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Research Article

Weighted H_∞ Filtering for a Class of Switched Linear Systems with Additive Time-Varying Delays

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This paper is concerned with the problem of weighted H_∞ filtering for a class of switched linear systems with two additive time-varying delays, which represent a general class of switched time-delay systems with strong practical background. Combining average dwell time (ADT) technique with piecewise Lyapunov functionals, sufficient conditions are established to guarantee the exponential stability and weighted H_∞ performance for the filtering error systems. The parameters of the designed switched filters are obtained by solving linear matrix inequalities (LMIs). A modification of Jensen integral inequality is exploited to derive results with less theoretical conservatism and computational complexity. Finally, two examples are given to demonstrate the effectiveness of the proposed method.

1. Introduction

Switched systems, as an important class of hybrid systems, have been extensively investigated during the last decades [1–4]. This is mainly motivated by the hybrid nature of many physical plants and the better performance under a controller switching strategy. Several methods, which are viewed as variations or generalizations of the Lyapunov stability theory, have been employed to cope with the performance analysis and control synthesis problems for switched systems, such as the common Lyapunov function method, the piecewise Lyapunov function method, the average dwell time (ADT) function method, the multiple Lyapunov function method, and the switched Lyapunov function method. Basic problems and recent progress are referred to [1, 2] and the references therein.

Switched systems with time delays in system states, control inputs, or switching signals can be modeled as switched delay systems. Due to the coexistence and interaction of the mode switchings and inherent time delays, the features of switched delay systems are very complicated which may even lead to instability and poor performance. Switched delay systems are also with strong practical background such as temperature control system, networked control systems (NCSs), and power systems [5–8]. That is why switched delay systems have gained growing popularity; following in the wake of this, much important progress has been made on the analysis and synthesis of switched delay systems [9–11], where the ADT approach has been proved to be a flexible and powerful tool for switched delay systems. When external noise signals appear in system models, state variables might not be available accurately which need to be estimated. One of the most effective methods for the state estimation problem is to design a filter, which has been concerned for switched delay systems with different performance indexes such as Kalman filtering, L_2–L_∞ filtering, H_2 filtering, and H_∞ filtering. Compared with others, H_∞ filtering [12, 13] permits the exogenous noises to be arbitrary with bounded energy or average power and without known precise statistics, and it is also more robust to the uncertainties in the external noise signals and system models. In the H_∞ filtering setting, a state estimator is designed to guarantee that the filter error system is stable in the absence of the external noise signals and its H_∞ performance from the external noise signals to the estimation error is below a prescribed level of noise attenuation. For some representative works on H_∞ filtering for switched delay

systems, to name a few, [14] investigated an exponential H_∞ filtering for continuous-time switched systems with interval time-varying delay to assure the exponential stability with a weighted H_∞ performance for the filtering error system via the free-weighting matrix (FWM) technique. A weighted H_∞ filter design procedure was developed by using the FWM technique for continuous-time switched time-varying delay systems to achieve the exponential stability with a weighted H_∞ performance for the filtering error system in [15]. Reference [16] designed a full-order switched H_∞ filter for a class of uncertain switched neutral systems subject to stochastic disturbance and time-varying delays, which guaranteed the robust mean-square exponential stability with a prescribed weighted H_∞ performance. On the basis of a filter with Luenberger observer type and a new integral inequality, [17] dealt with the H_∞ filtering problem for a class of switched linear neutral systems with time-varying delays. The ADT approach associated with the piecewise Lyapunov functional technology was employed in all of the above four results, and fast convergence and desirable accuracy were guaranteed by the exponential H_∞ filters in terms of reasonable error covariance of the filtering process.

On the other hand, among the great number of literatures concerning time delay, it is worth pointing out that [18, 19] proposed the additive time-varying delay as a new type of delay for nonswitched continuous-time systems. After that, additive time delay has attracted much attention in the past few years [20–24]. The significance of the additive delays lies in three aspects. First, the additive delay components may describe delays with sharply different properties in system modeling which might not be suitable to combine them together. As mentioned in [18], delays from sensor to controller and from controller to actuator in NCSs are of the case, where delays may not have identical properties. Second, regarding the sum of all additive delay components as one traditional single delay will be very conservative, it is not reasonable and necessary to treat the sum of maximum of single delays as the maximum of the sum of all delays at the same time. Finally, systems with additive delays are of strong application backgrounds such as remote control and NCSs. For instance, [19] utilized the continuous systems with two additive time-varying delays in state to investigate the sampled-data NCSs with network induced delays and data packet dropouts. Recently, by constructing new Lyapunov functionals to avoid some overly boundings, [23, 24] presented stability criteria with fewer matrix variables and less conservatism for systems with additive time-varying delays in control input and system state, respectively. Nevertheless, switched additive time delays systems have been rarely investigated due to the complexity of mode switching.

Inspired by the aforementioned discussion, this paper focuses on the weighted H_∞ filtering problem for a class of switched systems with additive time-varying delays. Delays under consideration are additive time-varying in the states. New criteria are presented to guarantee delay-dependent exponential stability and a weighted H_∞ performance for the filtering error system under ADT switching signals. Then, by solving the corresponding LMIs, the parameters of the designed switched H_∞ filters are obtained for all additive time-varying delays. Combining with a modification of Jensen integral inequality [24] instead of FWM technology, the derived conditions are with less theoretical conservatism and computational complexity. Two examples are provided to illustrate the effectiveness of the proposed method. To the best of our knowledge, little work has been addressed concerning stability analysis for switched systems with additive time-varying delays, not to mention the weighted H_∞ filtering problem.

The remainder of this paper is organized as follows. The weighted H_∞ filtering problem for switched systems with additive time-varying is formulated in Section 2. Section 3 presents delay-dependent sufficient conditions on the existence of weighted H_∞ filtering which guarantees exponential stability and H_∞ performance of the filtering error system, and a corresponding filter is designed. Section 4 gives two examples. Section 5 concludes this paper.

Notations. Some standard notations are used in this paper. R^n denotes the n dimensional Euclidean space; $P < 0$ ($\leq, >, \geq$) represents a real negative (negative-semidefinite, positive, positive-semidefinite) definite matrix P; $L_2[0,+\infty)$ denotes the space of square integrable vector functions on $[0,+\infty)$; the superscript T stands for matrix transposition; $\lambda_{\max}(P)$ presents the maximum eigenvalue of P; I is the identity matrix with compatible dimensions; asterisk $*$ denotes symmetric terms in symmetric term in a symmetric matrix.

2. Problem Formulation

Consider the following switched linear systems with two additive time-varying delays:

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + A_{d\sigma} x\left(t - d_1(t) - d_2(t)\right) + B_\sigma \omega(t), \\ y(t) &= C_\sigma x(t) + C_{d\sigma} x\left(t - d_1(t) - d_2(t)\right) + D_\sigma \omega(t), \\ z(t) &= E_\sigma x(t) + E_{d\sigma} x\left(t - d_1(t) - d_2(t)\right), \\ x(\theta) &= \varphi(\theta), \quad \theta \in [-d, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measured output, $z(t) \in R^p$ is the signal to be estimated, $\omega(t) \in R^q$ is the disturbance input which belongs to $L_2[0,+\infty)$, and $\varphi(\theta)$ is a differentiable vector-valued initial function. The piecewise constant function $\sigma(t) : [0,\infty) \to \psi_N \triangleq \{1,2,\ldots,N\}$ denotes a switching signal to be specified; corresponding switching sequence $\{x(t_0); (i_0,t_0), (i_1,t_1), \ldots, (i_k,t_k), \ldots \mid i_k \in \psi_N, k = 0,1,\ldots\}$ means that the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$. A_i, A_{di}, B_i, C_i, C_{di}, D_i, E_i, E_{di} are known constant matrices with appropriate dimensions; $d_1(t)$ and $d_2(t)$ represent two time-varying delay components satisfying

$$\begin{aligned} 0 \leq d_1(t) \leq d_1, \qquad 0 \leq d_2(t) \leq d_2, \\ \dot{d}_1(t) \leq h_1, \qquad \dot{d}_2(t) \leq h_2, \end{aligned} \tag{2}$$

where d_1, d_2, h_1, h_2 are constants. Set $d = d_1 + d_2$, $h = h_1 + h_2$, $d(t) = d_1(t) + d_2(t)$.

Our objective in this paper is to construct a full-order switched filter in the form of

$$\begin{aligned} \dot{x}_f(t) &= A_{f\sigma}x_f(t) + B_{f\sigma}y(t), \\ z_f(t) &= C_{f\sigma}x_f(t) + D_{f\sigma}y(t), \end{aligned} \tag{3}$$

where $x_f(t) \in R^n$ is the filter state, $z_f(t) \in R^p$ is the filter output estimating $z(t)$, the matrices A_{fi}, B_{fi}, C_{fi}, and $D_{fi}, i \in \psi_N$ are the filter parameters to be determined later. Filter (3) is assumed to switch synchronously according to the switching signal σ in system (1).

Set the state augmentation $\xi^T(t) = [x^T(t) \quad x_f^T(t)]$ and the estimation error $\epsilon(t) = z(t) - z_f(t)$, and thus augmenting system (1) to involve switched linear filter (3) gives the following filter error dynamic system:

$$\begin{aligned} \dot{\xi}(t) &= \overline{A}_\sigma \xi(t) + \overline{A}_{d\sigma}\xi(t-d(t)) + \overline{B}_\sigma \omega(t), \\ \epsilon(t) &= \overline{C}_\sigma \xi(t) + \overline{C}_{d\sigma}\xi(t-d(t)) + \overline{D}_\sigma \omega(t), \end{aligned} \tag{4}$$

where

$$\begin{gathered} \overline{A}_i = \begin{bmatrix} A_i & 0 \\ B_{fi}C_{fi} & A_{fi} \end{bmatrix}, \qquad \overline{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ B_{fi}C_{di} & 0 \end{bmatrix}, \\ \overline{B}_i = \begin{bmatrix} B_i \\ B_{fi}Di \end{bmatrix}, \\ \overline{C}_i = [E_i - D_{fi}C_i \quad -C_{fi}], \qquad \overline{C}_{di} = E_{di} - D_{fi}C_{di}, \\ \overline{D}_i = -D_{fi}D_i. \end{gathered} \tag{5}$$

The following definitions are addressed to derive the desired results.

Definition 1 (see [1]). The equilibrium $\xi^* = 0$ of the filtering error system (4) is exponentially stable under $\sigma(t)$, if the solution $\xi(t)$ of system (4) with $\omega(t) = 0$ satisfies $\|\xi(t)\| \le k\|\xi(t_0)\|_d e^{-\lambda(t-t_0)}$ for all $t \ge t_0$ for constants $k \ge 1$ and $\lambda \ge 0$, where $\|\cdot\|$ denotes the Euclidean norm, and $\|\xi(t)\|_d = \sup_{-d\le\theta\le 0}\{\xi(t+\theta), \dot{\xi}(t+\theta)\}$.

Definition 2 (see [1]). For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the number of switchings of $\sigma(t)$ over (T_1, T_2). If $N_\sigma(T_1, T_2) \le N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \ge 0$, then T_a is called the average dwell time. As commonly used in literature, choose $N_0 = 0$.

Definition 3. For $\alpha > 0$ and $\gamma > 0$, the weighted H_∞ filtering problem of system (1) is solvable if there exists a suitable filter of the form (3) and admissible ADT switching signals such that the filter error system (4) is exponentially stable when $\omega(t) = 0$; meanwhile, for any nonzero $\omega(t) \in L_2[0,\infty)$, system (4) has a weighted H_∞ performance (α, γ) under zero initial condition $\varphi(\theta) = 0, \theta \in [-d, 0]$; that is,

$$\int_0^\infty e^{-\alpha s}\epsilon^T(s)\epsilon(s)\,ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)\,ds. \tag{6}$$

The following modification of Jensen integral inequality is essential to develop the main results with less conservatism.

Lemma 4 (see [24]). *For any matrix $Z > 0$, scalars r, r_j $(j = 1, 2, 3, 4)$ satisfying $r > 0$, $0 \le r_2 - r_1 \le r$, and $r_4 = r - (r_2 - r_1) + r_3$, there exists a vector function ν such that the integrations concerned are well defined; then the following inequality is true:*

$$\begin{aligned} &\int_{r_1}^{r_2} \nu(s)^T Z \nu(s)\,ds + \int_{r_3}^{r_4} \nu(s)^T Z \nu(s)\,ds \\ &\quad \ge \frac{1}{r}\begin{bmatrix} \int_{r_1}^{r_2} \nu(s)\,ds \\ \int_{r_3}^{r_4} \nu(s)\,ds \end{bmatrix}^T \begin{bmatrix} Z & M \\ * & Z \end{bmatrix} \begin{bmatrix} \int_{r_1}^{r_2} \nu(s)\,ds \\ \int_{r_3}^{r_4} \nu(s)\,ds \end{bmatrix}, \end{aligned} \tag{7}$$

if there exists a matrix M such that

$$\begin{bmatrix} Z & M \\ * & Z \end{bmatrix} \ge 0. \tag{8}$$

3. Main Results

3.1. Exponential Stability and Weighted H_∞ Performance Analysis. First, we analyse the exponential stability for filtering error system (4) with $\omega(t) = 0$; that is,

$$\begin{aligned} \dot{\xi}(t) &= \overline{A}_\sigma \xi(t) + \overline{A}_{d\sigma}\xi(t-d(t)), \\ \epsilon(t) &= \overline{C}_\sigma \xi(t) + \overline{C}_{d\sigma}\xi(t-d(t)). \end{aligned} \tag{9}$$

Theorem 5. *Given constant $\alpha > 0$, if there exist matrices $P_i > 0$, $Q_{ki} > 0$, $R_{li} > 0$ and appropriately dimensioned matrices M_{li} such that*

$$\Phi^i = \begin{bmatrix} \Phi_{11}^i & \Phi_{12}^i & \Phi_{13}^i & \Phi_{14}^i & \Phi_{15}^i & 0 \\ * & \Phi_{22}^i & \Phi_{23}^i & 0 & \Phi_{25}^i & \Phi_{26}^i \\ * & * & \Phi_{33}^i & \Phi_{34}^i & \Phi_{35}^i & \Phi_{36}^i \\ * & * & * & \Phi_{44}^i & \Phi_{45}^i & \Phi_{46}^i \\ * & * & * & * & \Phi_{55}^i & \Phi_{56}^i \\ * & * & * & * & * & \Phi_{66}^i \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} R_{li} & M_{li} \\ * & R_{li} \end{bmatrix} \ge 0; \tag{11}$$

then the system (9) with (2) is exponentially stable under any ADT switching signal satisfying

$$T_a > T_a^* = \frac{\ln \mu}{\alpha}. \tag{12}$$

Moreover, an estimate of state decay is given by

$$\|\xi(t)\| \le \sqrt{\frac{b}{a}} e^{-\lambda(t-t_0)} \|\xi(t_0)\|_d, \tag{13}$$

where $\mu \geq 1$, $k = 1, 2, 3, 4$, $l = 1, 2, 3$ *satisfy*

$$P_i \leq \mu P_j, \qquad Q_{ki} \leq \mu Q_{kj}, \qquad R_{li} \leq \mu R_{lj}, \qquad \forall i, j \in \psi_N, \quad i \neq j, \tag{14}$$

$$\Phi_{11}^i = P_i \overline{A}_i + \overline{A}_i^T P_i + Q_{1i} + Q_{2i} + Q_{3i} + \alpha P_i + \overline{A}_i^T R_i \overline{A}_i - \frac{e^{-\alpha d_1}}{d_1} R_{1i} - \frac{e^{-\alpha d}}{d_1} R_{3i},$$

$$\Phi_{12}^i = P_i \overline{A}_{di} + \overline{A}_i^T R_i \overline{A}_{di} - \frac{e^{-\alpha d}}{d_1} M_{3i},$$

$$\Phi_{13}^i = \frac{e^{-\alpha d_1}}{d_1} (R_{1i} - M_{1i}) + \frac{e^{-\alpha d}}{d_1} R_{3i},$$

$$\Phi_{14}^i = \frac{e^{-\alpha d_1}}{d_1} M_{1i}, \qquad \Phi_{15}^i = \frac{e^{-\alpha d}}{d_1} M_{3i},$$

$$\Phi_{22}^i = \overline{A}_{di}^T R_i \overline{A}_{di} - e^{-\alpha d} \left[(1-h) Q_{1i} + \left(\frac{1}{d_1} + \frac{1}{d_2} \right) R_{3i} \right],$$

$$\Phi_{23}^i = \frac{e^{-\alpha d}}{d_1} M_{3i}^T + \frac{e^{-\alpha d}}{d_2} R_{3i},$$

$$\Phi_{25}^i = \frac{e^{-\alpha d}}{d_1} R_{3i} + \frac{e^{-\alpha d}}{d_2} M_{3i}^T, \qquad \Phi_{26}^i = -\frac{e^{-\alpha d}}{d_2} M_{3i}^T,$$

$$\Phi_{33}^i = -(1-h_1) e^{-\alpha d_1} Q_{2i} - \frac{2e^{-\alpha d_1}}{d_1} R_{1i} + \frac{e^{-\alpha d_1}}{d_1} \left(M_{1i}^T + M_{1i} \right) - \frac{e^{-\alpha d}}{d_1} R_{3i} - \frac{e^{-\alpha d}}{d_2} R_{3i},$$

$$\Phi_{34}^i = \frac{e^{-\alpha d_1}}{d_1} (R_{1i} - M_{1i}),$$

$$\Phi_{35}^i = -\frac{e^{-\alpha d}}{d_1} M_{3i} - \frac{e^{-\alpha d}}{d_2} M_{3i}^T, \qquad \Phi_{36}^i = \frac{e^{-\alpha d}}{d_2} M_{3i}^T,$$

$$\Phi_{44}^i = -e^{-\alpha d_1} (Q_{3i} - Q_{4i}) - \frac{e^{-\alpha d_1}}{d_1} R_{1i} - \frac{e^{-\alpha d_2}}{d_2} R_{2i},$$

$$\Phi_{45}^i = \frac{e^{-\alpha d_2}}{d_2} (R_{2i} - M_{2i}), \qquad \Phi_{46}^i = \frac{e^{-\alpha d_2}}{d_2} M_{2i},$$

$$\Phi_{55}^i = \frac{e^{-\alpha d_2}}{d_2} \left(M_{2i} + M_{2i}^T - 2R_{2i} \right) - \left(\frac{1}{d_1} + \frac{1}{d_2} \right) e^{-\alpha d} R_{3i},$$

$$\Phi_{56}^i = \frac{e^{-\alpha d_2}}{d_2} (R_{2i} - M_{2i}) + \frac{e^{-\alpha d}}{d_2} R_{3i},$$

$$\Phi_{66}^i = -e^{-\alpha d} \left(Q_{4i} + \frac{1}{d_2} R_{3i} \right) - \frac{e^{-\alpha d_2}}{d_2} R_{2i},$$

$$\lambda = \frac{1}{2} \left(\alpha - \frac{\ln \mu}{T_a} \right), \qquad R_i = d_1 R_{1i} + d_2 R_{2i} + d R_{3i},$$

$$a = \min_{\forall i \in \psi_N} \lambda_{\min} (P_i),$$

$$b = \max_{\forall i \in \psi_N} \lambda_{\max} (P_i) + \frac{3}{2} d^2 \max_{\forall i \in \psi_N} \lambda_{\max} (R_{1i} + R_{2i} + R_{3i}) + 4d \max_{\forall i \in \psi_N} \lambda_{\max} (Q_{1i} + Q_{2i} + Q_{3i} + Q_{4i}). \tag{15}$$

Proof. Define the piecewise Lyapunov-Krasoviskii functional candidate of the form

$$V_\sigma (\xi(t)) = V_{1\sigma} (\xi(t)) + V_{2\sigma} (\xi(t)) + V_{3\sigma} (\xi(t)), \tag{16}$$

where

$$\begin{aligned} V_{1\sigma} (\xi(t)) &= \xi^T(t) P_\sigma \xi(t) + \int_{t-d(t)}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{1\sigma} \xi(s)\, ds \\ &\quad + \int_{t-d_1(t)}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{2\sigma} \xi(s)\, ds \\ &\quad + \int_{t-d_1}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{3\sigma} \xi(s)\, ds, \\ V_{2\sigma} (\xi(t)) &= \int_{t-d}^{t-d_1} e^{\alpha(s-t)} \xi^T(s) Q_{4\sigma} \xi(s)\, ds, \\ V_{3\sigma} (\xi(t)) &= \int_{-d_1}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{\xi}(s)^T R_{1\sigma} \dot{\xi}(s)\, ds\, d\theta \\ &\quad + \int_{-d}^{-d_1} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{\xi}(s)^T R_{2\sigma} \dot{\xi}(s)\, ds\, d\theta \\ &\quad + \int_{-d}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{\xi}(s)^T R_{3\sigma} \dot{\xi}(s)\, ds\, d\theta. \end{aligned} \tag{17}$$

Then taking the time derivative of $V_i(\xi(t))$ along the trajectory of system (9) yields

$$\begin{aligned} \dot{V}_{1i} (\xi(t)) &\leq 2\xi^T(t) P_i \left(\overline{A}_i \xi(t) + \overline{A}_{di} \xi(t - d(t)) \right) \\ &\quad - \alpha \int_{t-d(t)}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{1i} \xi(s)\, ds \\ &\quad - (1-h) e^{-\alpha d} \xi^T(t - d(t)) Q_{1i} \xi(t - d(t)) \\ &\quad + \xi^T(t) (Q_{1i} + Q_{2i} + Q_{3i}) \xi(t) \\ &\quad - \alpha \int_{t-d_1(t)}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{2i} \xi(s)\, ds \\ &\quad - (1-h_1) e^{-\alpha d_1} \xi^T(t - d_1(t)) Q_{2i} \xi(t - d_1(t)) \\ &\quad - \alpha \int_{t-d_1}^{t} e^{\alpha(s-t)} \xi^T(s) Q_{3i} \xi(s)\, ds \\ &\quad - e^{-\alpha d_1} \xi^T(t - d_1) Q_{3i} \xi(t - d_1), \end{aligned}$$

$$\begin{aligned}
\dot{V}_{2i}(\xi(t)) &= -\alpha\int_{t-d}^{t-d_1} e^{\alpha(s-t)}\xi^T(s)Q_{4i}\xi(s)\,\mathrm{d}s \\
&\quad + e^{-\alpha d_1}\xi^T(t-d_1)Q_{4i}\xi(t-d_1) \\
&\quad - e^{-\alpha d}\xi^T(t-d)Q_{4i}\xi(t-d), \\
\dot{V}_{3i}(\xi(t)) &\le -\alpha\int_{-d_1}^{0}\int_{t+\theta}^{t} e^{\alpha(s-t)}\dot{\xi}(s)^T R_{1i}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad - \alpha\int_{-d}^{-d_1}\int_{t+\theta}^{t} e^{\alpha(s-t)}\dot{\xi}(s)^T R_{2i}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad - \alpha\int_{-d}^{0}\int_{t+\theta}^{t} e^{\alpha(s-t)}\dot{\xi}(s)^T R_{3i}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad - \int_{t-d}^{t} e^{-\alpha d}\dot{\xi}(s)^T R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad - \int_{t-d_1}^{t} e^{-\alpha d_1}\dot{\xi}(s)^T R_{1i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad - \int_{t-d}^{t-d_1} e^{-\alpha d_2}\dot{\xi}(s)^T R_{2i}\dot{\xi}(s)\,\mathrm{d}s + \dot{\xi}(t)^T R_i\dot{\xi}(t).
\end{aligned} \tag{18}$$

Lemma 4 and (11) give the following inequalities:

$$\begin{aligned}
&-\int_{t-d_1}^{t} e^{-\alpha d_1}\dot{\xi}(s)^T R_{1i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad = -e^{-\alpha d_1}\int_{t-d_1(t)}^{t}\dot{\xi}^T(s)R_{1i}\dot{\xi}(s)\,\mathrm{d}s \\
&\qquad - e^{-\alpha d_1}\int_{t-d_1}^{t-d_1(t)}\dot{\xi}^T(s)R_{1i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad \le -\frac{e^{-\alpha d_1}}{d_1}\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}^T\begin{bmatrix}R_{1i} & M_{1i}\\ * & R_{1i}\end{bmatrix}\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}, \\
&-\int_{t-d}^{t-d_1} e^{-\alpha d_2}\dot{\xi}(s)^T R_{2i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad = e^{-\alpha d_2}\int_{t-d_1-d_2(t)}^{t-d_1}\dot{\xi}^T(s)R_{2i}\dot{\xi}(s)\,\mathrm{d}s \\
&\qquad - e^{-\alpha d_2}\int_{t-d}^{t-d_1-d_2(t)}\dot{\xi}^T(s)R_{2i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad \le -\frac{e^{-\alpha d_2}}{d_2}\begin{bmatrix}\phi_3(t)\\ \phi_4(t)\end{bmatrix}^T\begin{bmatrix}R_{2i} & M_{2i}\\ * & R_{2i}\end{bmatrix}\begin{bmatrix}\phi_3(t)\\ \phi_4(t)\end{bmatrix}, \\
&-\int_{t-d}^{t} e^{-\alpha d}\dot{\xi}(s)^T R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad = e^{-\alpha d}\int_{t-d_1(t)}^{t}\dot{\xi}^T(s)R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\qquad - e^{-\alpha d}\int_{t-d(t)}^{t-d_1(t)}\dot{\xi}^T(s)R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\qquad - e^{-\alpha d}\int_{t-d_1-d_2(t)}^{t-d(t)}\dot{\xi}^T(s)R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\qquad - e^{-\alpha d}\int_{t-d}^{t-d_1-d_2(t)}\dot{\xi}^T(s)R_{3i}\dot{\xi}(s)\,\mathrm{d}s \\
&\quad \le -\frac{e^{-\alpha d}}{d_1}\begin{bmatrix}\phi_1(t)\\ \phi_5(t)\end{bmatrix}^T\begin{bmatrix}R_{3i} & M_{3i}\\ * & R_{3i}\end{bmatrix}\begin{bmatrix}\phi_1(t)\\ \phi_5(t)\end{bmatrix} \\
&\qquad - \frac{e^{-\alpha d}}{d_2}\begin{bmatrix}\phi_6(t)\\ \phi_4(t)\end{bmatrix}^T\begin{bmatrix}R_{3i} & M_{3i}\\ * & R_{3i}\end{bmatrix}\begin{bmatrix}\phi_6(t)\\ \phi_4(t)\end{bmatrix},
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\phi_1(t) &= \xi(t) - \xi(t-d_1(t)), \\
\phi_2(t) &= \xi(t-d_1(t)) - \xi(t-d_1), \\
\phi_3(t) &= \xi(t-d_1) - \xi(t-d_1-d_2(t)), \\
\phi_4(t) &= \xi(t-d_1-d_2(t)) - \xi(t-d), \\
\phi_5(t) &= \xi(t-d_1(t)) - \xi(t-d_1-d_2(t)), \\
\phi_6(t) &= \xi(t-d_1(t)) - \xi(t-d(t)).
\end{aligned} \tag{20}$$

Combining (10) and (16)–(19) leads to

$$\dot{V}_i(\xi(t)) + \alpha V_i(\xi(t)) \le \eta^T(t)\Phi^i\eta(t) < 0 \tag{21}$$

with $\eta^T(t) = [\xi^T(t)\xi^T(t-d(t))\xi^T(t-d_1(t))\xi^T(t-d_1)\xi^T(t-d_1-d_2(t))\xi^T(t-d)]$.

When $t \in [t_k, t_{k+1})$, integrating the above inequality from t_k to t yields

$$V_{\sigma(t)}(\xi(t)) \le e^{-\alpha(t-t_k)}V_{\sigma(t_k)}(\xi(t_k)). \tag{22}$$

Using (14) and (16), at switching instant t_k, we have

$$V_{\sigma(t_k)}(\xi(t_k)) \le \mu V_{\sigma(t_k^-)}(\xi(t_k^-)). \tag{23}$$

Therefore, it follows from (22), (23), and the relation $N_\sigma(t_0,t) \le (t-t_0)/T_a$ that

$$V_\sigma(\xi(t)) \le e^{-(\alpha-\ln\mu/T_a)(t-t_0)}V_{\sigma(t_0)}(\xi(t_0)). \tag{24}$$

Noticing that $a\|\xi(t)\|^2 \le V_{\sigma(t)}(\xi(t))$ and $V_{\sigma(t_0)}(\xi(t_0)) \le b\|\xi(t_0)\|_d^2$, we have

$$\|\xi(t)\|^2 \le \frac{1}{a}V_{\sigma(t)}(\xi(t)) \le \frac{b}{a}e^{-(\alpha-\ln\mu/T_a)(t-t_0)}\|\xi(t_0)\|_d^2, \tag{25}$$

which completes the proof. □

Remark 6. When $\mu = 1$, we have $T_a^* = 0$, which means that all subsystems employ a common Lyapunov functional, and the switching among them can be arbitrary. Meanwhile, let $\alpha = 0$ in (10); it gives asymptotical stability for (9) under arbitrary switching.

Remark 7. Due to the application of the modified Jensen integral inequality, the upper bound of $V_{3i}(\xi(t))$ is estimated more tightly, and fewer matrix variables are involved. Therefore, the theoretical conservatism and computational complexity are reduced for the derived conditions.

Next, we establish the following weighted H_∞ performance criteria for the filter error system (4).

Theorem 8. *For scalars $\alpha > 0$ and $\gamma > 0$, there exist matrices $P_i > 0$, $Q_{ki} > 0$, $R_{li} > 0$ and appropriately dimensioned matrices M_{li} such that*

$$\Pi^i = \begin{bmatrix} \Pi^i_{11} & \Pi^i_{12} \\ * & -I \end{bmatrix} < 0; \tag{26}$$

(11) and (14) hold, where $\mu \geq 1$, $k = 1, 2, 3, 4$, $l = 1, 2, 3$,

$$\begin{gathered} \Pi^i_{11} = \begin{bmatrix} \Phi^i & \Phi^i_{17} \\ * & \Phi^i_{77} \end{bmatrix}, \\ \Phi^i_{17} = \begin{bmatrix} P_i\overline{B}_i + A_i^T R_i B_i & A_{di}^T R_i B_i & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\ \Pi^i_{12} = \begin{bmatrix} \overline{C}_i^T & \overline{C}_{di}^T & 0 & 0 & 0 & 0 & \overline{D}_i^T \end{bmatrix}^T, \\ \Phi^i_{77} = -\gamma^2 I + B_i^T R_i B_i, \end{gathered} \tag{27}$$

$\Gamma(t) = \epsilon^T(t)\epsilon(t) - \gamma^2\omega^T(t)\omega(t)$; others are defined in Theorem 5. Then the filtering error system (4) with (2) is exponentially stable with weighted H_∞ performance (α, γ) under any ADT switching signal (12).

Proof. From Theorem 5, the filter error system (4) is exponentially stable with $\omega(t) = 0$. To show the weighted H_∞ performance, we choose the Lyapunov-Krosoviskii functional (16). According to Schur complement, (26) is equivalent to $\Pi^i_{11} + \Pi^i_{12}(\Pi^i_{12})^T < 0$ which means

$$\dot{V}_i(\xi(t)) \leq -\alpha V_i(\xi(t)) - \Gamma(t). \tag{28}$$

For any $t \in [t_k, t_{k+1})$, combining (23) and (28) yields

$$\begin{aligned} V_\sigma(\xi(t)) &\leq e^{-\alpha(t-t_k)} V_\sigma(\xi(t_k)) - \int_{t_k}^{t} e^{-\alpha(t-s)}\Gamma(s)\,ds \\ &\leq \mu e^{-\alpha(t-t_k)} V_\sigma(\xi(t_k^-)) - \int_{t_k}^{t} e^{-\alpha(t-s)}\Gamma(s)\,ds \\ &\leq \mu^{N_\sigma(0,t)} e^{-\alpha t} V_\sigma(\xi(0)) - \mu^{N_\sigma(0,t)} \\ &\quad \times \int_0^{t_1} e^{-\alpha(t-s)}\Gamma(s)\,ds - \cdots - \int_{t_k}^{t} e^{-\alpha(t-s)}\Gamma(s)\,ds \\ &\leq e^{-\alpha t + N_\sigma(0,t)\ln\mu} V_\sigma(\xi(0)) \\ &\quad - \int_0^t e^{-\alpha(t-s)+N_\sigma(s,t)\ln\mu}\Gamma(s)\,ds. \end{aligned} \tag{29}$$

Under the zero initial condition, multiplying both sides of the above inequality by $e^{-N_\sigma(0,t)\ln\mu}$ leads to

$$0 \leq -\int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\mu}\Gamma(s)\,ds, \tag{30}$$

which is equivalent to

$$\begin{aligned} &\int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\mu}\epsilon^T(s)\,\epsilon(s)\,ds \\ &\quad \leq \gamma^2 \int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\mu}\omega^T(s)\,\omega(s)\,ds. \end{aligned} \tag{31}$$

From (12), $N_\sigma(0, s)\ln\mu \leq \alpha s$ holds; then one gets

$$\int_0^t e^{-\alpha(t-s)-\alpha s}\epsilon^T(s)\,\epsilon(s)\,ds \leq \gamma^2 \int_0^t e^{-\alpha(t-s)}\omega^T(s)\,\omega(s)\,ds. \tag{32}$$

Integrating the above inequality from $t = 0$ to ∞ and exchanging the integration order lead to (6). This completes the proof. □

3.2. Weighted H_∞ Filter Design. Next, parameters of switched filter (3) can be determined by the following theorem.

Theorem 9. *For scalars $\alpha > 0$, $\gamma > 0$, there exist matrices $P_{1i} > 0$, $P_{3i} > 0$, $Q_{ki} > 0$, $R_{li} > 0$ and appropriately dimensioned matrices P_{2i}, N_i, M_{li}, $\overline{A}_{fi}$, $\overline{B}_{fi}$, $\overline{C}_{fi}$, $\overline{D}_{fi}$ such that*

$$\widehat{\Pi}^i = \begin{bmatrix} \widehat{\Pi}^i_{11} & \widehat{\Pi}^i_{12} \\ * & -I \end{bmatrix} < 0, \tag{33}$$

$$P_i \triangleq \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} > 0; \tag{34}$$

(11) and (14) hold for $\mu \geq 1$, $k = 1, 2, 3, 4$, $l = 1, 2, 3$ with

$$\widehat{\Pi}^i_{11} = \begin{bmatrix} \widehat{\Phi}^i_{11} & \widehat{\Phi}^i_{12} & \Phi^i_{13} & \Phi^i_{14} & \Phi^i_{15} & 0 & \widehat{\Phi}^i_{17} \\ * & \Phi^i_{22} & \Phi^i_{23} & 0 & \Phi^i_{25} & \Phi^i_{26} & 0 \\ * & * & \Phi^i_{33} & \Phi^i_{34} & \Phi^i_{35} & \Phi^i_{36} & \widehat{\Phi}^i_{37} \\ * & * & * & \Phi^i_{44} & \Phi^i_{45} & \Phi^i_{46} & 0 \\ * & * & * & * & \Phi^i_{55} & \Phi^i_{56} & 0 \\ * & * & * & * & * & \Phi^i_{66} & 0 \\ * & * & * & * & * & * & \widehat{\Phi}^i_{77} \end{bmatrix},$$

$$\widehat{\Pi}^i_{12} = \begin{bmatrix} (E_i - \overline{D}_{fi}C_i)^T & \overline{C}_{fi}^T & 0 & 0 & 0 & 0 & \overline{D}_i^T \end{bmatrix}^T,$$

$$\widehat{\Phi}^i_{11} = \begin{bmatrix} \Xi_i & \overline{A}_{fi}^T + A_i^T N_i + C_i^T \overline{B}_{fi} + \alpha N_i \\ * & \overline{A}_{fi} + \overline{A}_{fi}^T + \alpha N_i \end{bmatrix},$$

$$\widehat{\Phi}_{12}^{i} = \begin{bmatrix} P_{1i}A_{di} + \overline{B}_{fi}C_{di} + A_i^T R_i A_{di} \\ N_i A_{di} + \overline{B}_{fi}C_{di} \end{bmatrix},$$

$$\widehat{\Phi}_{17}^{i} = \begin{bmatrix} P_{1i}B_i + \overline{B}_{fi}D_i + A_i^T R_i B_i \\ N_i B_i + \overline{B}_{fi}D_i \end{bmatrix},$$

$$\widehat{\Phi}_{37}^{i} = A_{di}^T R_i B_i, \qquad \widehat{\Phi}_{77}^{i} = -\gamma^2 I,$$

$$\begin{aligned} \Xi_i = {} & P_{1i}A_i + A_i^T P_{1i} + \overline{B}_{fi}C_i + C_i^T \overline{B}_{fi}^T + \alpha P_{1i} + A_i^T R_i A_i \\ & + Q_{1i} + Q_{2i} + Q_{3i}; \end{aligned} \tag{35}$$

other symbols are mentioned earlier. Then there exists a filter of the form (3) such that the weighted H_∞ filtering problem of system (1) with (2) is solvable under any ADT switching signal (12). The filter matrices are constructed by

$$\begin{aligned} A_{fi} &= N_i^{-1}\overline{A}_{fi}, \qquad B_{fi} = N_i^{-1}\overline{B}_{fi}, \\ C_{fi} &= \overline{C}_{fi}, \qquad D_{fi} = \overline{D}_{fi}, \end{aligned} \tag{36}$$

or

$$\begin{aligned} A_{fi} &= \overline{A}_{fi}N_i^{-1}, \qquad B_{fi} = \overline{B}_{fi}, \\ C_{fi} &= \overline{C}_{fi}N_i^{-1}, \qquad D_{fi} = \overline{D}_{fi}. \end{aligned} \tag{37}$$

Proof. Set

$$\begin{aligned} N_i &= P_{2i}P_{3i}^{-1}P_{2i}^T, \qquad \overline{A}_{fi} = P_{2i}A_{fi}P_{3i}^{-1}P_{2i}^T, \\ \overline{B}_{fi} &= P_{2i}B_{fi}, \qquad \overline{C}_{fi} = P_{2i}P_{3i}^{-T}C_{fi}^T, \\ L &= \operatorname{diag}\left\{I, P_{2i}P_{3i}^{-1}, I, I, I, I, I, I\right\}, \qquad \overline{D}_{fi} = D_{fi}. \end{aligned} \tag{38}$$

Pre- and postmultiplying (26) by L and L^T, respectively, one can get that (33) is equivalent to (10). Thus, the filter error system (4) is exponentially stable with weighted H_∞ performance. Furthermore, (34) is equivalent to $0 < P_{1i} - P_{2i}P_{3i}^{-1}P_{2i}^T = P_{1i} - N_i$; then $N_i > 0$ and P_{2i} is nonsingular. Thus we have

$$\begin{bmatrix} A_{fi} & B_{fi} \\ C_{fi} & D_{fi} \end{bmatrix} = \begin{bmatrix} P_{2i}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{A}_{fi} & \overline{B}_{fi} \\ \overline{C}_{fi} & \overline{D}_{fi} \end{bmatrix} \begin{bmatrix} P_{2i}^{-T}P_{3i} & 0 \\ 0 & I \end{bmatrix}. \tag{39}$$

From the filter transfer function

$$\begin{aligned} T_i(s) &= C_{fi}\left(sI - A_{fi}\right)^{-1} B_{fi} + D_{fi} \\ &= \overline{C}_{fi}P_{2i}^T P_{3i}\left(sI - P_{2i}^{-1}\overline{A}_{fi}P_{2i}^{-T}P_{3i}\right)^{-1} P_{2i}^{-1}\overline{B}_{fi} + \overline{D}_{fi} \\ &= \overline{C}_{fi}\left(sN_i - \overline{A}_{fi}\right)^{-1}\overline{B}_{fi} + \overline{D}_{fi} \\ &= \overline{C}_{fi}\left(sI - N_i^{-1}\overline{A}_{fi}\right)^{-1} N_i^{-1}\overline{B}_{fi} + \overline{D}_{fi} \\ &= \overline{C}_{fi}N_i^{-1}\left(sI - \overline{A}_{fi}N_i^{-1}\right)^{-1}\overline{B}_{fi} + \overline{D}_{fi}, \end{aligned} \tag{40}$$

the filter matrices are readily established by (36) or (37). This completes the proof. □

Remark 10. For convenience, we only consider switched systems with two additive time-varying delays, but the proposed results in this paper can be extended to switched systems with multiple additive time-varying delay components.

Remark 11. When $d_1 = 0$ or $d_2 = 0$, (1) reduces to switched system with a single time-varying delay. Suppose that $d_2 = 0$ without loss of generality; by setting $Q_{2i} = Q_{4i} = 0$ and $R_{2i} = R_{3i} = 0$ in (16), criterions of stability with weighted H_∞ performance for switched systems with a single delay can be obtained similarly.

The following algorithm is helpful to derive filter matrices for the weighted H_∞ filtering problem under consideration.

Design Algorithm

Step 1. Scalars $\alpha > 0$, $\mu \geq 1$ are selected close to 1. $\gamma > 0$ is also prescribed. Then, LMIs in Theorem 9 are solved for given α, μ, and γ by LMI toolbox in MATLAB.

Step 2. If these LMIs have no solution, then there are two cases. Case 1: α and μ will be increased at a big step size; go back to Step 1. Case 2: the previous α and μ are proper, and go to Step 4.

Step 3. If these LMIs have feasible solutions, μ will be decreased at a small step size, and go to Step 1.

Step 4. α and μ are fixed. Decrease γ similarly until γ is the optimized value. According to (36) or (37), the filter matrices are established. Exit.

4. Example

Two examples are presented in this section to illustrate the proposed results.

Example 1. Consider switched system (1) with (2) consisting of two subsystems where subsystem 1 is described by

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 0 \\ -0.5 & 0.2 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -1 & 0 \\ 0 & -0.2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} -1 & 0.3 \end{bmatrix}, \\ C_{d1} &= \begin{bmatrix} 0.9 & -0.5 \end{bmatrix}, \qquad D_1 = 0.4, \\ E_1 &= \begin{bmatrix} 0.5 & -1.3 \end{bmatrix}, \qquad E_{d1} = \begin{bmatrix} -0.9 & 0.1 \end{bmatrix}, \end{aligned} \tag{41}$$

and subsystem 2 is described by

$$\begin{aligned} A_2 &= \begin{bmatrix} -0.9 & 0 \\ -0.8 & 0 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.4 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.5 \\ -0.7 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.5 & -0.9 \end{bmatrix}, \\ C_{d2} &= \begin{bmatrix} 0.3 & -1 \end{bmatrix}, \qquad D_2 = 0.2, \\ E_2 &= \begin{bmatrix} 0.2 & -1 \end{bmatrix}, \qquad E_{d2} = \begin{bmatrix} -0.8 & -0.5 \end{bmatrix}. \end{aligned} \tag{42}$$

Table 1: Minimum feasible γ for different cases.

d_1	0.6	0.6	0.6	0.7	0.7	0.7
d_2	0.3	0.5	0.8	0.3	0.5	0.8
$\gamma_{\min}$	1.2700	0.9397	0.0746	0.8918	0.2219	0.0026

Let $d_1(t) = 0.3 + 0.2\sin t$, $d_2(t) = 0.9 - 0.3\cos t$, $\alpha = 0.5$, $\gamma = 0.9$, $T_a = 0.8046$, $\mu = 1.3 > 1$. It can be easily checked that $d_1 = 0.5$, $d_2 = 0.6$, $h_1 = 0.2$, $h_2 = 0.3$, $T_a^* = \ln\mu/\alpha = 0.4807$. Solving LMIs (11), (14), (33), and (34) gets the designed filter with the following parameterized matrices:

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -0.5792 & -3.5696 \\ -1.2400 & 0.3236 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} 1.0124 \\ 0.1474 \end{bmatrix}, \\ C_{f1} &= \begin{bmatrix} -0.0371 & 0.3802 \end{bmatrix}, \qquad D_{f1} = 0.1262, \\ A_{f2} &= \begin{bmatrix} 0.3744 & 0.8232 \\ 1.0259 & -2.0503 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.2300 \\ 1.1451 \end{bmatrix}, \\ C_{f2} &= \begin{bmatrix} -0.0372 & 0.1669 \end{bmatrix}, \qquad D_{f2} = -0.0282. \end{aligned} \tag{43}$$

Under the initial condition $\xi(0) = [-1 \;\; 0.7 \;\; 0 \;\; 0.1]^T$ and $\omega(t) = 0.2\sin(0.1t)$ which belongs to $L_2[0,\infty)$, the state responses of the filter error system (4) are shown in Figure 1, and the estimation error is displayed in Figure 2. Figure 3 shows the corresponding ADT switching signal satisfying (12). The simulation results imply that the desired goal is well achieved. The admissible minimum feasible γ for different cases is shown in Table 1 to ensure the exponential stability for filtering error system.

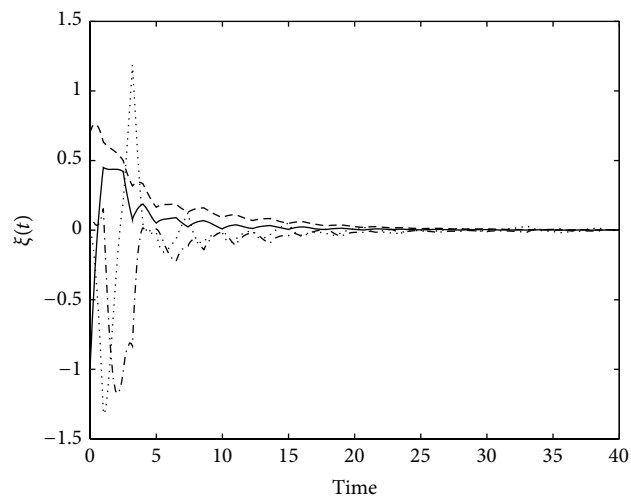

Figure 1: The state responses of the filtering error system.

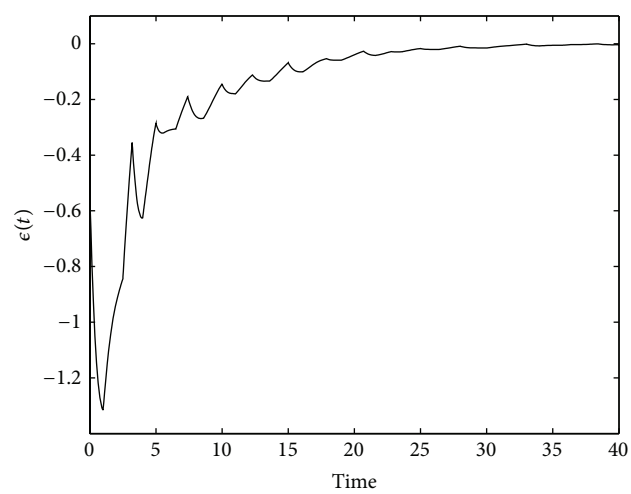

Figure 2: Filtering error.

Example 2. Consider a satellite yaw angles control system with noise perturbation, which is given by the following state-space representation [13]:

$$\begin{aligned} &\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot\theta_1(t) \\ \dot\theta_2(t) \\ \dot\delta_1(t) \\ \dot\delta_2(t) \end{bmatrix} \\ &\quad = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix} \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \delta_1(t) \\ \delta_2(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \omega(t), \\ &y(t) = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(t) & \theta_2(t) & \delta_1(t) & \delta_2(t) \end{bmatrix}^T. \end{aligned} \tag{44}$$

The satellite yaw angles system consists of two rigid bodies joined by a flexible link. This link is modelled as a spring with torque k and viscous damping f. Respectively, θ_1 and θ_2 denote the yaw angles for the main body and the instrumentation module of the satellite, $\delta_1 = \dot\theta_1$, $\delta_2 = \dot\theta_2$. $u(t)$ is the control torque, while J_1 and J_2 are the moments of inertia of two bodies. Here we choose $J_1 = J_2 = 1$, $k = 0.3$, $f = 0.004$, and $x(t) = [\theta_1(t) \;\; \theta_2(t) \;\; \delta_1(t) \;\; \delta_2(t)]^T$. According to the method proposed for this model in [19], the gain matrix for the state-feedback controller $u(t) = Kx(t - d_1 - d_2(t))$ is given by

$$K = \begin{bmatrix} 0.0087 & -1.0000 & 0.0610 & -0.5800 \\ -0.0046 & -0.00478 & 0.0018 & 1.0000 \end{bmatrix}. \tag{45}$$

Thus, the system is described by

$$\begin{aligned} \dot x(t) &= Ax(t) + A_d x(t - d_1 - d_2(t)) + B\omega(t), \\ y(t) &= Cx(t) + D\omega(t), \\ z(t) &= Ex(t), \end{aligned} \tag{46}$$

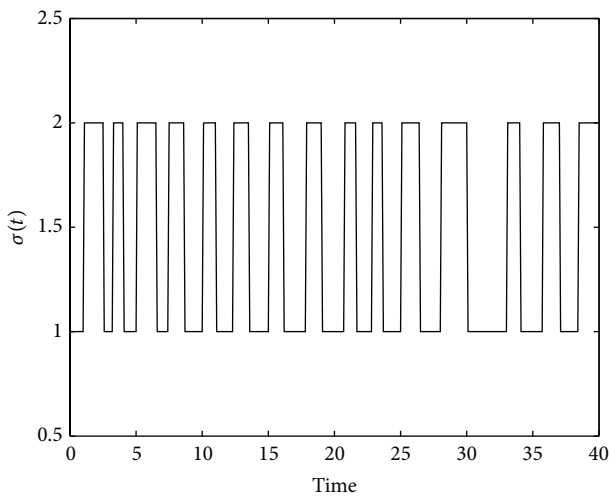

FIGURE 3: Switching signal.

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3 & 0.3 & -0.004 & 0.004 \\ 0.3 & -0.3 & 0.004 & -0.004 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0046 & -0.0048 & 0.0018 & 1.0000 \\ -0.0087 & 1.0000 & -0.0610 & 0.5800 \end{bmatrix}, \tag{47}$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix},$$

$$D = 0, \qquad E = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}.$$

We design a packet-loss-dependent filter of the following form [7]:

$$\begin{aligned} \dot{x}_f(t) &= A_{fi} x_f(t) + B_{fi} \hat{y}(t), \\ z_f(t) &= C_{fi} x(t), \quad i \in \{1, 2\}. \end{aligned} \tag{48}$$

When $i = 1$, it indicates that a packet is successfully received by the filter, and $\hat{y}(t) = y(t)$; then the filter model is described by

$$\begin{aligned} \dot{x}_f(t) &= A_{f1} x_f(t) + B_{f1} C x(t) + B_{f1} D \omega(t), \\ z_f(t) &= C_{f1} x(t). \end{aligned} \tag{49}$$

When $i = 2$, it means that a packet is lost, and $\hat{y}(t) = 0$; then the filter model is given by

$$\begin{aligned} \dot{x}_f(t) &= A_{f2} x_f(t), \\ z_f(t) &= C_{f2} x(t). \end{aligned} \tag{50}$$

Setting $\xi^T(t) = [x^T(t) \;\; x_f^T(t)]$, $\epsilon(t) = z(t) - z_f(t)$, we obtain the following filtering error system:

$$\begin{aligned} \dot{\xi}(t) &= \overline{A}_\sigma \xi(t) + \overline{A}_{d\sigma} \xi(t - d_1 - d_2(t)) + \overline{B}_\sigma \omega(t), \\ \epsilon(t) &= \overline{C}_\sigma \xi(t), \end{aligned} \tag{51}$$

where

$$\begin{aligned} &\overline{A}_1 = \begin{bmatrix} A & 0 \\ B_{f1} C & A_{f1} \end{bmatrix}, \qquad \overline{A}_2 = \begin{bmatrix} A & 0 \\ 0 & A_{f2} \end{bmatrix}, \\ &\overline{A}_{d1} = \overline{A}_{d2} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \qquad \overline{B}_1 = \begin{bmatrix} B \\ B_{f1} D \end{bmatrix}, \\ &\overline{B}_2 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \overline{C}_1 = \begin{bmatrix} E - C_{f1} & 0 \end{bmatrix}, \\ &\overline{C}_2 = \begin{bmatrix} E - C_{f2} & 0 \end{bmatrix}, \qquad \sigma = \{1, 2\}. \end{aligned} \tag{52}$$

Choose $d_1 = 0.1$, $d_2 = 0.4$, $h_1 = 0$, $h_2 = 0.2$, $\alpha = 0.7$, $\mu = 1.3$, $\gamma = 0.9$. By using Theorem 9, we obtain the following gain matrices of the packet-less-dependent filter:

$$A_{f1} = \begin{bmatrix} -0.5106 & -0.0411 & -0.0437 & -0.0195 \\ 1.3969 & -1.3594 & 0.4521 & -2.3196 \\ 0.8576 & -0.0883 & -2.7691 & -0.6975 \\ 1.0426 & 1.1355 & -1.2184 & -4.6745 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} -0.0946 \\ -1.7884 \\ -1.0630 \\ -2.9718 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} -0.5452 & -0.0923 & -0.1014 & 0.0485 \\ -0.3012 & -0.5121 & 0.3895 & -1.3274 \\ 0.5511 & -0.2017 & -2.0697 & -0.2610 \\ -0.9764 & 1.3459 & -1.6402 & -2.2371 \end{bmatrix}, \tag{53}$$

$$B_{f2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} 0.1103 & -0.7832 & -0.1077 & 0 \end{bmatrix},$$

$$C_{f2} = \begin{bmatrix} 0.1729 & -1.8773 & -0.3391 & 0 \end{bmatrix}.$$

Under the initial condition $\xi(0) = [-1 \;\; 0.7 \;\; 0 \;\; -0.5 \;\; -1 \;\; 0 \;\; 1 \;\; -1]^T$ and $\omega(t) = 0.1 \sin(\pi t/2)$ belonging to $L_2[0, \infty)$, the state responses of the filter error system (51) are shown in Figure 4, and the estimation error is displayed in Figure 5. Figure 6 shows the corresponding ADT switching signal satisfying (12).

5. Conclusion

The weighted H_∞ filtering problem for a class of switched linear system with two additive time-varying delays is

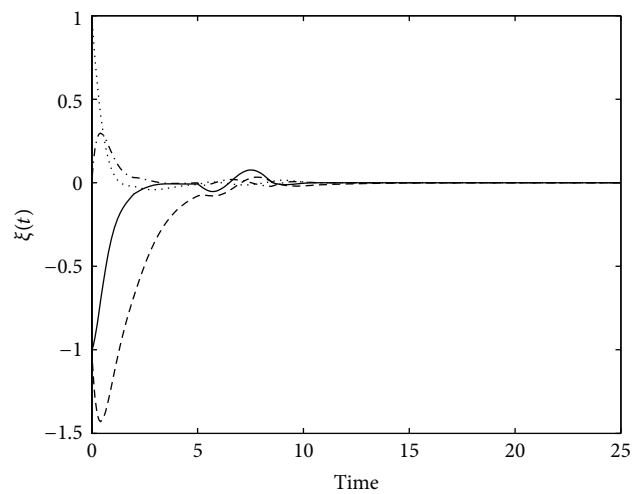

Figure 4: The state responses of the filtering error dynamic system.

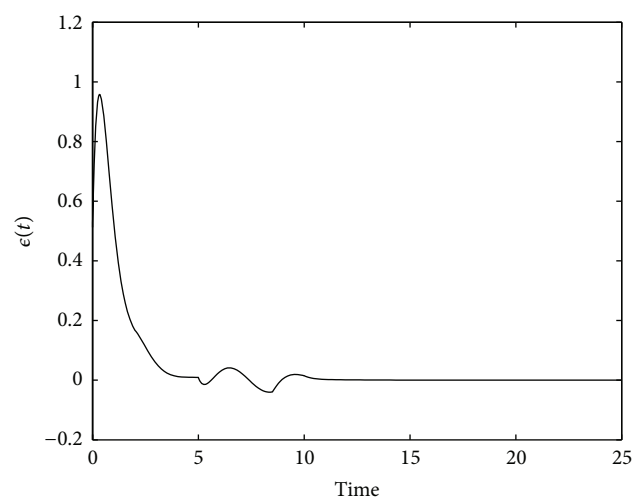

Figure 5: Filtering error.

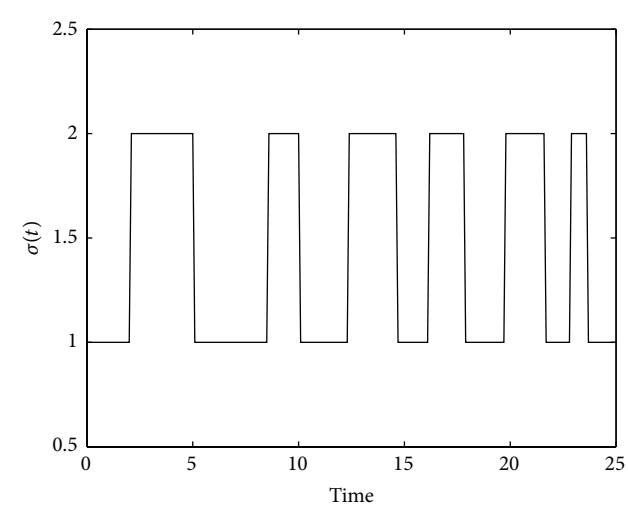

Figure 6: Switching signal.

investigated. First, new criterions are proposed to ensure exponential stability and a weighted H_∞ performance for the filtering error system under ADT switching signals. Then filter matrices of the desired switched filters are obtained. Less conservative and computational complex conditions are developed by applying a modification of Jensen integral inequality. Finally, two examples are given to show the effectiveness of the proposed method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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