Research Article

Prediction on Droplet Sauter Mean Diameter in Gas-Liquid Mist Flow Based on Droplet Fractal Theory

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We present a fractal model for droplet Sauter mean diameter in gas-liquid mist flow, based on the droplet fractal theory and the balance relationship between total droplet surface energy and total gas turbulent kinetic energy. The present model is expressed as functions of the droplet fractal dimension, gas superficial velocity, liquid superficial velocity, and other fluid characteristics. Agreement between the present model predictions and experimental measurements is obtained. Results verify the reliability of the present model.

1. Introduction

Droplet coalescence and breakup phenomena have obtained extensive attention in many physical and chemical process applications, that is, distillation, liquid-liquid extraction, emulsification, gas absorption, and multiphase reactions [1–3]. A lot of studies have shown that the tendency toward droplet coalescence or breakup depends on droplet size [4, 5]. Although a number of equations have been proposed to predict the effect of appropriate parameters on drop sizes, they are semiempirical [6–10].

Shavit and Chigier [11] firstly observed the droplet fractal characteristics in droplet separation and jet. Zhou et al. [12] measured the droplet fractal dimensions and studied the relationship between the fractal characteristics and droplet distribution. Zhou and Yu [13] proposed a multifractal model to describe droplet dissolution characteristics and verified this model using experimental result. Apte et al. [14] established a stochastic subgrid model for large-eddy simulation of atomizing spray. Gorokhovski and Saveliev [15] proposed a numerical method to describe stochastically droplet production and developed a droplet distribution model based on studying the breakup of droplets at the large Weber number. Liu et al. [16] introduced a finite stochastic breakup model for air-blast atomizers based on the self-similarity of droplet breakup theory.

2. Model

Azzopardi [17] and Andreussi et al. [18], however, found that the droplet diameter of liquid carried through gas flow was associated with the liquid rate of gas flow. Thus, droplet diameter could increase with the increase of liquid rate at a given gas flow rate. According to Taitel et al. [19], under bubble flow conditions, when the turbulent force of the liquid phase was strong enough to overcome gas-liquid interfacial tension, the turbulent force will reduce the gas phase into small gas bubbles. Similarly, in mist flow, the liquid phase is dispersed into spherical liquid droplets in gas flow. That is to say, there exist two types of mutual resistance imposed on liquid droplets: turbulent force, which breaks droplets, and surface tension, which keeps droplets intact (Figure 1).

It has been shown that the cumulative size distributions of liquid droplets whose sizes are greater than or equal to λ have been proven to follow the fractal scaling law [20, 21]:

$$N(L \geq \lambda) = \left(\frac{\lambda_{\max}}{\lambda} \right)^{D_f}, \quad (1)$$

where $\lambda_{\max}$ is the maximum droplet diameter. D_f is the droplet fractal dimension, $0 < D_f < 2$ in two-dimensional space, and $0 < D_f < 3$ in three-dimensional space.

FIGURE 1: Relationship between liquid droplet diameter and turbulent force.

Equation (1) can be approximately considered as a continuous and differentiable function. Solving the differential of (1), the liquid droplet number between λ and $\lambda + d\lambda$ can be obtained [22, 23]:

$$-dN = D_f \lambda_{\max}^{D_f} \lambda^{-(D_f+1)} d\lambda. \quad (2)$$

The negative variable in (2) indicates that liquid droplet number increases along with a decrease in droplet size [24], and $-dN > 0$.

Droplet Sauter mean diameter is defined as [25]

$$\lambda_{32} = \frac{-\int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^3 dN}{-\int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^2 dN}. \quad (3)$$

The numerator term of (3) can be expressed based on droplet fractal theory [26]:

$$-\int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^3 dN = \frac{D_f \lambda_{\max}^3}{(3 - D_f)} \left[1 - \left(\frac{\lambda_{\min}}{\lambda_{\max}} \right)^{3-D_f} \right]. \quad (4)$$

In gas-liquid mist flow real situation, $\lambda_{\min}/\lambda_{\max} < 10^{-6}$, and D_f is greater than 0.5, so $(\lambda_{\min}/\lambda_{\max})^{3-D_f} < 10^{-3}$, and (4) can be simplified as [27]

$$-\int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^3 dN = \frac{D_f \lambda_{\max}^3}{(3 - D_f)}. \quad (5)$$

Similarly, the denominator term of (3) can be expressed based on droplet fractal theory:

$$-\int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^2 dN = \frac{D_f \lambda_{\max}^2}{(2 - D_f)}. \quad (6)$$

Substituting (5) and (6) into (3), the relationship between the droplet Sauter mean diameter and the maximum droplet diameter in gas-liquid mist flow can be solved:

$$\lambda_{32} = \frac{(2 - D_f)}{(3 - D_f)} \lambda_{\max}. \quad (7)$$

According to the balance between the surface energy of the dispersed liquid droplets and the turbulent kinetic energy of the continuous gas flow, the Droplet Sauter mean diameter can be derived.

In gas-liquid mist flow, the relationship between liquid velocity, v_l , and liquid superficial velocity, v_{sl} , is [28]

$$v_{sl} = \frac{A_l}{A} v_l, \quad (8)$$

where

$$A = \frac{\pi D^2}{4}, \quad (9)$$

$$A_l = \frac{\pi D_f \lambda_{\max}^2}{4(2 - D_f)}.$$

The total free surface energy of dispersed droplets in continuous gas flow is

$$E_s = -v_l \sigma \int_{\lambda_{\min}}^{\lambda_{\max}} \pi \lambda dN = \frac{\pi D_f v_l \sigma \lambda_{\max}}{(1 - D_f)}. \quad (10)$$

It can be seen that the value of the droplet fractal dimension is $0 < D_f < 1$, when the total free surface energy of the dispersed droplets is greater than 0, $E_s > 0$.

Substituting (7) and (8) into (10), the total free surface energy of dispersed droplets in continuous gas flow can be expressed by droplet Sauter mean diameter using fractal theory:

$$E_s = \frac{\pi \sigma (2 - D_f)^2 D^2}{(1 - D_f)(3 - D_f) \lambda_{32}} v_{sl}. \quad (11)$$

The total turbulent kinetic energy of gas-liquid mist flow can be expressed [29]:

$$E_T = \frac{3}{16} \pi f_{sg} \rho_g A D^2 v_{sg}^3. \quad (12)$$

Total surface free energy of droplets in gas flow and the turbulent kinetic energy of gas are balanced [30, 31]:

$$E_s = E_T. \quad (13)$$

Substituting (11) and (12) into (13), droplet Sauter mean diameter can be obtained based on the balance between the surface energy of the dispersed liquid droplets and the turbulent kinetic energy of the continuous gas flow:

$$\lambda_{32} = \frac{16 \sigma v_{sl} (2 - D_f)^2}{3 f_{sg} \rho_g (1 - D_f) (3 - D_f) v_{sg}^3}. \quad (14)$$

Equation (14) cannot be directly solved by gas superficial velocity, v_{sg} , and liquid superficial velocity, v_{sl} , because droplet Sauter mean diameter, λ_{32} , and the droplet fractal dimension, D_f , are unknown parameters.

Based on mist flow characteristics, maximum droplet diameter can be expressed as [28]

$$\lambda_{\max} = \left[\frac{(2 - D_f) D^2 v_{sl}}{D_f (v_{sg} + v_{sl})} \right]^{1/2}. \quad (15)$$

Substituting (7) into (15), droplet Sauter mean diameter based on mist flow characteristics can be solved:

$$\lambda_{32} = \left[\frac{v_{sl} D^2 (2 - D_f)^3}{D_f (v_{sg} + v_{sl}) (3 - D_f)^2} \right]^{1/2}. \quad (16)$$

Substituting (14) into (16), the fractal expression for droplet Sauter mean diameter under gas-liquid mist flow conditions can be obtained:

$$\lambda_{32} = \frac{16\sigma v_{sl} (2 - D_f)^2}{3f_{sg}\rho_g (1 - D_f) (3 - D_f) v_{sg}^3}, \quad (17)$$

$$D_f = \frac{-B - \sqrt{B^2 - 4AC}}{2A},$$

where

$$A = (3Df_{sg}\rho_g v_{sg}^3)^2 + 256v_{sg}v_{sl}\sigma^2 + 256(v_{sl}\sigma)^2,$$

$$B = -2 \left[(3Df_{sg}\rho_g v_{sg}^3)^2 + 256v_{sg}v_{sl}\sigma^2 + 256(v_{sl}\sigma)^2 \right], \quad (18)$$

$$C = (3Df_{sg}\rho_g v_{sg}^3)^2.$$

Equation (17) is the analytical solution for droplet Sauter mean diameter in gas-liquid mist flow. The droplet Sauter mean diameter, λ_{32} , and the droplet fractal dimension, D_f , can be simultaneously solved using gas superficial velocity, v_{sg} , and liquid superficial velocity, v_{sl} . In this model, we explained the relationship between the droplet Sauter mean diameter and the gas-liquid mist flow situation based on the energy balance theory. When the gas superficial velocity, v_{sg} , increases, the turbulent kinetic energy of gas, E_T , increases. Because of the energy balance, the total surface free energy of droplets in gas flow, E_S , needs to increase too, which means the liquid phase has to divide into more droplets, leading to the increase of the surface between gas phase and liquid phase. That change of flow situation makes the droplet fractal dimension, D_f , be larger and droplet Sauter mean diameter, λ_{32} , be smaller. This is consistent with the physical situation.

3. Results and Discussion

Figure 2 shows a comparison between experimental results (solid points) measured by Fore and Dukler [32] and the present model predictions (full lines). The parameters $\sigma = 0.072 \text{ N/m}$ and $\mu_l = 1.05 \times 10^{-3} \text{ Pa}\cdot\text{s}$ are used in this experiment. It can be seen that droplet Sauter mean diameter decreases with an increase in gas superficial velocity, v_{sg} , and increases with an increase in liquid superficial velocity, v_{sl} . This is because when v_{sg} increases, the total turbulent kinetic energy increases, making droplet breakup into smaller droplet. And when the v_{sl} increases, the total free surface energy increases, making droplet coalescence into bigger droplet. Agreement between the predictions and the experimental results in different liquid viscosity and superficial velocity conditions is obtained. Results suggest that the present droplet Sauter mean diameter model is reliable.

4. Conclusion

Based on the droplet fractal theory and the balance relationship between total droplet surface energy and total gas turbulent kinetic energy, the fractal expression for droplet Sauter mean diameter in gas-liquid mist flow is derived. Agreement

FIGURE 2: Comparison between experimental results and present model predictions.

between the present model predictions and experimental measurements is obtained. Results verify the reliability of the present model.

Nomenclature

A :	Cross-sectional area of a conduit, m^2
A_l :	Cross-sectional area occupied by liquid, m^2
D :	Conduit diameter, m
D_f :	Droplet fractal dimensions
E_S :	Total surface free energy, W
E_T :	Total turbulent kinetic energy, W
N :	Fractal accumulative droplet number
v_l :	Liquid velocity, m/s
v_{sg} :	Gas superficial velocity, m/s
v_{sl} :	Liquid superficial velocity, m/s
λ :	Droplet diameter, m
$\lambda_{\max}$:	Maximum droplet diameter, m
$\lambda_{\min}$:	Minimum droplet diameter, m
λ_{32} :	Droplet Sauter mean diameter
ρ_g :	Gas density, kg/m^3
ρ_l :	Liquid density, kg/m^3
σ :	Interfacial tension, N/m .

Subscripts

g:	Gas phase
l:	Liquid phase
s:	Superficial
T:	Turbulent.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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