Research Article

Rotation and Magnetic Field Effect on Surface Waves Propagation in an Elastic Layer Lying over a Generalized Thermoelastic Diffusive Half-Space with Imperfect Boundary

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The aim of the present investigation is to study the effects of magnetic field, relaxation times, and rotation on the propagation of surface waves with imperfect boundary. The propagation between an isotropic elastic layer of finite thickness and a homogenous isotropic thermodiffusive elastic half-space with rotation in the context of Green-Lindsay (GL) model is studied. The secular equation for surface waves in compact form is derived after developing the mathematical model. The phase velocity and attenuation coefficient are obtained for stiffness, and then deduced for normal stiffness, tangential stiffness and welded contact. The amplitudes of displacements, temperature, and concentration are computed analytically at the free plane boundary. Some special cases are illustrated and compared with previous results obtained by other authors. The effects of rotation, magnetic field, and relaxation times on the speed, attenuation coefficient, and the amplitudes of displacements, temperature, and concentration are displayed graphically.

1. Introduction

The foundations of magnetoelasticity were presented by Knopoff [1] and Chadwick [2] and developed by Kaliski and Petykiewicz [3]. An increasing attention is devoted to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics, and related topics. All papers quoted above assumed that the interactions between the two fields take place by means of the Lorentz forces appearing in the equations of motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field. The most ideal interface model, as it is known, is called perfect bond interface where the displacement and traction are continuous across the interface. However, interfaces are seldom perfect. Therefore, various imperfect models such as three-phase and linear models like spring models have been

introduced by Yu et al. [4], Yu [5], and Benveniste [6]. Perhaps the most frequently studied imperfect interface model is the smooth bond interface, where the normal components of the displacements and traction are continuous across the interface, while the shear traction components are zero on the interface. Lord and Shulman [7] formulated a generalized theory of thermoelasticity with one thermal relaxation time, who obtained a wave equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [8] developed a temperature rate-dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. Hetnarski and Ignaczak [9] introduced a review and presentation of generalized theories of thermoelasticity. Diffusion can be defined as the random walk of an assemble of particles from regions of high concentration to that of low concentration. Nowadays, there is a great deal of interest in

the study of phenomena due to its applications in geophysics and electronic industry. In integrated circuit fabrication, diffusion is used to introduce “depants” in controlled amounts into semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, and the source/drain in metal oxide semiconductor (MOS) transistors and polysilicon gates in MOS transistors. In most of the applications, the concentration is calculated using Fick’s law. This is simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which introduced. Study of the diffusion phenomenon is used to improve the conditions of oil extractions. These days’ oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits. Until recently, thermodiffusion in solids, especially in metals, was considered as a quantity that is independent of body deformation. Practice, however, indicates that the process of thermodiffusion could have a very considerable influence on the deformation of the body. Thermodiffusion in elastic solid is due to the coupling of temperature, mass diffusion, and strain in addition to the exchange of heat and mass with the environment. Nowacki [10–13] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. This implies infinite speed of propagation of thermoelastic waves. Olesiak and Pyryev [14] investigated the theory of thermoelastic diffusion and coupled quasistationary problems of thermal diffusion for an elastic layer. They studied the influence of cross effects arising from the coupling of the fields of temperature, mass diffusion, and strain due to which the thermal excitation results in additional mass concentration and generates additional fields of temperature. Sherief et al. [15] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves.

Recently, Sherief and Saleh [16] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [17] discussed the reflection phenomena of waves from free surface of a thermoelastic diffusion with one relaxation time and with two relaxation times in [18]. Aouadi [19–21] investigated different problems in thermoelastic diffusion. Sharma and Walia [22, 23] discussed the effect of rotation on Rayleigh waves in the piezothermoelastic half-space. Kumar and Kansal [24] discussed the propagation of Rayleigh waves on free surface in transversely isotropic thermoelastic diffusion. Kumar and Kansal [25] derived the basic equations for generalized thermoelastic diffusion and discussed the Lamb waves. Dawn and Chakraborty [26] studied Rayleigh waves in Green-Lindsay’s model of generalized thermoelastic media. Kumar and Chawla [27] investigated the effect of rotation and stiffness on surface waves propagation in an elastic layer lying over a generalized thermodiffusive elastic half-space with imperfect boundary. New contributions on waves propagation in thermoelastic media have been discussed [28–32].

In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The linear model is simplified and idealized situation of imperfectly bonded

interface, where the discontinuities in displacements at interfaces have a linear relationship with the interface stresses. Taking these applications into account, the surface waves propagation at imperfect boundary between an isotropic elastic layer and isotropic thermodiffusive elastic half-space with rotation in the context of Green-Lindsay theory is investigated. The phase velocity and attenuation coefficients of wave propagation have been computed from the secular equations. The amplitudes of displacements, temperature, concentration, and specific loss are computed and depicted graphically to make clear the influence of magnetic field, rotation, stiffness, relaxation times, and diffusion on the phenomena and compare with the practical results.

2. Basic Equations

The basic governing equations for homogenous generalized thermodiffusive solid in the absence of heat and mass diffusion sources are as follows (Singh [17]).

(i) Constitutive relations are

$$\sigma_{ji} = 2\mu\epsilon_{ji} + [\lambda\epsilon_{kk} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau'\dot{C})]\delta_{ji}. \quad (1)$$

(ii) Equation of motion in the rotating frame of reference is

$$\begin{aligned} \mu u_{i,jj} + (\lambda + \mu) u_{j,ji} - \beta_1(T + \tau_1\dot{T})_{,i} \\ - \beta_2(C + \tau'\dot{C})_{,i} + F_i \\ = \rho [\ddot{u} + \vec{\Omega} \times \vec{\Omega} \times \vec{u} + 2\vec{\Omega} \times \dot{\vec{u}}]_i, \end{aligned} \quad (2)$$

where $\vec{\Omega} \times \vec{\Omega} \times \vec{u}$ is the centripetal acceleration due to the time varying motion only and $2\vec{\Omega} \times \dot{\vec{u}}$ is the Coriolis acceleration:

$$\begin{aligned} \vec{\Omega} \times \vec{\Omega} \times \vec{u} &= (-\Omega^2 u, 0, -\Omega^2 w) = -\Omega^2 (u, 0, w) \\ 2\vec{\Omega} \times \dot{\vec{u}} &= 2(\Omega \dot{w}, 0, -\Omega \dot{u}) = 2\Omega (\dot{w}, 0, -\dot{u}), \end{aligned} \quad (3)$$

where

$$\vec{F} = \vec{J} \times \vec{B}. \quad (4)$$

Consider that the medium is a perfect electric conductor; we take the linearized Maxwell equations governing the electromagnetic field, taking into account absence of the displacement current (SI):

$$\begin{aligned} \text{curl } \vec{h} &= \vec{J}, & \text{curl } \vec{E} &= -\mu_e \frac{\partial \vec{h}}{\partial t}, \\ \text{div } \vec{h} &= 0, & \text{div } \vec{E} &= 0, \end{aligned} \quad (5)$$

where

$$\vec{h} = \text{curl}(\vec{u} \times \vec{H}_0), \quad (6)$$

FIGURE 1: Schematic of the problem.

where we have used

$$\vec{H} = \vec{H}_0 + \vec{h}(x, z, t), \quad \vec{H}_0 = (0, H, 0) \quad (7)$$

$$\vec{B} = \mu_e \vec{H} = \mu_e H \vec{j}; \quad (8)$$

then

$$\begin{aligned} F_x &= BJ_z = [\mu_e H] H \left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_1 \partial x_3} \right) \\ &= \mu_e H^2 \left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_1 \partial x_3} \right), \\ F_z &= BJ_x = [\mu_e H] H \left(\frac{\partial^2 u}{\partial x_3 \partial x_1} + \frac{\partial^2 w}{\partial x_3^2} \right) \\ &= \mu_e H^2 \left(\frac{\partial^2 u}{\partial x_3 \partial x_1} + \frac{\partial^2 w}{\partial x_3^2} \right). \end{aligned} \quad (9)$$

(iii) Equation of heat conduction is

$$\rho c_E (\dot{T} + \tau_0 \ddot{T}) + \beta_1 T_0 \dot{\epsilon}_{kk} + a T_0 (\dot{C} + \tau_0 \ddot{C}) = K T_{,ii}. \quad (10)$$

(iv) Equation of mass diffusion is

$$D \beta_2 \epsilon_{kk,ii} + Da \left(T + \tau_1 \frac{\partial T}{\partial t} \right)_{,ii} + \dot{C} - Db \left(C + \tau' \frac{\partial C}{\partial t} \right)_{,ii} = 0. \quad (11)$$

Here, the medium is rotating with angular velocity $\Omega = \Omega \underline{n}$, where $\underline{n}$ is the unit vector along the axis of rotation and this equation of motion includes two additional terms, namely,

- (i) the centripetal acceleration $\vec{\Omega} \times \vec{\Omega} \times \vec{u}$ due to time-varying motion,
- (ii) the Coriolis acceleration $2\vec{\Omega} \times \dot{\vec{u}}$,

where $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$, λ and μ are Lamé's constants, α_t is the coefficient of linear thermal expansion, ρ and C_E are, respectively, the density and specific heat at constant strain, a , b are, respectively, coefficient describing the measure of thermoelastic diffusion effects and of diffusion effects, T_0 is the reference temperature assumed to be such that $|T/T_0| \ll 1$, τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 > 0$ and τ_0^0, τ_1^0 are diffusion relaxation times with $\tau_1^0 \geq \tau_0^0 > 0$, and u_i are components of displacement vector. $T(x, y, z)$ is the temperature change and C is the concentration; $\sigma_{ij} (= \sigma_{ji})$, $\epsilon_{ij} = (u_{i,j} + u_{j,i})/2$ are, respectively, the components of stress and strain tensor.

The symbols correspond to partial derivative and time derivative, respectively.

Following Bullen [33], the equations of motion and constitutive relations in isotropic elastic medium are given by

$$(\lambda^e + \mu^e) u_{j,ij}^e + \mu^e u_{i,jj}^e + F_i^e = \rho^e \ddot{u}_i^e, \quad (12)$$

$$\sigma_{ji}^e = \lambda^e \theta^e \delta_{ij} + 2\mu^e \epsilon_{ij}^e, \quad i = j = 1, 2, 3, \quad (13)$$

where

$$\Theta^e = u_{k,k}^e, \quad (14)$$

$$\epsilon_{ij}^e = \frac{u_{i,j}^e + u_{j,i}^e}{2}, \quad i = j = 1, 2, 3, \quad (15)$$

and $u^e = (u_1^e, u_2^e, u_3^e)$ is the displacement vector, ρ^e is the density of the isotropic medium and λ^e , μ^e are Lamé's constants, $\sigma_{ji}^e (= \sigma_{ij}^e)$ are components of stress tensor, and δ_{ij} is the Kronecker delta.

3. Formulation of the Problem

As shown in Figure 1, we consider an isotropic elastic layer (Medium M_1) of thickness H overlaying a homogeneous, isotropic, generalized thermodiffusive elastic half-space in rotating frame of reference (Medium M_2). The origin of the coordinate system (x, y, z) is taken at any point on

the horizontal surface and x_2 -axis in the direction of wave propagation and x_3 -axis taking vertically downward into half-space, so that all particles on a line parallel to x_2 -axis are equally displaced. Therefore, all the field quantities will be independent of x_2 -axis coordinate. The interface between isotropic elastic layer and thermodiffusive elastic half-space with rotation has been taken at an imperfect boundary. The displacement vector u , temperature T , concentration C , and rotation for medium M_2 are taken as

$$\begin{aligned}\vec{u} &= (u_1, 0, u_3), T(x_1, x_3, t), C(x_1, x_3, t), \\ \Omega &= (0, \Omega, 0)\end{aligned}\quad (16)$$

and displacement vector $\vec{u}^e$ for the layer (Medium M_1) is taken as

$$\vec{u}^e = (u_1^e, 0, u_3^e). \quad (17)$$

We define the dimensionless quantities

$$\begin{aligned}x'_i &= \frac{\omega_1^* x_i}{v_1}, & u'_i &= \frac{\omega_1^* u_i}{v_1}, & t' &= \omega_1^* t, & \tau'_1 &= \omega_1^* \tau, \\ T' &= \frac{\beta_1 T}{\rho v_1^2}, & C' &= \frac{\beta_2 C}{\rho v_1^2}, & \tau'_0 &= \omega_1^* \tau_0, & \tau'^0 &= \omega_1^* \tau^0, \\ \tau'' &= \omega_1^* \tau', & \sigma'_{ij} &= \frac{\sigma_{ij}}{a_1 T_0}, & \Omega' &= \frac{\Omega}{\omega_1^*}, \\ K'_n &= \frac{v_1 k_n}{\beta_1 T_0 \omega_1^*}, & K'_1 &= \frac{v_1 k_1}{\beta_1 T_0 \omega_1^*}, \\ v_1^2 &= \frac{\lambda + 2\mu + \mu_e H^2}{\rho}, & \omega_1^* &= \frac{\rho c_E v_1^2}{k_1}.\end{aligned}\quad (18)$$

Upon introducing the quantities in (1)–(3), (8), and (14)–(16), after suppressing the primes, with the aid of (17) and (18), we obtain

$$\begin{aligned}u_{1,11} + \delta_2 u_{3,13} + \delta_1 u_{1,33} - \tau_\theta T_{,1} - \tau'_c C_{,1} \\ = [\ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3], \\ \delta_1 u_{3,11} + \delta_2 u_{1,31} + u_{3,33} - \tau_\theta T_{,3} - \tau'_c C_{,3} \\ = [\ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1], \\ \nabla^2 T = \tau_\theta^0 \dot{T} + \chi_1 \tau_f^0 \dot{C} + \chi_2 \dot{e} \\ q_1^* \nabla^2 e + q_2^* \tau_\theta' \nabla^2 T - q_3^* \tau_c' \nabla^2 + \dot{C} = 0,\end{aligned}\quad (19)$$

$$\begin{aligned}\frac{(u_{1,11}^e + u_{3,13}^e)}{\delta_3^2} + \frac{\nabla^2 u_1^e}{\delta_4^2} &= \ddot{u}_1^e, \\ \frac{(u_{1,13}^e + u_{3,33}^e)}{\delta_3^2} + \frac{\nabla^2 u_3^e}{\delta_4^2} &= \ddot{u}_3^e,\end{aligned}\quad (20)$$

where

$$\begin{aligned}\delta_1 &= \frac{\mu}{(\lambda + 2\mu + \mu_e H)}, & \delta_2 &= \frac{\lambda + \mu + \mu_e H}{(\lambda + 2\mu + \mu_e H)}, \\ \chi_1 &= \frac{a T_0 v_1^2 \beta_1}{\omega_1^* k \beta_2}, & \chi_2 &= \frac{T_0 \beta_1^2}{\rho k \omega_1^*}, \\ q_1^* &= \frac{D \omega_1^* \beta_2^2}{\rho v_1^2}, & q_2^* &= \frac{D \omega_1^* b}{v_1^2}, \\ q_3^* &= \frac{D \omega_1^* \beta_2 a}{\beta_1 v_1^2}, & \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \\ e &= \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, & \delta_4^2 &= \frac{\rho^e v_1^2}{\mu^e}, \\ \delta_3^2 &= \frac{\rho^e v_1^2}{\mu_e H^2 + \lambda^e + \mu^e}, & \delta_5^2 &= \frac{\rho^e v_1^2}{\lambda^e + 2\mu^e \mu_e H^2}, \\ \tau'_\theta &= 1 + \tau_1 \frac{\partial}{\partial t}, & \tau_\theta^0 &= 1 + \tau_0 \frac{\partial}{\partial t}, \\ \tau'_c &= 1 + \tau' \frac{\partial}{\partial t}, & \tau_f^0 &= 1 + \tau^0 \frac{\partial}{\partial t}, \\ R_H^2 &= \frac{\mu_e H}{\lambda + 2\mu}.\end{aligned}\quad (21)$$

For an isotropic elastic layer, we introduce potential functions Φ and Ψ through the relations

$$u_1^e = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3}, \quad u_3^e = \frac{\partial \Phi}{\partial x_3} + \frac{\partial \Psi}{\partial x_1}. \quad (22)$$

Substituting from (22) into (20), we have

$$\Phi_{,11} + \Phi_{,33} - \delta_5^2 \ddot{\Phi} = 0, \quad (23)$$

$$\Psi_{,11} + \Psi_{,33} - \delta_4^2 \ddot{\Psi} = 0. \quad (24)$$

4. Solution of the Problem

To solve (19), (22), and (23), we assume the solution in the form

$$\begin{aligned}(u_1, u_2, T, C, \Phi, \Psi) \\ = (1, W, S, R, P_1, P_2) U \exp [i\xi (x_1 + m x_3 - ct)],\end{aligned}\quad (25)$$

where $c = \omega/\xi$ is the nondimensional phase velocity, ω is the frequency, m is still parameter 1, and W , S , and R are, respectively, the amplitude ratio of u_1 , u_3 , T , and C with

respect to u_1 . Substituting the values of u_1 , u_3 , T , and C from (25) in (19), we obtain

$$\begin{aligned}
 & 1 + \delta_1 m^2 - c^2 (1 + \Lambda^2) + (\delta_2 m - 2i\Lambda c^2) W \\
 & + i c \omega^{-1} (\tau_{11} S + \tau_{21} R) = 0, \\
 & (\delta_2 m - 2\Lambda i c^2) + (\delta_1 + m^2 - c^2 (1 + \Lambda^2)) W \\
 & + i c \omega^{-1} m (\tau_{11} S + \tau_{21} R) = 0, \\
 & \chi_2 C + \chi_2 C m W - i \omega^{-1} c^2 \chi_1 \tau_{20} R \\
 & - i \omega^{-1} (i \omega (1 + m^2) + c^2 \tau_{10}) S = 0, \\
 & q_1^* (1 + m^2) + q_1^* (1 + m^2) m \omega - i q_2^* (1 + m^2) \tau_{11} c \omega^{-1} S \\
 & + [i q_3^* (1 + m^2) \tau_{21} c \omega^{-1} + c^3 \omega^{-2}] R = 0,
 \end{aligned} \tag{26}$$

where

$$\begin{aligned}
 \Lambda &= \Omega \omega^{-1}, \quad \tau_{11} = 1 - i \omega \tau_1, \quad \tau_{21} = 1 - i \omega \tau_1', \\
 \tau_{10} &= 1 - i \omega \tau_0, \quad \tau_{20} = 1 - i \omega \tau_0'.
 \end{aligned} \tag{27}$$

The system of (26) has a nontrivial solution if the determinant of the coefficient $[1, W, S, R]^T$ vanishes, which yield to the following polynomial characteristic equation:

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0. \tag{28}$$

The constants A^* , B^* , C^* , and D^* are given in Appendix A. The characteristic equation (28) is biquadratic in m^2 and hence possesses four roots, m_p^2 ; $p = 1, 2, 3, 4$ corresponding to four roots; there exist three types of quasi-longitudinal waves and one quasitransverse wave. The formal expression for displacement, temperature, and concentration satisfying the radiation condition $\text{Re}(m_p) \geq 0$ can be written as

$$(u_1, u_3, T, C) = \sum_{p=1}^4 (1, n_{1p}, n_{2p}, n_{3p}) \exp [i \xi (x + i m_p z - ct)]. \tag{29}$$

Substituting the values of Φ and Ψ from (25) in (23) and (24) and with the aid of (22), we obtain

$$\begin{aligned}
 u_1^e &= i \xi (B_1 c_5 + B_2 s_5) + \xi m_6 (D_1 s_6 - D_2 c_6) \exp i \xi (x - ct), \\
 u_3^e &= \xi m_5 (B_2 c_5 - B_1 s_5) + i \xi (D_1 c_6 + D_2 s_6) \exp i \xi (x - ct),
 \end{aligned} \tag{30}$$

where

$$\begin{aligned}
 m_5 &= \sqrt{c^2 \delta_5^2 - 1}, \quad m_6 = \sqrt{c^2 \delta_4^2 - 1}, \\
 c_5 &= \cos(\xi m_5 x_3), \quad c_6 = \cos(\xi m_5 x_3), \\
 s_5 &= \sin(\xi m_5 x_3), \quad s_6 = \sin(\xi m_6 x_3),
 \end{aligned} \tag{31}$$

where A_p ($p = 1, 2, 3, 4$), B_1, B_2, D_1, D_2 are arbitrary constants. The coupling constants n_{1p}, n_{2p}, n_{3p} ($p = 1, 2, 3, 4$) are given in Appendix B.

5. Boundary Conditions

In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The boundary conditions are the vanishing of the normal stress; Maxwell's electromagnetic stress tensor τ_{ij} is given by

$$\tau_{ij} = \mu_e [H_i h_j + H_j h_i - H_k h_k \delta_{ij}] \tag{32}$$

and tangential stress at free surface. The discontinuities in displacements have linear relations with stresses, continuity of normal, Maxwell's electromagnetic stress tensor and tangential stress, vanishing of the gradient of temperature, and concentration at the interface between the isotropic elastic layer and isotropic thermodiffusive elastic half-space. Mathematically, these can be written as follows.

(i) Mechanical conditions:

$$\begin{aligned}
 (\sigma_{33}^e)_{M_1} + \tau_{33}^e &= 0 \quad (\sigma_{31}^e)_{M_1} + \tau_{31}^e = 0, \\
 x_3 &= -H, \\
 (\sigma_{33}^e)_{M_1} + \tau_{33}^e &= (\sigma_{33}^e)_{M_2} + \tau_{33} \\
 (\sigma_{31}^e)_{M_1} + \tau_{31}^e &= (\sigma_{31}^e)_{M_2} + \tau_{31} \\
 (\sigma_{33}^e)_{M_1} + \tau_{33}^e &= k_n [(u_3)_{M_2} - (u_3)_{M_1}] \\
 (\sigma_{31}^e)_{M_1} + \tau_{31}^e &= k_t [(u_3)_{M_2} - (u_3)_{M_1}]; \\
 x_3 &= 0.
 \end{aligned} \tag{33}$$

(ii) Temperature condition

$$\frac{\partial T}{\partial x_3} = 0, \quad x_3 = 0. \tag{34}$$

(iii) Concentration condition

$$\frac{\partial C}{\partial x_3} = 0, \quad x_3 = 0, \tag{35}$$

where k_n and k_t are the normal and transverse stiffness of layer which have dimension Nm^{-3} .

6. Derivation of the Secular Equations

Substituting the value of $u_1, u_3, T, C, u_1^e, u_3^e$ from (29)-(30) into (33)-(35), with the aid of (1), (10) and (13)-(14), after simplification we obtain

$$b_5 \tan(\xi m_5 H) \Delta_1 + b_5 \Delta_2 + \frac{b_6 s_6}{c_5} \Delta_3 - \frac{b_6 c_6}{c_5} \Delta_4 = 0, \tag{36}$$

where $\Delta_1 = |R_{ij}|_{7 \times 7}$ the entries R_{ij} of the determinant are given in Appendix C and Δ_2 is obtained by replacing the first column of Δ_1 by $[R_{11}^* \ 0 \ 0 \ R_{41}^* \ R_{51}^* \ R_{61}^* \ R_{71}^*]^T$, Δ_3 is obtained by replacing the second column of Δ_2 by $[R_{12}^* \ 0 \ 0 \ R_{42}^* \ R_{52}^* \ R_{62}^* \ R_{72}^*]^T$, and Δ_4 is obtained by replacing

the third column of Δ_3 by $[R_{13}^* \ 0 \ 0 \ R_{43}^* \ R_{53}^* \ R_{63}^* \ R_{73}^*]^T$. The entries of Δ_p ($p = 1, 2, 3, 4$) are given in Appendix C.

If we write

$$c^{-1} = v^{-1} + i\omega^{-1}G, \quad (37)$$

then $\xi = F + iG$ where $F = \omega/v$ and G are real numbers. Also the roots of characteristic equations are in general complex.

Hence, assume that $m_p = p_p + iq_p$ so that exponent in the plane wave solutions in (25) becomes

$$iF(x_1 - m_p^A x_3 - vt) - F\left(\frac{G}{F}x_1 + m_p^R x_3\right), \quad (38)$$

where

$$m_p^A = \frac{G}{F}p_p + q_p, \quad m_p^R = p_p - i\frac{G}{F}q_p. \quad (39)$$

This shows that v is the propagation velocity and G is the attenuation coefficient of the wave. Upon using the representation (37) in secular equation (36), the values of propagation speed v and attenuation coefficient G of wave propagation can be obtained.

7. Particular Cases

(i) *Normal Stiffness*. In this case, $k_n \neq 0$, $k_t \rightarrow \infty$, and the secular equation (36) remains the same. But the following will be replaced in the values of Δ_p ($p = 1, 2, 3, 4$):

$$\begin{aligned} R_{71} &= \frac{-i\xi}{\beta_1 T_0}, & R_{72} &= 0, & R_{73} &= \frac{\xi m_6}{\beta_1 T_0}, \\ R_{74} &= R_{75} = R_{76} = R_{77} = 1, \\ R_{71}^* &= 0, & R_{72}^* &= -i\xi, & R_{73}^* &= 0. \end{aligned} \quad (40)$$

(ii) *Tangential Stiffness*. In this case $k_t \neq 0$, $k_n \rightarrow \infty$, and the secular equation (36) remains the same with the change of values of Δ_p ($p = 1, 2, 3, 4$) by taking

$$\begin{aligned} R_{61} &= -\frac{i\xi}{\beta_1 T_0}, & R_{62} &= 0, & R_{63} &= \xi m_6 \frac{1}{\beta_1 T_0}, \\ R_{64} &= n_{12}, & R_{65} &= n_{11}, & R_{66} &= n_{13}, & R_{67} &= n_{14}, \\ R_{61}^* &= 0, & R_{62}^* &= -\xi m_5 \frac{1}{\beta_1 T_0}, & R_{63}^* &= -i\xi \frac{1}{\beta_1 T_0}. \end{aligned} \quad (41)$$

(iii) *Welded Contact*. In this case, $k_n \rightarrow \infty$, $k_t \rightarrow \infty$, and the secular equation (36) remains the same, but the value of Δ_p ($p = 1, 2, 3, 4$) is given by replacing

$$\begin{aligned} R_{61} &= -i\xi \frac{1}{\beta_1 T_0}, & R_{62} &= 0, & R_{63} &= \xi m_6 \frac{1}{\beta_1 T_0}, \\ R_{64} &= n_{12}, & R_{65} &= n_{11}, & R_{66} &= n_{13}, & R_{67} &= n_{14}, \end{aligned}$$

$$R_{61}^* = 0, \quad R_{72}^* = -\xi m_5 \frac{1}{\beta_1 T_0}, \quad R_{63}^* = -i\xi \frac{1}{\beta_1 T_0},$$

$$R_{61} = -i\xi \frac{1}{\beta_1 T_0}, \quad R_{62} = 0, \quad R_{63} = \xi m_6 \frac{1}{\beta_1 T_0}$$

$$R_{64} = n_{12}, \quad R_{65} = n_{11}, \quad R_{66} = n_{13}, \quad R_{67} = n_{14},$$

$$R_{61}^* = 0, \quad R_{72}^* = -\xi m_5 \frac{1}{\beta_1 T_0}, \quad R_{63}^* = -i\xi \frac{1}{\beta_1 T_0}. \quad (42)$$

8. Special Cases

Case (i). If we take $\Omega = 0$, that is, in the absence of rotation effect, the frequency equation (36) will reduce to the frequency equation for an isotropic elastic layer and a homogenous isotropic thermodiffusive elastic half-space without rotation.

Case (ii). If we take $H = 0$, $\Omega = 0$, $\tau' > 0$, and $\tau_1 > 0$ and in the absence of diffusion effect, that is, $b_1 = b_3 = a = b = 0$, (36) will reduce to the frequency equation for Rayleigh wave

$$\begin{aligned} \left(2 - \frac{c^2}{\delta_1^2}\right)^2 (\bar{m}_1^2 + \bar{m}_2^2 + \bar{m}_1 \bar{m}_2 - 1 + c^2) \\ - 4\bar{m}_1 \bar{m}_2 \bar{m}_3 (\bar{m}_1 + \bar{m}_2) = 0. \end{aligned} \quad (43)$$

Here, $m_3 = \sqrt{1 - c^2/\delta_1^2}$ and $\bar{m}_p$ ($p = 1, 2$) are the roots of (28), obtained by taking $b_1 = b_3 = a = b = 0$ and $\tau' > 0$, $\tau_1 > 0$. The resulting equation (43) is similar to (22) as given by Dawn and Chakraborty [26].

Case (iii). In the absence of isotropic elastic layer and thermal and diffusion effects, we obtain the frequency equation corresponding to isotropic elastic half-space by changing the dimensionless quantities into physical quantities as

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4 \left(1 - \frac{c^2}{c_1^2}\right)^{1/2} \left(1 - \frac{c^2}{c_2^2}\right)^{1/2}, \quad (44)$$

where $c_1^2 = (\lambda + \mu + \mu^e)/\rho$, $c_2^2 = \mu/\rho$.

The frequency equation (36) is the same as derived in Ewing et al. [34].

9. Surface Displacements, Temperature Change, and Concentration

The amplitude of surface displacements, temperature change, and concentration at the surface $z = 0$ during Rayleigh wave propagation in the cases of stress free, vanishing of

the gradient of temperature, and concentration of the half-space are

$$\begin{aligned} u_1 &= \bar{E}A \exp[iF(x_1 - vt)], \\ u_3 &= \bar{R}A \exp[iF(x_1 - vt)], \\ T &= \bar{W}A \exp[iF(x_1 - vt)], \\ C &= \bar{L}A \exp[iF(x_1 - vt)], \end{aligned} \quad (45)$$

where

$$\begin{aligned} A &= A_1 \exp(-Fx_1), \\ \bar{E} &= \frac{F_1^* - F_2^* + F_3^* - F_4^*}{F_1^*}, \\ \bar{R} &= \frac{(n_{11}F_1^* - n_{12}F_2^* + n_{13}F_3^* - n_{14}F_4^*)}{F_1^*}, \\ \bar{W} &= \frac{(n_{21}F_1^* - n_{22}F_2^* + n_{23}F_3^* - n_{24}F_4^*)}{F_1^*}, \\ \bar{L} &= \frac{(n_{31}F_1^* - n_{32}F_2^* + n_{33}F_3^* - n_{34}F_4^*)}{F_1^*} \end{aligned} \quad (46)$$

and F_p^* ($p = 1, 2, 3, 4$) are given in Appendix D.

10. Specific Loss

The specific loss is the ratio of energy (Δw) dissipated in taking a specimen through a stress cycle, to the elastic energy (w) stored in the specimen when the strain is maximum. The specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, Kolsky [35] shows that the specific loss $\Delta w/w$ equals 4π times the absolute value of imaginary part of ξ to the real part of ξ ; that is,

$$\begin{aligned} \frac{\Delta w}{w} &= 4\pi \left| \frac{\text{Im}(\xi)}{\text{Re}(\xi)} \right| = 4\pi \left| \frac{iG}{F} \right| = 4\pi \left| \frac{iG}{\omega/\nu} \right| \\ &= 4\pi \left| \frac{i\nu G}{\omega} \right| = 4\pi \left| \frac{1}{F} (iG) \right|. \end{aligned} \quad (47)$$

11. Numerical Results and Discussion

Following Sherief and Saleh [16], we take the following values of relevant parameter for the copper material:

$$\begin{aligned} \lambda &= 7.76 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, \\ T_0 &= 0.293 \times 10^3 \text{ K} \quad \rho = 8.954 \times 10^3 \text{ kg m}^{-3}, \\ a &= 1.2 \times 10^4 \text{ m}^2 \text{ s}^{-2} \text{ k}^{-1}, \quad b = 9 \times 10^5 \text{ m}^5 \text{ s}^{-2} \text{ g k}^{-1} \end{aligned}$$

FIGURE 2: Variation of concentration with respect to the wave number with variation of rotation.

$$\begin{aligned} c_E &= 3831 \times 10^3 \text{ J Kg}^{-1} \text{ K}^{-1}, \quad \alpha_t = 1.78 \times 10^{-5} \text{ k}^{-1}, \\ \alpha_c &= 1.98 \times 10^{-4} \text{ k}^{-1} \text{ m}^3 \text{ g} \quad K = 0.383 \times 10^3 \text{ W m}^{-1} \text{ k}^{-1}, \\ D &= 0.85 \times 10^{-8} \text{ Sm}^{-3} \text{ g}^{-1}, \quad \tau_0 = 0.07 \text{ s}, \\ \tau^1 &= 0.03, \quad \tau^0 = 0.04, \quad \tau_1 = 0.07 \text{ s}. \end{aligned} \quad (48)$$

The elastic constants for granite are given by Bullen [33]:

$$\begin{aligned} \lambda^e &= \mu^e = 2.238 \times 10^3 \text{ J} \cdot \text{Kg m}^{-1} \text{ K}^{-1}, \\ \rho^e &= 2.65 \times 10^3 \text{ J} \cdot \text{kg m}^{-3}. \end{aligned} \quad (49)$$

The concentration change, phase velocity and attenuation coefficient of wave propagation, displacement, stresses, temperature in the context of Green Lindsay (GL) theory of thermoelastic diffusion with variations of magnetic field, and rotation in 2D and 3D have been computed for various values of nondimensional wave number and calculated numerically and represented graphically in Figures 2–22.

Figure 2 displays the variation of concentration change with different values of rotation with respect to the wave number; it appears that the concentration change increases with an increasing of the wave number and if there is no rotation and small wave number takes larger values than in the presence of rotation and large values of the wave number. Figures 3 and 4 clear the phase velocity and attenuation coefficients variations with respect to the wave number with variation of the rotation; it is concluded that the phase velocity begins from 1.2 but the attenuation coefficient begins from zero if the wave number equals zero and tends to zero if the wave number tends to infinity; also, it is seen that they increase and decrease periodically with an increasing of the wave number and rotation that indicate the interruption of the phase velocity and attenuation coefficient with the largest values of the wave number and rotation that agree

FIGURE 3: Variation of phase velocity with respect to the wave number with variation of rotation.

FIGURE 4: Variation of attenuation coefficient with respect to the wave number with variation of rotation.

with the practical results. Figure 5 shows the variation of the displacement component with respect to the wave number with variation of the rotation; it is concluded that it takes various values for zeros value of the wave number and tends to zero if the wave number tends to infinity; also, it seems that it increases and decreases periodically with an increasing of the wave number and rotation.

Figures 6 and 7 make clear the variation of the stresses components with respect to the wave number with variation of the rotation; it appears that it begins from zero for zeros value of the wave number and tends to zero if the wave number tends to infinity; also, it is shown that it increases and decreases periodically with an increasing of the wave number and rotation. Figure 8 displays the variation of the temperature with various values of the wave number and rotation; one can see that it begins from zero for zeros wave

FIGURE 5: Variation of the displacement u with respect to the wave number with variation of rotation.

FIGURE 6: Variation of normal stress σ_{xx} with respect to the wave number with variation of rotation.

FIGURE 7: Variation of shear stress σ_{xy} with respect to the wave number with variation of rotation.

FIGURE 8: Variation of the temperature θ with respect to the wave number with variation of rotation.

FIGURE 9: Variation of the concentration change with respect to the wave number with variation of the magnetic field.

number and tends to zero, decreases and increases tends to zero if the wave number tends to infinity; it is clear that the temperature decreases with increasing rotation values that physically indicates the negative influence of rotation on the temperature that takes into consideration engineering and structures. Also, it is seen that, in the absence of rotation, all values of the concentration change, phase velocity, attenuation coefficient, displacement, stresses components, and temperature take a smooth behavior comparing with the corresponding values in the presence of rotation that take harmonic behavior.

Figure 9 shows the variation of concentration change with various values of magnetic field with respect to the wave number; it appears that the concentration change increases with an increasing of the wave number and decreases and arrives to the unity if the wave number tends to infinity; if the magnetic field is absent, the concentration change smallest

FIGURE 10: Variation of phase velocity with respect to the wave number with variation of the magnetic field.

FIGURE 11: Variation of the attenuation coefficient with respect to the wave number with variation of the magnetic field.

than the corresponding values for small values of the wave number and inverses to the largest values for the large values of the wave number. Figures 10 and 11 make obvious the phase velocity and attenuation coefficients variations with respect to the wave number with variation of the magnetic field; it is concluded that the phase velocity begins from 1.2 but the attenuation coefficient begins from zero if the wave number equals zero and tends to zero if the wave number tends to infinity; also, it appears that they increase and decrease periodically with an increasing of the wave number and magnetic field that indicate the interruption of the phase velocity and attenuation coefficient with the largest values of the wave number and magnetic field that agree with the practical results. From Figure 12, it is concluded that the displacement component takes various values for zeros value of the wave number and tends to zero if the wave

FIGURE 12: Variation of the displacement u with respect to the wave number with variation of magnetic field.

FIGURE 13: Variation of shear stress σ_{xx} with respect to the wave number with variation of the magnetic field.

number tends to infinity; also, it is shown that it increases and decreases with an increasing of the wave number and magnetic field.

Figures 13 and 14 display the variation of the normal and shear stresses with respect to the wave number with variation of the magnetic field; it appears that it begins from zero for zeros value of the wave number and tends to zero if the wave number tends to infinity; also, it is shown that the normal stress decreases and then increases arriving to zero with the large values of the wave number and magnetic field. It is seen that the shear stress increases and periodically decreases with the variation of the wave number and magnetic field and begins from zero at zeros value of the wave number and tends again to zero if the wave number tends to infinity. Figure 15 dispatches the variation of the temperature with various values of the wave number and magnetic field; we can show that it begins from zero for zeros wave number

FIGURE 14: Variation of shear stress σ_{xy} with respect to the wave number with variation of the magnetic field.

FIGURE 15: Variation of the temperature θ with respect to the wave number with variation of magnetic field.

FIGURE 16: Variation of the concentration charge with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 17: Variation of the phase velocity with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 20: Variation of the stress σ_{xx} with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 18: Variation of the attenuation coefficient with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 21: Variation of the stress σ_{xy} with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 19: Variation of displacement u with respect to the wave number and x -axis.

$$H_0 = 10^5 \text{ and } \Omega = 0.1$$

FIGURE 22: Variation of the temperature with respect to the wave number and x -axis.

and tends to zero for infinity wave number and increases and decreases with variation of the wave number and magnetic field.

Totally, it is clear that the temperature decreases with increasing rotation values that physically indicates the negative influence of rotation on the temperature that takes into consideration engineering and structures.

Figures 16–22 display the 3D variation on the concentration change, phase velocity, attenuation coefficient, displacement, stresses components, and temperature, respectively, with respect to the wave number and x -axis if $H_o = 10^5$ and $\Omega = 0.1$.

It is obvious from Figure 16 that the concentration change decreases to arrive to zero with the increased values of the wave number; it increases and decreases to its minimum value and after that increases with an increasing of x -axis. From Figures 17 and 18, it appears that the phase velocity begins from 1.2 but the attenuation coefficient begins from zero if the wave number equals zero and tends to zero if the wave number tends to infinity; also, it appears that it increases and decreases periodically and arrives to zero with an increasing of x -axis and the wave number. Figure 19 shows that the displacement decreases, increases periodically with the variation of the wave number, and tends to zero as the wave number tends to infinity. It appears that the displacement decreases and increases with the increased values of x -axis.

From Figure 20, we concluded that the normal stress decreases and then increases arriving to zero as the wave number tends to zero and increases with the small values of x -axis and after that decreases to tend to zero. Figure 21 shows that the shear stress component increases and decreases periodically with the variation of the wave number but increases and decreases with an increasing of x -axis.

Finally, it appears that the temperature increases and decreases arrive to zero as the wave number tends to infinity but decreases with an increasing of the small x -axis and after that increases.

12. Concluding Remarks

Surface waves at imperfect boundary between isotropic elastic layer of finite thickness and isotropic thermodiffusive elastic half-space with magnetic field, stiffness, and rotation with two thermal relaxation times (GL) model are illustrated. The secular equation in compact form has been derived. The concentration change, phase velocity, attenuation coefficient, displacement, stresses, and temperature are displayed graphically. The amplitudes of displacements, temperature, and concentration are computed at the free plane boundary and presented graphically. Specific loss of energy is obtained and depicted graphically.

The analysis to be carried will be useful in the design and construction of rotating sensors, engineering, structures, and surface acoustic waves devices and the following remarks have been concluded.

- (1) If there is no rotation, small wave number takes larger values than in the presence of rotation and large values of the wave number.

- (2) The phase velocity, attenuation coefficient, displacement, stresses components, and temperature begin from zero for zeros value of the wave number and tend to zero if the wave number tends to infinity; also, it is seen that they increase and decrease periodically with an increasing of the wave number and rotation and magnetic field.
- (3) The temperature decreases with increasing rotation values that physically indicates the negative influence of rotation on the temperature that takes into consideration engineering and structures and acoustic and rotating sensors.

Appendices

A. Constant of (28)

Consider the following:

$$\begin{aligned}
 h_1 &= c^2 (1 + \Lambda^2), & h_2 &= i\omega^{-1} c \tau_{11}, & h_3 &= i\omega^{-1} c \tau_{21}, \\
 h_5 &= \chi_2 c, & h_7 &= -i\omega^{-1} c^2 \chi_1 \tau_{10} \\
 l_1 &= 2i\Lambda c^2, & l_2 &= i\omega^{-1} c \tau_{11}, & l_3 &= -i\omega^{-1} c \tau_{21} q_3^*, \\
 l_4 &= \omega^{-2} c^3, & l_1^* &= 1 - i\omega^{-1} c^2 \tau_{20}, \\
 d_1 &= (l_1^* (l_4 - l_3) + h_7 l_2), & d_2 &= (l_4 + h_7 l_2 - l_3 l_1^* - l_3), \\
 d_3 &= (l_4 h_5 - h_7 q_1^* - h_5 l_3), \\
 d_4 &= (l_2 h_5 + l_1^* q_1^*), & d_5 &= (l_3 h_5 + h_7 q_1^*), \\
 d_6 &= (l_2 h_5 + l_1^* q_1^* + q_1^*), \\
 d_7 &= (l_3 (1 + R_H^2) + h_3 q_1^*), & d_8 &= (\delta_2 d_2 + h_2 d_5), \\
 d_9 &= (h_3 q_1^* + \delta_2) l_3, \\
 f_1 &= d_2 - (\delta_1 - h_1) l_3 + (h_2 d_5 - h_3 d_6) \delta_1, \\
 f_2 &= \delta_2 d_9, & f_3 &= (1 - h_1) d_7 - h_3 q_1^* - h_3 \delta_2 q_1^*, \\
 f_4 &= (d_1 + d_2 (\delta_1 - h_1) + h_2 d_3 - h_3 d_4) \delta_1, \\
 f_5 &= \delta_2^2 (d_8 - h_3 d_6), \\
 f_7 &= h_3 ((\delta_1 - h_1) q_1^* - d_6 + d_6 \delta_2), \\
 f_8 &= \delta_1 (\delta_1 - h_1) d_1, & f_9 &= (\delta_2 d_1 - h_2 d_3 - d_4 h_3) \delta_2, \\
 g_1 &= \delta_2 d_3 + (\delta_1 - h_1) d_5 - d_3, \\
 g_2 &= (\delta_1 - h_1) d_6 + d_4 - \delta_2 d_4, \\
 g_3 &= (1 - h_1) (\delta_1 - h_1), & g_4 &= l_1^2 d_1, \\
 g_5 &= h_2 (\delta_1 - h_1), & g_6 &= h_3 (\delta_1 - h_1), \\
 A^* &= \frac{-(f_1 + f_2 + f_3)}{\delta_1 d_7},
 \end{aligned}$$

$$\begin{aligned}
B^* &= \frac{-(1-h_1)f_1 + f_4\delta_1 - f_5 - f_7 - l_3l_1^2}{\delta_1 d_7}, \\
C^* &= -\frac{(1-h_1)f_4 - f_9 - h_2g_1 - h_3g_2 + f_8 + l_1^2 d_2}{\delta_1 d_7}, \\
D^* &= -\frac{[d_1g_3 + g_4 - d_3g_5 - d_4g_6]}{\delta_1 d_7}.
\end{aligned} \tag{A.1}$$

B. Constant of (29)

Consider the following:

$$\begin{aligned}
n_{1p} &= -\left[(\Lambda_1 + i\Lambda_2 m_p - \Lambda_3 m_p^2 - i\Lambda_4 m_p^3 \right. \\
&\quad \left. - \Lambda_5 m_p^4 - i\Lambda_6 m_p^5) \times (E_p^*)^{-1}\right], \\
n_{2p} &= \left[\frac{\Lambda_7 + \Lambda_8 m_p + \Lambda_9 m_p^2 - \Lambda_{10} m_p^3 + \Lambda_{11} m_p^4}{E_p^*}\right], \\
n_{3p} &= -\left[(\Lambda_{12} - \Lambda_{13} m_p + \Lambda_{14} m_p^2 \right. \\
&\quad \left. - \Lambda_{15} m_p^3 + \Lambda_{16} m_p^4 - \Lambda_{17} m_p^5 \right. \\
&\quad \left. + \Lambda_{18} m_p^6) \times (E_p^*)^{-1}\right]
\end{aligned} \tag{B.1}$$

$$E_p^* = G_1 - G_2 m_p^2 - G_3 m_p^6 + G m_p^8, \quad p = 1, 2, 3, 4,$$

where

$$\begin{aligned}
G_1 &= (\delta_1 - h_1) d_1, \\
G_2 &= (d_1 + (\delta_1 - h_1) d_2 - h_2 d_3 - h_3 d_4), \\
G_3 &= (d_2 - (\delta_1 - h_1) l_3 + h_2 d_5 - h_3 d_6), \\
G_4 &= d_7, \quad \Lambda_1 = l_1 d_1, \\
\Lambda_2 &= (\delta_2 + R_H^2) d_1 - h_2 d_3 - h_3 d_4, \\
\Lambda_3 &= l_1 d_2, \quad \Lambda_4 = d_8 - h_3 d_6, \\
\Lambda_5 &= -l_1 l_3, \quad \Lambda_6 = d_9, \\
\Lambda_7 &= -(\delta_1 - h_1) d_3, \quad \Lambda_8 = l_1 d_3, \\
\Lambda_9 &= (\delta_2 d_3 + (\delta_1 - h_1) d_5 - d_3), \\
\Lambda_{10} &= l_1 d_5, \quad \Lambda_{11} = d_5 - \delta_2 d_5, \\
\Lambda_{12} &= (\delta_1 - h_1) d_4, \quad \Lambda_{13} = l_1 d_4, \\
\Lambda_{14} &= d_4 + (\delta_1 - h_1) d_6 - \delta_2 d_4, \\
\Lambda_{15} &= l_1 d_6, \quad \Lambda_{16} = d_6 + (\delta_1 - h_1) q_1^* - \delta_2 d_6, \\
\Lambda_{17} &= q_1^* l_1, \quad \Lambda_{18} = q_1^* (1 - \delta_2).
\end{aligned} \tag{B.2}$$

C. Determinant Elements of (36)

Consider the following:

$$\begin{aligned}
R_{14} &= R_{15} = R_{16} = R_{17} = R_{21} = R_{22} = R_{23} = R_{31} \\
&= R_{32} = R_{33} = R_{42} = R_{51} = R_{53} = 0, \\
R_{11} &= w_8, \quad R_{12} = -y_7, \quad R_{13} = w_7, \\
R_{24} &= b_2, \quad R_{25} = b_1, \quad R_{26} = b_3, \quad R_{27} = b_4, \\
R_{34} &= q_2, \quad R_{35} = q_1, \quad R_{36} = q_3, \\
R_{37} &= q_4, \quad R_{41} = (b_5 + b_9), \quad R_{43} = -b_6, \\
R_{44} &= J_2, \quad R_{45} = J_1, \quad R_{46} = J_3, \\
R_{47} &= J_4, \quad R_{52} = b_7, \quad R_{54} = s_2^*, \\
R_{55} &= s_1^*, \quad R_{56} = s_3^*, \quad R_{57} = s_4^*, \\
R_{16} &= [b_5 + b_9], \quad R_{62} = -i\xi \frac{k_n}{\beta_1 T_0}, \quad R_{63} = -b_6, \\
R_{64} &= r_2, \quad R_{65} = r_1, \quad R_{66} = r_3, \\
R_{67} &= r_4, \quad R_{71} = -i\xi \frac{k_1}{\beta_1 T_0}, \quad R_{72} = b_7, \\
R_{73} &= \xi m_6 \frac{k_1}{\beta_1 T_0}, \quad R_{74} = k_1, \\
R_{75} &= k_1, \quad R_{76} = k_1, \quad R_{77} = k_1, \\
R_{11}^* &= y_8, \quad R_{41}^* = 0, \quad R_{51}^* = b_8, \\
R_{61}^* &= -\xi m_5 \frac{k_n}{\beta_1 T_0}, \quad R_{71}^* = -b_8, \quad R_{12}^* = w_8, \\
R_{52}^* &= 0, \quad R_{42}^* = (b_5 + b_9) = R_{62}^*, \quad R_{72}^* = -i\xi \frac{k_1}{\beta_1 T_0}, \\
R_{13}^* &= -y_7, \quad R_{43}^* = 0, \quad R_{53}^* = -b_7, \\
R_{63}^* &= -i\xi \frac{k_n}{\beta_1 T_0}, \quad R_{73}^* = b_7, \\
w_8 &= b_8 \sin(\xi m_5 H), \quad w_5 = b_5 \cos(\xi m_5 H), \\
w_7 &= b_7 \sin(\xi m_6 H), \quad w_6 = b_6 \sin(\xi m_6 H), \\
b_8 &= 2im_5 \Gamma_1^e, \quad b_7 = (1 - m_6^2) \Gamma_1^e, \\
b_5 &= \frac{((\lambda^e + \mu_e H_Y^2)(1 + m_5^2) + 2\mu_e^e m_5^2) \xi^2}{a_1 T_0}, \\
b_6 &= \frac{2i\mu_e^e m_6 \xi}{a_1 T_0}, \\
y_7 &= -b_7 \cos(\xi m_6 H), \quad y_5 = b_5 \sin(\xi m_5 H), \\
y_8 &= b_8 \cos(\xi m_5 H), \quad y_6 = b_6 \cos(\xi m_6 H),
\end{aligned}$$

$$\begin{aligned}
J_p &= i\Gamma_2\xi - \Gamma_3n_{2p} - \Gamma_4n_{3p} - \Gamma_5\xi n_{1p}m_p \\
&\quad + \frac{\mu_e H_Y^2}{\beta_1 T_0} i\xi (i - n_{1p}m_p), \\
b_p &= n_{2p}m_p, \quad q_p = n_{3p}m_p, \\
r_p &= k_n n_{1p}, \quad \Gamma_5 = \frac{\lambda + 2\mu}{a_1 T_0}, \quad \Gamma_2 = \frac{\lambda}{\beta_1 T_0}, \\
\Gamma_3 &= \frac{\rho v_1^2 \tau_{11}}{\beta_1 T_0}, \quad \Gamma_4 = \frac{\rho v_1^2 \tau_{21}}{\beta_1 T_0}, \\
\Gamma_1^e &= \frac{\mu^e}{\beta_1 T_0}, \quad \Gamma_1 = \frac{\mu}{\beta_1 T_0}, \\
s_p^* &= [i\xi n_{1p} - \xi m_p] \Gamma_1.
\end{aligned} \tag{C.1}$$

D. F_p^* ($P = 1, 2, 3, 4$) Given in (46)

Consider the following:

$$\begin{aligned}
F_1^* &= q_2 (J_3 s_4^* - J_4 s_3^*) - q_3 (J_2 s_4^* - J_4 s_2^*) + q_4 (J_2 s_3^* - J_3 s_2^*), \\
F_2^* &= q_1 (J_3 s_4^* - J_4 s_3^*) - q_3 (J_1 s_4^* - J_4 s_1^*) + q_4 (J_1 s_3^* - J_3 s_1^*), \\
F_3^* &= q_1 (J_2 s_4^* - J_4 s_2^*) - q_2 (J_1 s_4^* - J_1 s_2^*) + q_4 (J_2 s_1^* - J_1 s_2^*), \\
F_4^* &= q_1 (J_2 s_3^* - J_3 s_2^*) - q_2 (J_1 s_3^* - J_3 s_1^*) + q_3 (J_1 s_2^* - J_2 s_3^*).
\end{aligned} \tag{D.1}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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