Research Article

Observer-Based Adaptive Iterative Learning Control for a Class of Nonlinear Time Delay Systems with Input Saturation

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This paper presents an adaptive iterative learning control scheme for the output tracking of a class of nonlinear systems with unknown time-varying delays and input saturation nonlinearity. An observer is presented to estimate the states and linear matrix inequality (LMI) method is employed for observer design. The assumption of identical initial condition for ILC is relaxed by introducing boundary layer function. The possible singularity problem is avoided by introducing hyperbolic tangent function. The uncertainties with time-varying delays are compensated for by the combination of appropriate Lyapunov-Krasovskii functional and Young's inequality. Both time-varying and time-invariant radial basis function neural networks are employed to deal with system uncertainties. On the basis of a property of hyperbolic tangent function, the system output is proved to converge to a small neighborhood of the desired trajectory by constructing Lyapunov-like composite energy function in two cases, while keeping all the closed-loop signals bounded. Finally, a simulation example is presented to verify the effectiveness of the proposed approach.

1. Introduction

Over the past decades, tremendous research efforts have been made aiming at the development of systematic design methods for the iterative learning control (ILC) of nonlinear systems performing control task over a finite interval repeatedly. ILC has become the most suitable and effective control scheme for such repeatable control tasks because of its capacity of achieving perfect tracking by learning mechanism along iteration. Generally, according to the stability analysis tool, ILC can be classified into two categories: traditional ILC [1–4] and adaptive ILC (AILC) [5–10]. The basic principle of traditional ILC is to use information collected from previous execution to form the control action for current operation by a learning mechanism for purpose of improving performances from iteration to iteration. Furthermore, the stability conclusion of traditional ILC is usually obtained by using contraction mapping theorem and fixed point theorem. However, traditional ILC requires for the global Lipschitz continuous condition, which makes it difficult to apply it to certain nonlinear systems. Besides, traditional ILC uses contraction mapping theorem rather than Lyapunov method as the key principle of stability analysis, which makes it

difficult to relax the global Lipschitz condition to local Lipschitz or even non-Lipschitz condition and cooperate with the mainstream methods of nonlinear control theory, such as adaptive control and neural control. To relax the constraints of traditional ILC and extend it to a broader range, some researchers tried to introduce the idea of adaptive control into ILC and proposed adaptive iterative learning control (AILC). AILC takes advantage of both adaptive control and ILC, which successfully overcomes the restriction of global Lipschitz condition; thus it enables us to use fuzzy logic systems or neural networks as approximators to deal with nonlinear uncertainties. In general, the control parameters of AILC methods are tuned along the iteration axis, and the so-called composite energy function (CEF) [5] is usually constructed to analyze the stability and convergence property of the closed-loop systems. The past decade has witnessed great progress in AILC of uncertain nonlinear systems [6–10].

In practice, control of systems with time delays has always been a meaning research, since time delay can be often encountered in a wide range of physical systems and devices, such as turbojet engines, aircraft systems, microwave oscillators, nuclear reactors, and chemical processes [11, 12]. The existence of time delays in a system may degrade the

control performance and even at worst may become a source of instability. Thus, the investigation of time delay in systems has always been an active topic for control engineers. Consequently, stabilization problem of control systems with time delay has received much attention for several decades and a large number of research results have been reported in the literature that deal with various analysis and design problems [11–16]. However, in the field of AILC, only a few results are available for nonlinear systems with time delays [17–19]. In [17], an AILC strategy was developed for a class of scalar systems with unknown time-varying delay and then extended to a class of high-order systems with both time-varying and time-invariant parameters, where the unknown time-varying parameter was estimated in the iterative learning process. However, the proposed controller in [17] requires that the uncertainties in the system satisfy local Lipschitz condition and nonlinear parameterized condition such that adaptive learning laws can be used to estimate the unknown time-varying parameters. In [18, 19], we designed an AILC scheme for a class of nonlinearly parameterized systems and an RBF NN-based AILC for class of unparameterized systems, respectively, where the systems in two papers are with both unknown time-varying delays and unknown dead zone input. However, all of the aforementioned results are on systems with time delay states. As for systems with time delay outputs, to the best of our knowledge, there are no works reported in the literature.

Other than time delay, another challenging problem in control of nonlinear systems lies in the existence of nonsmooth and nonlinear characteristics such as dead zone, hysteresis, saturation, and backlash. Among them, the significance of controller design for systems with saturation can be overemphasized, as any control systems depending on actuators have physical limitations, for example, mechanical actuators and aircraft. The existence of saturation can severely limit system performances and usually leads to undesirable inaccuracies and even instability [20]. Therefore, the control design for nonlinear systems preceded by input saturation is a challenging but worthwhile and necessary issue. For control systems with input saturation, many results have been published in the past several decades [20–26]. To address such a problem, the principal work is to model the input saturation nonlinearity for controller design. Generally, saturation is modeled as $u = \text{sat}(v) = v + \Delta u$, where v and u are the input and output of saturation nonlinearity, respectively, and Δu denotes the difference between the actual control effect u and input v . In [20], the hyperbolic tangent function is employed in the model of input saturation. Like control design for systems with time delays, only a few results are available at present stage from the viewpoint of ILC. In [7], an iterative learning control scheme is designed for a class of nonlinear uncertain systems with input saturation and the analysis of convergence in the iteration domain was presented on the basis of composite energy function consisting of both input and output information. In [27] an adaptive iterative learning control strategy for high-speed trains with unknown speed delays and control input saturation is designed to address speed trajectory tracking problem. It turns out that control design in the presence of both input saturation and

time delays is even more difficult. As far as we know, there are few works reported from the viewpoint of AILC to deal with nonlinear systems with saturation nonlinearity and time delays in the literature at present stage.

In the control community, state feedback control is very powerful for nonlinear systems as the full information of the state vectors is assumed to be accessible for feedback. However, in many real world control systems, only the measured output information, rather than the full state information, is available for feedback. For systems with only output measurable, observer-based output feedback controller has been proven to be the most effective scheme, which is a dynamic output feedback controller that estimates the system states online. Over the past decades, there has been a considerable development in various observer design methodologies using different approaches [28–35]. Although so many results have been developed, only a few results are available from the point of AILC [36–42]. How to design an AILC for nonlinear systems using only output measurement is an interesting and challenging issue. Tayebi and Xu proposed an observer-based iterative learning control scheme for the tracking problem of a class of time-varying nonlinear systems and gave the sufficient conditions for the boundedness and the convergence to zero of the estimation error by using contraction mapping method [36]. In [37], a learning control approach combined with an observer was developed for output tracking of a class of nonlinear systems with time-varying parametric uncertainties and the Lyapunov-like energy function was employed to facilitate convergence analysis. Wang and Chien introduced an error observer to design an iterative learning controller for robotic systems, where a robust learning component using a filtered fuzzy neural network was presented to solve the problem of unknown nonlinearities [38]. Subsequently, the results in [38] were extended to SISO nonlinear system [39], MIMO nonlinear systems [40], and MIMO nonlinear systems with delayed output [41]. Chen et al. extended the result in [36] and proposed an observer-based AILC for nonlinear systems with unknown time-varying parametric uncertainties and the delayed output, where the Lyapunov-Krasovskii-like composite energy function was constructed to prove the boundedness of all closed-loop signals and the convergence of output tracking error [42]. But unfortunately the system nonlinearities must satisfy global Lipschitz continuous condition and the observer design depends on the known system's nonlinear functions. Even though the AILC problem for nonlinear systems using only output measurement has been investigated in the above works, it is noted that the dynamics of nonlinear systems in these works need to satisfy some special structures or conditions.

Motivated by the above observations, in this paper we consider the observer-based AILC problem for a class of nonlinear systems with unknown time-varying delays preceded by input saturation. To the best of our knowledge, up till now no works have been reported in the field of AILC to deal with such kinds of systems. In the proposed AILC scheme, both time-varying and time-invariant radial basis function neural network are utilized to form the iterative learning controller and a CEF along both time and iteration axes is constructed

to analyze the stability and convergence property. Theoretical analysis and simulation example show that the proposed approach can guarantee that all the signals are bounded and tracking errors converge to a small neighbourhood of the origin. The main contributions of the proposed AILC scheme are highlighted as follows. (1) As far as we know, it is the first time, in the literature, that observer-based AILC problem of unknown nonlinear systems with unknown time-varying time delays and input saturation is investigated. (2) Both time-varying and time-invariant RBF neural networks are used to approximate the unknown time-varying functions and uncertainty that is independent of time variable. (3) Hyperbolic tangent function is employed to handle the singularity problem encountered in Lyapunov synthesis. (4) The boundary layer function is introduced to remove the requirement for identical initial condition which is required for the majority of ILC schemes.

The organization of this paper is as follows. The problem formulation and preliminaries are given in Section 2. In Section 3, the observer and AILC scheme are developed. Results of extensive simulation studies are presented to demonstrate the validity of the proposed scheme in Section 4, followed by conclusions in Section 5.

Throughout this paper, the following notations will be used. (1) σ denotes the integral variable. (2) $|\cdot|$ denotes the absolute value of a scalar function. (3) $\|\cdot\|$ denotes the Euclidean norm or any other consistent norm of a vector or matrix. (4) N is the set of nonnegative integers. (5) For a signal vector $r_k(t)$, define $\|r_k(t)\|_{L_T^\infty} = \max_{(k,t) \in N \times [0,T]} \|r_k(t)\|$ and $\|r_k(t)\|_{L_T^2} = \int_0^T \|r_k(\sigma)\|^2 d\sigma$. If $\|r_k(t)\|_{L_T^\infty} < \infty$, we say that $r_k(t)$ is bounded in L_T^∞ -norm, which is denoted by $r_k(t) \in L_T^\infty$. Similarly, we denote the boundedness of $r_k(t)$ in L_T^2 -norm by $r_k(t) \in L_T^2$. Obviously, the boundedness in L_T^∞ -norm implies the boundedness in L_T^2 -norm because $\|r_k(t)\|_{L_T^2}^2 \leq T \|r_k(t)\|_{L_T^\infty}^2$.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation. In this paper, we consider a class of nonlinear time delay systems with input saturation which runs on a finite time interval $[0, T]$ repeatedly

$$\begin{aligned} \dot{x}_{i,k}(t) &= x_{i+1,k}(t), \quad i = 1, \dots, n-1, \\ \dot{x}_{n,k}(t) &= f(x_k(t), t) + h(y_{\tau,k}(t), t) + u_k(v_k(t)) \\ &\quad + d(t), \\ y_k(t) &= x_{1,k}(t), \quad t \in [0, T], \\ y_k(t) &= \omega(t), \quad t \in [-\tau_{\max}, 0), \end{aligned} \quad (1)$$

where $k \in N$ denotes the index of iteration; $y_k(t) \in R$ and $x_{i,k}(t) \in R$, $i = 1, \dots, n$, are the system output and states, respectively; $x_k(t) \triangleq [x_{1,k}(t), \dots, x_{n,k}(t)]^T$ are the delay-free state vectors; $\tau_i(t)$ are unknown time-varying delays and $y_{\tau,k} \triangleq y_k(t - \tau_i(t))$, $i = 2, \dots, n$, and $y_{\tau,k}(t) = [y_{\tau_{1,k}}(t), \dots, y_{\tau_{n,k}}(t)]^T$; $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$ are unknown smooth

functions; $h(\cdot, \cdot)$ is an unknown smooth function of time delay output with upper bound. $d(t)$ is unknown bounded external disturbance. $u_k(v_k(t))$ denotes the saturation nonlinearity. $\omega(t)$ denotes the initial function for delayed output. The states are assumed to be unavailable for measurement and only output is measurable. The system is bounded-input-bounded-output (BIBO) stable. In the subsequent parts, the variable t will be omitted when no confusions arise.

Our design objective is to determine an adaptive iterative controller for system (1) to steer the output y_k following a desired reference signal y_d over $[0, T]$ as $k \rightarrow \infty$, while ensuring that all the signals in the closed-loop system remain bounded.

Define the desired trajectory as $x_d = [y_d, \dot{y}_d, \dots, y_d^{(n-1)}]^T$, which is a vector of y_d up to its $(n-1)$ th time derivative. To facilitate control system design, the following reasonable assumptions are made.

Assumption 1. The unknown time-varying delays $\tau_i(t)$ satisfy $0 \leq \tau_i(t) \leq \tau_{\max}$, $\dot{\tau}_i(t) \leq \kappa < 1$, $i = 1, 2, \dots, n$, where $\tau_{\max}$ is known and κ is an unknown positive constant.

Assumption 2. The unknown smooth function $h(\cdot, \cdot)$ satisfies the following inequality:

$$|h(y_{\tau,k}, t)| \leq \sum_{j=1}^n \rho_j(y_{\tau_j,k}(t)), \quad (2)$$

where $\rho_j(\cdot)$ are unknown positive smooth functions.

Assumption 3. The desired reference signal $y_d(t)$ up to its n th derivative is continuous, bounded, and available.

Assumption 4. The unknown external disturbance $d(t)$ is bounded; that is, $|d(t)| \leq D_1$, with D_1 as an unknown constant.

Assumption 5. The initial state vector is chosen as $\omega(t) = y_d(t)$, $t \in [-\tau_{\max}, 0)$.

Remark 6. Assumption 1 is usually necessary in the control problem of systems with unknown time-varying delays, which ensures that the time delay terms can be compensated for by using Lyapunov-Krasovskii functional. And this assumption is more relaxed than that in [17] which requires for a priori knowledge of κ .

Remark 7. Compared with the assumption of time-varying parameterization with known upper bound functions in [17], Assumption 2 is much more relaxed and can be easily satisfied.

Remark 8. Assumption 5 is only for analytical purpose and of no practical meaning.

2.2. Input Saturation. In this paper, the output of a control u_k with input v_k subjected to the condition of saturation is given by

$$u_k = \begin{cases} v_k, & |v_k| < u_M \\ u_M \operatorname{sgn}(v_k), & |v_k| \geq u_M, \end{cases} \quad (3)$$

where u_M is the bound of input v_k . For convenience of design, we rewrite the saturation nonlinearity as

$$u_k = \beta(v_k) + d_1(v_k) \quad (4)$$

with $\beta(v_k) = u_M \times \tanh(v_k/u_M)$. The part of the control effect that cannot be implemented is defined as

$$d_1(v_k) = u_k - \beta(v_k). \quad (5)$$

Obviously, $d_1(v_k)$ is bounded by

$$\begin{aligned} |d_1(v_k)| &= |\operatorname{sat}(v_k) - \beta(v_k)| \leq u_M (1 - \tanh(1)) \\ &= D_2. \end{aligned} \quad (6)$$

A graphic presentation of saturation model is shown in Figure 1.

2.3. RBF Neural Networks. In control engineering, two types of artificial neural networks are usually used to approximate unknown smooth functions, which specifically are linearly parameterized neural networks (LPNNs) and multilayer neural networks (MNNs). As a kind of LPNNs, the radial basis function (RBF) neural network (NN) [43] is usually used as a tool to model unknown nonlinear functions owing to its nice approximation capabilities. The RBF NN can be seen as a two-layer network in which hidden layer performs a fixed nonlinear transformation with no adjustable parameters; that is, the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Generally, the RBF NN approximates the continuous function $Q(Z) : R^q \rightarrow R$ as follows:

$$Q_{nn}(Z) = W^T \phi(Z), \quad (7)$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, $W = [w_1, w_2, \dots, w_l]^T \in R^l$ is the weight vector, and the NN node number $l > 1$; and $\phi(Z) = [\phi_1(Z), \dots, \phi_l(Z)]^T$, with $\phi_i(Z)$ as the commonly used Gaussian functions; that is, $\phi_i(Z) = e^{-(Z-\mu_i)^T(Z-\mu_i)/\sigma_i^2}$, $i = 1, \dots, l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{iq}]$ is the center of the receptive field and σ_i is the width of the Gaussian function. It has been proven that if l is chosen sufficiently large, $W^T \phi(Z)$ can approximate any continuous function, $Q(Z)$, over a compact set $\Omega_Z \subset R^q$ to any arbitrary accuracy in the form of $Q(Z) = W^{*T} \phi(Z) + \varepsilon(Z)$, for all $Z \in \Omega_Z \subset R^q$ where W^* is the ideal constant weight vector and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set; that is, $|\varepsilon(Z)| \leq \varepsilon^*$, for all $Z \in \Omega_Z$, where $\varepsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an “artificial” quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset R^q$; that is, $W^* := \operatorname{argmin}_{W \in R^l} \{\sup_{Z \in \Omega_Z} |Q(Z) - W^T \phi(Z)|\}$.

When neural networks are used to approximate unknown functions, updating laws for estimating the weight vector

FIGURE 1: Saturation model.

need to be designed. In the early stage, the gradient-based back-propagation algorithms and their variants are the most popular algorithms for training neural networks. Along with the applications in traditional adaptive control framework, differential type updating laws were developed in the control scheme design and stability analysis by using Lyapunov method. Over the past few decades, large numbers of adaptive neural control schemes have been presented [11–13, 21, 24, 26]. However, difficulties arise when using (7) to approximate unknown time-varying function $Q(Z, t)$, especially when $Q(Z, t)$ is fast time-varying. In order to deal with this problem, a kind of time-varying neural networks is proposed to approximate the unknown time-varying functions in the form of $Q(Z, t) = W^{*T}(t)\phi(Z) + \varepsilon(Z, t)$, whose ideal weight vector $W^*(t)$ is time-varying [44, 45]. Difference type learning law is designed and the named CEF method is exploited to obtain the convergence and stability results.

In this paper, we will employ both time-varying RBF neural networks proposed in [44, 45] and time-invariant RBF NNs (7) as the approximator to overcome the design problem from the uncertainties.

2.4. Schur Complement Lemma [46]. In this paper, the following lemma is used.

Lemma 9. *The LMI*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} < 0, \quad (8)$$

where $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$, is equivalent to

$$\begin{aligned} S_{22} &< 0, \\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T &< 0. \end{aligned} \quad (9)$$

3. Observer and Adaptive Neural Iterative Learning Controller Design

3.1. *Observer Design.* Rewrite system (1) as

$$\dot{x}_k = Ax_k + K_0 y_k + B[f(x_k, t) + h(y_{\tau,k}, t) + \beta(v_k) + d_1(v_k) + d(t)] \quad (10)$$

with

$$\begin{aligned} K_0 &= [k_1, k_2, \dots, k_n]^T, \\ A &= \begin{bmatrix} -k_1 & & & \\ -k_2 & I_{n-1} & & \\ \vdots & & \ddots & \\ -k_n & 0 & \dots & 0 \end{bmatrix}, \\ B &= [0, \dots, 0, 1]^T, \end{aligned} \quad (11)$$

where I_{n-1} is a unit square matrix with $n - 1$ dimensions. $K_0 \in R^n$ can be selected such that A is a strict Hurwitz matrix. Thus, given a matrix $Q > 0$, there exists $P > 0$ satisfying the following inequality:

$$A^T P + PA + \frac{n+3}{\lambda} PP^T < -Q, \quad (12)$$

where λ is a positive scalar.

Remark 10. In this paper, to solve inequality (12), we decompose $A = \bar{A} + K_0 \bar{B}$ where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \dots & 0 \end{bmatrix}, \\ \bar{B} &= [-1, 0, \dots, 0]. \end{aligned} \quad (13)$$

Based on Lemma 9, (12) is equivalent to the following LMI:

$$\begin{bmatrix} P\bar{A} + M\bar{B} + \bar{B}^T M^T + \bar{A}^T P + Q & P \\ P & -\frac{\lambda}{n+3} I_n \end{bmatrix} < 0, \quad (14)$$

where Q is a given positive definite matrix and I_n is a unit matrix. Furthermore, it should be noted that P , M , and λ can be computed simultaneously via MATLAB LMI toolbox, and the observer gain matrix is obtained by $K_0 = P^{-1}M$.

For simplicity, define $\bar{d}(t) = d_1(v_k) + d(t)$. Then it is obvious that $|\bar{d}(t)| \leq D_0$ with $D_0 = D_1 + D_2$. To estimate the states of system (10), design the observer as

$$\begin{aligned} \dot{\hat{x}}_k &= A\hat{x}_k + K_0 y_k + B[\Psi_k + v_k], \\ \hat{y}_k &= \hat{x}_{1,k}, \end{aligned} \quad (15)$$

where Ψ_k will be given later. For subsequent design, we define $\Delta v_k = \beta(v_k) - v_k$ which describes the effect of input saturation and can be effectively approximated by using a dynamic neural network.

Define $z_k \triangleq [z_{1,k}, z_{2,k}, \dots, z_{n,k}] = x_k - \hat{x}_k$; then we can obtain the dynamic of observer error by subtracting (15) from (10):

$$\dot{z}_k = Az_k + B[F(x_k, t) + h(y_{\tau,k}, t) + \bar{d}(t) - \Psi_k], \quad (16)$$

where $F(x_k, t) = f(x_k, t) + \Delta v_k$. Choose a positive function of observer error as $V_{z_k} = z_k^T P z_k$; taking the time derivative of V_{z_k} yields

$$\begin{aligned} \dot{V}_{z_k} &= z_k^T (A^T P + PA) z_k \\ &\quad + 2z_k^T PB[F(x_k, t) + h(y_{\tau,k}, t) + \bar{d}(t) - \Psi_k]. \end{aligned} \quad (17)$$

Considering Assumption 2 and utilizing Young's inequality, we can have

$$\begin{aligned} 2z_k^T PBh(y_{\tau,k}, t) &\leq 2|z_k^T PB| |h(y_{\tau,k}, t)| \\ &\leq \frac{n}{\lambda} z_k^T PP^T z_k + \lambda \sum_{j=1}^n \rho_j^2(y_{\tau,j,k}(t)), \end{aligned} \quad (18)$$

$$2z_k^T PB\bar{d}(t) \leq \frac{1}{\lambda} z_k^T PP^T z_k + \lambda D_0^2. \quad (19)$$

To compensate for the time delay term, consider the following Lyapunov-Krasovskii functional:

$$V_{U_k}(t) = \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \int_{t-\tau_j(t)}^t \rho_j^2(y_k(\sigma)) d\sigma. \quad (20)$$

Recalling Assumption 1, differentiating (20) with respect to time leads to

$$\begin{aligned} \dot{V}_{U_k}(t) &= \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) - \lambda \sum_{j=1}^n \frac{1-\dot{\tau}_j(t)}{(1-\kappa)} \rho_j^2(y_{\tau_j,k}) \\ &\leq \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) - \lambda \sum_{j=1}^n \rho_j^2(y_{\tau_j,k}). \end{aligned} \quad (21)$$

Combining (17)–(19) and (21), it follows that

$$\begin{aligned} \dot{V}_{z_k} + \dot{V}_{U_k} &\leq z_k^T \left(A^T P + PA + \frac{n+1}{\lambda} \right) z_k \\ &\quad + 2z_k^T PB[F(x_k, t) - \Psi_k] \\ &\quad + \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) + \lambda D_0^2. \end{aligned} \quad (22)$$

To overcome the difficulty from unknown time-varying function $F(x_k, t)$, we utilize time-varying RBF NN to approximate $F(x_k, t)$ on the compact set $\Omega = \{x_k\} \subset R^n$ which is specified by

$$F(x_k, t) = W^{*T}(t) \phi(x_k) + \varepsilon(x_k, t), \quad (23)$$

where the Gaussian basis function is given by $\phi(x_k) = [\varphi_1(x_k), \varphi_2(x_k), \dots, \varphi_l(x_k)]^T \in R^l$ with $\varphi_i(x_k) = \exp(-\|x_k - \mu_i\|^2/\sigma_i^2)$, $\mu_i \in R^n$ and $\sigma_i \in R$, denoting the centers and widths, respectively. $W^*(t)$ and $\varepsilon(x_k, t)$ are bounded by $\sup_{t \in [0, T]} \{\|W^*(t)\|\} \leq \varepsilon_W$ and $\sup_{t \in [0, T]} \{\|\varepsilon(x_k, t)\|\} \leq \varepsilon_0$. Consequently, we can determine that

$$\Psi_k = \widehat{W}_k^T(t) \phi(\widehat{x}_k). \quad (24)$$

Then we can have

$$\begin{aligned} 2z_k^T PB [F(x_k, t) - \Psi_k] &= 2z_k^T PB [W^{*T}(t) \phi(x_k) \\ &+ \varepsilon(x_k, t) - \widehat{W}_k^T(t) \phi(\widehat{x}_k)] = 2z_k^T PB [W^{*T}(t) \phi(x_k) \\ &- W^{*T}(t) \phi(\widehat{x}_k) + \varepsilon(x_k, t) + W^{*T}(t) \phi(\widehat{x}_k) \\ &- \widehat{W}_k^T(t) \phi(\widehat{x}_k)] = 2z_k^T PB [W^{*T}(t) \tilde{\phi}(x_k, \widehat{x}_k) \\ &+ \varepsilon(x_k, t) - \widehat{W}_k^T(t) \phi(\widehat{x}_k)], \end{aligned} \quad (25)$$

where $\widehat{W}_k = \widehat{W}_k - W^*$ denotes the estimation error, $\tilde{\phi}(x_k, \widehat{x}_k) = \phi(x_k) - \phi(\widehat{x}_k)$. $\tilde{\phi}(x_k, \widehat{x}_k)$ is bounded by $\tilde{\phi}^T(x_k, \widehat{x}_k) \tilde{\phi}(x_k, \widehat{x}_k) \leq 4l$. Then by using Young's inequality it is clear that

$$\begin{aligned} 2z_k^T PB [W^{*T}(t) \tilde{\phi}(x_k, \widehat{x}_k) + \varepsilon(x_k, t)] \\ \leq \frac{2}{\lambda} z_k^T P P^T z_k + 4\lambda \varepsilon_W^2 + \lambda \varepsilon_0^2. \end{aligned} \quad (26)$$

Substituting (25) and (26) back into (22) and applying (12) we have

$$\begin{aligned} \dot{V}_{z_k} + \dot{V}_{U_k} &\leq z_k^T \left(A^T P + PA + \frac{n+3}{\lambda} P P^T \right) z_k \\ &- 2z_k^T PB \widehat{W}_k^T(t) \phi(\widehat{x}_k) + \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) \\ &+ \lambda D_0^2 + 4\lambda \varepsilon_W^2 + \lambda \varepsilon_0^2 \\ &\leq -z_k^T Q z_k - 2z_k^T PB \widehat{W}_k^T(t) \phi(\widehat{x}_k) \\ &+ \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda \varepsilon_W^2 \\ &+ \lambda \varepsilon_0^2 \\ &\leq -\lambda_{\min}(Q) \|z_k\|^2 - 2z_k^T PB \widehat{W}_k^T(t) \phi(\widehat{x}_k) \\ &+ \frac{\lambda}{(1-\kappa)} \sum_{j=1}^n \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda \varepsilon_W^2 \\ &+ \lambda \varepsilon_0^2. \end{aligned} \quad (27)$$

3.2. Adaptive Neural Iterative Learning Controller Design. Define errors $e_{1,k} = \widehat{x}_{1,k} - y_d$, $e_{i,k} = \widehat{x}_{i,k} - y_d^{(i-1)}$, $i = 2, \dots, n-1$, and $e_{n,k} = \widehat{x}_{n,k} - y_d^{(n-1)}$. Then we make the assumptions on initial errors.

Assumption 11. Identical initial condition is satisfied for original system (1) and observer (15); that is, $z_{i,k}(0) = 0$, $i = 1, 2, \dots, n$.

Assumption 12. Identical initial condition is not necessary for $e_{i,k}$; that is, the initial state errors $e_{i,k}(0)$ at each iteration are not necessarily zero small and fixed but are assumed to be bounded.

Define the filtered tracking error as $e_{sk} = [\Lambda^T \ 1] e_k$, where $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_{n-1}]^T$ and $\lambda_1, \dots, \lambda_{n-1}$ are chosen such that the polynomial $H(s) = s^{n-1} + \lambda_{n-1}s^{n-2} + \dots + \lambda_1$ is a Hurwitz polynomial. It is obvious that if e_{sk} approaches zero as $k \rightarrow \infty$, then $\|e_k\|$ will converge to the origin asymptotically.

Based on Assumption 12, there exist known constants ε_i , such that $|e_{i,k}(0)| \leq \varepsilon_i$, $i = 1, 2, \dots, n$, for any $k \in N$. In order to overcome the uncertainty from initial tracking error, we define an auxiliary function [9, 10] as

$$\begin{aligned} s_k &= e_{sk} - \eta(t) \text{sat}\left(\frac{e_{sk}}{\eta(t)}\right), \\ \eta(t) &= \varepsilon e^{-Kt}, \end{aligned} \quad (28)$$

where $\varepsilon = [\Lambda^T \ 1] [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T$, $K > 0$. The saturation function $\text{sat}(\cdot)$ is defined as

$$\text{sat}\left(\frac{e_{sk}}{\eta(t)}\right) = \begin{cases} 1, & \text{if } e_{sk} > \eta(t), \\ \frac{e_{sk}}{\eta(t)}, & \text{if } |e_{sk}| \leq \eta(t), \\ -1, & \text{if } e_{sk} < -\eta(t). \end{cases} \quad (29)$$

Remark 13. $\eta(t)$ is called time-varying boundary layer function. It is obvious that $\eta(t)$ decreases along time axis, which satisfies initial condition $\eta(0) = \varepsilon$ and $0 < \eta(T) \leq \eta(t)$, for all $t \in [0, T]$. If $s_k(t)$ can converge to zero for all $t \in [0, T]$ as $k \rightarrow \infty$, then e_{sk} will be bounded by $\eta(t)$.

According to initial condition, we can easily obtain

$$\begin{aligned} |e_{sk}(0)| &= |\lambda_1 e_{1,k}(0) + \lambda_2 e_{2,k}(0) + \dots + e_{n,k}(0)| \\ &\leq \lambda_1 |e_{1,k}(0)| + \lambda_2 |e_{2,k}(0)| + \dots + |e_{n,k}(0)| \\ &\leq \lambda_1 \varepsilon_1 + \lambda_2 \varepsilon_2 + \dots + \varepsilon_n = \eta(0) \end{aligned} \quad (30)$$

which implies that $s_k(0) = e_{sk}(0) - \eta(0)(e_{sk}(0)/\eta(0)) = 0$ is satisfied for any $k \in N$. For further use, we give the following equality:

$$\begin{aligned} s_k \text{sat}\left(\frac{e_{sk}}{\eta(t)}\right) &= \begin{cases} 0, & \text{if } \left|\frac{e_{sk}}{\eta(t)}\right| \leq 1 \\ s_k \text{sgn}(e_{sk}), & \text{if } \left|\frac{e_{sk}}{\eta(t)}\right| > 1 \end{cases} \\ &= s_k \text{sgn}(s_k) = |s_k|, \end{aligned} \quad (31)$$

where the sign function is defined as

$$\text{sgn}(\cdot) = \begin{cases} 1, & \text{if } \cdot > 0 \\ 0, & \text{if } \cdot = 0 \\ -1, & \text{if } \cdot < 0. \end{cases} \quad (32)$$

To continue the design procedure, we give the dynamic of e_k as follows:

$$\begin{aligned} \dot{e}_{i,k} &= e_{i+1,k} + k_i z_{1,k}, \quad i = 1, \dots, n-1, \\ \dot{e}_{n,k} &= k_n z_{1,k} + v_k + \widehat{W}_k^T \phi(\widehat{x}_k) - y_d^{(n)}. \end{aligned} \quad (33)$$

Define a Lyapunov function as

$$V_{s_k} = \frac{1}{2} s_k^2. \quad (34)$$

Taking the derivative of V_{s_k} with respect to time yields

$$\begin{aligned} \dot{V}_{s_k} &= s_k \dot{s}_k = \begin{cases} s_k (\dot{e}_{sk} - \dot{\eta}(t)), & \text{if } e_{sk} > \eta(t) \\ 0, & \text{if } |e_{sk}| \leq \eta(t) \\ s_k (\dot{e}_{sk} + \dot{\eta}(t)), & \text{if } e_{sk} < -\eta(t) \end{cases} \\ &= s_k (\dot{e}_{sk} - \dot{\eta}(t) \text{sgn}(s_k)) \\ &= s_k \left[\sum_{j=1}^{n-1} (\lambda_j e_{j+1,k} + \lambda_j k_j z_{1,k}) + K\eta(t) \text{sgn}(s_k) \right. \\ &\quad \left. - y_d^{(n)} + k_n z_{1,k} + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k \right] \\ &= s_k \left[\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} + [\Lambda^T \ 1] K_0 z_{1,k} + K e_{sk} - y_d^{(n)} \right. \\ &\quad \left. + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k \right] - K s_k^2, \end{aligned} \quad (35)$$

where we utilize the following relation:

$$\begin{aligned} &s_k (-K e_{sk} + K\eta(t) \text{sgn}(s_k)) \\ &= s_k \left(-K s_k - K\eta(t) \text{sat}\left(\frac{e_{sk}}{\eta(t)}\right) + K\eta(t) \text{sgn}(s_k) \right) \\ &= -K s_k^2 - K\eta(t) |s_k| + K\eta(t) |s_k| = -K s_k^2. \end{aligned} \quad (36)$$

Choose the Lyapunov candidate for the whole closed-loop system as $V_k = V_{z_k} + V_{U_k} + V_{s_k}$. Combining (27) and (35) we can obtain the derivative of V_k as follows:

$$\begin{aligned} \dot{V}_k &\leq -\lambda_{\min}(Q) \|z_k\|^2 - 2z_k^T P B \widehat{W}_k^T \phi(\widehat{x}_k) + \frac{\lambda}{(1-\kappa)} \\ &\quad \cdot \sum_{j=1}^n \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda l \varepsilon_W^2 + \lambda \varepsilon_0^2 \\ &\quad + s_k \left[\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} + [\Lambda^T \ 1] K_0 z_{1,k} + K e_{sk} - y_d^{(n)} \right. \\ &\quad \left. + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k \right] - K s_k^2. \end{aligned} \quad (37)$$

For the convenience of expression, denote $\Xi(y_k) \triangleq (\lambda/(1-\kappa)) \sum_{j=1}^n \rho_j^2(y_k) + \lambda D_0^2 + 4\lambda l \varepsilon_W^2 + \lambda \varepsilon_0^2$; then (37) can be simplified as

$$\begin{aligned} \dot{V}_k &\leq -\lambda_{\min}(Q) \|z_k\|^2 - 2z_k^T P B \widehat{W}_k^T \phi(\widehat{x}_k) \\ &\quad + s_k \left[\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} + [\Lambda^T \ 1] K_0 z_{1,k} + K e_{sk} - y_d^{(n)} \right. \\ &\quad \left. + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k + \frac{\Xi(y_k)}{s_k} \right] - K s_k^2. \end{aligned} \quad (38)$$

However, note that $\Xi(y_k)/s_k$ is not well defined at $s_k = 0$ and may lead to controller singularity problem if we use $\Xi(y_k)/s_k$ in the controller design. In order to overcome this problem, we exploit the following property.

Lemma 14 (see [47]). *For any constant $\eta > 0$ and any variable $p \in \mathbb{R}$,*

$$\lim_{p \rightarrow 0} \frac{\tanh^2(p/\eta)}{p} = 0. \quad (39)$$

Employing the hyperbolic tangent function, (38) becomes

$$\begin{aligned} \dot{V}_k &\leq -\lambda_{\min}(Q) \|z_k\|^2 - 2z_k^T P B \widehat{W}_k^T \phi(\widehat{x}_k) \\ &\quad + s_k \left[\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} + [\Lambda^T \ 1] K_0 z_{1,k} + K e_{sk} - y_d^{(n)} \right. \\ &\quad \left. + \widehat{W}_k^T \phi(\widehat{x}_k) + v_k + \frac{b \tanh^2(s_k/\eta(t)) \Xi(y_k)}{s_k} \right] \\ &\quad + \left[1 - b \tanh^2\left(\frac{s_k}{\eta(t)}\right) \right] \Xi(y_k) - K s_k^2. \end{aligned} \quad (40)$$

From Lemma 14, we know that $\lim_{s_k \rightarrow 0} b \tanh^2(s_k/\eta(t)) \Xi(y_k)/s_k = 0$. Hence, $b \tanh^2(s_k/\eta(t)) \Xi(y_k)/s_k$ is defined at $s_k = 0$ and the problem of possible singularity is avoided.

Apparently, $(\lambda/(1-\kappa)) \sum_{j=1}^n \rho_j^2(y_k)$ is continuous and well-defined over compact set $\Omega_y = \{y_k\} \subset R$, so it can be approximated by a time-invariant neural network to arbitrary accuracy as $(\lambda/(1-\kappa)) \sum_{j=1}^n \rho_j^2(y_k) = W_{\Xi}^{*T} \phi_{\Xi}(y_k) + \varepsilon_{\Xi}(y_k)$, where $|\varepsilon_{\Xi}(y_k)| \leq \bar{\varepsilon}_{\Xi}$ is the approximation error, $W_{\Xi}^* \in R^{l_{\Xi}}$ are unknown ideal time-invariant weight vector, and $\phi_{\Xi}(y_k) \in R^{l_{\Xi}}$ is the basis functions, where l_{Ξ} denotes the NN node number. Also, we make the following finiteness assumption for W_{Ξ}^* .

Assumption 15. The ideal weight vector W_{Ξ}^* is bounded; that is,

$$\|W_{\Xi}^*\| \leq \varepsilon_{W_{\Xi}}, \quad (41)$$

where $\varepsilon_{W_{\Xi}}$ is unknown positive constant.

Then $\Xi(y_k)$ can be expressed as

$$\begin{aligned} \Xi(y_k) &= W_{\Xi}^{*T} \phi_{\Xi}(y_k) + \varepsilon_{\Xi}(y_k) + \lambda D_0^2 + 4\lambda \varepsilon_W^2 \\ &\quad + \lambda \varepsilon_0^2. \end{aligned} \quad (42)$$

For convenience of presentation, denote $\mu \triangleq \varepsilon_{\Xi}(y_k) + \lambda D_0^2 + 4\lambda \varepsilon_W^2 + \lambda \varepsilon_0^2$. Furthermore, we rewrite $\Xi(y_k)$ as $\Xi(y_k) = W_2^{*T} \phi_2(y_k)$, with $W_2^* = [W_{\Xi}^{*T}, \mu]^T$ and $\phi_2(y_k) = [\phi_{\Xi}^T(y_k), 1]^T$.

Until now, we can design the output feedback controller as follows:

$$\begin{aligned} v_k &= -\sum_{j=1}^{n-1} \lambda_j e_{j+1,k} - [\Lambda^T \ 1] K_0 z_{1,k} - K e_{s_k} + y_d^{(n)} \\ &\quad - \widehat{W}_k^T(t) \phi(\widehat{x}_k) \\ &\quad - \frac{b \widehat{W}_{2,k}^T \phi_2(y_k) \tanh^2(s_k/\eta(t))}{s_k}, \end{aligned} \quad (43)$$

where $\widehat{W}_k(t)$ and $\widehat{W}_{2,k}$ are the estimates of $W^*(t)$ and W_2^* , respectively.

The difference type and differential-difference type update algorithms are designed, respectively, for $\widehat{W}_k(t)$ and $\widehat{W}_{2,k}$ as follows:

$$\begin{aligned} \widehat{W}_k(t) &= \widehat{W}_{k-1}(t) + 2q_1 z_k^T P B \phi(\widehat{x}_k), \\ \widehat{W}_0(t) &= 0, \\ t &\in [0, T], \\ (1-\gamma) \widehat{W}_{2,k} &= -\gamma \widehat{W}_{2,k} + \gamma \widehat{W}_{2,k-1} \\ &\quad + q_2 b \tanh^2\left(\frac{s_k}{\eta(t)}\right) \phi_2(y_k), \\ \widehat{W}_{2,k}(0) &= \widehat{W}_{2,k-1}(T), \\ \widehat{W}_{2,0}(t) &= 0, \end{aligned} \quad (44)$$

$$t \in [0, T],$$

where $q_1, q_2 > 0$ and $0 < \gamma < 1$ are design parameters.

Define the estimation errors as $\widetilde{W}_k(t) = \widehat{W}_k(t) - W^*(t)$ and $\widetilde{W}_{2,k} = \widehat{W}_{2,k} - W_{2,k}^*$. Then, substituting the controller (43) back into (40) yields

$$\begin{aligned} \dot{V}_k &\leq -\lambda_{\min}(Q) \|z_k\|^2 - 2z_k^T P B \widehat{W}_k^T \phi(\widehat{x}_k) \\ &\quad - b \widehat{W}_{2,k}^T \phi_2(y_k) \tanh^2\left(\frac{s_k}{\eta(t)}\right) \\ &\quad + \left[1 - b \tanh^2\left(\frac{s_k}{\eta(t)}\right)\right] \Xi(y_k) - K s_k^2. \end{aligned} \quad (46)$$

For subsequent discussion, we rewrite (46) as

$$\begin{aligned} &2z_k^T P B \widehat{W}_k^T \phi(\widehat{x}_k) + b \widehat{W}_{2,k}^T \phi_2(y_k) \tanh^2\left(\frac{s_k}{\eta(t)}\right) \\ &\leq -\dot{V}_k - \lambda_{\min}(Q) \|z_k\|^2 \\ &\quad + \left[1 - b \tanh^2\left(\frac{s_k}{\eta(t)}\right)\right] \Xi(y_k) - K s_k^2. \end{aligned} \quad (47)$$

For further analysis, we exploit the following property of the tangent hyperbolic function.

Lemma 16. Consider the set Ω_{s_k} defined by $\Omega_{s_k} := \{s_k \mid |s_k| \leq m\eta(t)\}$. Then, for any $s_k \notin \Omega_{s_k}$, the following inequality is satisfied:

$$1 - b \tanh^2\left(\frac{s_k}{\eta(t)}\right) < 0, \quad (48)$$

where $b > 1$, $m = \ln(\sqrt{b/(b-1)} + \sqrt{1/(b-1)})$.

Proof. See Appendix A. $\square$

The convergence and boundedness property of the proposed observer-based AILC scheme are summarized in the following theorem.

Theorem 17. Considering the closed-loop system consisting of plant (1) and input saturation model (3) under Assumptions 1–5, 11, 12 and 15, design state observer (15) and adaptive iterative learning controller (43) with parameter update algorithms (44) and (45); the following properties can be guaranteed: (i) all the signals of the closed-loop system are bounded in L_T^∞ -norm; (ii) the error $e_{s_k}(t)$ converges to a small region around zero as $k \rightarrow \infty$; that is, $\lim_{k \rightarrow \infty} \int_0^T (e_{s_k}(\sigma))^2 d\sigma \leq \varepsilon_{esk}$, $\varepsilon_{esk} = (1/2K)(1+m)^2 \varepsilon^2$; and (iii) transient performance $\lim_{k \rightarrow \infty} |y_k(t) - y_d(t)| \leq \min\{k_0 \sum_{i=1}^{n-1} \sqrt{\varepsilon_i^2} e^{-\lambda_0 t} + (1+m)\varepsilon k_0(1/(\lambda_0 - K))(e^{-Kt} - e^{-\lambda_0 t}), [p_1 + (1/k_1)(Kp_1 + p_2)]\varepsilon e^{-Kt}\}$, where λ_0 , k_0 , p_1 , and p_2 are positive constants and will be given later.

Proof. See Appendix B. $\square$

4. Simulation Study

In this section, we present a simulation example to verify the effectiveness of proposed control scheme. Consider the

following second-order nonlinear time delay system with input saturation:

$$\begin{aligned}\dot{x}_{1,k}(t) &= x_{2,k}(t), \\ \dot{x}_{2,k}(t) &= f(x_k(t), t) + h(x_{\tau,k}(t), t) + u_k(t) + d(t), \quad (49) \\ y_k(t) &= x_{1,k}(t),\end{aligned}$$

where $f(X_k(t), t) = -x_{1,k}(t)x_{2,k}(t)\sin(x_{1,k}(t)x_{2,k}(t))$ and $h(y_{\tau,k}, t) = 0.5\sin(t)e^{-|\cos(0.5t)|}(y_{\tau_1}\sin(y_{\tau_1}) + y_{\tau_2}\sin(y_{\tau_2}))$, with $\tau_1 = 0.5(1 + \sin(0.3t))$ and $\tau_2 = 0.8(1 - \sin(0.5t))$. The external disturbance is given by $d = 0.1 * \text{rand} * \sin t$, where rand presents Gaussian noise which takes a random value on $[0, 1]$. We choose $Q = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.002 \end{bmatrix}$. By using LMI toolbox, we can obtain $K_0 = [3.2894, 2.9764]^T$ and $P = \begin{bmatrix} 0.4741 & -0.2848 \\ -0.2848 & 0.4741 \end{bmatrix}$. Next, we give the simulation study in three cases.

Case 1. The desired trajectory is given by $x_d(t) = [\sin t, \cos t]^T$. The systems run on $[0, 4\pi]$ repetitively. The design parameters are chosen as $\varepsilon_1 = \varepsilon_2 = 1$, $\lambda = 2$, $K = 2$, $\gamma = 0.5$, $q_1 = 0.5$, $q_2 = 1$, $\varepsilon = \lambda\varepsilon_1 + \varepsilon_2 = 3$, and $u_M = 1.3$. The parameters for two NNs are chosen as $l = 30$, $\mu_j = (1/l)(2j - l)[2, 3, 2, 3]$, $\sigma_j = 1.5$, $j = 1, 2, \dots, l$, and $l_{\Xi} = 15$, $\mu_{\Xi j} = (2/l_{\Xi})(2j - l_{\Xi})$, $\sigma_{\Xi j} = 1$, $j = 1, 2, \dots, l_{\Xi}$. The upper bound of control input is $u_M = 1.3$. Parts of the simulation results are shown in Figures 2–9. Figures 2 and 6 express the trajectories of the observer output and the desired reference signal of the first iteration and the tenth iteration, respectively; Figures 3 and 7 show the trajectories of $x_{1,k}$ and $\hat{x}_{1,k}$ of the first iteration ($k = 1$) and the tenth iteration ($k = 10$), respectively; Figures 4 and 7 show the effects of input saturation. Obviously, these signals are bounded. Figures 8 and 9 show the convergence of $\int_0^T z_{1,k}^2(t)dt$ and $\int_0^T s_k^2(t)dt$ along the iteration axis, which indicates that the proposed approach achieves perfect tracking by iterative learning.

Case 2. To show the control performance for more complicated desired trajectory, we choose the desired trajectory as $x_d(t) = [\sin t + \sin(0.5t), \cos t + 0.5\cos(0.5t)]^T$. The system runs on $[0, 8\pi]$ repetitively. The design parameters are chosen as the same as Case 1. The control input is bounded by $u_M = 4$. The systems run for ten times repeatedly. Parts of the simulation results are shown in Figures 10–17. Figures 10–12 show the results of the first iteration ($k = 1$). It can be seen that the tracking performance of the first iteration is not so good. Figures 13–15 present the simulation results of the tenth iteration ($k = 10$). Obviously, the tracking performance has been improved greatly compared with that of the first iteration, which can be also demonstrated by convergence of $\int_0^T z_{1,k}^2(t)dt$ and $\int_0^T s_k^2(t)dt$ along the iteration domain in Figures 16 and 17. The results indicate the validity of the proposed method for more complicated design trajectory.

Case 3. Finally, the contribution of the proposed observer-based AILC scheme is shown by comparing the proposed controller with traditional adaptive neural network controller

FIGURE 2: Reference signal y_d on observer state $\hat{x}_{1,k}$ ($k = 1$) in Case 1.

FIGURE 3: $x_{1,k}$ on $\hat{x}_{1,k}$ ($k = 1$) in Case 1.

[48]. The controller is the same, but the adaptive laws using σ -modification for adaptive laws of parameters are given by

$$\begin{aligned}\hat{W} &= \Gamma_W (z_k^T P B \phi(\hat{x}_k) - \delta_W \hat{W}), \\ \hat{W}_2 &= q_2 s_k \phi_2(y_k) - \gamma \hat{W}_2.\end{aligned}\quad (50)$$

The design parameters are chosen as $\Gamma_W = \text{diag}\{0.2, \dots, 0.2\}$, $\delta_W = 0.5$, $q_2 = 1$, $\gamma = 0.5$, and $u_M = 3.5$. Since traditional adaptive NN controller runs in time domain, the notation k in this case does not have any practical meaning. Figures 18–20 provide simulation results. From the simulation results shown in Figures 18–20, it is obvious that the adaptive NN

FIGURE 4: Control output u_k and input v_k ($k = 1$) in Case 1. FIGURE 6: $x_{1,k}$ on $\hat{x}_{1,k}$ ($k = 10$) in Case 1. FIGURE 5: Reference signal y_d on observer state $\hat{x}_{1,k}$ ($k = 10$) in Case 1. FIGURE 7: Control output u_k and input v_k ($k = 10$) in Case 1.

controller is unable to achieve perfect tracking performance and performs much worse than the proposed approach.

According to the simulation results, it is confirmed that the proposed AILC can guarantee fairly good control performance for uncertain nonlinear systems with unknown time-varying delays and control input saturation in the presence of external disturbance. Moreover, it is verified that our control scheme is more suitable than robust adaptive neural network control methods for finite time repeated problem.

5. Conclusions

An adaptive iterative learning control scheme has been proposed for a class of nonlinear time-varying systems

with unknown time-varying delays and input saturation running on a finite time interval repeatedly. An observer has been designed to estimate the states. The uncertainties from unknown time-varying delays have been compensated for by using appropriate Lyapunov-Krasovskii functional in the Lyapunov function candidate, such that control law is delay-independent. Both time-varying and time-invariant RBF neural networks are employed to deal with the systems uncertainties. Theoretical analysis by constructing Lyapunov-like CEF has shown that the proposed systematic design is able to guarantee the convergence of tracking errors to a small residual domain around the origin and the boundedness of all the closed-loop signals. Simulation results have been

FIGURE 8: $\int_0^T z_{1,k}^2(t)dt$ versus the number of iterations in Case 1. FIGURE 9: $\int_0^T s_k^2(t)dt$ versus the number of iterations in Case 1.

provided to show the effectiveness the proposed control scheme.

Appendices

A. Proof of Lemma 16

Denote $x = s_k(t)/\eta(t)$. The inequality (48) in Lemma 16 can be expressed as

$$\frac{1}{b} < \tanh^2(x) = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2 = 1 - \left(\frac{2}{e^x + e^{-x}} \right)^2. \quad (\text{A.1})$$

Since e^x and e^{-x} are positive, we can know

$$e^x + e^{-x} > 2\sqrt{\frac{b}{b-1}}. \quad (\text{A.2})$$

FIGURE 10: Reference signal y_d on observer state $\hat{x}_{1,k}$ ($k = 1$) in Case 2. FIGURE 11: $x_{1,k}$ on $\hat{x}_{1,k}$ ($k = 1$) in Case 2.

Further, we can obtain

$$e^{2x} - 2\sqrt{\frac{b}{b-1}}e^x + 1 > 0. \quad (\text{A.3})$$

Solving quadratic inequality (A.3), we get

$$0 < e^x < \sqrt{\frac{b}{b-1}} - \sqrt{\frac{1}{b-1}} \quad (\text{A.4})$$

$$\text{or } e^x > \sqrt{\frac{b}{b-1}} + \sqrt{\frac{1}{b-1}}.$$

FIGURE 12: Control output u_k and input v_k ($k = 1$) in Case 2. FIGURE 14: $x_{1,k}$ on $\hat{x}_{1,k}$ ($k = 10$) in Case 2. FIGURE 13: Reference signal y_d on observer state $\hat{x}_{1,k}$ ($k = 10$) in Case 2. FIGURE 15: Control output u_k and input v_k ($k = 10$) in Case 2.

On the other hand, from $|s_k(t)| > m\eta(t)$, we know

$$\begin{aligned} x &< -m \\ \text{or } x &> m \end{aligned} \quad (\text{A.5})$$

which implies

$$\begin{aligned} 0 < e^x &< \frac{1}{\sqrt{b/(b-1)} + \sqrt{1/(b-1)}} \\ &= \sqrt{\frac{b}{(b-1)}} - \sqrt{\frac{1}{(b-1)}} \end{aligned}$$

$$\text{or } e^x > \sqrt{\frac{b}{(b-1)}} + \sqrt{\frac{1}{(b-1)}}. \quad (\text{A.6})$$

Obviously, Lemma 16 holds.

B. Proof of Theorem 17

The term $[1 - b \tanh^2(s_k/\eta(t))]\Xi(y_k)$ in (47) may be positive or negative, which depends on the size of s_k . Thus, two cases must be considered for subsequent analysis of stability.

Case 1 ($s_k \in \Omega_{s_k}$). If $s_k \in \Omega_{s_k}$, then $|s_k| \leq m\eta(t)$ is satisfied. We divide it into three cases. (1) If $s_k = 0$, we know that e_{s_k} is

FIGURE 16: $\int_0^T z_{1,k}^2(t)dt$ versus the number of iterations in Case 2. FIGURE 17: $\int_0^T s_k^2(t)dt$ versus the number of iterations in Case 2. FIGURE 18: Reference signal y_d on observer state $\hat{x}_1$ in Case 3. FIGURE 19: x_1 on $\hat{x}_1$ in Case 3. FIGURE 20: Control output u_k and input v_k in Case 3.

bounded by $\eta(t)$; that is, $|e_{sk}| \leq \eta(t)$; (2) if $s_k > 0$, we know $s_k = e_{sk} - \eta(t)$; then from $|s_k| \leq m\eta(t)$ we can obtain $s_k = e_{sk} - \eta(t) \leq m\eta(t)$ which further implies $0 < e_{sk} \leq (1+m)\eta(t)$; (3) similarly, if $s_k < 0$, we have $s_k = e_{sk} + \eta(t) \geq -m\eta(t)$ which implies $0 > e_{sk} \geq -(1+m)\eta(t)$. Synthesizing the foregoing discussion we know that $|e_{sk}| \leq (1+m)\eta(t)$ holds. Thus it implies the boundedness of $\hat{x}_{i,k}$ since $x_d(t)$ is bounded in L_T^∞ -norm. From updating laws (44) and (45), we know that $\widehat{W}_k(t) \in L_T^\infty$ and $\widehat{W}_{2,k}(t) \in L_T^\infty$. Following this chain of reasoning, the boundedness of z_k and x_k in L_T^∞ -norm can be deduced. Then, we can achieve the boundedness of v_k . As such, all closed-loop signals are bounded L_T^∞ -norm. Furthermore, from $|e_{sk}| \leq (1+m)\eta(t)$ and the definition of e_k , it can be easily derived that $e_{1,k}$ and $e_{2,k}$ are in a small

ball of $p_1\eta(t)$ and $p_2\eta(t)$, respectively, with p_1 and p_2 as small positive constants, which, in other words, are $|e_{1,k}| \leq p_1\eta(t)$ and $|e_{2,k}| \leq p_2\eta(t)$. Then it can be derived that $|\dot{e}_{1,k}| \leq p_1K\eta(t)$. From (33), it follows that

$$k_1 |z_{1,k}| \leq |e_{2,k}| + |\dot{e}_{1,k}| = (Kp_1 + p_2) \varepsilon e^{-Kt}. \quad (\text{B.1})$$

Then it implies that

$$\begin{aligned} |y_k - y_d| &= |z_{1,k} + e_{1,k}| \leq |z_{1,k}| + |e_{1,k}| \\ &\leq \left[p_1 + \frac{1}{k_1} (Kp_1 + p_2) \right] \varepsilon e^{-Kt}. \end{aligned} \quad (\text{B.2})$$

In this case, it is obvious that all the signals of the closed system are bounded.

Case 2 ($s_k(t) \notin \Omega_{s_k}$). According to Lemma 16, it is clear that $[1 - b \tanh^2(s_k/\eta(t))]\Xi(y_k)$ can be removed from the right-hand side of (47). Therefore, it follows from (47) that

$$\begin{aligned} 2z_k^T PB \bar{W}_k^T \phi(\hat{x}_k) + b \bar{W}_{2,k}^T \phi_2(y_k) \tanh^2\left(\frac{s_k}{\eta(t)}\right) \\ \leq -\dot{V}_k - \lambda_{\min}(Q) \|z_k\|^2 - K s_k^2. \end{aligned} \quad (\text{B.3})$$

Next, we will check the stability of the system by using CEF-based analysis. Define the Lyapunov-like CEF as follows:

$$\begin{aligned} E_k(t) &= \frac{1}{2q_1} \int_0^t \bar{W}_k^T \bar{W}_k d\sigma + \frac{\gamma}{2q_2} \int_0^t \bar{W}_{2,k}^T \bar{W}_{2,k} d\sigma \\ &\quad + \frac{(1-\gamma)}{2q_2} \bar{W}_{2,k}^T \bar{W}_{2,k}. \end{aligned} \quad (\text{B.4})$$

The subsequent derivations consist of five parts.

(1) *Difference of $E_k(t)$* . Computing the difference of $E_k(t)$, we obtain

$$\begin{aligned} \Delta E_k(t) &= E_k(t) - E_{k-1}(t) \\ &= \frac{1}{2q_1} \int_0^t [\bar{W}_k^T \bar{W}_k - \bar{W}_{k-1}^T \bar{W}_{k-1}] d\sigma \\ &\quad + \frac{\gamma}{2q_2} \int_0^t [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] d\sigma \\ &\quad + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}]. \end{aligned} \quad (\text{B.5})$$

Recalling difference type adaptive learning law (44) and utilizing the algebraic relation $(a-b)^T(a-b) - (a-c)^T(a-c) = (c-b)^T[2(a-b) + (b-c)]$, the first term on the right-hand side of (B.5) can be transformed as

$$\begin{aligned} \frac{1}{2q_1} \int_0^t [\bar{W}_k^T \bar{W}_k - \bar{W}_{k-1}^T \bar{W}_{k-1}] d\sigma \\ = \int_0^t 2z_k^T PB \bar{W}_k^T \phi(\hat{x}_k) d\sigma \\ - 2q_1 \int_0^t (z_k^T PB)^2 \|\phi(\hat{x}_k)\|^2 d\sigma. \end{aligned} \quad (\text{B.6})$$

Taking mixed type adaptive law (45) into consideration, we can obtain

$$\begin{aligned} \frac{\gamma}{2q_2} \int_0^t [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] \\ = \frac{\gamma}{2q_2} \int_0^t [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] d\sigma \\ + \frac{(1-\gamma)}{q_2} \int_0^t \bar{W}_{2,k}^T \dot{\bar{W}}_{2,k} d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T(0) \bar{W}_{2,k}(0) - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] \\ = \int_0^t b \tanh^2\left(\frac{s_k}{\eta}\right) \bar{W}_{2,k}^T \phi_2(y_k) d\sigma \\ - \frac{\gamma}{q_2} \int_0^t \bar{W}_{2,k}^T (\bar{W}_{2,k} - \bar{W}_{2,k-1}) d\sigma \\ + \frac{\gamma}{2q_2} \int_0^t [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T(0) \bar{W}_{2,k}(0) - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] \\ = \int_0^t b \tanh^2\left(\frac{s_k}{\eta}\right) \bar{W}_{2,k}^T \phi_2(y_k) d\sigma \\ - \frac{\gamma}{q_2} \int_0^t \bar{W}_{2,k}^T (\bar{W}_{2,k} - \bar{W}_{2,k-1}) d\sigma \\ + \frac{\gamma}{2q_2} \int_0^t [\bar{W}_{2,k}^T \bar{W}_{2,k} - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T(0) \bar{W}_{2,k}(0) - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] \\ = \int_0^t b \tanh^2\left(\frac{s_k}{\eta}\right) \bar{W}_{2,k}^T \phi_2(y_k) d\sigma \\ - \frac{\gamma}{2q_2} \int_0^t (\bar{W}_{2,k} - \bar{W}_{2,k-1})^T (\bar{W}_{2,k} - \bar{W}_{2,k-1}) d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T(0) \bar{W}_{2,k}(0) - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}] \\ \leq \int_0^t b \tanh^2\left(\frac{s_k}{\eta}\right) \bar{W}_{2,k}^T \phi_2(y_k) d\sigma \\ + \frac{(1-\gamma)}{2q_2} [\bar{W}_{2,k}^T(0) \bar{W}_{2,k}(0) - \bar{W}_{2,k-1}^T \bar{W}_{2,k-1}]. \end{aligned} \quad (\text{B.7})$$

Substituting (B.6) and (B.7) back into (B.5) and using (B.3), it follows that

$$\begin{aligned} \Delta E_k(t) \\ \leq \int_0^t 2z_k^T PB \bar{W}_k^T \phi(\hat{x}_k) d\sigma \end{aligned}$$

$$\begin{aligned}
& + \int_0^t b \tanh^2 \left(\frac{s_k}{\eta} \right) \widetilde{W}_{2,k}^T \phi_2(y_k) d\sigma \\
& + \frac{(1-\gamma)}{2q_2} \left[\widetilde{W}_{2,k}^T(0) \widetilde{W}_{2,k}(0) - \widetilde{W}_{2,k-1}^T \widetilde{W}_{2,k-1} \right] \\
& \leq -V_k(t) + V_k(0) - K \int_0^t s_k^2 d\sigma \\
& - \lambda_{\min}(Q) \int_0^t \|z_k\|^2 d\sigma \\
& + \frac{(1-\gamma)}{2q_2} \left[\widetilde{W}_{2,k}^T(0) \widetilde{W}_{2,k}(0) - \widetilde{W}_{2,k-1}^T \widetilde{W}_{2,k-1} \right] \\
& \leq V_k(0) \\
& + \frac{(1-\gamma)}{2q_2} \left[\widetilde{W}_{2,k}^T(0) \widetilde{W}_{2,k}(0) - \widetilde{W}_{2,k-1}^T \widetilde{W}_{2,k-1} \right].
\end{aligned} \tag{B.8}$$

From Assumptions 5, 11, and 12, it is obvious that $V_k(0) = 0$. Letting $t = T$ in (B.8), according to $\widetilde{W}_{2,k}(0) = \widetilde{W}_{2,k-1}(T)$, $\widetilde{W}_{2,1}(0) = 0$, we can have

$$\begin{aligned}
\Delta E_k(T) & \leq -V_k(T) - K \int_0^T s_k^2 d\sigma \\
& - \lambda_{\min}(Q) \int_0^T \|z_k\|^2 d\sigma \\
& \leq -K \int_0^T s_k^2 d\sigma - \lambda_{\min}(Q) \int_0^T \|z_k\|^2 d\sigma \leq 0.
\end{aligned} \tag{B.9}$$

Inequality (B.9) indicates that $E_k(T)$ is decreasing along iteration axis. Thus the boundedness of $E_k(T)$ can be ensured provided that $E_1(T)$ is finite.

(2) *The Finiteness of $E_1(T)$.* According to (B.4), we know

$$\begin{aligned}
E_1(t) & = \frac{1}{2q_1} \int_0^t \widetilde{W}_1^T \widetilde{W}_1 d\sigma + \frac{\gamma}{2q_2} \int_0^t \widetilde{W}_{2,1}^T \widetilde{W}_{2,1} d\sigma \\
& + \frac{(1-\gamma)}{2q_2} \widetilde{W}_{2,1}^T \widetilde{W}_{2,1}.
\end{aligned} \tag{B.10}$$

Taking the time derivative of $E_1(t)$ yields

$$\begin{aligned}
\dot{E}_1(t) & = \frac{1}{2q_1} \widetilde{W}_1^T \dot{\widetilde{W}}_1 + \frac{\gamma}{2q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} \\
& + \frac{(1-\gamma)}{q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1}.
\end{aligned} \tag{B.11}$$

Recalling parameter adaptive learning laws (44) and (45), we have $\dot{\widetilde{W}}_1(t) = 2q_1 z_1^T PB\phi(\hat{x}_1)$ and $(1-\gamma)\dot{\widetilde{W}}_{2,1} = -\gamma\dot{\widetilde{W}}_{2,1} + q_2 b \tanh^2(s_1/\eta(t))\phi_2(y_1)$; then we obtain

$$\begin{aligned}
\frac{1}{2q_1} \widetilde{W}_1^T \dot{\widetilde{W}}_1 & = \frac{1}{2q_1} \left[\widetilde{W}_1^T \dot{\widetilde{W}}_1 - 2\widetilde{W}_1^T \dot{\widetilde{W}}_1 \right] + \frac{1}{q_1} \widetilde{W}_1^T \dot{\widetilde{W}}_1 \\
& = \frac{1}{2q_1} \left[(\widetilde{W}_1^T - W^*)^T (\dot{\widetilde{W}}_1 - \dot{W}^*) \right. \\
& \quad \left. - 2(\widetilde{W}_1^T - W^*)^T \dot{\widetilde{W}}_1 \right] + 2q_1 z_1^T PB\widetilde{W}_1^T \phi(\hat{x}_1) \\
& = \frac{1}{2q_1} \left[-\widetilde{W}_1^T \dot{\widetilde{W}}_1 + W^{*T} \dot{W}^* \right] \\
& + 2q_1 z_1^T PB\widetilde{W}_1^T \phi(\hat{x}_1) \leq \frac{1}{2q_1} W^{*T} \dot{W}^* \\
& + 2q_1 z_1^T PB\widetilde{W}_1^T \phi(\hat{x}_1),
\end{aligned} \tag{B.12}$$

$$\begin{aligned}
\frac{\gamma}{2q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} & + \frac{(1-\gamma)}{q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} = \frac{\gamma}{2q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} \\
& - \frac{\gamma}{q_2} \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} + b \tanh^2 \left(\frac{s_1}{\eta(t)} \right) \widetilde{W}_{2,1}^T \phi_2(y_1) \\
& = \frac{\gamma}{2q_2} \left[\widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} - 2\widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} + \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} \right] - \frac{\gamma}{2q_2} \\
& \cdot \widetilde{W}_{2,1}^T \dot{\widetilde{W}}_{2,1} + b \tanh^2 \left(\frac{s_1}{\eta(t)} \right) \widetilde{W}_{2,1}^T \phi_2(y_1) \\
& \leq \frac{\gamma}{2q_2} \left[\widetilde{W}_{2,1} - \dot{\widetilde{W}}_{2,1} \right]^T \left[\widetilde{W}_{2,1} - \dot{\widetilde{W}}_{2,1} \right] \\
& + b \tanh^2 \left(\frac{s_1}{\eta(t)} \right) \widetilde{W}_{2,1}^T \phi_2(y_1) = \frac{\gamma}{2q_2} W_2^{*T} \dot{W}_2^* \\
& + b \tanh^2 \left(\frac{s_1}{\eta(t)} \right) \widetilde{W}_{2,1}^T \phi_2(y_1).
\end{aligned} \tag{B.13}$$

Combining (B.12) and (B.13), it follows from (B.11) that

$$\begin{aligned}
\dot{E}_1(t) & \leq \frac{1}{2q_1} W^{*T} \dot{W}^* + 2q_1 z_1^T PB\widetilde{W}_1^T \phi(\hat{x}_1) \\
& + \frac{\gamma}{2q_2} W_2^{*T} \dot{W}_2^* \\
& + b \tanh^2 \left(\frac{s_1}{\eta(t)} \right) \widetilde{W}_{2,1}^T \phi_2(y_1) \\
& \leq -\dot{V}_1 - \lambda_{\min}(Q) \|z_1\|^2 - Ks_1^2 + \frac{1}{2q_1} W^{*T} \dot{W}^* \\
& + \frac{\gamma}{2q_2} W_2^{*T} \dot{W}_2^*.
\end{aligned} \tag{B.14}$$

Denote $c_{\max} = \max_{t \in [0, T]} \{(1/2q_1)W^{*T}(t)W^*(t) + (\gamma/2q_2)W_2^{*T}W_2^*\}$. Integrating (B.14) over $[0, t]$ leads to

$$\begin{aligned} E_1(t) - E_1(0) &\leq -V_1(t) + V_1(0) \\ &\quad - \lambda_{\min}(Q) \int_0^t \|z_1\|^2 d\sigma \\ &\quad - \int_0^t K s_1^2(\sigma) d\sigma + t \cdot c_{\max}. \end{aligned} \quad (\text{B.15})$$

According to $\widehat{W}_{2,1}(0) = 0$, we get

$$E_1(0) = \frac{(1-\gamma)}{2q_2} \widehat{W}_{2,1}^T(0) \widehat{W}_{2,1}(0) = \frac{(1-\gamma)}{2q_2} \|W_2^*\|^2. \quad (\text{B.16})$$

Substituting (B.16) back into (B.15) results in

$$\begin{aligned} E_1(t) &\leq t \cdot c_{\max} + \frac{(1-\gamma)}{2q_2} \|W_2^*\|^2 \\ &< T \cdot c_{\max} + \frac{(1-\gamma)}{2q_2} \|W_2^*\|^2, \quad t \in [0, T] \end{aligned} \quad (\text{B.17})$$

which implies the boundedness of $E_1(t)$ on $[0, T]$. Letting $t = T$, we can obtain the boundedness of $E_1(T)$:

$$E_1(T) \leq T \cdot c_{\max} + \frac{(1-\gamma)}{2q_2} \|W_2^*\|^2 < \infty. \quad (\text{B.18})$$

Thus, recalling Part (1) we obtain the finiteness of $E_k(T)$.

(3) *The Finiteness of $E_k(t)$.* Next we will prove the boundedness of the signals by induction. Firstly, we separate $E_k(t)$ into two parts

$$\begin{aligned} E_k^1(t) &= \frac{1}{2q_1} \int_0^t \widehat{W}_k^T \widehat{W}_k d\sigma + \frac{\gamma}{2q_2} \int_0^t \widehat{W}_{2,k}^T \widehat{W}_{2,k} d\sigma, \\ E_k^2(t) &= \frac{(1-\gamma)}{2q_2} \widehat{W}_{2,k}^T \widehat{W}_{2,k}. \end{aligned} \quad (\text{B.19})$$

The boundedness of $E_k^1(T)$ and $E_k^2(T)$ is guaranteed for all iterations. Consequently, for all $k \in N$, there exist two constants M_1 and M_2 satisfying

$$\begin{aligned} E_k^1(t) &\leq E_k^1(T) \leq M_1 < \infty, \\ E_k^2(T) &\leq M_2. \end{aligned} \quad (\text{B.20})$$

Then, we have

$$E_k(t) = E_k^1(t) + E_k^2(t) \leq M_1 + E_k^2(t). \quad (\text{B.21})$$

On the other hand, from (45) and $E_{k+1}^2(0) = E_k^2(T)$, we obtain

$$\begin{aligned} \Delta E_{k+1}(t) &< \frac{(1-\gamma)}{2q_2} [\widehat{W}_{2,k+1}^T(0) \widehat{W}_{2,k+1}(0) - \widehat{W}_{2,k}^T \widehat{W}_{2,k}] \\ &\leq M_2 - E_k^2(t). \end{aligned} \quad (\text{B.22})$$

Combining (B.21) and (B.22) results in

$$E_{k+1}(t) = E_k(t) + \Delta E_{k+1}(t) \leq M_1 + M_2. \quad (\text{B.23})$$

As we have proven that $E_1(t)$ is bounded, therefore $E_k(t)$ is finite. Furthermore, we can obtain the boundedness of $\widehat{W}_k(t)$ and $\widehat{W}_{2,k}$.

(4) *Learning Convergence Property.* Applying (B.9) repeatedly, we have

$$\begin{aligned} E_k(T) &= E_1(T) + \sum_{j=2}^k \Delta E_j(T) \\ &\leq E_1(T) - K \sum_{j=2}^k \int_0^T s_j^2 d\sigma \\ &\quad - \lambda_{\min}(Q) \sum_{j=2}^k \int_0^T \|z_j\|^2 d\sigma. \end{aligned} \quad (\text{B.24})$$

We rewrite inequality (B.24) as

$$\begin{aligned} \sum_{j=2}^k \int_0^T s_j^2 d\sigma &\leq \frac{(E_1(T) - E_k(T))}{K} \leq \frac{E_1(T)}{K}, \\ \sum_{j=2}^k \int_0^T \|z_j\|^2 d\sigma &\leq \frac{(E_1(T) - E_k(T))}{\lambda_{\min}(Q)} \leq \frac{E_1(T)}{\lambda_{\min}(Q)}. \end{aligned} \quad (\text{B.25})$$

Taking the limitation of (B.25), it follows that

$$\begin{aligned} \lim_{k \rightarrow \infty} \sum_{j=2}^k \int_0^T s_j^2(\sigma) d\sigma &\leq \frac{E_1(T)}{K}, \\ \lim_{k \rightarrow \infty} \sum_{j=2}^k \int_0^T \|z_j\|^2 d\sigma &\leq \frac{E_1(T)}{\lambda_{\min}(Q)}. \end{aligned} \quad (\text{B.26})$$

Since $E_1(T)$ is bounded, according to the convergence theorem of the sum of series, $\lim_{k \rightarrow \infty} \int_0^T s_k^2(\sigma) d\sigma = 0$ and $\lim_{k \rightarrow \infty} \int_0^T \|z_k\|^2 d\sigma = 0$. It is obvious that $\lim_{k \rightarrow \infty} \int_0^T (y_k - \widehat{y}_k)^2 d\sigma \leq \lim_{k \rightarrow \infty} \int_0^T \|z_k\|^2 d\sigma = 0$, for all $t \in [0, T]$. Besides, $\lim_{k \rightarrow \infty} \int_0^T s_k^2(\sigma) d\sigma = 0$ is equivalent to $\lim_{k \rightarrow \infty} \int_0^T |s_k(\sigma)| d\sigma = 0$ which means that $\lim_{k \rightarrow \infty} \int_0^T |e_{sk}(\sigma)| d\sigma \leq \int_0^T \eta(\sigma) d\sigma$ and $\lim_{k \rightarrow \infty} \int_0^T (e_{sk}(\sigma))^2 d\sigma \leq \int_0^T \eta^2(\sigma) d\sigma$. From $\int_0^T s_k^2(\sigma) d\sigma \leq \int_0^T s_k^2(\sigma) d\sigma$ and $\int_0^t \|z_k(\sigma)\|^2 d\sigma \leq \int_0^T \|z_k(\sigma)\|^2 d\sigma$, we can get the boundedness of $s_k(t)$ and $\|z_k\|$ in L_T^2 -norm, which further implies that $x_k(t)$ and $\widehat{x}_k(t)$ are bounded. Based on the above reasoning, we can arrive at the fact that $v_k(t)$ is bounded.

Synthesizing the discussions in two cases, we can draw the conclusion that, for two cases, the proposed control algorithm is able to guarantee that all closed-loop signals are bounded, and $\lim_{k \rightarrow \infty} \int_0^T \|z_k\|^2 d\sigma = 0$ and $\lim_{k \rightarrow \infty} \int_0^T (e_{sk}(\sigma))^2 d\sigma \leq$

$\varepsilon_e, \varepsilon_e = \int_0^T ((1+m)\eta_1(\sigma))^2 d\sigma = (1/2K)(1+m)^2 \varepsilon^2 (1 - e^{-2KT}) \leq (1/2K)(1+m)^2 \varepsilon^2 = \varepsilon_{esk}$. Furthermore, the bound of $e_{sco}(t)$ will satisfy $\lim_{k \rightarrow \infty} |e_{sk}(t)| = e_{sco}(t) = (1+m)\varepsilon e^{-Kt}$, for all $t \in [0, T]$.

(5) *Transient Performance.* Next, we determine an upper bound of the error vector $e_k(t)$. Define the vector $\zeta_k(t) = [e_{1,k}(t), e_{2,k}(t), \dots, e_{n-1,k}(t)]^T$; then a state representation of $e_{sk}(t) = [\Lambda^T \ 1] e_k(t)$ can be expressed as

$$\dot{\zeta}_k(t) = A_s \zeta_k(t) + b_s e_{sk}(t) + K_s z_{1,k}, \quad (\text{B.27})$$

where

$$A_s = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{n-1} \end{bmatrix} \in R^{(n-1) \times (n-1)},$$

$$b_s = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{n-1}, \quad (\text{B.28})$$

$$K_s = \begin{bmatrix} k_1 \\ \vdots \\ k_{n-1} \end{bmatrix},$$

with A_s as a stable matrix. In addition, there are two constants $k_0 > 0$ and $\lambda_0 > 0$ such that $\|e^{A_s t}\| \leq k_0 e^{-\lambda_0 t}$ [48]. The solution for $\zeta_k(t)$ is

$$\begin{aligned} \zeta_k(t) &= e^{A_s t} \zeta_k(0) + \int_0^t e^{A_s(t-\sigma)} b_s e_{sk}(\sigma) d\sigma \\ &\quad + \int_0^t e^{A_s(t-\sigma)} K_s z_{1,k}(\sigma) d\sigma. \end{aligned} \quad (\text{B.29})$$

Consequently, it follows that

$$\begin{aligned} \|\zeta_k(t)\| &\leq k_0 \|\zeta_k(0)\| e^{-\lambda_0 t} \\ &\quad + k_0 \int_0^t e^{-\lambda_0(t-\sigma)} |e_{sk}(\sigma)| d\sigma \\ &\quad + k_0 \|K_s\| \int_0^t e^{-\lambda_0(t-\sigma)} |z_{1,k}(\sigma)| d\sigma. \end{aligned} \quad (\text{B.30})$$

When we choose suitable parameters such that $\lambda_0 > K$, from $\lim_{k \rightarrow \infty} |e_{sk}(t)| \leq (1+m)\eta(t)$, we can have

$$\begin{aligned} \|\zeta_\infty(t)\| &= k_0 \|\zeta_\infty(0)\| e^{-\lambda_0 t} \\ &\quad + k_0 \int_0^t e^{-\lambda_0(t-\sigma)} |e_{sco}(\sigma)| d\sigma \\ &\quad + k_0 \|K_s\| \int_0^t e^{-\lambda_0(t-\sigma)} |z_{1,\infty}(\sigma)| d\sigma \\ &\leq k_0 \|\zeta_\infty(0)\| \\ &\quad + (1+m)\varepsilon k_0 \int_0^t e^{-\lambda_0(t-\sigma)} e^{-K\sigma} d\sigma + 0 \\ &= k_0 \|\zeta_\infty(0)\| \\ &\quad + (1+m)\varepsilon k_0 \frac{1}{\lambda_0 - K} (e^{-Kt} - e^{-\lambda_0 t}) \\ &\leq k_0 \|\zeta_\infty(0)\| + \frac{1}{\lambda_0 - K} (1+m)\varepsilon k_0. \end{aligned} \quad (\text{B.31})$$

Noting $e_{sk}(t) = [\Lambda^T \ 1] e_k(t)$ and $e_k(t) = [\zeta_k^T(t) \ e_{n,k}(t)]^T$, we have

$$\begin{aligned} \|e_k(t)\| &\leq \|\zeta_k(t)\| + |e_{n,k}(t)| \\ &= \|\zeta_k(t)\| + |e_{sk}(t) - \Lambda^T \zeta_k(t)| \\ &\leq (1 + \|\Lambda\|) \|\zeta_k(t)\| + |e_{sk}(t)|. \end{aligned} \quad (\text{B.32})$$

Considering the two inequalities above, we can obtain

$$\begin{aligned} \|e_\infty(t)\| &\leq (1 + \|\Lambda\|) \|\zeta_\infty(t)\| + |e_{sco}(t)| \\ &\leq (1 + \|\Lambda\|) \left(k_0 \|\zeta_\infty(0)\| + \frac{1}{\lambda_0 - K} (1+m)\varepsilon k_0 \right) \\ &\quad + (1+m)\eta(t) \\ &\leq (1 + \|\Lambda\|) \left(k_0 \sum_{i=1}^{n-1} \sqrt{\varepsilon_i^2} + \frac{1}{\lambda_0 - K} (1+m)\varepsilon k_0 \right) \\ &\quad + (1+m)\eta(t). \end{aligned} \quad (\text{B.33})$$

Since $\zeta_k(t) = [e_{1,k}(t), e_{2,k}(t), \dots, e_{n-1,k}(t)]^T$, then it follows from (B.28) that

$$\begin{aligned} |e_{1,k}(t)| &\leq \|\zeta_k(t)\| \\ &\leq k_0 \|\zeta_k(0)\| e^{-\lambda_0 t} \\ &\quad + k_0 \int_0^t e^{-\lambda_0(t-\sigma)} |e_{sk}(\sigma)| d\sigma. \end{aligned} \quad (\text{B.34})$$

When $k \rightarrow \infty$, it can be shown that

$$\begin{aligned}
 |e_{1,\infty}(t)| &\leq \|\zeta_{\infty}(t)\| \\
 &\leq k_0 \|\zeta_{\infty}(0)\| e^{-\lambda_0 t} \\
 &\quad + k_0 \int_0^t e^{-\lambda_0(t-\sigma)} |e_{s,\infty}(\sigma)| d\sigma \\
 &\leq k_0 \|\zeta_{\infty}(0)\| e^{-\lambda_0 t} \\
 &\quad + (1+m) \varepsilon k_0 \int_0^t e^{-\lambda_0(t-\sigma)} e^{-K\sigma} d\sigma \\
 &= k_0 \|\zeta_{\infty}(0)\| e^{-\lambda_0 t} \\
 &\quad + (1+m) \varepsilon k_0 \frac{1}{\lambda_0 - K} (e^{-Kt} - e^{-\lambda_0 t}) \\
 &\leq k_0 \sum_{i=1}^{n-1} \sqrt{\varepsilon_i^2} e^{-\lambda_0 t} \\
 &\quad + (1+m) \varepsilon k_0 \frac{1}{\lambda_0 - K} (e^{-Kt} - e^{-\lambda_0 t}).
 \end{aligned} \tag{B.35}$$

Define $\chi_k = [\chi_{1,k}, \dots, \chi_{n,k}]^T = x_k - x_d$; then we can know that $\chi_k = z_k + e_k$. From $\lim_{k \rightarrow \infty} \int_0^T \|z_k\|^2 d\sigma = 0$ we can have $\lim_{k \rightarrow \infty} \|z_k(t)\| = 0$; that is, $\|z_{\infty}(t)\| = 0, t \in [0, T]$. Consequently, it is obvious that

$$\begin{aligned}
 \|\chi_{\infty}\| &\leq \|z_{\infty}\| + \|e_{\infty}\| \\
 &\leq (1 + \|\Lambda\|) \left(k_0 \sum_{i=1}^{n-1} \sqrt{\varepsilon_i^2} + \frac{1}{\lambda_0 - K} (1+m) \varepsilon k_0 \right) \\
 &\quad + (1+m) \eta(t), \\
 |\chi_{1,\infty}| &= |y_{\infty} - y_d| \leq |z_{1,\infty}| + |e_{1,\infty}| \\
 &\leq k_0 \sum_{i=1}^{n-1} \sqrt{\varepsilon_i^2} e^{-\lambda_0 t} \\
 &\quad + (1+m) \varepsilon k_0 \frac{1}{\lambda_0 - K} (e^{-Kt} - e^{-\lambda_0 t}).
 \end{aligned} \tag{B.36}$$

This concludes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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