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Research Article

Piecewise H_∞ Static Output Feedback Controller Design for Nonlinear Systems Based on T-S Affine Fuzzy Models

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This paper is concerned with the problem of designing H_∞ controllers via static output feedback controller for a class of complex nonlinear systems, which is approximated by continuous-time affine fuzzy models. A decomposition method is presented to divide the output-space into different operating regions and interpolation regions. Based on this partition, a novel piecewise controller with affine terms via static output feedback is designed. By using a dilated linear matrix inequality (LMI) characterization, some nonconvex conditions are converted into convex ones to make the asymptotic stability and H_∞ performance of the closed-looped system. The effectiveness of the proposed method is illustrated by a numerical example.

1. Introduction

As a well universal approximator, Takagi-Sugeno (T-S) fuzzy models provide an effective way to deal with nonlinearities and uncertainties in practical engineering areas since 1985 [1–3]. Some locally linear time-invariant systems in the form of IF-THEN rules are used to describe the prominent feature of T-S fuzzy models. During the past few years, the T-S fuzzy control systems have been studied extensively, and many significant advances have been achieved [4–9].

Both linear fuzzy models and affine fuzzy models are usually used to describe T-S Fuzzy systems [10–12]. Compared with linear fuzzy models, T-S affine fuzzy dynamic system with offset terms has much improved function approximation capabilities [3]. This has ignited enormous research activities in search of new methodologies to investigated affine T-S fuzzy systems (see [13, 14] and the references therein).

Most of the above results are mainly focused on state feedback control, which requires all state variables to be accessible. This may simplify the considered problem but cause the obtained results to be restrictive in engineering. Since only a partial information through a measured output can be available in many practical nonlinear control systems, many researchers focus their attention on the output stabilization [15, 16]. The main advantage of the static output feedback is the simplicity of its implementation and reduction of real time computational cost when implementing practical applications. Recently, static output feedback (SOF) problem for T-S fuzzy systems has attracted considerable attention of many researchers. Until now, most of the existing fuzzy SOF control results are obtained for linear fuzzy systems. Many efforts have been done to convert the nonconvex problem to convex problem, such as iterative linear matrix inequality (ILMI) approaches (see [17–19]) and linear matrix inequality (LMI) approaches [20–22].

Compared with linear fuzzy systems, the constant bias terms in affine fuzzy systems bring more challenge to the controller design. For continuous-time affine fuzzy systems, most existing results about controller design via output feedback are derived in the framework of bilinear matrix inequalities (BMIs) [12, 23, 24]. In [25], some sufficient conditions for the robust H_∞ output feedback controller design problem for discrete-time affine fuzzy systems are presented in the frameworks of LMIs. However, they supposed the transitions of both the plant and the controller could happen among all regions, having the controller with conservative designing conditions and heavy computations. To the best of our knowledge, few results have been developed about H_∞ controller design via static output feedback for continuous-time affine fuzzy systems based on output partition.

In this paper, the considered nonlinear system is described by affine fuzzy models, and the system outputs are chosen as premise variables. There is no doubt that this modeling approach may be coarse to represent a complex nonlinear system because less information can be used for modeling. However, it is more convenient and less conservative in the case where state variables are not all available, for example, static output feedback problem. Moreover, static output feedback is simply to be implemented. Based on such fuzzy models, the H_∞ static output feedback control design problem is discussed in this paper. A partition is presented to divide the output-space into different operating regions and interpolation regions. Then based on this partition, a new piecewise static output feedback controller with affine terms is proposed, whose design conditions are derived in the framework of LMIs by introducing additional slack matrix variables. Finally, a numerical example is given to illustrate the effectiveness and merits of the obtained method.

The rest of this paper is organized as follows. Section 2 introduces the preliminaries and design objective. The H_∞ static output feedback controller design problem is addressed in Section 3, and a solution of this problem is given in terms of LMIs. A numerical example is given in Section 4 to show the effectiveness of proposed method. Finally, conclusions are drawn in Section 5.

The following notations are used throughout the paper. The superscript "T" stands for matrix transposition. Then notation $X > 0$ where X is symmetric matrix indicates that X is positive definite. In block symmetric matrices or long matrix expressions, the symbol $*$ is used to represent a term that is induced by symmetry.

2. Preliminaries and Problem Statement

2.1. System Model. The following continuous-time affine T-S model is described by fuzzy IF-THEN rules, whose collection represents the approximation of a complex nonlinear system.

Plant Rule i:

if $y_1(t)$ is $M_{i1}, \ldots,$ and $y_s(t)$ is M_{is}, then

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \mu_i + B_{1i} u(t) + B_{2i} w(t), \\ z(t) &= C_1 x(t), \\ y(t) &= C_2 x(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state space vector, $u(t) \in R^p$ is the control input, $w(t) \in L_2[0,\infty)$ denotes the external disturbance, $y(t) = [y_1(t), \ldots, y_s(t)]$ is the measurable premise variable, and $z(t) \in R^q$ is the regulated output. A_i, μ_i, B_{1i}, B_{2i}, C_i, C_2 $(i = 1, 2, \ldots, r)$ are constant matrices with appropriate dimension. $M_{i1}, M_{i2}, \ldots, M_{is}$ are the fuzzy sets that are characterized by membership function; s is the number of premise variables; and r is the number of IF-THEN rules.

The overall affine fuzzy model achieved by fuzzy-blending of each individual plant rule (local model) is given by

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(y(t)) \left(A_i x(t) + \mu_i + B_{1i} u(t) + B_{2i} w(t)\right), \\ z(t) &= C_1 x(t), \\ y(t) &= C_2 x(t), \end{aligned} \tag{2}$$

where

$$w_i(y(t)) = \prod_{j=1}^{s} M_{ij}\left(y_j(t)\right). \tag{3}$$

Denote

$$h_i(y(t)) = \frac{w_i(y(t))}{\sum_{i=1}^{r} w_i(y(t))}, \tag{4}$$

and then

$$0 \le h_i(y(t)) \le 1, \quad \sum_{i=1}^{r} h_i(y(t)) = 1. \tag{5}$$

$M_{ij}(y_j(t))$ is the grade of membership of $y_j(t)$ in M_{ij}, and $h_i(y(t))$ is the normalized membership function.

Remark 1. Similar to [12], a constant affine term μ_i is included in (1) to approximate the original nonlinear system, which makes the function approximation capabilities of fuzzy systems improve substantially.

Assumption 2. For each i from 1 to r, assume that B_{1i} are of full column rank.

A decomposition method will be used to divide the output-space into different operating regions and interpolation regions. In a certain operating region, it follows $h_i(y(t)) = 1$ and $h_j(y(t)) = 0$ $(j \neq i)$, and then the system can be described by $\dot{x}(t) = A_i x(t) + \mu_i + B_{1i} u(t) + B_{2i} w(t)$. Between different operating regions, that is, interpolation regions, $0 < h_i(y(t)) < 1$; then a convex combination of different affine systems can describe the system.

Denote $\{\bar{S}_i\}_{i \in G} \subseteq R^s$ as a polyhedral partition of the output, and F as the set of region indexes. $G_0 \subseteq G$ and $G_1 \subseteq G$ are the set of indexes for region that contain or do not contain the origin, respectively. For each local region S_i, the set $K(i)$ contains the indexes for the system matrices used in the interpolation within that region. For operating regions, $K(i)$ only contains a single element. By a blending of $m \in K(i)$ subsystems, the global system in (2) is represented as follows in each cell:

$$\begin{aligned} \dot{x}(t) &= \widetilde{A}_i x(t) + \widetilde{\mu}_i + \widetilde{B}_{1i} u(t) + \widetilde{B}_{2i} w(t), \\ z(t) &= C_1 x(t), \\ y(t) &= C_2 x(t), \quad y(t) \in \bar{S}_i, \ i \in G, \end{aligned} \tag{6}$$

where

$$\begin{aligned}\widetilde{A}_i &= \sum_{m\in K(i)} h_m(y(t))\,A_m,\\ \widetilde{\mu}_i &= \sum_{m\in K(i)} h_m(y(t))\,\mu_m,\\ \widetilde{B}_{1i} &= \sum_{m\in K(i)} h_m(y(t))\,B_{1m},\\ \widetilde{B}_{2i} &= \sum_{m\in K(i)} h_m(y(t))\,B_{2m},\\ h_m(y(t)) &> 0, \qquad \sum_{m\in K(i)} h_m(y(t)) = 1.\end{aligned} \tag{7}$$

The range of $y(t)$ is represented by the following system of at most s linear inequalities while $y(t) \in \overline{S}_i$:

for $y_1(t)$, $y_1(t) \le \alpha_{i1}$ or $y_1(t) \ge \beta_{i1}$ or $\alpha_{i1} \le y_1(t) \le \beta_{i1}$,
for $y_2(t)$, $y_2(t) \le \alpha_{i2}$ or $y_2(t) \ge \beta_{i2}$ or $\alpha_{i2} \le y_2(t) \le \beta_{i2}$,
$\vdots$
for $y_s(t)$, $y_s(t) \le \alpha_{is}$ or $y_s(t) \ge \beta_{is}$ or $\alpha_{is} \le y_s(t) \le \beta_{is}$.

2.2. Problem Statement. Based on output partition, the following affine piecewise controller via static output feedback is chosen:

$$u(t) = K_i y(t) + \eta_i, \quad y(t) \in \overline{S}_i,\ i \in G, \tag{8}$$

where K_i, η_i are the controller gains to be designed later. Here we assume that the equilibrium point of closed-loop system is located at the origin and therefore we have $\eta_i = 0$ for all $i \in G_0$. Applying the piecewise static output feedback controller (8) to system (6), the resulting closed-loop system is given by the following:

$$\begin{aligned}\dot{x}(t) &= \left(\widetilde{A}_i + \widetilde{B}_{1i}K_iC_2\right)x(t) + \widetilde{B}_{1i}\eta_i + \widetilde{\mu}_i + \widetilde{B}_{2i}w(t),\\ z(t) &= C_1x(t), \qquad y(t) \in \overline{S}_i, \quad i \in G.\end{aligned} \tag{9}$$

Remark 3. Since in many practical control systems, the system sates $x(t)$ of system (6) are not completely available. Thus, the static output feedback controller is more applicable than that via state feedback controller.

Problem 4. For the closed-loop system (9), design a piecewise affine H_∞ via static output feedback in the form of (8), such that the following requirements are satisfied.

(1) The closed-loop system (9) is asymptotically stable when $w(t) \equiv 0$.

(2) Under zero initial conditions, the L_2-norm of the operator from disturbance $w(t)$ to the controlled output $z(t)$ is less than γ,

$$\|z(t)\|_2 < \gamma\,\|w(t)\|_2, \tag{10}$$

for any external disturbance $w(t) \in L_2$.

3. Main Results

In this section, we will give some sufficient conditions to solve H_∞ static output feedback controller design problem for the closed-loop system (9).

The following lemma will be used to derive the main results of this paper.

Lemma 5. *The following two statements are equivalent for the closed-loop affine fuzzy system (9) being asymptotically stable and satisfying an H_∞ performance index γ.*

(1) *If there exist a symmetric matrix $P > 0$ and scalars $\tau_{iq} \ge 0$ $(q = 1, 2, \ldots, n)$ such that*

for $i \in G_0$, $m \in K(i)$,

$$\begin{bmatrix} \Upsilon_{im} & PB_{2m} \\ B_{2m}^T P & -\gamma^2 I \end{bmatrix} < 0, \tag{11}$$

where $\Upsilon_{im} = (A_m + B_{1m}K_iC_2)^T P + P(A_m + B_{1m}K_iC_2) + C_1^T C_1$,

and for $i \in G_1$, $m \in K(i)$,

$$\begin{bmatrix} \Delta_{im} & PB_{2m} & P\mu_m - \sum_{q=1}^{s} \tau_{iq} C_2^T u_{iq} \\ * & -\gamma^2 I & 0 \\ * & * & -\sum_{q=1}^{s} \tau_{iq} v_{iq} \end{bmatrix} < 0, \tag{12}$$

where $\Delta_{im} = \Upsilon_{im} - \sum_{q=1}^{s} \tau_{iq} C_2^T T_{iq} C_2$ and T_{iq}, u_{iq}, v_{iq} are similar to those in [12] such that $\Gamma_{iq}(y(t)) \equiv y^T(t)T_{iq}y(t) + 2u_{iq}u)y(t) + v_{iq} < 0$ while $y(t) \in \overline{S}_i$, $i \in G$.

(2) *For large enough positive constants α, k, if there exist $P > 0$, matrices F_i, W_i, and scalars $\tau_{ij} \ge 0$ $(j = 1, 2, \ldots, n)$ such that*

for $i \in G_0$, $m \in K(i)$

$$\begin{bmatrix} \Phi - 2\alpha\overline{P}_1 & \overline{P}_1 + (\Pi_{im} + \alpha I)^T W_i \\ \overline{P}_1 + W_i^T(\Pi_{im} + \alpha I) & -W_i - W_i^T \end{bmatrix} < 0, \tag{13}$$

and for $i \in G_1$, $m \in K(i)$

$$\begin{bmatrix} \Phi_i - 2k\overline{P}_2 & \overline{P}_2 + (\Gamma_{im} + kI)^T F_i \\ \overline{P}_2 + F_i^T(\Gamma_{im} + kI) & -F_i - F_i^T \end{bmatrix} < 0, \tag{14}$$

where

$$\Phi = \begin{bmatrix} C_1^T C_1 & 0 \\ 0 & -\gamma^2 I \end{bmatrix},$$

$$\Phi_i = \begin{bmatrix} \Phi_{i11} & 0 & -\sum_{q=1}^{n} \tau_{iq} C_2^T u_{iq} \\ * & -\gamma^2 I & 0 \\ * & * & -\sum_{q=1}^{s} \tau_{iq} v_{iq} \end{bmatrix},$$

$$\Phi_{i11} = C_1^T C_1 - \sum_{q=1}^{s} \tau_{iq} C_2^T T_{iq} C_2,$$

$$\overline{P}_1 = \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix}, \qquad \overline{P}_2 = \begin{bmatrix} P & 0 & 0 \\ * & I & 0 \\ * & * & 1 \end{bmatrix}, \tag{15}$$

$$\Pi_{im} = \begin{bmatrix} A_m + B_{1m} K_i C_2 & B_{2m} \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_{im} = \begin{bmatrix} A_m + B_{1m} K_i C_2 & B_{2m} & \mu_m + B_{1m} \eta_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Proof. Firstly, we prove that if the statement (1) holds, then the closed-loop affine fuzzy system (9) is asymptotically stable and satisfying an H_∞ performance index γ.

Chose a Lyapunov function as

$$V(x(t)) = x^T(t) P x(t). \tag{16}$$

Then from the derivative of $V(t)$ along system (9), it follows

$$\begin{aligned} &\dot{V}(x(t)) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) \\ &= \begin{bmatrix} x(t) \\ w(t) \\ 1 \end{bmatrix}^T \begin{bmatrix} \Delta_i^1 & P\widetilde{B}_{2i} & P\widetilde{\mu}_i \\ * & -\gamma^2 I & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ 1 \end{bmatrix} \end{aligned} \tag{17}$$

with $\Delta_i^1 = (\widetilde{A}_i + \widetilde{B}_{1i} K_i C_2)^T P + P(\widetilde{A}_i + \widetilde{B}_{1i} K_i C_2) + C_1^T C_1$.

For $i \in G_1$, it can be easily seen that the following inequality implies $\dot{V}(x(t)) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0$. Consider

$$\begin{bmatrix} \Delta_i^2 & P\widetilde{B}_{2i} & P\widetilde{\mu}_i - \sum_{q=1}^{s} \tau_{iq} C_2^T u_{iq} \\ * & -\gamma^2 I & 0 \\ * & * & -\sum_{q=1}^{s} \tau_{iq} v_{iq} \end{bmatrix} < 0, \tag{18}$$

where $\Delta_i^2 = (\widetilde{A}_i + \widetilde{B}_{1i} K_i C_2)^T P + P(\widetilde{A}_i + \widetilde{B}_{1i} K_i C_2) + C_1^T C_1 - \sum_{q=1}^{s} \tau_{iq} C_2^T T_{iq} C_2$.

Then condition (12) implies (18), by expanding the fuzzy-basis functions. For the case $i \in G_0$, condition (11) can be easily obtained without using the S-procedure, where $\widetilde{\mu}_i = 0$.

Secondly, (1) $\Rightarrow$ (2): for the case $i \in G_0$ if (11) is satisfied, then rewrite (11) as $\Phi + \overline{P}_1 \Pi_{im} + \Pi_{im}^T \overline{P}_1 < 0$. It implies that there exists a sufficiently large positive scalar α_1 such that $\forall \alpha > \alpha_1$:

$$\Phi + \overline{P}_1 \Pi_{im} + \Pi_{im}^T \overline{P}_1 + \frac{1}{2\alpha} \Pi_{im}^T \overline{P}_1 \Pi_{im} < 0. \tag{19}$$

The above-mentioned inequality can be rewritten as

$$\begin{aligned} &\Phi - 2\alpha \overline{P}_1 + \frac{\alpha}{2} \left(\overline{P}_1 + \frac{1}{\alpha} (\Pi_{im} + \alpha I)^T \overline{P}_1 \right) \overline{P}_1^{-1} \\ &\quad \times \left(\overline{P}_1 + \frac{1}{\alpha} \overline{P}_1 (\Pi_{im} + \alpha I) \right) < 0. \end{aligned} \tag{20}$$

Applying Schur complement, we get

$$\begin{bmatrix} \Phi - 2\alpha \overline{P}_1 & \overline{P}_1 + \frac{1}{\alpha} (\Pi_{im} + \alpha I)^T \overline{P}_1 \\ \overline{P}_1 + \frac{1}{\alpha} \overline{P}_1 (\Pi_{im} + \alpha I) & -\frac{2}{\alpha} \overline{P}_1 \end{bmatrix} < 0. \tag{21}$$

By taking $W_i = (1/\alpha) \overline{P}_1$, then condition (13) is obtained. In the same way, if (12) is satisfied for the case $i \in G_1$, we can still prove (13) holds.

(2) $\Rightarrow$ (1) For the case $i \in G_0$, multiplying (13) by $[I \;\; (\Pi_{im} + \alpha I)^T]$ and its transpose from the left side and the right one implies (11) holds. Similarly for the case of $i \in G_1$, the conclusion that (14) implies (12) holds.

Then the proof is completed. □

Notice that conditions (13) and (14) in Lemma 5 are nonconvex. Next theorem will give some convex conditions for the affine H_∞ controller design problem via static output feedback for (6).

Theorem 6. *Consider the affine fuzzy system (6) with the designed static output feedback controller (8), for large enough positive constants α, k, if there exist $P > 0$, matrices W_i, F_{i11}, F_{i21}, F_{i22}, F_{i31}, F_{i32}, and F_{i33} with appropriate dimensions, where $F_{i11} = \begin{bmatrix} F_{i1} & 0 \\ F_{i2} & F_{i3} \end{bmatrix}$, $W_i = \begin{bmatrix} W_{i1} & 0 \\ W_{i2} & W_{i3} \end{bmatrix}$, $Y_i \in R^{m \times s}$, $V_i \in R^{m \times 1}$, and scalars $\tau_{iq} \geq 0$ $(q = 1, 2, \ldots, n)$ such that*

for $i \in G_0$, $m \in K(i)$,

$$\begin{bmatrix} C_1^T C_1 - 2\alpha P & * & * & * \\ 0 & -\gamma^2 I - 2\alpha I & * & * \\ \Lambda_{31}^{im} & W_{i1}^T T_m B_{2m} + \alpha W_{i2}^T & -T_m^T W_{i1} - W_{i1}^T T_m & * \\ 0 & I + \alpha W_{i3}^T & -W_{i2} & -W_{i3} - W_{i3}^T \end{bmatrix} < 0, \tag{22}$$

and for $i \in G_1$, $m \in K(i)$,

$$\begin{bmatrix} \xi_{11}^i & * & * & * & * & * \\ 0 & -\gamma^2 I - 2kI & * & * & * & * \\ \xi_{31}^i & 0 & \xi_{33}^i & * & * & * \\ \xi_{41}^{im} & \xi_{42}^{im} & \xi_{43}^{im} & -T_m^T F_{i11} - F_{i11}^T T_m & \xi_{41}^{im} & * \\ 0 & I + kF_{i22}^T & kF_{i32}^T & -F_{i21} & -F_{i22} - F_{i22}^T & * \\ 0 & 0 & 1 + kF_{i33}^T & -F_{i31} & -F_{i32} & -F_{i33} - F_{i33}^T \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} \xi_{11}^i &= C_1^T C_1 - \sum_{q=1}^{n} \tau_{iq} C_2^T T_{iq} C_2 - 2kP, \\ \xi_{31}^i &= -\sum_{q=1}^{n} \tau_{iq} u_{iq}^T C_2, \qquad \xi_{33}^i = -\sum_{q=1}^{n} \tau_{iq} v_{iq} - 2k, \\ \xi_{41}^{im} &= P + kF_{i11}^T T_m + F_{i11}^T T_m A_m + \begin{bmatrix} Y_i \\ 0 \end{bmatrix} C_2, \\ \xi_{43}^{im} &= F_{i11}^T T_m \mu_m + \begin{bmatrix} V_i \\ 0 \end{bmatrix} + kF_{i31}^T, \\ \xi_{42}^{im} &= F_{i11}^T T_m B_{2m} + kF_{i21}^T, \\ \Lambda_{31}^{im} &= P + kW_{i1}^T T_m + W_{i1}^T T_m A_m + \begin{bmatrix} Y_i \\ 0 \end{bmatrix} C_2. \end{aligned} \tag{24}$$

The closed-loop system (9) is asymptotically stable with a H_∞ *performance index* γ. *Moreover, the controller gains are given by*

$$\begin{aligned} K_i &= W_{i1}^{-T} Y_i, \quad i \in G_0; \\ K_i &= F_{i1}^{-T} Y_i, \quad \eta_i = F_{i1}^{-T} V_i, \quad i \in G_1. \end{aligned} \tag{25}$$

Proof. Let W_i and F_i have the following form:

$$\begin{aligned} W_i &= \begin{bmatrix} \sum_{l \in K(i)} \theta_l T_l^T W_{i11} & 0 \\ W_{i21} & W_{i22} \end{bmatrix}, \\ F_i &= \begin{bmatrix} \sum_{l \in K(i)} \theta_l T_l^T F_{i11} & 0 & 0 \\ F_{i21} & F_{i22} & F_{i23} \\ F_{i31} & F_{i32} & F_{i33} \end{bmatrix}, \end{aligned} \tag{26}$$

with

$$W_{i11} = \begin{bmatrix} W_{i1} & 0 \\ W_{i2} & W_{i3} \end{bmatrix}, \qquad F_{i11} = \begin{bmatrix} F_{i1} & 0 \\ F_{i2} & F_{i3} \end{bmatrix}. \tag{27}$$

Based on Assumption 2, we can take some invertible matrices $T_i = \begin{bmatrix} (B_{1i}^T B_{1i})^{-1} B_{1i}^T \\ \Omega \end{bmatrix}$ for $i = 1, 2, \ldots, r$ such that

$$T_i B_{1i} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \tag{28}$$

Here the rows of Ω are the ones which are mutually independent and perpendicular to the columns of B_{1i}. It can be easily seen that Ω is not unique. Denote that function θ_l is defined as

$$\theta_l = \begin{cases} 1, & l = m, \\ 0, & \text{else}. \end{cases} \tag{29}$$

After some matrix computation in (13) and (14), (22) and (23) are easily obtained. □

Remark 7. When α and k in (22) and (23) are set to be fixed parameters, the problems become convex and can be solved by employing the LMI Toolbox. To find the optimal values of corresponding parameters, in this paper, we will first solve the feasibility problem of LMIs in (22) and (23) by using LMI Toolbox under a set of initial scaling parameters. Then, for the optimization problem, apply a numerical optimization algorithm, such as the program *fminsearch*, and then a locally convergent solution to the problem is obtained.

When $\eta_i \equiv 0$ in (8), a linear static output feedback controller can be given as follows:

$$u(t) = K_i y(t), \quad y(t) \in \overline{S}_i, \ i \in G. \tag{30}$$

Based on Theorem 6, the following corollary can be easily obtained.

Corollary 8. *Consider the affine fuzzy system (6) with the designed static output feedback controller (30), for large enough positive constants α, k, if there exist $P > 0$, matrices W_i, F_{i11}, F_{i21}, F_{i22}, F_{i31}, F_{i32}, and F_{i33}, where $F_{i11} = \begin{bmatrix} F_{i1} & 0 \\ F_{i2} & F_{i3} \end{bmatrix}$, $W_i = \begin{bmatrix} W_{i1} & 0 \\ W_{i2} & W_{i3} \end{bmatrix}$, $Y_i \in R^{m\times s}$, and scalars $\tau_{iq} \geq 0$ $(q = 1, 2, \ldots, n)$ such that*

$$\text{for } i \in G_0,\ m \in K(i),$$

$$\begin{bmatrix} C_1^T C_1 - 2\alpha P & * & * & * \\ 0 & -\gamma^2 I - 2\alpha I & * & * \\ \Lambda_{31}^{im} & W_{i1}^T T_m B_{2m} + \alpha W_{i2}^T & -T_m^T W_{i1} - W_{i1}^T T_m & * \\ 0 & I + \alpha W_{i3}^T & -W_{i2} & -W_{i3} - W_{i3}^T \end{bmatrix} < 0, \tag{31}$$

and for $i \in G_1$, $m \in K(i)$,

$$\begin{bmatrix} \xi_{11}^i & * & * & * & * & * \\ 0 & -\gamma^2 I - 2kI & * & * & * & * \\ \xi_{31}^i & 0 & \xi_{33}^i & * & * & \\ \xi_{41}^{im} & \xi_{42}^{im} & \chi_{43}^{im} & -T_m^T F_{i11} - F_{i11}^T T_m & \xi_{41}^{im} & * \\ 0 & I + kF_{i22}^T & kF_{i32}^T & -F_{i21} & -F_{i22} - F_{i22}^T & * \\ 0 & 0 & 1 + kF_{i33}^T & -F_{i31} & -F_{i32} & -F_{i33} - F_{i33}^T \end{bmatrix} < 0, \tag{32}$$

where $\chi_{43}^{im} = F_{i11}^T T_m \mu_m + kF_{i31}^T$.

The corresponding closed-loop system is asymptotically stable with a H_∞ performance index γ. Moreover, The controller gains are given by

$$\begin{aligned} K_i &= W_{i1}^{-T} Y_i, \quad i \in G_0; \\ K_i &= F_{i1}^{-T} Y_i, \quad i \in G_1. \end{aligned} \tag{33}$$

Remark 9. Notice that the results given in this paper are obtained based on the assumption that the input matrix B_{1i} is of full column rank, which leads to much wider scope of the applicability than the common input matrix. Nevertheless, if there exist invertible matrices T_i, $i = 1, 2, \ldots, r$, such that

$$T_i B_{1i} = \begin{bmatrix} I_{n_c \times n_c} & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{for } i = 1, 2, \ldots, r, \tag{34}$$

where $n_c < m$, which means that the input matrices are not full column rank, then a novel piecewise affine state feedback controller is desirable with the form

$$u(t) = \widetilde{K}_i y(t) + \widetilde{\sigma}_i, \quad y(t) \in \overline{S}_i, \ i \in F, \tag{35}$$

where $\widetilde{K}_i = \begin{bmatrix} K_i^{n_c} \\ 0 \end{bmatrix}$, $\widetilde{\sigma}_i = \begin{bmatrix} \sigma_i^{n_c} \\ 0 \end{bmatrix}$. The corresponding static output feedback control synthesis method is proposed as follows.

Corollary 10. *The continuous affine fuzzy system (6) is quadratically stabilizable at large by the piecewise controller (35) if there exist $P > 0$, matrices W_i, F_{i11}, F_{i21}, F_{i22}, F_{i31}, F_{i32}, F_{i33} with appropriate dimensions, where $F_{i11} = \begin{bmatrix} F_{i1} & 0 \\ F_{i2} & F_{i3} \end{bmatrix}$, $W_i = \begin{bmatrix} W_{i1} & 0 \\ W_{i2} & W_{i3} \end{bmatrix}$, and $Y_i \in R^{n_c \times n}$, $V_i \in R^{n_c \times 1}$, and scalars $\tau_{ij} \geq 0$ $(i \in F; j = 1, 2, \ldots, n)$ such that for given positive constants α, k, the linear matrix inequalities in (31) and (32) are satisfied, and the local gains of the fuzzy controller are given by*

$$\begin{aligned} K_i^{n_c} &= W_{i1}^{-T} Y_i, \quad i \in F_0; \\ K_i^{n_c} &= F_{i11}^{-T} Y_i, \quad \sigma_i^{n_c} = F_{i11}^{-T} V_i, \quad i \in F_1. \end{aligned} \tag{36}$$

4. Numerical Example

In order to demonstrate the effectiveness and merits of the proposed piecewise affine static output feedback controller over the existing results, the following numerical example is considered.

R_i: if $y_1(t)$ is M_i, then

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \mu_i + B_{1i} u(t) + B_{2i} w(t); \quad i = 1, 2, 3, \\ z(t) &= C_1 x(t); \\ y(t) &= C_2 x(t), \end{aligned} \tag{37}$$

where

$$\begin{aligned} A_1 = A_3 &= \begin{bmatrix} 1.3017 & 0.15 & 0 \\ -1.5107 & -0.5789 & -1.3171 \\ 2.0211 & 0.5320 & -0.2348 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1.8183 & 1 & -0.5362 \\ -1.2883 & -1 & -1 \\ 2.9549 & 1 & -0.5326 \end{bmatrix}, \\ \mu_1 = -\mu_3 &= \begin{bmatrix} -0.5031 \\ 1.0347 \\ -1.5836 \end{bmatrix}, \qquad \mu_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\ B_{11} = B_{13} &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.5 & 1 & 0 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.5 & 0.1 & 0 \\ 1 & 0.5 & 1 \end{bmatrix}, \\ B_{21} &= \begin{bmatrix} 0.2 \\ 0 \\ 0.1 \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix}, \qquad B_{23} = \begin{bmatrix} 0.3 \\ 0.1 \\ 1 \end{bmatrix}. \end{aligned} \tag{38}$$

Here, the system state is assumed to be not completely available for feedback and thus cannot be chosen as the premise of the fuzzy model. The membership functions M_1, M_2, and M_3 are depicted in Figure 1.

Due to the fact that system state cannot be used for controlling the system, the existing controller design results for T-S affine fuzzy systems in [14, 24] are not suitable for this class of fuzzy systems. Besides, the existing output feedback controller design method in [25] is proposed for discrete-time affine fuzzy systems and not suitable for continuous-time affine fuzzy systems.

In this simulation, both the piecewise affine static output feedback controller (8) and the piecewise linear static output feedback controller (30) are used to guarantee the resulting closed-loop system asymptotically stable with H_∞ performance γ under $w(t) = 15e^{-0.1t}\sin(20\pi t)$ and zero initial conditions. By applying Theorem 6 and the *fminsearch*, the controller gains of piecewise static output feedback controller are obtained as follows:

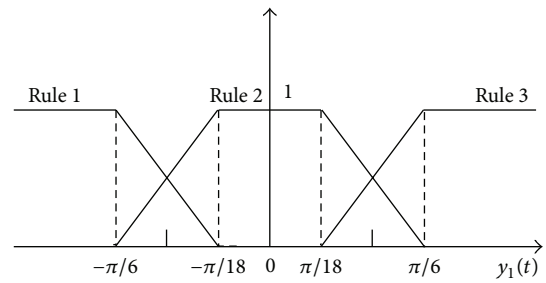

Figure 1: Membership functions.

Table 1: H_∞ performance γ.

Approach	Performance
Theorem 6	7.6819
Corollary 8	9.4937

$$\begin{aligned} K_1 &= \begin{bmatrix} -14.2556 & -0.0095 \end{bmatrix}, & \sigma_1 &= 2.1926, \\ K_2 &= \begin{bmatrix} -27.7366 & -0.9305 \end{bmatrix}, & \sigma_2 &= 2.1872, \\ K_3 &= \begin{bmatrix} -22.7194 & -0.9619 \end{bmatrix}, & \sigma_3 &= 0, \\ K_4 &= \begin{bmatrix} -27.3177 & -0.9648 \end{bmatrix}, & \sigma_4 &= -2.3762, \\ K_5 &= \begin{bmatrix} -15.9958 & -0.1540 \end{bmatrix}, & \sigma_5 &= -2.4800. \end{aligned} \tag{39}$$

Table 1 shows the minimum H_∞ performance γ based on the two abovementioned approaches (Theorem 6 and Corollary 8), respectively, with scaling parameters $\alpha = 1.2559$ and $k = 0.8062$. It can be seen that the proposed piecewise affine static output feedback controller (8) can guarantee a smaller optimal H_∞ performance index over the piecewise linear static output feedback controller (30). The state responses curves of the closed-loop system via the proposed static output feedback controller design method are given in Figures 2–4. The square root of ratio of the regulated output energy to the disturbance input noise energy is shown in Figure 5.

5. Conclusion

This paper addressed the H_∞ static output feedback controller design problem for a class of continuous-time affine fuzzy systems. Based on a partition of output-space technique, a new piecewise controller with affine terms via static output feedback is designed. The corresponding design conditions to guarantee the closed-loop system asymptotic stable with H_∞ performance are derived in a dilated linear matrix inequality (LMI) characterization. Finally, an example has been given to illustrate the effectiveness of the proposed approach.

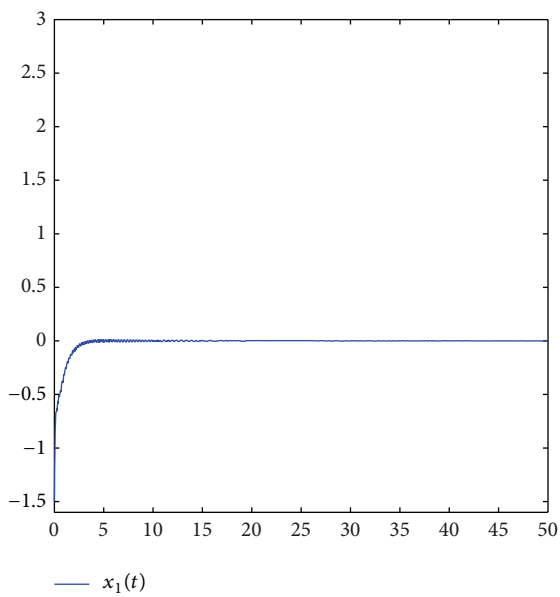

Figure 2: Response curve of $x_1(t)$.

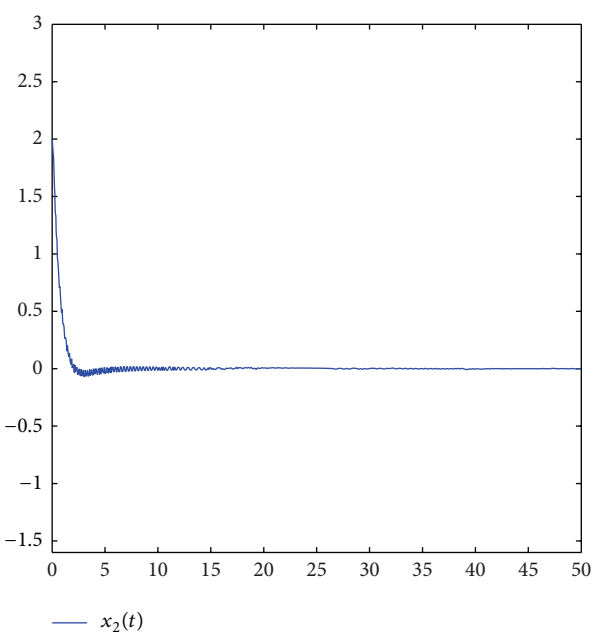

Figure 3: Response curve of $x_2(t)$.

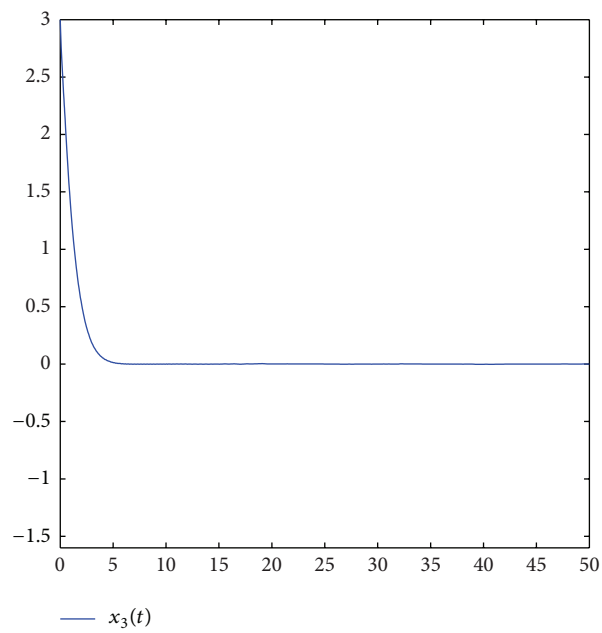

Figure 4: Response curve of $x_3(t)$.

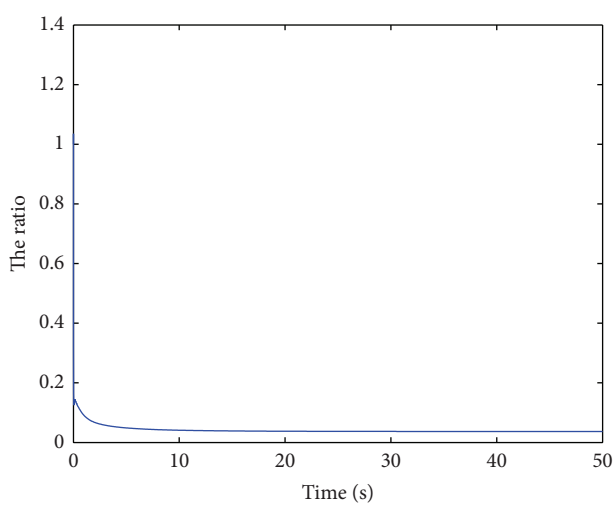

Figure 5: The ratio of $\sqrt{\int_0^\infty z^T(t)z(t)dt / \int_0^\infty w^T(t)w(t)dt}$.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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