taxi-searching times in different zones are also different as shown in Figure 6. This is mainly because customer demands and network conditions (e.g., the travel time to traverse OD pair) are different in six zones. Figure 6 shows that, in terms of the total taxi-searching time, zone 4 is maximal, zone 6 is minimal, and zone 3 is nearly equal to zone 6. And moreover, Figure 6 indicates that the maximal gap of total taxi-searching time in different zones reaches at least thirty minutes considering the taxi fleet size. This means the characteristic of a traffic zone directly influences taxi's availability and further the level of service quality in taxi market.

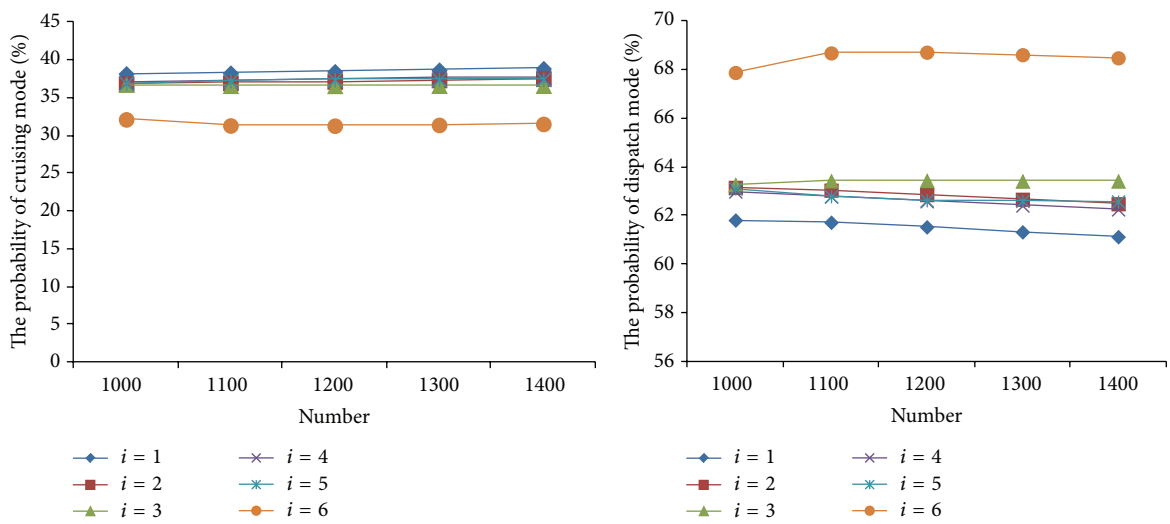
Figure 7 gives the relationship of the customer-waiting time in cruising mode versus the taxi fleet size in different zones. Figure 7 shows that the customer-waiting times in six zones are all positive and decrease nonlinearly with an increase in the taxi fleet size. This indicates that the marginal effect of advancing the taxi fleet size in improving service quality is diminishing. In terms of a given taxi fleet size, the customer-waiting times of cruising mode are different in six zones as shown in Figure 7. Figure 7 shows that, in terms of the customer-waiting time, zone 3 is maximal, zone 4 is minimal, and zone 4 is nearly equal to zone 5. And, moreover, Figure 7 indicates that the maximal gap of the customer-waiting time in different zones reaches at least three minutes. And, furthermore, this gap in six zones is decreasing with an increase in taxi fleet size, which means that the distribution and utilization of taxis in six zones will become similar with an increase in the taxi fleet size.

Figure 8 presents the relationship between the customer-waiting time of dispatch mode and an increase in the taxi fleet size in six zones. Figure 8 shows that the customer-waiting times in six zones are all positive and decrease nonlinearly with an increase in the taxi fleet size. In terms of a single taxi fleet size, the customer-waiting time of dispatch mode in zone 6 is maximal and the customer-waiting time of zone 4 is minimal. The maximal gap of this index in different zones reaches about six minutes. The tendency that the gap of customer-waiting time in dispatch mode in six zones is decreasing with an increase in the taxi number is then reflected.

7. Conclusions

This paper considered the effect of the integrated service mode including cruising and dispatch modes and the travel time uncertainty on taxi network equilibrium. To facilitate this aim the cumulative prospect theory was adopted to study the trip mode choice problem. Based on the elasticity of customer demand, a mathematical programming model was established to describe the supply-demand equilibrium relationship of taxi service on a network with respect to the integrated service mode. From this an iterative algorithm was designed to present the optimal solutions of cruising mode's parameters, dispatch mode's parameters, and taxis network's parameters in an equilibrium state. Finally, a numerical example was used to present the achievement of supply-demand equilibrium state and determine the outcomes of key parameters.

The results provided interesting insights and valuable recommendations for improving taxi market operation. Specifically, using taxi network equilibrium, the taxi-searching and customer-waiting time in six zones were obtained with an increase in the taxi fleet size. We found that the taxi-searching times of both dispatch mode and cruising mode in zones were positive and increased with an increase in the taxi fleet size, while the customer-waiting times of both dispatch mode and cruising mode in zones were also positive but decreased with an increase in the taxi fleet size. Moreover, the taxi-searching time of cruising mode was greater than



(a) The probability of cruising mode chosen

(b) The probability of dispatch mode chosen

Figure 5: The probability of different modes chosen versus taxi fleet size in six zones.

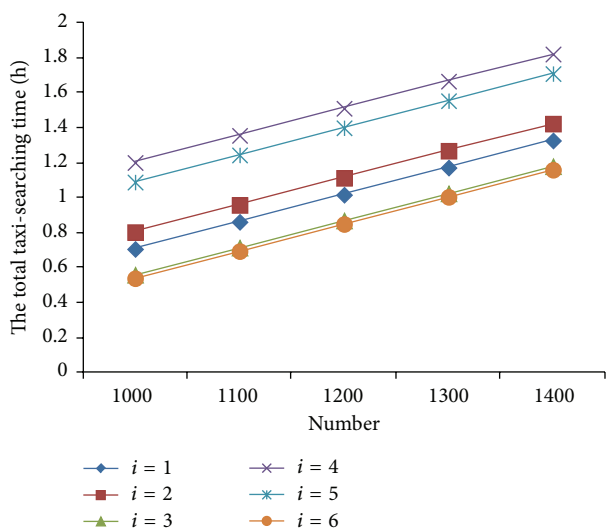


Figure 6: The total taxi-searching time versus taxi fleet size in six zones.

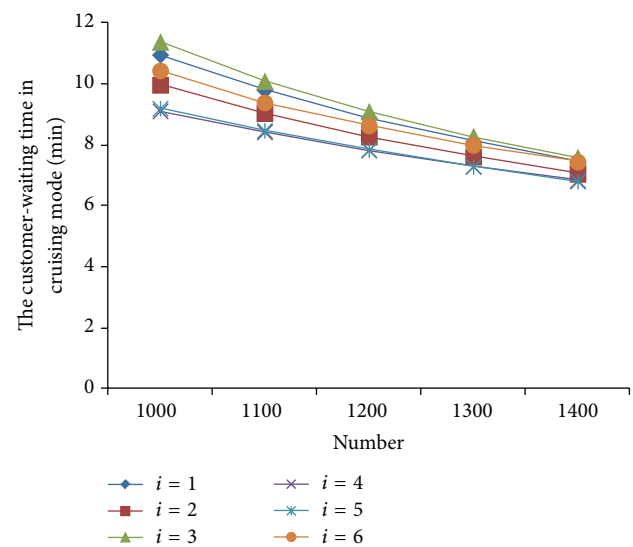


Figure 7: Customer-waiting time in cruising mode versus taxi fleet size in zones.

the one of dispatch mode, while the customer-waiting time of cruising mode was also longer than the one of dispatch mode in six zones. Further, the algorithm indicated that the probability of dispatch mode being chosen by a customer was greater than cruising mode in the six zones considered. Finally, based on the above results from different service modes, the total taxi-searching times of integrated taxi mode were also obtained. Thus, from the practical point of view, these results should enable policymakers to know current operation situations in taxi market and to further adjust and formulate the reasonable operation policies in terms of integrated service mode.

However, to make this mathematical model more suitable for reality, several relationships in the model still need to be further improved. Firstly, the dispatch policy used in the model is a little simple so it cannot offer further insights into the interactions of cruising and dispatch modes. Therefore, applying a complicated dispatch policy will be more meaningful. Secondly, in the application of CPT, the reference points in those designed zones are only considered as constants instead of relevant functions, which need a suitable method to replace the current situation. And furthermore, an uncertainty of customers choosing trip mode should be reflected by adopting a continuous probability distribution function instead of the discrete function, which will be attempted to make further improvement.

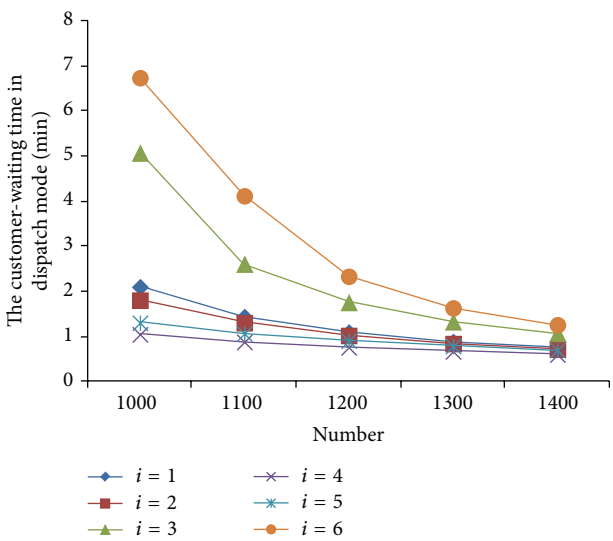


FIGURE 8: Customer-waiting time in dispatch mode versus taxi fleet size in zones.

Conflict of Interests

None of the authors has a financial relationship with a commercial entity that has an interest in the subject of this paper.

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