Research Article

Quantized Feedback Control Design of Nonlinear Large-Scale Systems via Decentralized Adaptive Integral Sliding Mode Control

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Received 14 April 2015; Revised 19 June 2015; Accepted 12 July 2015

Academic Editor: Sarah K. Spurgeon

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A novel decentralized adaptive integral sliding mode control law is proposed for a class of nonlinear uncertain large-scale systems subject to quantization mismatch between quantizer sensitivity parameters. Firstly, by applying linear matrix inequality techniques, integral-type sliding surface functions are derived for ensuring the stability of the whole sliding mode dynamics and obtaining the prescribed bounded $\mathcal{L}_2$ gain performance. Secondly, the decentralized adaptive sliding mode control law is developed to ensure the reachability of the sliding manifolds in the presence of quantization mismatch, interconnected model uncertainties, and external disturbances. Finally, an example is shown to verify the validity of theoretical results.

1. Introduction

It is well known that quantization phenomena are frequently encountered in numerous practical engineering systems; as a result, the control design considering signal quantization has received much attention since 1998. Numerous well-known results have been issued, such as stability analysis of linear and nonlinear continuous-time systems [1, 2], distributed coordination of multiagent systems [3], event-triggered control, H_∞ or stabilization for networked control systems [4–6], H_∞ filter design for linear continuous-time systems [7], robust or fault-tolerant control using sliding mode control technique [8–12], and H_2 control of continuous-time linear systems [13] subject to input quantization and matched external disturbances.

In this paper, the input quantization problem will be considered. Though similar discussions are also investigated in existing results, such as in [13–15], it should be pointed out that the research works there and in [1, 2, 4, 7, 16] are all built on the assumption that quantizer sensitivity parameters, which are cultivated separately at the coder and decoder sides,

are identical all the time or the ratio of them keeps unchanged [17–19]. This assumption actually requires that the adjustment synchronization of sensitivity parameters is enforced at every time step in practical engineering applications; thus it might not be implementably induced by hardware imperfections of coder and decoder. For coping with this problem, a time-varying ratio model with known lower and upper boundaries is first introduced in [20]. By utilizing robust control technique, the H_2 control design for a class of uncertain systems subject to input quantization mismatch is investigated there.

On the other hand, the large-scale systems are widely used in modern industrial systems, and thus decentralized control design has been well studied in the past two decades; see [21–23] and the references therein. In [21, 22], based on sliding mode control technique, the decentralized output feedback of nonlinear interconnected systems is well investigated. Specially, since the large-scale systems usually are linked via networks, the decentralized control involving signal quantization has aroused the attention of scholars [24, 25]. However, to the best of our knowledge, no results

have considered the decentralized control design via sliding mode control technique for large-scale systems subject to input quantization mismatch.

Motivated by all the mentioned above, we will address the problems of quantized feedback decentralized adaptive integral sliding mode control design for a class of nonlinear uncertain large-scale systems with input quantization mismatch. The purpose is to contribute to the development of quantized feedback sliding mode control theory and the bounded $\mathcal{L}_2$ gain performance analysis for nonlinear large-scale uncertain system. The main contribution includes two aspects. First, applying linear matrix inequality technique, sufficient conditions are derived for guaranteeing the robust stability of sliding mode dynamics with bounded $\mathcal{L}_2$ gain performance. Second, considering the established *boundary-unknown* time varying ratio relation model of the quantization sensitivity parameters, the decentralized adaptive integral sliding mode control strategy is proposed to eliminate the effect of the mismatch of the quantization sensitivity parameters, model uncertainties, and external disturbances.

This paper is organized as follows. Section 2 provides a system description, relation model for the mismatch of the quantization sensitivity parameters, and some preliminary results. The main results are presented in Section 3. In Section 4, an example is given to illustrate the results and this paper is concluded in Section 5 finally.

The following notations are used in this paper. $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; A^T denotes the transpose of matrix A ; and I and 0 represent the identity matrix and a zero matrix in appropriate dimension, respectively. $X > 0$ ($X \geq 0$) means that X is real symmetric and positive definite (semipositive definite). The symbol $\text{He}(X)$ represents $X + X^T$. $|x|_p$ denotes the p -norm of the vector x ; that is, $|x|_p = (|x_1|^p + |x_2|^p + \dots + |x_n|^p)^{1/p}$, $p \geq 1$. When $p = \infty$, $|x|_\infty = \max_{1 \leq i \leq n} |x_i|$. Specially, the notation $|\cdot|$ denotes the standard Euclidean norm of a vector, or the induced norm of a matrix, respectively. In symmetric block matrices, we use a notation $*$ to represent a term that is inferred by symmetry.

2. Problem Statement and Preliminaries

In this paper, the following class of nonlinear uncertain large-scale systems with N interconnected subsystems is considered:

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \Delta A_i(t)) x_i(t) + B_i Q_i(u_i(t)) \\ &\quad + B_i f_i(t, x, p) + D_i \omega_i(t), \end{aligned} \quad (1)$$

$$z_i(t) = C_i x_i(t) + H_i \omega_i(t), \quad (2)$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $Q_i(u_i) \in \mathbb{R}^{m_i}$, and $z_i \in \mathbb{R}^{p_i}$ are the state, the control input, quantized control input, and controlled output of the i th subsystem, respectively; $i = 1, 2, \dots, N$. $\omega_i \in \mathbb{R}^{q_i}$, belonging to $\mathcal{L}_2[0, T]$, is the exogenous input of the i th subsystem. $\Delta A_i(t) \in \mathbb{R}^{n_i \times n_i}$ is the unknown mismatched time varying model uncertainty in the i th subsystem. $f_i(t, x, p) \in \mathbb{R}^{m_i}$ describes the nonlinear interconnected term affecting the i th subsystem,

where $x = [x_1^T, x_2^T, \dots, x_N^T]^T$ and $p \in \mathbb{R}^r$ is uncertain system parameter. Some assumptions are made in the following.

Assumption 1. Time varying uncertainties $\Delta A_i(t)$ satisfy

$$\begin{aligned} \Delta A_i(t) &= F_i \Delta_i(t) E_i, \\ \Delta_i(t) \Delta_i^T(t) &\leq I, \end{aligned} \quad (3)$$

where F_i and E_i are known real constant matrices with appropriate dimensions, $\Delta_i(t)$ is an unknown time varying matrix function.

Assumption 2. The interconnected term $f_i(t, x, p)$ satisfies

$$|f_i(t, x, p)| \leq k_i^o + \sum_{j=1}^N k_{ij}^1 |x_j| + \sum_{j=1}^N k_{ij}^2 |x_i| |x_j|, \quad (4)$$

where k_i^o , k_{ij}^1 , and k_{ij}^2 are unknown positive constants.

Now we show the general description of the quantizer $Q(\cdot)$. For the variable $s_i \in \mathbb{R}^{q_i}$, the quantizer operator $Q_i(s_i)$ is defined by a mathematical function $\text{round}(s_i)$ that rounds the elements of s_i towards the nearest integers; namely,

$$\begin{aligned} Q_i(s_i) &= \tau_{id}(t) \cdot \text{round}\left(\frac{s_i}{\tau_{ic}(t)}\right) \\ &:= \tau_{id}(t) q_i\left(\frac{s_i}{\tau_{ic}(t)}\right), \end{aligned} \quad (5)$$

where $\tau_{ic}(t)$ and $\tau_{id}(t)$ are the quantization sensitivity parameters at the coder and decoder sides of the i th subsystem, respectively. During the operation process, the information of quantized measurement $q_i(s_i/\tau_{ic}(t))$ is generated at the coder side of i th subsystem; then it is sent to the decoder side over a communication channel. While at the decoder side of i th subsystem, under the assumption that the channel is ideal, the value of quantized measurement $q_i(s_i/\tau_{ic}(t))$ is received, and the quantization sensitivity parameter of the quantizer $\tau_{id}(t)$ is adopted; then the decoder of i th subsystem generates the signal $\tau_{id}(t) q_i(s_i/\tau_{ic}(t))$, that is, the quantization operator $Q_i(s_i)$.

Usually as done in [1, 2], it is assumed that the quantization parameters $\tau_{ic}(t)$ and $\tau_{id}(t)$ are equal. In [17], the robustness of quantized control systems with respect to the mismatch of quantizer parameters is firstly presented. However, the adjustments of $\tau_{ic}(t)$ and $\tau_{id}(t)$ are required to be synchronized at each instant. That is to say, $\tau_{id}(t)/\tau_{ic}(t) = r_i$, where r_i is a time-invariant parameter. In actual control engineering, both of the requirements are obviously quite strict and hard to be implemented due to the hardware imperfections. For this, we establish the following *time-varying* ratio parameter model for the i th subsystem:

$$r_i(t) := \frac{\tau_{id}(t)}{\tau_{ic}(t)}, \quad (6)$$

where $r_i(t) \in (r_{\min}, r_{\max})$, $r_{\min}$, and $r_{\max}$, satisfying $r_{\max} \geq r_{\min}$, are unknown positive scalars.

Compared with our existing result in our previous result [20], the established model is more general since the lower and upper boundaries cannot be known.

Remark 3. Usually, besides quantization, signals transmitted over the channels also assume discretization on time, such as in [4, 14]. To focus on the control design for the mismatch problem of quantizer sensitivity, only signal quantization is considered in this paper. Similar discusses were also made in [13, 15, 19].

The main aim of this paper is to form decentralized adaptive integral sliding mode control strategy with $\mathcal{L}_2$ performance for the nonlinear large-scale system (1) subject to the quantization mismatch as described in (6).

Now considering $Q_i(u_i(t))$ in (1) and the ratio in (6), we have

$$\begin{aligned} Q_i(u_i(t)) &= \tau_{id}(t) q_i\left(\frac{u_i(t)}{\tau_{ic}(t)}\right) \\ &= \frac{\tau_{id}(t)}{\tau_{ic}(t)} \tau_{ic}(t) q_i\left(\frac{u_i(t)}{\tau_{ic}(t)}\right) \\ &= r_i(t) \tau_{ic}(t) q_i\left(\frac{u_i(t)}{\tau_{ic}(t)}\right) \\ &= r_i(t) (u_i(t) + e_{\tau_{ic}}(t)), \end{aligned} \quad (7)$$

where $e_{\tau_{ic}}(t) := q_{\tau_{ic}(t)}(u_i(t)) - u_i(t)$ and $q_{\tau_{ic}(t)}(u_i(t)) := \tau_{ic}(t) q_i(u_i(t)/\tau_{ic}(t))$. And $e_{\tau_{ic}}(t)$ satisfies

$$|e_{\tau_{ic}}(t)| \leq \nabla_i \tau_{ic}, \quad (8)$$

with $\nabla_i = \sqrt{m_i}/2$; m_i is the dimension of the vector u_i .

Definition 4. $\mathcal{L}_2$ Gain Performance. Given positive scalars γ_i , the nonlinear large-scale system in (1)-(2) with $u_i(t) = 0$ is said to be robust stable with a bounded $\mathcal{L}_2$ gain performance; if it is robust stable for $\omega_i(t) = 0$, and, under zero initial condition, for nonzero $\omega_i(t) \in \mathcal{L}_2[0, \infty)$, it holds that

$$\sum_{i=1}^N \int_0^\infty z_i^T(t) z_i(t) dt < \sum_{i=1}^N \gamma_i^2 \int_0^\infty \omega_i^T(t) \omega_i(t) dt. \quad (9)$$

Remark 5. Denote that

$$\begin{aligned} z(t) &= [z_1^T(t), z_2^T(t), \dots, z_N^T(t)]^T, \\ \omega(t) &= [\omega_1^T(t), \omega_2^T(t), \dots, \omega_N^T(t)]^T, \end{aligned} \quad (10)$$

and letting $\gamma_i = \gamma$, $i = 1, 2, \dots, N$, one can easily obtain that

$$\int_0^\infty z^T(t) z(t) dt < \gamma^2 \int_0^\infty \omega^T(t) \omega(t) dt. \quad (11)$$

This is the usually considered $\mathcal{L}_2$ gain performance for simple linear system as in [26].

For the considered problem, the following lemma will be used.

Lemma 6 (see [27]). Given a symmetric matrix Θ and matrices X, Y with appropriate dimensions, then

$$\Theta + X \Xi(t) Y + Y^T \Xi^T(t) X^T < 0 \quad (12)$$

for all $\Xi(t)$ satisfying $\Xi^T(t) \Xi(t) \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that the following inequality holds:

$$\Theta + \epsilon X X^T + \epsilon^{-1} Y^T Y < 0. \quad (13)$$

3. Main Results

In this paper, the following integral-type sliding surface function is considered:

$$\sigma_i(t) = G_i x_i(t) - \int_0^t G_i (A_i - B_i K_i) x_i(\theta) d\theta, \quad (14)$$

where A_i and B_i are system matrices defined in large-scale system (1). $G_i \in \mathbb{R}^{m_i \times n_i}$ and $K_i \in \mathbb{R}^{m_i \times n_i}$ are real matrices to be designed. In particular, the matrix G_i is selected such that $G_i B_i$ is nonsingular. Without loss of generality, as done in [28, 29], G_i is designed to be $G_i = B_i^+$, the pseudoinverse of B_i for the convenience of the proof.

By the theory of sliding mode control [30, 31], when the trajectory of large-scale system (1) is kept on the sliding dynamics, it has $\sigma_i(t) = 0$ and $\dot{\sigma}_i(t) = 0$. Thus, by taking $\dot{\sigma}_i(t) = 0$, one can obtain the equivalent control for the i th subsystem of large-scale system (1):

$$\begin{aligned} Q_{ieq}(u_i(t)) &= -[G_i B_i]^{-1} \{G_i \Delta A_i(t) x_i(t) \\ &\quad + G_i B_i f_i(t, x, p) + G_i B_i K_i x_i(t) + G_i D_i \omega_i(t)\} \\ &= -(G_i B_i)^{-1} G_i \Delta A_i(t) x_i(t) - f_i(t, x, p) - K_i x_i(t) \\ &\quad - (G_i B_i)^{-1} G_i D_i \omega_i(t). \end{aligned} \quad (15)$$

Substituting (15) into (1), one can get that

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \Delta A_i(t)) x_i(t) \\ &\quad + B_i \{- (G_i B_i)^{-1} G_i \Delta A_i(t) x_i(t) - f_i(t, x, p) \\ &\quad - K_i x_i(t) - (G_i B_i)^{-1} G_i D_i \omega_i(t)\} + B_i f_i(t, x, p) \\ &\quad + D_i \omega_i(t) = [(A_i - B_i K_i) \\ &\quad + (I - B_i (G_i B_i)^{-1} G_i) \Delta A_i(t)] x_i(t) + [I \\ &\quad - B_i (G_i B_i)^{-1} G_i] D_i \omega_i(t). \end{aligned} \quad (16)$$

Denoting $\bar{G}_i = I - B_i (G_i B_i)^{-1} G_i$, $\bar{A}_i = A_i - B_i K_i$, $\Delta \bar{A}_i(t) = \bar{G}_i \Delta A_i(t)$, and $\bar{D}_i = \bar{G}_i D_i$, one can see that (16) can be represented as

$$\dot{x}_i(t) = [\bar{A}_i + \Delta \bar{A}_i(t)] x_i(t) + \bar{D}_i \omega_i(t), \quad (17)$$

where $\Delta \bar{A}_i(t) = \bar{G}_i \Delta A_i(t) = \bar{G}_i F_i \Delta_i(t) E_i$.

We have the following theorem for the sliding mode dynamics in (17).

Theorem 7. Given scalars $\gamma_i > 0$, the sliding mode dynamics in (17) is quadratically stable with bounded $\mathcal{L}_2$ gain performance γ_i , if there exist a symmetric positive definite matrix X_i , a general matrix W_i , and a positive scalar ε_i such that the following linear matrix inequality holds:

$$\begin{bmatrix} \text{He}\{A_i X_i - B_i W_i\} + \varepsilon_i \bar{G}_i F_i F_i^T \bar{G}_i^T & X_i C_i^T & X_i E_i^T & \bar{D}_i + X_i C_i^T H_i \\ * & -I & 0 & 0 \\ * & * & -\varepsilon_i I & 0 \\ * & * & * & H_i^T H_i - \gamma_i^2 I \end{bmatrix} < 0. \quad (18)$$

Moreover, if the LMI condition has a feasible solution, then the matrices K_i in (14) can be derived by

$$K_i = W_i X_i^{-1}. \quad (19)$$

Proof. Consider a Lyapunov function candidate as

$$\bar{V}(t) = \sum_{i=1}^N \bar{V}_i(t) = \sum_{i=1}^N x_i^T(t) P_i x_i(t), \quad (20)$$

where $P_i > 0$. Along the solution of sliding mode dynamics (17), one can get

$$\begin{aligned} \dot{\bar{V}}(t) &= \sum_{i=1}^N \dot{\bar{V}}_i = \sum_{i=1}^N x_i^T(t) P_i \dot{x}_i(t) + \sum_{i=1}^N \dot{x}_i^T(t) P_i x_i(t) \\ &= \sum_{i=1}^N x_i^T(t) P_i \{[\bar{A}_i + \Delta \bar{A}_i(t)] x_i(t) + \bar{D}_i \omega_i(t)\} \\ &\quad + \sum_{i=1}^N \{[\bar{A}_i + \Delta \bar{A}_i(t)] x_i(t) + \bar{D}_i \omega_i(t)\}^T P_i x_i(t). \end{aligned} \quad (21)$$

Therefore,

$$\begin{aligned} \dot{\bar{V}}(t) &+ \sum_{i=1}^N z_i^T(t) z_i(t) - \sum_{i=1}^N \gamma_i^2 \omega_i^T(t) \omega_i(t) = \sum_{i=1}^N x_i^T(t) \\ &\cdot \left[(\bar{A}_i + \Delta \bar{A}_i(t))^T P_i + P_i (\bar{A}_i + \Delta \bar{A}_i(t)) \right] x_i(t) \end{aligned}$$

$$\begin{bmatrix} \text{He}\{[A_i - B_i K_i + G_i F_i \Delta_i(t) E_i] P_i^{-1}\} + P_i^{-1} C_i^T C_i P_i^{-1} & \bar{D}_i + P_i^{-1} C_i^T H_i \\ * & H_i^T H_i - \gamma_i^2 I \end{bmatrix} < 0. \quad (27)$$

Defining $X_i = P_i^{-1}$ and $W_i = K_i X_i$, we have

$$\begin{bmatrix} \Theta_{11i} & \bar{D}_i + X_i C_i^T H_i \\ * & H_i^T H_i - \gamma_i^2 I \end{bmatrix} < 0, \quad (28)$$

$$\begin{aligned} &+ \sum_{i=1}^N 2x_i^T(t) P_i \bar{D}_i \omega_i(t) + \sum_{i=1}^N (C_i x_i(t) + H_i \omega_i(t))^T \\ &\cdot (C_i x_i(t) + H_i \omega_i(t)) - \sum_{i=1}^N \gamma_i^2 \omega_i^T(t) \omega_i(t) \\ &= \sum_{i=1}^N \xi_i^T(t) \Gamma_i \xi_i(t), \end{aligned} \quad (22)$$

where $\xi := [x_i^T(t) \ \omega_i^T(t)]^T$ and

$$\Gamma_i = \begin{bmatrix} \gamma_{11i} & \gamma_{12i} \\ * & \gamma_{22i} \end{bmatrix} \quad (23)$$

with $\gamma_{11i} = \text{He}[(\bar{A}_i + \Delta \bar{A}_i(t))^T P_i] + C_i^T C_i$, $\gamma_{12i} = P_i \bar{D}_i + C_i^T H_i$, and $\gamma_{22i} = H_i^T H_i - \gamma_i^2 I$.

Suppose that $\xi_i^T(t) \Gamma_i \xi_i(t) < 0$, we have

$$\dot{\bar{V}}(t) + \sum_{i=1}^N z_i^T(t) z_i(t) - \sum_{i=1}^N \gamma_i^2 \omega_i^T(t) \omega_i(t) < 0. \quad (24)$$

Integrating both sides of the inequality (24) with respect to time t from 0 to T yields

$$\begin{aligned} \bar{V}(T) - \bar{V}(0) &+ \sum_{i=1}^N \int_0^T z_i^T(t) z_i(t) dt \\ &- \sum_{i=1}^N \gamma_i^2 \int_0^T \omega_i^T(t) \omega_i(t) dt < 0. \end{aligned} \quad (25)$$

Under the zero initial conditions, we have $\bar{V}(0) = 0$ and $\bar{V}(T) = \sum_{i=1}^N x_i^T(T) P_i x_i(T) \geq 0$; one can see that

$$\sum_{i=1}^N \int_0^T z_i^T(t) z_i(t) dt < \sum_{i=1}^N \gamma_i^2 \int_0^T \omega_i^T(t) \omega_i(t) dt. \quad (26)$$

Taking the limit as $T \rightarrow +\infty$ on both sides of (26) gives rise to (9).

Performing a congruence transformation to (23) by matrix $\text{diag}\{P_i^{-1}, I\}$, we have

where $\Theta_{11i} = \text{He}\{A_i X_i - B_i W_i\} + G_i F_i \Delta_i(t) E_i X_i + X_i E_i^T \Delta_i(t) F_i^T G_i^T + X_i C_i^T C_i X_i$. Furthermore, according to Lemma 6, one can see that (28) is equivalent to

$$\begin{bmatrix} \Xi_{11i} & \bar{D}_i + X_i C_i^T H_i \\ * & H_i^T H_i - \gamma_i^2 I \end{bmatrix} < 0, \quad (29)$$

where $\Xi_{11i} = \text{He}\{A_i X_i - B_i W_i\} + \varepsilon_i G_i F_i F_i^T G_i^T + (1/\varepsilon_i) X_i E_i^T E_i X_i + X_i C_i^T C_i X_i$. Applying Schur complement technique, we can further get (18).

This completes the proof. $\square$

$$u_i^j(t) = \begin{cases} -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} + u_i^0, & \text{if } \sigma_i^j(t) < 0 \\ 0, & \text{if } \sigma_i^j(t) = 0 \\ -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} - u_i^0, & \text{if } \sigma_i^j(t) > 0, \end{cases} \quad (30)$$

where $u_i^0 = \nabla_i \tau_{ic}(t)$, $\beta_i > 1$, $i = 1, 2, \dots, N$; $j = 1, 2, \dots, m_i$. The parameters $\widehat{l}_i$, $\widehat{\rho}_i > 0$, and $\widehat{\gamma}_i > 0$ are the estimations of l_i , ρ_i , and r_i^{-1} , respectively. And they are derived from the designed adaptive laws:

$$\dot{\widehat{l}}_i(t) = \frac{1}{\eta_{i1}} |\sigma_i(t)|, \quad (31)$$

$$\dot{\widehat{\rho}}_i(t) = \frac{1}{\eta_{i2}} |\sigma_i(t)|, \quad (32)$$

$$\dot{\widehat{\gamma}}_i = \frac{1}{\eta_{i3}} (\widehat{l}_i + \widehat{\rho}_i + [|G_i F_{i1}| |E_i| + |K_i|] |x_i|) \widehat{\gamma}_i^3 |\sigma_i(t)| \quad (33)$$

with the initial values $\widehat{l}_i(0) = \widehat{l}_{i0} > 0$, $\widehat{\gamma}_i(0) = \widehat{\gamma}_{i0} > 0$, and $\widehat{\rho}_i(0) = \widehat{\rho}_{i0} > 0$, and the parameters η_{i1} , η_{i2} , and η_{i3} are selected positive scalars; $i = 1, 2, \dots, N$. l_i and ρ_i will be given later.

Remark 8. The controller form as (31)–(33) can be often seen in existing theory results, for instance, [23, 32]. However, it is hard to expect $|\sigma(t)|$ to be exactly zero in practice engineering; thus how to solve the gain increase problem in theory completely is a very interesting research topic. In addition, one can see from the simulation part later, by choosing sufficiently large parameters η_{i1} , η_{i2} , and η_{i3} , the gains cannot increase indefinitely in a finite time. So they can be used in some engineering applications.

Remark 9. The solution for systems with discontinuous right-hand side as (30) can be interpreted in the sense of [33–35].

To clarify the proof of the main result, a lemma is first summarized in the following.

Lemma 10. For the quantized control input $Q_i(u_i)$ satisfying the mismatched relation shown in (6), the control law in (30) guarantees the establishment of the inequality:

$$\sigma_i^T(t) Q_i(u_i) \leq -r_i(t) \cdot [\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|] |x_i|] \widehat{\gamma}_i |\sigma_i(t)|. \quad (34)$$

Proof. It is obvious that the above inequality (34) holds when $\sigma_i(t) = 0$. And when $\sigma_i^j(t) < 0$, we have $u_i^j = -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i (\sigma_i^j(t)/|\sigma_i(t)|) + u_i^0$. Combining with

In what follows, let us denote that $\sigma_i(t) = [\sigma_i^1(t), \sigma_i^2(t), \dots, \sigma_i^{m_i}(t)]^T$ and $u_i(t) = [u_i^1(t), u_i^2(t), \dots, u_i^{m_i}(t)]^T$ for convenience. The following decentralized adaptive integral sliding mode control law is proposed:

the mismatched relation in (6), the definition of quantization error, one can see that

$$\begin{aligned} (u_i^j - u_i^0) Q_i(u_i^j) &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} Q_i(u_i^j) \\ &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \\ &\cdot r_i(t) (u_i^{(j)} + e_{\tau_{ic}}^j(t)) = -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) \\ &\cdot \left[-[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \right. \\ &\left. + \nabla_i \tau_{ic}(t) + e_{\tau_{ic}}^j(t) \right] \\ &= \left\{ -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \right. \\ &\cdot \left. \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \right\}^2 r_i(t) - [\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) [\nabla_i \tau_{ic}(t) \\ &+ e_{\tau_{ic}}^j(t)]. \end{aligned} \quad (35)$$

By the design of the parameters, $\nabla_i \tau_{ic} \geq |e_{\tau_{ic}}^j(t)|$ and $\sigma_{ij}(t) < 0$, one can see that

$$\begin{aligned} &-[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_{i1}| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) \\ &\cdot [\nabla_i \tau_{ic}(t) + e_{\tau_{ic}}^j(t)] \geq 0. \end{aligned} \quad (36)$$

Combining (35) with (36), we have

$$\begin{aligned} (u_i^j - u_i^0) Q_i(u_i^j) &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} Q_i(u_i^j) \geq r_i(t) \\ &\cdot \left\{ -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \right\}^2. \end{aligned} \quad (37)$$

Since $-[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i < 0$, it is not difficult to get that

$$\begin{aligned} \sigma_i^j(t) Q_i(u_i^j) &\leq -r_i(t) \\ &\cdot [\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \\ &\cdot \frac{(\sigma_i^j(t))^2}{|\sigma_i(t)|}. \end{aligned} \quad (38)$$

Similarly, when $\sigma_i^j(t) > 0$, we have $u_i^j = -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i (\sigma_i^j(t)/|\sigma_i(t)|) - u_i^0$. Combining with the mismatched relation in (6), the definition of quantization error again, one can see that

$$\begin{aligned} (u_i^j + u_i^0) Q_i(u_i^j) &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} Q_i(u_i^j) \\ &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \\ &\cdot r_i(t) (u_i^j(t) + e_{\tau_{ic}}^j(t)) = -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) \\ &\cdot \left\{ -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \right. \\ &\left. - \nabla_i \tau_{ic}(t) + e_{\tau_{ic}}^j(t) \right\} \\ &= \left\{ -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \right. \\ &\cdot \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \left. \right\}^2 r_i(t) + \{\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) [\nabla_i \tau_{ic}(t) \\ &- e_{\tau_{ic}}^j(t)] \}. \end{aligned} \quad (39)$$

According to the design of the parameters, $\nabla_i \tau_{ic} \geq |e_{\tau_{ic}}^j(t)|$ and $\sigma_i^j(t) > 0$, one can see that

$$\begin{aligned} &[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} r_i(t) \\ &\cdot [\nabla_i \tau_{ic}(t) - e_{\tau_{ic}}^j(t)] \geq 0. \end{aligned} \quad (40)$$

Combining (39) with (40), we have

$$\begin{aligned} (u_i^j + u_i^0) Q_i(u_i^j) &= -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \\ &\cdot Q_i(u_i^j) \\ &\geq \left[-[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{\sigma_i^j(t)}{|\sigma_i(t)|} \right]^2 \\ &\cdot r_i(t). \end{aligned} \quad (41)$$

Furthermore, we have

$$\begin{aligned} \sigma_i^j(t) Q_i(u_i^j) &\leq -r_i(t) \\ &\cdot [\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \\ &\cdot \frac{(\sigma_i^j(t))^2}{|\sigma_i(t)|}. \end{aligned} \quad (42)$$

Therefore, the following inequality holds:

$$\begin{aligned} \sigma_i^T(t) Q_i(u_i) &= \sum_{j=1}^m \sigma_{ij}(t) Q_i(u_i^j) \leq -\sum_{j=1}^m r_i(t) \\ &\cdot [\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i \frac{(\sigma_i^j(t))^2}{|\sigma_i(t)|} \\ &\leq -[\beta_i(\widehat{l}_i + \widehat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|]] \\ &\cdot \widehat{\gamma}_i \sum_{j=1}^m \left[\frac{(\sigma_i^j(t))^2}{|\sigma_i(t)|} \right] = -r_i(t) [\beta_i(\widehat{l}_i + \widehat{\rho}_i) \\ &+ [|G_i F_i| |E_i| + |K_i|]] \widehat{\gamma}_i |\sigma_i(t)|. \end{aligned} \quad (43)$$

The proof has been achieved completely. $\square$

The main result of this paper is summarized as follows.

Theorem 11. Consider the nonlinear large-scale system (1) subject to Assumptions 1-2 with the mismatch between the quantization sensitivity parameters; if the control law is designed as shown in (30) with adaptive laws in (31)–(33), then the trajectories of system (1) can be driven onto the sliding surfaces asymptotically.

Proof. Let us first consider $V_i(t) = (1/2)\sigma_i^T(t)\sigma_i(t)$, then the time derivative along the system trajectories is

$$\begin{aligned} \dot{V}_i(t) = & \sigma_i^T(t) \{G_i \Delta A_i(t) x_i + G_i B_i Q_i(u_i) \\ & + G_i B_i f_i(t, x, p) + G_i B_i K_i x_i + G_i D_i \omega_i(t)\}. \end{aligned} \quad (44)$$

According to the design of the switching vector $G_i = B_i^+$, one can see that the above equation can be rewritten to be

$$\begin{aligned} \dot{V}_i(t) = & \sigma_i^T(t) Q_i(u_i(t)) + \sigma_i^T(t) \\ & \cdot \{G_i \Delta A_i(t) x_i + f_i(t, x, p) + K_i x_i + G_i D_i \omega_i(t)\}. \end{aligned} \quad (45)$$

By Assumptions 1-2, we have

$$\begin{aligned} \dot{V}_i(t) \leq & \sigma_i^T(t) Q_i(u_i(t)) + |\sigma_i(t)| |G_i F_i| |E_i| |x_i| \\ & + |\sigma_i(t)| |K_i| |x_i| + |\sigma_i(t)| |G_i D_i| |\omega_i(t)| \\ & + |\sigma_i(t)| \left[k_i^0 + \sum_{j=1}^N k_{ij}^1 |x_j| + \sum_{j=1}^N k_{ij}^2 |x_i| |x_j| \right]. \end{aligned} \quad (46)$$

Applying the basic inequality $ab \leq (1/2)(a^2 + b^2)$, $a \in \mathbb{R}$, $b \in \mathbb{R}$, one can see that

$$\begin{aligned} \dot{V}_i(t) & \leq \sigma_i^T(t) Q_i(u_i(t)) + |\sigma_i(t)| |G_i F_i| |E_i| |x_i| \\ & + |\sigma_i(t)| |K_i| |x_i| + |\sigma_i(t)| |G_i D_i| |\omega_i(t)| \\ & + |\sigma_i(t)| \left[k_i^0 + \sum_{j=1}^N k_{ij}^1 |x_j| + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 (x_i^2 + x_j^2) \right] \\ & = \sigma_i^T(t) Q_i(u_i(t)) + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| \\ & + |\sigma_i(t)| \left[|G_i D_i| |\omega_i(t)| + k_i^0 + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 x_i^2 \right] \\ & + |\sigma_i(t)| \left[\sum_{j=1}^N k_{ij}^1 |x_j| + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 x_j^2 \right]. \end{aligned} \quad (47)$$

Since k_i^0 , k_{ij}^1 , and k_{ij}^2 are unknown parameters, for convenience, similar to what is done in [23], we denote that unknown parameters l_i and ρ_i satisfy

$$\begin{aligned} l_i & \geq |G_i D_i| |\omega_i(t)| + k_i^0 + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 x_i^2, \\ \rho_i & \geq \sum_{j=1}^N k_{ij}^1 |x_j| + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 x_j^2, \end{aligned} \quad (48)$$

then (47) can be rewritten as

$$\begin{aligned} \dot{V}_i(t) & \leq \sigma_i^T(t) Q_i(u_i(t)) + |\sigma_i(t)| |G_i F_i| |E_i| |x_i| \\ & + |\sigma_i(t)| |K_i| |x_i| + |\sigma_i(t)| |G_i D_i| |\omega_i(t)| \\ & + |\sigma_i(t)| \left[k_i^0 + \sum_{j=1}^N k_{ij}^1 |x_j| + \frac{1}{2} \sum_{j=1}^N k_{ij}^2 (x_i^2 + x_j^2) \right] \\ & = \sigma_i^T(t) Q_i(u_i(t)) + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| \\ & + |\sigma_i(t)| l_i + |\sigma_i(t)| \rho_i. \end{aligned} \quad (49)$$

Take Lyapunov function candidate as

$$V(t) = \sum_{i=1}^N V_i(t) + \frac{1}{2} \sum_{i=1}^N \eta_{i1} \tilde{l}_i^2 + \frac{1}{2} \sum_{i=1}^N \eta_{i2} \tilde{\rho}_i^2 + \frac{1}{2} \sum_{i=1}^N \eta_{i3} \tilde{\gamma}_i^2, \quad (50)$$

where $\tilde{l}_i = l_i - \hat{l}_i$, $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$, and $\tilde{\gamma}_i = \tilde{\gamma}_i^{-1} - r_i$. Thus we have

$$\begin{aligned} \dot{V}(t) & = \sum_{i=1}^N \dot{V}_i(t) + \sum_{i=1}^N \eta_{i1} \dot{\tilde{l}}_i + \sum_{i=1}^N \eta_{i2} \dot{\tilde{\rho}}_i + \sum_{i=1}^N \eta_{i3} \dot{\tilde{\gamma}}_i \\ & = \sum_{i=1}^N \{ \sigma_i^T(t) Q_i(u_i(t)) \\ & + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| l_i \\ & + |\sigma_i(t)| \rho_i \} + \sum_{i=1}^N \eta_{i1} \dot{\tilde{l}}_i + \sum_{i=1}^N \eta_{i2} \dot{\tilde{\rho}}_i + \sum_{i=1}^N \eta_{i3} \dot{\tilde{\gamma}}_i. \end{aligned} \quad (51)$$

According to the design of the adaptive laws (31)–(33) and combining with $\tilde{l}_i = l_i - \hat{l}_i$, $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$, and $\tilde{\gamma}_i = \tilde{\gamma}_i^{-1} - r_i$, we have

$$\begin{aligned} \dot{V}(t) & = \sum_{i=1}^N \dot{V}_i(t) + \sum_{i=1}^N \eta_{i1} \dot{\tilde{l}}_i + \sum_{i=1}^N \eta_{i2} \dot{\tilde{\rho}}_i + \sum_{i=1}^N \eta_{i3} \dot{\tilde{\gamma}}_i \\ & = \sum_{i=1}^N \{ \sigma_i^T(t) Q_i(u_i(t)) \\ & + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| \hat{l}_i \\ & + |\sigma_i(t)| \hat{\rho}_i \} + \sum_{i=1}^N \eta_{i3} \tilde{\gamma}_i \left(-\frac{1}{\tilde{\gamma}_i^2} \right) \dot{\tilde{\gamma}}_i \\ & = \sum_{i=1}^N \{ \sigma_i^T(t) Q_i(u_i(t)) \\ & + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| \hat{l}_i \\ & + |\sigma_i(t)| \hat{\rho}_i \} - \sum_{i=1}^N \tilde{\gamma}_i \tilde{\gamma}_i (\hat{l}_i + \hat{\rho}_i) \\ & + [|G_i F_i| |E_i| + |K_i|] |x_i| |\sigma_i(t)|. \end{aligned} \quad (52)$$

According to Lemma 10, one can see that

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^N \left\{ -r_i \hat{\gamma}_i \left(\beta_i (\hat{l}_i + \hat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|] |x_i| \right) \right. \\ &\quad \cdot |\sigma_i(t)| + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| \hat{l}_i \\ &\quad + |\sigma_i(t)| \hat{\rho}_i \left. - \sum_{i=1}^N \tilde{\gamma}_i \hat{\gamma}_i \left(\hat{l}_i + \hat{\rho}_i + [|G_i F_i| |E_i| + |K_i|] \right) \right. \\ &\quad \cdot |x_i| \left. \right\} |\sigma_i(t)|. \end{aligned} \quad (53)$$

Furthermore, noticing that $\tilde{\gamma}_i = \hat{\gamma}_i^{-1} - \gamma_i$, $\beta_i > 1$, and $\hat{l}_i > 0$, $\hat{\rho}_i > 0$, one can easily obtain that

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^N \left\{ - (r_i + \tilde{\gamma}_i) \right. \\ &\quad \cdot \hat{\gamma}_i \left(\beta_i (\hat{l}_i + \hat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|] |x_i| \right) |\sigma_i(t)| \\ &\quad + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| \hat{l}_i + |\sigma_i(t)| \\ &\quad \cdot \hat{\rho}_i \left. \right\} = \sum_{i=1}^N \left\{ - (\beta_i (\hat{l}_i + \hat{\rho}_i) + [|G_i F_i| |E_i| + |K_i|] |x_i|) \right. \\ &\quad \cdot |\sigma_i(t)| + |\sigma_i(t)| [|G_i F_i| |E_i| + |K_i|] |x_i| + |\sigma_i(t)| \hat{l}_i \\ &\quad + |\sigma_i(t)| \hat{\rho}_i \left. \right\} = - \sum_{i=1}^N (\beta_i - 1) (\hat{l}_i + \hat{\rho}_i) |\sigma_i(t)| < 0. \end{aligned} \quad (54)$$

Integrating both sides of the above equation from 0 to t gives rise to

$$V(t) - V(0) \leq \sum_{i=1}^N \int_0^t (1 - \beta_i) (\hat{l}_i + \hat{\rho}_i) |\sigma_i(s)| ds. \quad (55)$$

One can easily obtain that

$$\sum_{i=1}^N \int_0^t (\beta_i - 1) (\hat{l}_i + \hat{\rho}_i) |\sigma_i(s)| ds \leq V(0). \quad (56)$$

Taking the limit as $t \rightarrow +\infty$ on both sides of (56) yields

$$\sum_{i=1}^N \int_0^{\infty} (\beta_i - 1) (\hat{l}_i + \hat{\rho}_i) |\sigma_i(s)| ds \leq V(0) < +\infty. \quad (57)$$

According to Barbalat's lemma, one can see that

$$\lim_{t \rightarrow +\infty} \sum_{i=1}^N (\beta_i - 1) (\hat{l}_i + \hat{\rho}_i) |\sigma_i(s)| = 0. \quad (58)$$

Since $\beta_i > 1$, $\hat{l}_i > 0$, and $\hat{\rho}_i > 0$ are positive increasing functions in terms of the adaptive control laws in (31) and (32), thus we have $\sigma_i(t) \rightarrow 0$ as $t \rightarrow +\infty$.

Hence the proof is obtained completely. $\square$

4. Illustrative Example

Consider the nonlinear uncertain large-scale system (1) with the following data:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 3 \\ 2 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \\ F_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ E_1 &= [0 \ 1], \\ E_2 &= [1 \ 1], \\ D_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \\ C_1 &= [1 \ 2], \\ C_2 &= [2 \ 1], \\ H_1 &= 0.1, \\ H_2 &= 0.2. \end{aligned} \quad (59)$$

In addition, the interconnected terms $f_1(t, x, p)$ and $f_2(t, x, p)$ and the exogenous disturbance terms $\omega_1(t)$ and $\omega_2(t)$ are selected as

$$\begin{aligned} f_1(t, x, p) &= 0.2x_{12}x_{22} + 0.1x_{21} + 0.2x_{22}, \\ f_2(t, x, p) &= 0.2x_{12}x_{22} + 0.2x_{11} + 0.2x_{12}, \\ \omega_1 &= 0.2e^{-t} \cos(3t), \\ \omega_2(t) &= 0.1e^{-3t} \sin(2t). \end{aligned} \quad (60)$$

Our aim here is to verify the effectiveness of the proposed theoretical results in the previous sections. Combining with the selection of parameters $G_1 = [0.2 \ 0.4]$ and $G_2 = [-0.8 \ 0.4]$, and solving LMI condition in Theorem 7, one can obtain that $K_1 = [1.4552 \ 3.6645]$ and

FIGURE 1: The response curves of system states.

FIGURE 2: The response curves of the switching functions.

$K_2 = [-6.1215 \ -5.3471]$. Moreover, the integral-type sliding surface functions in (14) can be computed as

$$\begin{aligned} \sigma_1 &= [0.2 \ 0.4] x_1 - \int_0^t [-1.0552 \ -2.6645] x_1(\theta) d\theta, \\ \sigma_2 &= [-0.8 \ 0.4] x_2 - \int_0^t [6.9215 \ 3.7471] x_2(\theta) d\theta. \end{aligned} \quad (61)$$

In order to illustrate the effect caused by the mismatch of the quantization sensitivity parameters, the evolutions of $\tau_{ic}(t)$ and $\tau_{id}(t)$, $i = 1, 2$, are selected to be of the following form:

$$\begin{aligned} \tau_{1c}(t) &= \begin{cases} 1.2, & \text{if } t \leq 1, \\ 0.8, & \text{if } 1 < t \leq 6, \\ 0.5, & \text{if } 6 < t \leq 15, \\ 0.3, & \text{if } t > 15, \end{cases} \\ \tau_{1d}(t) &= \begin{cases} 1.4, & \text{if } t \leq 4, \\ 0.35, & \text{if } 4 < t \leq 12, \\ 0.15, & \text{if } 12 < t \leq 25, \\ 0.068, & \text{if } t > 25. \end{cases} \\ \tau_{2c}(t) &= \begin{cases} 1, & \text{if } t \leq 2, \\ 0.6, & \text{if } 2 < t \leq 8, \\ 0.3, & \text{if } 8 < t \leq 20, \\ 0.2, & \text{if } t > 20, \end{cases} \end{aligned}$$

$$\tau_{2d}(t) = \begin{cases} 1.1, & \text{if } t \leq 5, \\ 0.3, & \text{if } 5 < t \leq 15, \\ 0.1, & \text{if } 15 < t \leq 25, \\ 0.063, & \text{if } t > 25. \end{cases} \quad (62)$$

Choosing the parameters $\beta_1 = 2$, $\beta_2 = 4$, $\eta_{11} = 180$, $\eta_{21} = 300$, $\eta_{12} = 80$, $\eta_{22} = 100$, $\eta_{13} = 20$, and $\eta_{23} = 50$ and the initial values of system states $x_1(0) = [5 \ 3]$, $x_2(0) = [-4 \ 3]$, $\hat{l}_1(0) = 0.03$, $\hat{l}_2(0) = 0.02$, $\hat{\rho}_1(0) = 0.001$, $\hat{\rho}_2(0) = 0.006$, $\hat{\gamma}_1(0) = 4$, and $\hat{\gamma}_2(0) = 3$, and applying the proposed decentralized adaptive integral sliding mode control strategy proposed in (30)–(33), the simulation results are shown in Figures 1–8. One can see from Figures 1 and 2 that the proposed decentralized adaptive sliding mode control law can effectively cope with the effect of quantization mismatch, model uncertainties, and external disturbances and guarantee that the system trajectories of the two subsystems can be driven into the desired switching surfaces and convergence to the origin asymptotically. It can be also observed from Figures 3–8 that $\hat{l}_i$, ρ_i , and r_i can be well estimated and the estimation error converges into zero in a finite time.

5. Conclusions

This paper has been concerned with the problem of decentralized adaptive integral sliding mode control design for a class of nonlinear uncertain large-scale systems subject to quantization mismatch. The LMI conditions are derived for the design of the switching surfaces of each subsystem

FIGURE 3: The response curves of $\hat{l}_1$ and $\hat{l}_2$. FIGURE 4: The response curves of $\hat{\rho}_1$ and $\hat{\rho}_2$. FIGURE 5: The response curves of $\hat{\gamma}_1$ and $\hat{\gamma}_2$. FIGURE 6: The response curves of $\tilde{l}_1$ and $\tilde{l}_2$. FIGURE 7: The response curves of $\tilde{\rho}_1$ and $\tilde{\rho}_2$.

FIGURE 8: The response curves of $\tilde{r}_1$ and $\tilde{r}_2$.

and guaranteeing the bounded $\mathcal{L}_2$ gain performance. Furthermore, the decentralized adaptive integral sliding mode control laws are established for ensuring the reachability of the sliding manifold. Finally, an example is given for illustrating the effectiveness of the theoretical results. How to form sliding-mode output feedback control or observed-based sliding mode control for solving the more kinds of disturbances, including the constant $w(t)$ as special case, will be an important and interesting research topic in our future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Grants nos. 11301276, 61403207, and 61273155) and by the Natural Science Foundation of Jiangsu Province (BK20130984 and BK20131000).

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