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Research Article

Robust H_∞ Filtering for Discrete-Time Markov Jump Linear System with Missing Measurements

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The problem of robust H_∞ filtering is investigated for discrete-time Markov jump linear system (DMJLS) with uncertain parameters and missing measurements. The missing measurements process is modelled as a Bernoulli distributed sequence. A robust H_∞ filter is designed and sufficient conditions are established in terms of linear matrix inequalities via a mode-dependent Lyapunov function approach, such that, for all admissible uncertain parameters and missing measurements, the resulting filtering error system is robustly stochastically stable and a guaranteed H_∞ performance constraint is achieved. Furthermore, the optimal H_∞ performance index is subsequently obtained by solving a convex optimisation problem and the missing measurements effects on the H_∞ performance are evaluated. A numerical example is given to illustrate the feasibility and effectiveness of the proposed filter.

1. Introduction

Discrete-time Markov jump linear system (DMJLS) is an important type of stochastic hybrid system, of which the parameters jumping is governed according to a finite state Markov chain. Therefore, DMJLS has both obvious continuous and discrete dynamics and is commonly used to model problems with abrupt variations in practical engineering system structures and parameters, which may be caused by random failures or sudden violent environments changes in a wide variety of areas, such as electrical engineering, signal processing, target tracking, and multifault diagnosis (see [1–5] and references therein). During the past decades, DMJLS has attracted more and more increasing research interest, among which the state estimation comprises an important research issue and has found many practical applications. Take a multifault diagnosis problem, for example. The system parameters may change under different fault condition and the fault switching always presents Markovian characteristic. Thus, the problem of state estimation for a multifault system can be modelled as filtering for DMJLS.

Among the DMJLS state estimation methods, the interacting multiple model (IMM) method, proposed in [6], is considered to be one of the most representative suboptimal estimators and performs much more cost-effectively than the other recursive methods. However, the parametric uncertainties often occur in many practical engineering systems. Unfortunately, IMM has some considerable drawbacks lying in its inevitable dependence on exact system model and noise statistic restriction with known Gaussian distribution. Nowadays, more and more attention has been focused on the H_∞ filter, which can deal with the state estimation problem for DMJLS with uncertain models and unknown but energy bounded external noises [7]. The H_∞ filter is briefly described as the design of a filter for a given system such that the L_2-induced gain from the exogenous disturbance to the filter error is below a prescribed level. Due to its loose requirement on the system model and noise statistics, more and more attention has been attracted and significant advances have been made in the H_∞ filtering method.

On the other hand, besides the parametric uncertainties, in practical applications, the measured outputs are also usually subject to the uncertainties of randomly occurring missing phenomenon due to kinds of reasons, such as the sensor fault, external disturbance, or network data transmission loss [8, 9], where only noise is regarded as the

original measurements to motivate the estimator, without the prior knowledge of whether the true signals are contained or not. As to the problem of state estimation with missing measurements, a Bernoulli distribution is usually adopted to describe the missing behavior, which is considered to be firstly proposed in [10], wherein an optimal recursive filter is presented for systems with missing measurements. Another model for the missing measurements phenomenon is a Markovian jumping process. A jump linear estimator in terms of a predictor is proposed in [11], which can select a corrector gain at each time. In [12], an IMM based distributed filter for DMJLS with missing measurements is derived by describing the switching of system mode and missing measurements state with two independent Markov chains, and thus, a new overall Markov chain can be obtained by the product of the above two mode sets.

Although more and more efforts have been tried on the problem of filtering with missing measurements, almost all existing results are concerned with linear system [13–16]. The issues involved with DMJLS have not been fully investigated. While taking the parameter uncertainties into account simultaneously, the results of filtering for DMJLS are still scarce up to now, to the best of the authors' knowledge. The main difficulty to deal with such a problem lies in how to incorporate the probabilistic missing measurement into a robust estimation framework. Thus, this intuition motivates this paper to initiate the research on considering parametric uncertainties, missing measurements in the state estimation for DMJLS.

This paper is concerned with the problem of H_∞ filtering for DMJLS with parametric uncertainties and missing measurements. The missing measurements considered in this paper follow a Bernoulli switching process. A robust H_∞ filter is designed and sufficient conditions for the existence of a feasible solution to the problem are discussed by using a mode-dependent Lyapunov function approach, such that for all possible uncertain parameters and missing measurements, the resulting filtering error system is robustly stochastically stable and the estimation error is bounded with a guaranteed H_∞ attenuation level. Furthermore, the optimal H_∞ performance index is subsequently obtained by solving a convex optimisation problem and the missing measurements effects on H_∞ performance are evaluated. The main contribution of this paper lies in considering the impacts of uncertain parameters and missing measurements simultaneously in the problem of state estimation for DMJLS and both the stochastic and deterministic problem are dealt with in a unified framework, which is important and challenging in both practice and theory.

The rest of the paper is organized as follows. In Section 2, the main problem is formulated and some preliminaries are given. In Section 3, the performance of the robust mode-dependent H_∞ filter is analyzed. The filter design problem is tackled in Section 4. Some illustrative simulation results are given in Section 5. Lastly, the conclusion is drawn in Section 6.

Notation 1. In this paper, the notations used are standard. The subscript "T" denotes matrix transposition. Both $\mathfrak{R}^{n\times 1}$ and $\mathfrak{R}^n$ denote the n-dimensional Euclidean space. $\|\cdot\|$ represents the Euclidean norm. I is the identity matrix with appropriate dimensions. $l_2[0\ +\infty)$ is the space of square-summable vector functions over $[0\ +\infty)$. $*$ means the symmetric terms in a symmetric matrix. For a real matrix P, $P > 0$ means that P is symmetric and positive definite. $E\{x\}$ denotes the mathematical statistical expectation of a stochastic variable x and $E\{x \mid y\}$ means the conditional expectation of x given y.

2. Problem Formulation and Preliminary

For a given probability space (Ω, F, P), where Ω represents the sample space, F is the algebra of events, and P is the probability measure defined on F, consider the following DMJLS with parametric uncertainties and missing measurements:

$$\begin{aligned} x_{k+1} &= \left[A(\theta_k) + \Delta A(k,\theta_k)\right] x_k \\ &\quad + \left[B(\theta_k) + \Delta B(k,\theta_k)\right] w_k, \\ y_k &= \xi_k C(\theta_k) x_k + D(\theta_k) w_k, \\ z_k &= L(\theta_k) x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathfrak{R}^{n\times 1}$ is the system state vector; $y_k \in \mathfrak{R}^{m\times 1}$ denotes the measured output; w_k is the external noise which belongs to $l_2[0\ +\infty)$; $z_k \in \mathfrak{R}^{p\times 1}$ is the objective signal to be estimated; and $\{\theta_k, k \in N\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $S = \{1, \ldots, s\}$ with transition probability $\pi_{ij} = \mathrm{Prob}(\theta_k = j \mid \theta_{k-1} = i)$ for all $i, j \in S$.

$\Delta A(k,\theta_k)$ and $\Delta B(k,\theta_k)$, denoting the norm-bounded parameter uncertainties, are unknown real-valued matrices with appropriate dimensions corresponding to each mode satisfying

$$\begin{aligned} &\left[\Delta A(k,\theta_k) \quad \Delta B(k,\theta_k)\right] \\ &\quad = M(\theta_k) F(k,\theta_k) \left[N_a(\theta_k) \quad N_b(\theta_k)\right], \\ &F(k,\theta_k) F^T(k,\theta_k) \le I, \\ &\qquad\qquad \forall k > 0, \end{aligned} \tag{2}$$

where $M(\theta_k), N_a(\theta_k)$, and $N_b(\theta_k)$ are known real constant matrices of appropriate dimensions corresponding to each mode and $F(k,\theta_k)$ is unknown matrix representing the mode-dependent uncertainties. The model of parametric uncertainties actually comes from the tolerance of physical system parameters, which can be modelled in norm-bounded form. This form of uncertainties is commonly used in robust filtering and control [17, 18].

When the system is in mode $i \in S$, that is, $\theta_k = i$, the mode-dependent system matrices are notated for simplification as follows:

$$\begin{aligned} A_i &:= A(\theta_k = i), \\ B_i &:= B(\theta_k = i), \\ C_i &:= C(\theta_k = i), \end{aligned}$$

$$\begin{aligned} D_i &:= D(\theta_k = i), \\ L_i &:= L(\theta_k = i); \\ \Delta A_{ik} &:= \Delta A(k, \theta_k = i), \\ \Delta B_{ik} &:= \Delta B(k, \theta_k = i), \\ F_{ik} &:= F(k, \theta_k = i); \\ M_i &:= M(\theta_k = i), \\ N_{ai} &:= N_a(\theta_k = i), \\ N_{bi} &:= N_b(\theta_k = i). \end{aligned} \tag{3}$$

Different from the Markov process model in [11, 12], ξ_k, denoting the measurements state (missing or normal), is a Bernoulli distributed sequence [10] taking the values of 1 and 0 with

$$\begin{aligned} \text{Prob}\{\xi_k = 1\} &= E\{\xi_k\} = \beta, \\ \text{Prob}\{\xi_k = 0\} &= 1 - E\{\xi_k\} = 1 - \beta, \end{aligned} \tag{4}$$

and $\beta \in [0 \quad 1]$ is a known positive scalar, noting that [15]

$$\begin{aligned} E\{\xi_k - \beta\} &= 0, \\ E\left\{(\xi_k - \beta)^2\right\} &= \beta(1 - \beta). \end{aligned} \tag{5}$$

It is obvious that $\xi_k = 0$ directly describes the missing measurements phenomenon. It is worth mentioning that the filter cannot distinguish whether the valid measurement is missing or not.

Consider the following mode-dependent H_∞ filtering system, which is a DMJLS adaptation for the filter model in [15]:

$$\hat{x}_{k+1} = G_i \hat{x}_k + K_i (y_k - \beta C_i \hat{x}_k), \tag{6}$$

$$\hat{z}_k = H_i \hat{x}_k, \tag{7}$$

where $\hat{x}_k \in \mathfrak{R}^{n\times 1}$ denotes the filter state vector, $y_k \in \mathfrak{R}^{m\times 1}$ is the input of the filter, and $\hat{z}_k$ is the estimation of z_k and it is mentioned that G_i, K_i, and H_i are the mode-dependent H_∞ filter parameters to be determined.

Augment the model of (1) to include the states of the filter (6)-(7), and the filtering error system is obtained below:

$$\overline{x}_{k+1} = \overline{A}_i \overline{x}_k + (\xi_k - \beta)\overline{A}_{1i}\overline{x}_k + \overline{B}_i w_k, \tag{8}$$

$$e_k = \overline{L}_i \overline{x}_k, \tag{9}$$

where $\overline{x}_k = \left[x_k^T \quad \hat{x}_k^T\right]^T$, $\overline{z}_k = z_k - \hat{z}_k$, and the parameters of the augmented systems are given by

$$\begin{aligned} \overline{A}_i &= \begin{bmatrix} A_i + \Delta A_{ik} & 0 \\ \beta K_i C_i & G_i - \beta K_i C_i \end{bmatrix} \\ &= \begin{bmatrix} A_i & 0 \\ \beta K_i C_i & G_i - \beta K_i C_i \end{bmatrix} + \begin{bmatrix} M_i \\ 0 \end{bmatrix} F_{ik} \begin{bmatrix} N_{ai} & 0 \end{bmatrix} \\ &= \widetilde{A}_i + \Delta\widetilde{A}_{ik}, \\ \overline{A}_{1i} &= \begin{bmatrix} 0 & 0 \\ K_i C_i & 0 \end{bmatrix}, \\ \overline{B}_i &= \begin{bmatrix} B_i + \Delta B_{ik} \\ K_i D_i \end{bmatrix} = \begin{bmatrix} B_i \\ K_i D_i \end{bmatrix} + \begin{bmatrix} \Delta B_{ik} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} B_i \\ K_i D_i \end{bmatrix} + \begin{bmatrix} M_i \\ 0 \end{bmatrix} F_{ik} N_{bi} = \widetilde{B}_i + \widetilde{M}_i F_{ik} \widetilde{N}_{bi} \\ &= \widetilde{B}_i + \Delta\widetilde{B}_{ik}, \\ \overline{L}_i &= \begin{bmatrix} L_i & -H_i \end{bmatrix}, \\ \widetilde{A}_i &= \begin{bmatrix} A_i & 0 \\ \beta K_i C_i & G_i - \beta K_i C_i \end{bmatrix}, \\ \Delta\widetilde{A}_{ik} &= \widetilde{M}_i F_{ik} \widetilde{N}_{ai}, \\ \widetilde{B}_i &= \begin{bmatrix} B_i \\ K_i D_i \end{bmatrix}, \\ \Delta\widetilde{B}_{ik} &= \widetilde{M}_i F_{ik} \widetilde{N}_{bi}, \\ \widetilde{M}_i &= \begin{bmatrix} M_i \\ 0 \end{bmatrix}, \\ \widetilde{N}_{ai} &= \begin{bmatrix} N_{ai} & 0 \end{bmatrix}, \\ \widetilde{N}_{bi} &= N_{bi}. \end{aligned} \tag{10}$$

Definition 1 (see [7]). System (8)-(9) is said to have robustly stochastic stability if in the case of $w_k \equiv 0$ for every initial condition $x_0 \in R^n$ and $\theta_0 \in S$, the following inequality holds:

$$E\left\{\sum_{k=0}^{\infty} \|\overline{x}_k\|^2 \mid \overline{x}_0, \theta_0\right\} < \infty. \tag{11}$$

Definition 2 (see [7]). Given a scalar $\gamma > 0$, system (8)-(9) satisfies an H_∞ noise attenuation performance index γ under the robustly stochastic stability condition, if it is robustly stochastically stable and, under zero initial condition, for all nonzero $w_k \in l_2[0 \quad +\infty)$ the following inequality holds:

$$E\left\{\sum_{k=0}^{\infty} \|\overline{z}_k\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} \|w_k\|^2. \tag{12}$$

To proceed further, a well-known lemma is introduced to tackle the uncertainties as follows.

Lemma 3 (see [19]). *For real matrices $Q = Q^T$, M, F, and N with appropriate dimensions. Suppose that F satisfies $FF^T \leq I$, and the following inequality holds:*

$$Q + MFN + N^T F^T M^T < 0. \tag{13}$$

If and only if there exists a positive scalar $\varepsilon > 0$ holding

$$Q + \varepsilon^{-1} MM^T + \varepsilon N^T N < 0, \tag{14}$$

then the problem in this paper is addressed below.

Problem 4. With Definitions 1 and 2, the objective in this paper is to design a robust mode-dependent H_∞ filter of the form (7)-(8) for system (1), such that given a prescribed level of noise attenuation $\gamma > 0$, for all possible missing measurements, the filtering error system (8)-(9) is robustly stochastically stable for the whole uncertain domain and satisfies the H_∞ robustness performance (12).

3. H_∞ Error Performance Analysis

In this section, the performance analysis of the mode-dependent H_∞ filter approach is given. Theorem 5 presents the sufficient conditions for the existence of the H_∞ filter.

Theorem 5. *Consider the DMJLS (1). Given a constant $\gamma > 0$ as the H_∞ performance index, the filtering error system (8)-(9) is robustly stochastically stable and satisfies the H_∞ robustness performance, if there exists a matrix $P_i > 0$ satisfying the following LMIs:*

$$\Psi_i = \begin{bmatrix} -\overline{P}_i & 0 & 0 & \overline{P}_i\overline{A}_i & \overline{P}_i\overline{B}_i \\ * & -\overline{P}_i & 0 & \alpha\overline{P}_i\overline{A}_{1i} & 0 \\ * & * & -I & \overline{L}_i & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad \text{for } i = 1, 2, \ldots, s, \tag{15}$$

where $\alpha = [(1-\beta)\beta]^{1/2}$, $\overline{P}_i = \sum_{j=1}^{s} \lambda_{i,j} P_j$, and P_j is a stochastic mode-dependent Lyapunov matrix for the filtering error system.

Proof. For the filtering error system, construct a stochastic mode-dependent Lyapunov function as

$$V(\overline{x}_k, i) = \overline{x}_k^T P_i \overline{x}_k, \quad \text{for } i = 1, 2, \ldots, s. \tag{16}$$

Then

$$\begin{aligned} E\{\Delta V(\overline{x}_k, i)\} &= E\{V(\overline{x}_{k+1}, i) \mid \overline{x}_k, i\} - V(\overline{x}_k, i) \\ &= E\left\{\overline{x}_{k+1}^T \sum_{j=1}^{s} \pi_{i,j} P_j \overline{x}_{k+1}\right\} - \overline{x}_k^T P_i \overline{x}_k \\ &= E\{\overline{x}_{k+1}^T \overline{P}_i \overline{x}_{k+1}\} - \overline{x}_k^T P_i \overline{x}_k \\ &= \overline{x}_k^T \left(\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i\right) \overline{x}_k \\ &\quad + E\{(\xi_k - \beta)^2\} \overline{x}_k^T \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i} \overline{x}_k \\ &\quad + 2\overline{x}_k^T \overline{A}_i^T \overline{P}_i \overline{B}_i w_k + w_k^T \overline{B}_i^T \overline{P}_i \overline{B}_i w_k \\ &= \overline{x}_k^T \left(\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i + \alpha^2 \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i}\right) \overline{x}_k \\ &\quad + 2\overline{x}_k^T \overline{A}_i^T \overline{P}_i \overline{B}_i w_k + w_k^T \overline{B}_i^T \overline{P}_i \overline{B}_i w_k. \end{aligned} \tag{17}$$

From (15), according to the basic matrix manipulations, $\begin{bmatrix} -\overline{P}_i & 0 & \overline{P}_i\overline{A}_i \\ * & -\overline{P}_i & \alpha\overline{P}_i\overline{A}_{1i} \\ * & * & -P_i \end{bmatrix} < 0$ holds. Thus, $\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i + \alpha^2 \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i} < 0$ by Schur complement.

Therefore, with $w_k = 0$,

$$\begin{aligned} \Delta V(\overline{x}_k, i) &= \overline{x}_k^T \left(\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i + \alpha^2 \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i}\right) \overline{x}_k \\ &\leq -\mu \overline{x}_k^T \overline{x}_k < 0, \end{aligned} \tag{18}$$

with $\mu = \lambda_{\min}\{-(\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i + \alpha^2 \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i})\}$ as the minimum eigenvalue of $-(\overline{A}_i^T \overline{P}_i \overline{A}_i - P_i + \alpha^2 \overline{A}_{1i}^T \overline{P}_i \overline{A}_{1i})$.

Then,

$$\begin{aligned} E\left[\sum_{k=0}^{M} \Delta V(\overline{x}_k, i)\right] &= E[V(\overline{x}_{M+1}, i) \mid \overline{x}_k, i] \\ &\quad - E[V(\overline{x}_0, 0)] \\ &\leq -\mu E\left[\sum_{k=0}^{M} \overline{x}_k^T \overline{x}_k\right]. \end{aligned} \tag{19}$$

Thus,

$$\begin{aligned} &E\left\{\sum_{k=0}^{M} \|\overline{x}_k\|^2\right\} \\ &\quad \leq \frac{1}{\mu}\{E[V(\overline{x}_0, 0)] - E[V(\overline{x}_{k+1}, k+1) \mid \overline{x}_k, i]\} \\ &\quad \leq \frac{1}{\mu} E[V(\overline{x}_0, i)] < \infty. \end{aligned} \tag{20}$$

The filtering error system (8)-(9) is robustly stochastically stable according to Definition 1.

Assume the zero initial condition $V(\overline{x}_0, i) = 0$, and one has that

$$E\left[\sum_{k=0}^{M} \Delta V(\overline{x}_k, i)\right] = V(\overline{x}_M, i) \geq 0. \tag{21}$$

Consider the performance index:

$$J = E\left\{\sum_{k=0}^{\infty}\left[\|\overline{z}_k\|^2 - \gamma^2\|w_k\|^2\right]\right\}. \tag{22}$$

Therefore,

$$\begin{aligned} J &\le E\left\{\sum_{k=0}^{\infty}\left[\|\overline{z}_k\|^2 - \gamma^2\|w_k\|^2\right] + \Delta V(\overline{x}_k, i)\right\} \\ &= \sum_{k=0}^{\infty}\left[\overline{z}_k^T\overline{z}_k - \gamma^2 w_k^T w_k + \Delta V(\overline{x}_k, i)\right] \\ &= \sum_{k=0}^{\infty}\left\{\left[\overline{L}(\theta_k)\overline{x}_k\right]^T\left[\overline{L}(\theta_k)\overline{x}_k\right] - \gamma^2 w_k^T w_k\right. \\ &\quad + \overline{x}_k^T\left(\overline{A}_i^T\overline{P}_i\overline{A}_i - P_i + \alpha^2\overline{A}_{1i}^T\overline{P}_i\overline{A}_{1i}\right)x_k \\ &\quad \left. + 2\overline{x}_k^T\overline{A}_i^T\overline{P}_i\overline{B}_i w_k + w_k^T\overline{B}_i^T\overline{P}_i\overline{B}_i w_k\right\} = \sum_{k=0}^{\infty}\eta_k^T\Phi_i\eta_k, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \eta_k &= \left[\overline{x}_k^T \quad w_k^T\right]^T, \\ \Phi_i &= \begin{bmatrix} \overline{A}_i^T\overline{P}_i\overline{A}_i - P_i + \alpha^2\overline{A}_{1i}^T\overline{P}_i\overline{A}_{1i} + \overline{L}_i^T\overline{L}_i & \overline{A}_i^T\overline{P}_i\overline{B}_i \\ * & \overline{B}_i^T\overline{P}_i\overline{B}_i - \gamma^2 I \end{bmatrix}. \end{aligned} \tag{24}$$

According to Schur complement, (15) guarantees $\Phi_i < 0$, which means $\|\overline{z}_k\|^2 < \gamma^2\|w_k\|^2$ and $J < 0$. The proof is completed. □

In order to design an admissible H_∞ filter to eliminate the cross coupling of matrix product terms between the Lyapunov matrix and system matrix among different subsystems, when solving (15), the following theorem is presented inspired by [20].

Theorem 6. *Consider the DMJLS (1). The filtering error system (8)-(9) is robustly stochastically stable and satisfies the given H_∞ performance index $\gamma > 0$, if there exist matrices $P_i > 0$, X_i satisfying the following LMIs:*

$$\begin{bmatrix} -X_i - X_i^T + \overline{P}_i & 0 & 0 & X_i\overline{A}_i & X_i\overline{B}_i \\ * & -X_i - X_i^T + \overline{P}_i & 0 & \alpha X_i\overline{A}_{1i} & 0 \\ * & * & -I & \overline{L}_i & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad \text{for } i = 1, 2, \ldots, s, \tag{25}$$

where $\alpha = [(1-\beta)\beta]^{1/2}$, $\overline{P}_i = \sum_{j=1}^{s}\lambda_{i,j}P_j$, and P_j is a stochastic mode-dependent Lyapunov matrix for the filtering error system.

Proof. Define a transformation matrix Ξ as

$$\Xi = \begin{bmatrix} \overline{A}_i^T & \alpha\overline{A}_{1i}^T & 0 & I & 0 \\ \overline{B}_i^T & 0 & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 \end{bmatrix}. \tag{26}$$

Perform the congruence transformation Ξ to (25) leading to

$$\begin{bmatrix} \overline{A}_i^T\overline{P}_i\overline{A}_i + (1-\beta)\beta\overline{A}_{1i}^T\overline{P}_i\overline{A}_{1i} - P_i & \overline{A}_i^T\overline{P}_i\overline{B}_i & \overline{L}_i^T \\ * & \overline{B}_i^T\overline{P}_i\overline{B}_i - \gamma^2 I & 0 \\ * & * & -I \end{bmatrix}. \tag{27}$$

Apply Schur complement and consider Theorem 5. Then the proof is concluded. □

In order to solve the problem of parametric uncertainties in Theorem 6, the following sufficient condition for the filtering error system (8)-(9) to be robustly stochastically stable and achieve the H_∞ constraint is proved in Theorem 7.

Theorem 7. *Consider the DMJLS (1) with parameter uncertainties and missing measurements. Given a constant $\gamma > 0$ as the H_∞ performance index, the filtering error system (8)-(9) is robustly stochastically stable and satisfies the H_∞ performance, for the whole uncertain domain of the parameters as (2), if there exist matrices $P_i > 0$, X_i, and a positive scalar ε_i satisfying the LMIs below:*

$$\begin{bmatrix} -X_i - X_i^T + \overline{P} & 0 & 0 & X_i\widetilde{A}_i & X_i\widetilde{B}_i & X_i\widetilde{M}_i \\ * & -X_i - X_i^T + \overline{P} & 0 & \alpha X_i\overline{A}_{1i} & 0 & 0 \\ * & * & -I & \overline{L}_i & 0 & 0 \\ * & * & * & -P_i + \varepsilon_i\widetilde{N}_{ai}^T\widetilde{N}_{ai} & \varepsilon_i\widetilde{N}_{ai}^T\widetilde{N}_{bi} & 0 \\ * & * & * & * & -\gamma^2 I + \varepsilon_i\widetilde{N}_{bi}^T\widetilde{N}_{bi} & 0 \\ * & * & * & * & * & -\varepsilon_i I \end{bmatrix}, \quad \text{for } i = 1, 2, \ldots, s, \tag{28}$$

where $\alpha = [(1-\beta)\beta]^{1/2}$.

Proof. Note that (25) can be written in the form of (13) as

$$Q_i + \Gamma_{1i} F_{ik} \Gamma_{2i} + \Gamma_{2i}^T F_{ik}^T \Gamma_{1i}^T < 0, \tag{29}$$

with

$$Q_i = \begin{bmatrix} -X_i - X_i^T + \overline{P}_i & 0 & 0 & X_i \widetilde{A}_i & X_i \widetilde{B}_i \\ * & -X_i - X_i^T + \overline{P}_i & 0 & \alpha X_i \overline{A}_{1i} & 0 \\ * & * & -I & \overline{L}_i & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}, \tag{30}$$

$$\Gamma_{1i} = \begin{bmatrix} \widetilde{M}_i^T X_i^T & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Gamma_{2i} = \begin{bmatrix} 0 & 0 & 0 & \widetilde{N}_{ai} & \widetilde{N}_{bi} \end{bmatrix}^T.$$

By applying Schur complement and Lemma 3 to (29), the LMIs (25) hold, if and only if there exists positive scalar ε_i satisfying the LMIs (28), which completes the proof. □

4. H_∞ Filter Design

Theorem 8 below presents the solvability for the addressed robust H_∞ filter design problem in Theorem 7 and whether the LMIs (28) are feasible for all the possible parametric uncertainties and missing measurements.

Theorem 8. *Consider the DMJLS (1) with parameter uncertainties and missing measurements. The filtering error system (8)-(9) is robustly stochastically stable and satisfies a prescribed H_∞ performance index $\gamma > 0$, if there exist matrices $P_{1i}, P_{2i}, P_{3i}, \widetilde{G}_i, \widetilde{K}_i, \widetilde{H}_i, U_i, S_i$, and R_i and scalar $\varepsilon_i > 0$ satisfying the following LMIs:*

$$\begin{bmatrix} \Pi_{11i} & \Pi_{12i} & 0 & 0 & 0 & \Pi_{16i} & \Pi_{17i} & \Pi_{18i} & U_i M_i \\ * & \Pi_{22i} & 0 & 0 & 0 & \Pi_{26i} & \Pi_{27i} & \Pi_{28i} & S_i M_i \\ * & * & \Pi_{33i} & \Pi_{34i} & 0 & \alpha \widetilde{K}_i C_i & 0 & 0 & 0 \\ * & * & * & \Pi_{44i} & 0 & \alpha \widetilde{K}_i C_i & 0 & 0 & 0 \\ * & * & * & * & -I & L_i & -H_i & 0 & 0 \\ * & * & * & * & * & \Pi_{66i} & -P_{2i} & \varepsilon_i N_{ai}^T N_{bi} & 0 \\ * & * & * & * & * & * & -P_{3i} & 0 & 0 \\ * & * & * & * & * & * & * & \Pi_{88i} & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_i I \end{bmatrix}, \tag{31}$$

$$\text{for } i = 1, 2, \ldots, s,$$

$$\begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} > 0, \tag{32}$$

where $\alpha = [(1-\beta)\beta]^{1/2}$ and the elements

$$\begin{aligned} \Pi_{11i} &= \Pi_{33i} = -U_i - U_i^T + \sum_{j=1}^{s} \lambda_{ij} P_{1j}, \\ \Pi_{12i} &= \Pi_{34i} = -R_i - S_i^T + \sum_{j=1}^{s} \lambda_{ij} P_{2j}, \\ \Pi_{16i} &= U_i A_i + \beta \widetilde{K}_i C_i, \\ \Pi_{17i} &= \Pi_{27i} = \widetilde{G}_i - \beta \widetilde{K}_i C_i, \\ \Pi_{18i} &= U_i B_i + \widetilde{K}_i D_i, \\ \Pi_{22i} &= \Pi_{44i} = -R_i - R_i^T + \sum_{j=1}^{s} \lambda_{ij} P_{3j}, \\ \Pi_{26i} &= S_i A_i + \beta \widetilde{K}_i C_i, \\ \Pi_{28i} &= S_i B_i + \widetilde{K}_i D_i, \\ \Pi_{66i} &= -P_{1i} + \varepsilon_i N_{ai}^T N_{ai}, \\ \Pi_{88i} &= -\gamma^2 I + \varepsilon_i N_{bi}^T N_{bi}. \end{aligned} \tag{33}$$

Moreover, if the LMIs (31)-(32) have a feasible solution, the parameters of an admissible filter can be given by

$$\begin{aligned} G_i &= R_i^{-1} \widetilde{G}_i, \\ K_i &= R_i^{-1} \widetilde{K}_i, \\ H_i &= \widetilde{H}_i. \end{aligned} \tag{34}$$

Proof. Assume the matrices P_i and X_i in Theorem 7 in the form of

$$\begin{aligned} P_i &= \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}, \\ X_i &= \begin{bmatrix} U_i & R_i \\ S_i & R_i \end{bmatrix}, \end{aligned} \tag{35}$$

where R_i is assumed to be nonsingular. After substituting the above matrices into (28) and replacing with $\widetilde{G}_i = R_i G_i$, $\widetilde{K}_i = R_i K_i$, and $\widetilde{H}_i = H_i$, it is easily shown that (31)-(32) are equivalent to (28). Therefore, the filtering error system is guaranteed to be robustly stochastically stable and achieves the prescribed H_∞ performance constraint. The proof is finally concluded. □

Remark 9. By denoting $\varsigma = \gamma^2$, the minimum H_∞ noise attenuation performance bound and the corresponding filter

parameter design can be obtained by solving the following convex optimisation problem [21]:

$$\begin{aligned} &\min_{P_{1i},P_{2i},P_{3i},\widetilde{G}_i,\widetilde{K}_i,\widetilde{H}_i,U_i,S_i,R_i,\varepsilon_i} \quad \varsigma \\ &\text{subject to} \quad \text{LMIs (33)} \\ &\text{over} \quad \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} > 0, \quad \varepsilon_i > 0. \end{aligned} \tag{36}$$

Therefore, the optimal performance of the admissible H_∞ filter can be obtained with the minimum H_∞ performance constraint.

5. Numerical Example

In this section, a numerical example is presented to demonstrate the effectiveness and the feasibility of the proposed filter. Consider the DMJLS with two subsystems and the following parameters:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & -0.45 \\ 0.81 & 0.81 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & -0.27 \\ 0.81 & 1.23 \end{bmatrix}; \\ B_1 &= \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.3 \\ 0.8 \end{bmatrix}; \\ C_1 &= \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}; \\ D_1 &= 0.8, \\ D_2 &= 0.9; \\ L_1 &= L_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}; \\ M_1 &= M_2 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \\ N_{a1} &= N_{a2} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \\ N_{b1} &= N_{b2} = 0.1. \end{aligned} \tag{37}$$

F_{1k} and F_{2k} are selected randomly between -1 and 1 at each time step. The transition probability matrix is

$$\Lambda = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}. \tag{38}$$

The probability of the missing measurements is $\beta = 0.6$.

The minimum value of γ is $\gamma_{\min} = 0.5487$ by solving the corresponding convex optimisation problem (36) with the help of Matlab LMI Toolbox. By solving (31)-(32), the parameters of the H_∞ filter can be obtained below:

$$\begin{aligned} G_1 &= \begin{bmatrix} 0.1171 & -0.3809 \\ 0.5549 & 0.4219 \end{bmatrix}, \\ K_1 &= \begin{bmatrix} 0.0711 \\ -0.6840 \end{bmatrix}, \\ H_1 &= \begin{bmatrix} -0.1000 & -0.2000 \end{bmatrix}; \\ G_2 &= \begin{bmatrix} 0.1660 & -0.1318 \\ 0.2093 & 0.8124 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.2376 \\ -1.5140 \end{bmatrix}, \\ H_2 &= \begin{bmatrix} -0.0996 & -0.1999 \end{bmatrix}. \end{aligned} \tag{39}$$

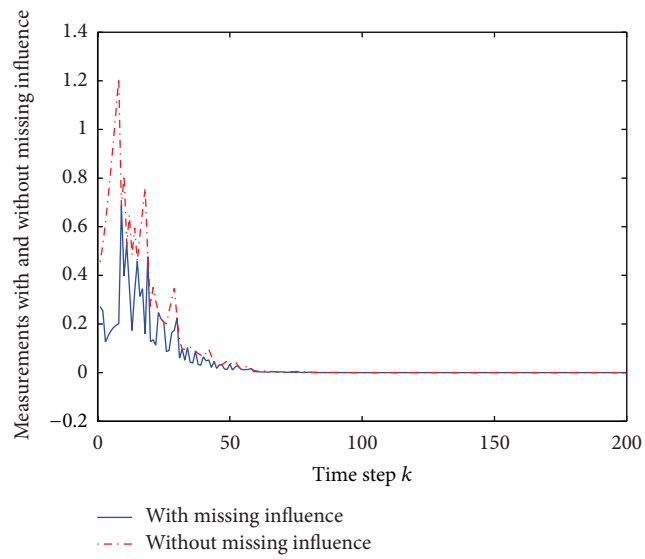

FIGURE 1: Measurements y_k with ($\xi_k = 0$) and without ($\xi_k = 1$) missing influence.

Figure 1 shows the measurements without and with missing influence after running 200 steps each simulation.

The corresponding realization of the jumping mode is plotted in Figure 2.

By setting the initial condition $x_0 = [0.4 \ \ 0.6]^T$ and choosing the energy bounded noise $w_k = 0.5\exp(-0.1k)\sin(0.01\pi k)$, the error response of the resulting error system is given in Figure 3. It is clearly observed that the filtering error is stable against the parametric uncertainties and missing measurements effects, which implies the feasibility and effectiveness of the proposed filter.

Figure 4 illustrates the missing measurements effects on the optimal H_∞ performance index, by comparing the minimum H_∞ performance index $\gamma_{\min}$ under different level of probability β, indicating that the H_∞ performance deteriorates with the increasing of the missing probability.

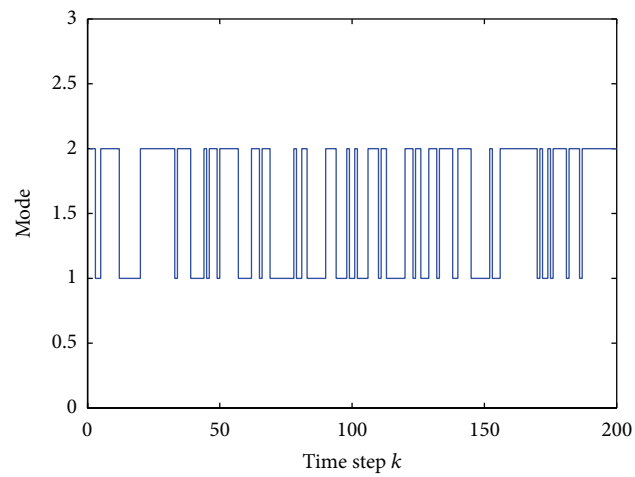

Figure 2: Jumping modes.

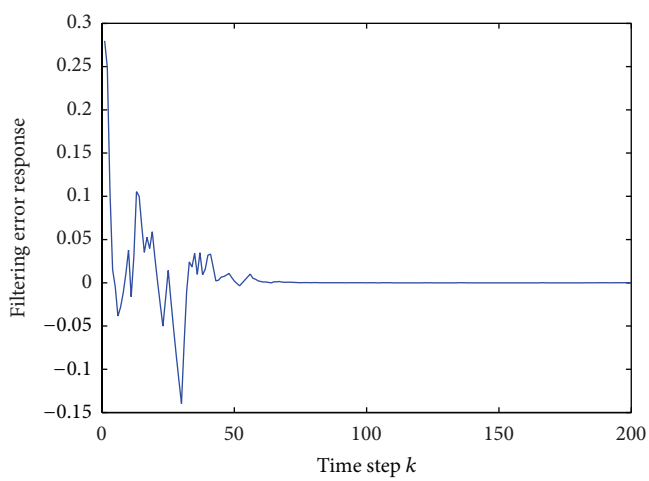

Figure 3: The filtering error response in terms of $\overline{z}_k = z_k - \widehat{z}_k$.

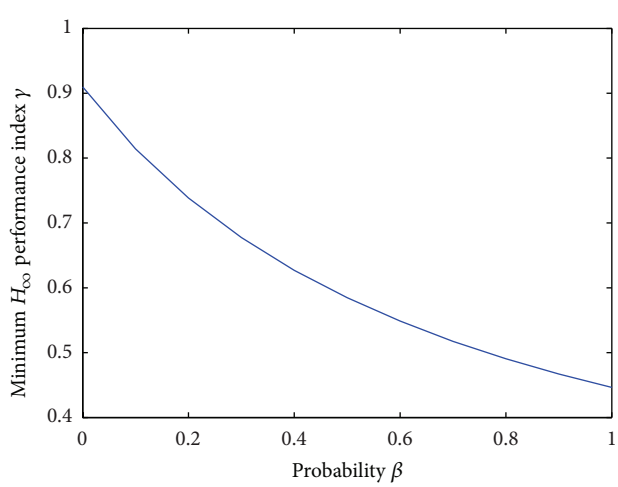

Figure 4: Minimum H_∞ performance index $\gamma_{\min}$ versus probability β.

6. Conclusion

In this paper, the analysis and design of robust H_∞ filtering for DMJLS with parametric uncertainties and missing measurements are investigated. The missing measurements are assumed to follow a Bernoulli distributed sequence. With a mode-dependent Lyapunov function, the H_∞ filter is designed and sufficient conditions are established to ensure the robustly stochastic stability and H_∞ attenuation level of the filtering error system. The numerical example implies the effectiveness and feasibility of the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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