Research Article

Exponential Stabilization of a Class of Time-Varying Delay Systems with Nonlinear Perturbations

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This paper addresses the problem of exponential stabilization of a class of time-varying delay systems with nonlinear perturbations. These perturbations are related not only with current state $x(t)$ and the delayed state $x(t - h(t))$ but also with $\beta(t)$, where $\beta(t)$ is a continuous function defined on $[0, +\infty)$. With the delay interval divided into two equidistant subintervals, a novel Lyapunov functional is introduced, and several new exponential stabilization criteria are derived in terms of linear matrix inequalities (LMIs) by employing reciprocally convex approach. Two examples are given to illustrate the effectiveness of the main results.

1. Introduction

Time delay is commonly encountered in various physical and engineering systems such as aircraft, biological systems, and networked control systems. Since the existence of time delays causes poor performance, oscillation, or even instability, it is very important to investigate stability analysis for systems with time delays before designing control systems. On the other hand, the systems almost present some uncertainties because it is not easy to obtain an exact mathematical model due to environmental noise, uncertain or slowly varying parameters, and so forth. Therefore, considerable amounts of efforts have been done to the stability and stabilization of time-delay systems and time-delay systems with nonlinear perturbations; see, for example, [1–25] and the references cited therein.

Recently, Zhang et al. [12] considered interval time-varying delay systems and obtained some delay-dependent conditions by employing Finsler's lemma. Combining the descriptor model transformation and the integral inequality method, Han [3] investigated the robust stability of linear systems with time-varying delay and nonlinear perturbations and obtained several improved stability conditions. On

the basis of free weighting matrices technique, robust stabilization criteria for neutral systems with nonlinear perturbations were reported in [9]. Wang et al. [6] introduced a new parameter in the Lyapunov functional for the time-varying delay systems with nonlinear perturbations and obtained less conservative results, whereas the range of the time delays considered in the paper was assumed from zero to an upper bound. Note that the stability investigated in the above-mentioned papers was primarily focused on asymptotic stability. Using delay decomposition method and Finsler's lemma, Liu et al. [24] studied the exponential stability of neutral systems with interval time-varying delays and nonlinear perturbations. So far, there are few articles concerning the problem of exponential stabilization of time-varying delay systems with nonlinear perturbations. Thuan et al. [16] provided a detailed analysis for the problem of designing state feedback controllers to exponential stabilization of time-delay systems with nonlinear perturbations by using the integral inequality method and constructing a Lyapunov functional containing the triple integral terms. However, there still exists a gap for reducing both the conservatism and the number of decision variables.

In this paper, we study the exponential stabilization of a class of time-delay systems with nonlinear perturbations. The main contributions of this paper can be summarized as follows: (i) a novel Lyapunov functional containing the center point of time-delay interval is constructed; (ii) compared with the systems studied in [3, 8, 16], the nonlinear perturbations of (1) are related not only with the current state $x(t)$ and the delayed state $x(t - h(t))$ but also with $\beta(t)$, where $\beta(t)$ is a continuous function satisfying $\int_0^{+\infty} \beta^2(s) e^{2\alpha s} ds < +\infty$ and α is a positive constant; new sufficient conditions are obtained that ensure the stability of a closed-loop system, which extend and improve the main results of [16]. Furthermore, the stabilization conditions are shown to be less conservative than those reported in Zhang et al. [12] when there are no nonlinear perturbations in the system. Finally, two numerical examples are presented to demonstrate the effectiveness and advantages of the main results.

Notation. Throughout the paper, R^n denotes the n -dimensional Euclidean space with vector norm $\|\cdot\|$, and $R^{n \times m}$ is the set of all $n \times m$ -dimensional real matrices. I denotes the identity matrix of appropriate dimensions, and the superscript “ T ” stands for matrix transposition. The notation $P > 0$ (≥ 0) means that P is symmetric and positive (semipositive) definite. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of A , respectively. In addition, in symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry.

2. Problem Description and Preliminaries

Consider the following system with a nonlinear perturbation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Dx(t - h(t)) \\ &\quad + f(t, x(t), x(t - h(t))) + Bu(t), \quad t \geq 0, \quad (1) \\ x(t) &= \phi(t), \quad t \in [-h_2, 0], \end{aligned}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $A, D \in R^{n \times n}$, and $B \in R^{n \times m}$, $\phi(t) \in C([-h_2, 0], R^n)$ with $\|\phi\| = \sup_{t \in [-h_2, 0]} \{\|\phi(t)\|, \|\dot{\phi}(t)\|\}$, where $C([-h_2, 0], R^n)$ is the Banach space of continuous functions. The delay $h(t)$ is time-varying and satisfies

$$0 \leq h_1 \leq h(t) \leq h_2, \quad \dot{h}(t) \leq \mu, \quad (2)$$

where μ is a positive constant and h_1 and h_2 are constants representing the lower and upper bounds of the delay, respectively. $f(t, x(t), x(t - h(t)))$ is a nonlinear perturbation satisfying

$$\begin{aligned} &\|f(t, x(t), x(t - h(t)))\| \\ &\leq a \|x(t)\| + b \|x(t - h(t))\| + \beta(t), \end{aligned} \quad (3)$$

where a and b are positive scalars and $\beta(t)$ satisfies $\int_0^{+\infty} \beta^2(s) e^{2\alpha s} ds < +\infty$.

Remark 1. In [3, 8, 16], the authors assumed that the nonlinear terms satisfy

$$\begin{aligned} f^T(t, x(t)) f(t, x(t)) &\leq a^2 x^T(t) F^T F x(t), \\ g^T(t, x(t - h(t))) g(t, x(t - h(t))) &\leq b^2 x^T(t - h(t)) G^T G x(t - h(t)), \end{aligned} \quad (4)$$

where F and G are constant matrices and a and b are positive scalars. It is obvious that the assumptions on the nonlinear terms given in (2) and (3) are more general.

The following definitions and lemmas will be used for providing the main results in the sequel.

Definition 2. Given a scalar $\alpha > 0$: system (1) with $u(t) = 0$ is α -stable if there exists a positive number $\gamma > 0$ such that every solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq \gamma e^{-\alpha t}, \quad t \geq 0. \quad (5)$$

Definition 3. Given a scalar $\alpha > 0$: system (1) is α -stabilizable if there exists a state feedback control $u(t) = Kx(t)$ such that the closed-loop system

$$\begin{aligned} \dot{x}(t) &= (A + BK)x(t) + Dx(t - h(t)) \\ &\quad + f(t, x(t), x(t - h(t))), \quad t \geq 0, \quad (6) \\ x(t) &= \phi(t), \quad t \in [-h_2, 0] \end{aligned}$$

is α -stable.

Lemma 4 (see [25]). For any $x, y \in R^n$ and a positive symmetric definite matrix $N \in R^{n \times n}$,

$$\pm 2y^T x \leq x^T N^{-1} x + y^T N y. \quad (7)$$

Lemma 5 (lower bound lemma for reciprocal convexity; see [20]). Let $f_1, f_2, \dots, f_N : R^m \rightarrow R$ have positive values in an open subset D of R^m . Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t) \quad (8)$$

subject to

$$\left\{ g_{i,j} : R^m \rightarrow R, \quad g_{j,i}(t) \triangleq g_{i,j}(t), \quad \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}. \quad (9)$$

3. Main Results

We use the following notation for the convenience:

$$\begin{aligned}
 \bar{h} &= \frac{h_1 + h_2}{2}, \\
 \delta &= \frac{h_2 - h_1}{2}, \\
 \lambda &= \lambda_{\min}(P^{-1}), \\
 \Gamma &= \lambda_{\max}(P^{-1}) + h_1 \lambda_{\max}(P^{-1}Q_1P^{-1}) + (h_2 - h_1) \lambda_{\max}(P^{-1}Q_2P^{-1}) + (h_2 - h_1) \lambda_{\max} \begin{bmatrix} P^{-1}R_{11}P^{-1} & P^{-1}R_{12}P^{-1} \\ * & P^{-1}R_{22}P^{-1} \end{bmatrix} \\
 &\quad + \frac{(3h_1 + h_2)(h_2 - h_1)^2}{16} \lambda_{\max}(P^{-1}S_1P^{-1}) + \frac{(h_1 + 3h_2)(h_2 - h_1)^2}{16} \lambda_{\max}(P^{-1}S_2P^{-1}) + \frac{h_1^3}{2} \lambda_{\max}(P^{-1}S_3P^{-1}), \\
 \Lambda &= \Gamma \|\phi\|^2 + 6 \int_0^{+\infty} \beta^2(s) e^{2\alpha s} ds, \\
 \xi^T(t) &= [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h_1) \quad x^T(t-\bar{h}) \quad x^T(t-h_2) \quad x^T(t)].
 \end{aligned} \tag{10}$$

The following theorem presents an exponential stabilization condition for (1).

Theorem 6. Let $\alpha > 0$ and assume that conditions (2) and (3) are satisfied. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $S_i > 0$ ($i = 1, 2, 3$), $\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$, M_1 , M_2 , and Y such that the following LMIs hold

$$\begin{aligned}
 \Sigma_1 &= \begin{bmatrix} \Sigma_{11} & DP & \Sigma_{13} & 0 & 0 & PA^T + Y^T B^T & a^2 P & 0 \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & 0 & PD^T & 0 & b^2 P \\ * & * & \Sigma_{33} & \Sigma_{34} & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{44} & \Sigma_{45} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{6}a^2 I & 0 \\ * & * & * & * & * & * & 0 & -\frac{1}{6}b^2 I \end{bmatrix} \tag{11}
 \end{aligned}$$

< 0 ,

$$\begin{aligned}
 \Sigma_2 &= \begin{bmatrix} \Sigma_{11} & DP & \Sigma_{13} & 0 & 0 & PA^T + Y^T B^T & a^2 P & 0 \\ * & \bar{\Sigma}_{22} & 0 & \bar{\Sigma}_{24} & \bar{\Sigma}_{25} & PD^T & 0 & b^2 P \\ * & * & \Sigma_{33} & \bar{\Sigma}_{34} & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{44} & \bar{\Sigma}_{45} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{6}a^2 I & 0 \\ * & * & * & * & * & * & 0 & -\frac{1}{6}b^2 I \end{bmatrix} \tag{12}
 \end{aligned}$$

< 0 ,

$$\begin{bmatrix} S_i & M_i \\ * & S_i \end{bmatrix} \geq 0 \quad (i = 1, 2), \tag{13}$$

where

$$\begin{aligned}
 \Sigma_{11} &= AP + PA^T + BY + Y^T B^T + 2\alpha P + I + Q_1 \\
 &\quad - e^{-2\alpha h_1} S_3, \\
 \Sigma_{22} &= -(1 - \mu) e^{-2\alpha h_2} Q_2 + e^{-2\alpha \bar{h}} M_1 + e^{-2\alpha \bar{h}} M_1^T \\
 &\quad - 2e^{-2\alpha \bar{h}} S_1, \\
 \Sigma_{33} &= e^{-2\alpha h_1} Q_2 + e^{-2\alpha h_1} R_{11} - e^{-2\alpha h_1} Q_1 - e^{-2\alpha \bar{h}} S_1 \\
 &\quad - e^{-2\alpha h_1} S_3, \\
 \Sigma_{44} &= e^{-2\alpha h_1} R_{22} - e^{-2\alpha \bar{h}} R_{11} - e^{-2\alpha \bar{h}} S_1 - e^{-2\alpha h_2} S_2, \\
 \Sigma_{55} &= -e^{-2\alpha \bar{h}} R_{22} - e^{-2\alpha h_2} S_2, \\
 \Sigma_{66} &= h_1^2 S_3 + \delta^2 S_2 + \delta^2 S_1 + I - 2P, \\
 \Sigma_{13} &= e^{-2\alpha h_1} S_3, \\
 \Sigma_{23} &= e^{-2\alpha \bar{h}} (S_1 - M_1), \\
 \Sigma_{24} &= e^{-2\alpha \bar{h}} (S_1 - M_1^T), \\
 \Sigma_{34} &= e^{-2\alpha h_1} R_{12} + e^{-2\alpha \bar{h}} M_1^T, \\
 \Sigma_{45} &= -e^{-2\alpha \bar{h}} R_{12} + e^{-2\alpha h_2} S_2, \\
 \bar{\Sigma}_{22} &= -(1 - \mu) e^{-2\alpha h_2} Q_2 + e^{-2\alpha h_2} M_2 + e^{-2\alpha h_2} M_2^T \\
 &\quad - 2e^{-2\alpha h_2} S_2,
 \end{aligned}$$

$$\begin{aligned}
\bar{\Sigma}_{24} &= e^{-2\alpha h_2} (S_2 - M_2), \\
\bar{\Sigma}_{25} &= e^{-2\alpha h_2} (S_2 - M_2^T), \\
\bar{\Sigma}_{34} &= e^{-2\alpha h_1} R_{12} + e^{-2\alpha \bar{h}} S_1, \\
\bar{\Sigma}_{45} &= -e^{-2\alpha \bar{h}} R_{12} + e^{-2\alpha h_2} M_2^T,
\end{aligned} \tag{14}$$

then system (1) is robustly α -stabilizable, the state feedback control $u(t) = YP^{-1}x(t)$, and the solution $x(t, \phi)$ of the closed-loop system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\Lambda}{\lambda}} e^{-\alpha t}, \quad t \geq 0. \tag{15}$$

Proof. Let us denote

$$\begin{aligned}
\bar{Q}_i &= P^{-1} Q_i P^{-1}, \\
\bar{M}_i &= P^{-1} M_i P^{-1}, \quad i = 1, 2; \\
\bar{S}_i &= P^{-1} S_i P^{-1}, \quad i = 1, 2, 3; \\
\bar{R}_{11} &= P^{-1} R_{11} P^{-1}, \\
\bar{R}_{12} &= P^{-1} R_{12} P^{-1}, \\
\bar{R}_{22} &= P^{-1} R_{22} P^{-1}.
\end{aligned} \tag{16}$$

Define a Lyapunov functional by

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{17}$$

where

$$\begin{aligned}
V_1(x_t) &= x^T(t) P^{-1} x(t), \\
V_2(x_t) &= \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) \bar{Q}_1 x(s) ds \\
&\quad + \int_{t-h(t)}^{t-h_1} e^{2\alpha(s-t)} x^T(s) \bar{Q}_2 x(s) ds \\
&\quad + \int_{t-\bar{h}}^{t-h_1} e^{2\alpha(s-t)} \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix}^T \\
&\quad \cdot \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix} ds, \\
V_3(x_t) &= \delta \int_{-\bar{h}}^{-h_1} \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) \bar{S}_1 \dot{x}(\tau) d\tau ds \\
&\quad + \delta \int_{-h_2}^{-\bar{h}} \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) \bar{S}_2 \dot{x}(\tau) d\tau ds \\
&\quad + h_1 \int_{-h_1}^0 \int_{t+s}^t e^{2\alpha(\tau-t)} \dot{x}^T(\tau) \bar{S}_3 \dot{x}(\tau) d\tau ds.
\end{aligned} \tag{18}$$

Calculating the time derivative of $V(x_t)$ along the trajectories of (6), we conclude that

$$\begin{aligned}
\dot{V}_1(x_t) &= x^T(t) \left[P^{-1} (A + BK) + (A + BK)^T P^{-1} \right] x(t) \\
&\quad + 2x^T(t) P^{-1} D x(t-h(t)) \\
&\quad + 2x^T(t) P^{-1} f(t, x(t), x(t-h(t))),
\end{aligned} \tag{19}$$

$$\begin{aligned}
\dot{V}_2(x_t) &\leq -2\alpha V_2(x_t) + x^T(t) \bar{Q}_1 x(t) \\
&\quad - (1-\mu) e^{-2\alpha h_2} x^T(t-h(t)) \bar{Q}_2 x(t-h(t)) \\
&\quad + e^{-2\alpha h_1} x^T(t-h_1) [\bar{Q}_2 - \bar{Q}_1] x(t-h_1) \\
&\quad + e^{-2\alpha h_1} \begin{bmatrix} x(t-h_1) \\ x(t-\bar{h}) \end{bmatrix}^T \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{22} \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-\bar{h}) \end{bmatrix} \\
&\quad - e^{-2\alpha \bar{h}} \begin{bmatrix} x(t-\bar{h}) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{22} \end{bmatrix} \begin{bmatrix} x(t-\bar{h}) \\ x(t-h_2) \end{bmatrix},
\end{aligned} \tag{20}$$

$$\begin{aligned}
\dot{V}_3(x_t) &\leq -2\alpha V_3(x_t) + \dot{x}^T(t) (h_1^2 \bar{S}_3 + \delta^2 \bar{S}_2 + \delta^2 \bar{S}_1) \dot{x}(t) \\
&\quad - \delta e^{-2\alpha h_2} \int_{t-h_2}^{t-\bar{h}} \dot{x}^T(s) \bar{S}_2 \dot{x}(s) ds \\
&\quad - \delta e^{-2\alpha \bar{h}} \int_{t-\bar{h}}^{t-h_1} \dot{x}^T(s) \bar{S}_1 \dot{x}(s) ds \\
&\quad - h_1 e^{-2\alpha h_1} \int_{t-h_1}^t \dot{x}^T(s) \bar{S}_3 \dot{x}(s) ds.
\end{aligned} \tag{21}$$

By virtue of Lemma 4, we have

$$\begin{aligned}
&2x^T(t) P^{-1} f(t, x(t), x(t-h(t))) \\
&\leq x^T(t) P^{-1} P^{-1} x(t) \\
&\quad + f^T(t, x(t), x(t-h(t))) f(t, x(t), x(t-h(t))) \\
&= x^T(t) P^{-1} P^{-1} x(t) + \|f(t, x(t), x(t-h(t)))\|^2.
\end{aligned} \tag{22}$$

Using (3) and the inequality $(H + J + L)^2 \leq 3H^2 + 3J^2 + 3L^2$, where H , J , and L are constants, we obtain

$$\begin{aligned}
&\|f(t, x(t), x(t-h(t)))\|^2 \\
&\leq (a \|x(t)\| + b \|x(t-h(t))\| + \beta(t))^2 \\
&\leq 3a^2 \|x(t)\|^2 + 3b^2 \|x(t-h(t))\|^2 + 3\beta^2(t).
\end{aligned} \tag{23}$$

It follows from (22) and (23) that

$$\begin{aligned} & 2x^T(t)P^{-1}f(t, x(t), x(t-h(t))) \\ & \leq x^T(t)(P^{-1}P^{-1} + 3a^2I)x(t) \\ & \quad + 3b^2x^T(t-h(t))x(t-h(t)) + 3\beta^2(t). \end{aligned} \quad (24)$$

Combining (19) and (24), we get

$$\begin{aligned} \dot{V}_1(x_t) & \leq -2\alpha V_1(x_t) + x^T(t)[P^{-1}(A+BK) \\ & \quad + (A+BK)^TP^{-1} + 2\alpha P^{-1} + P^{-1}P^{-1} + 3a^2I]x(t) \\ & \quad + 2x^T(t)P^{-1}Dx(t-h(t)) + 3b^2x^T(t-h(t))x(t-h(t)) \\ & \quad + 3\beta^2(t). \end{aligned} \quad (25)$$

Now, we estimate the upper bounds of the last three integral terms in inequality (21) as follows.

(i) Assume first that $h_1 \leq h(t) \leq \bar{h}$. From Jensen's inequality [19] and Lemma 5,

$$\begin{aligned} & -\delta \int_{t-\bar{h}}^{t-h_1} \dot{x}^T(s)\bar{S}_1\dot{x}(s)ds \\ & = -\delta \int_{t-\bar{h}}^{t-h(t)} \dot{x}^T(s)\bar{S}_1\dot{x}(s)ds \\ & \quad -\delta \int_{t-h(t)}^{t-h_1} \dot{x}^T(s)\bar{S}_1\dot{x}(s)ds \\ & \leq -\frac{\delta}{\bar{h}-h(t)}\left(\int_{t-\bar{h}}^{t-h(t)} \dot{x}(s)ds\right)^T \\ & \quad \cdot \bar{S}_1\left(\int_{t-\bar{h}}^{t-h(t)} \dot{x}(s)ds\right) \\ & \quad -\frac{\delta}{h(t)-h_1}\left(\int_{t-h(t)}^{t-h_1} \dot{x}(s)ds\right)^T \\ & \quad \cdot \bar{S}_1\left(\int_{t-h(t)}^{t-h_1} \dot{x}(s)ds\right) \\ & \leq -\begin{bmatrix} x(t-h(t)) - x(t-\bar{h}) \\ x(t-h_1) - x(t-h(t)) \end{bmatrix}^T \\ & \quad \cdot \begin{bmatrix} \bar{S}_1 & \bar{M}_1 \\ * & \bar{S}_1 \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-\bar{h}) \\ x(t-h_1) - x(t-h(t)) \end{bmatrix}, \end{aligned} \quad (26)$$

and so

$$\begin{aligned} & -\delta e^{-2\alpha\bar{h}} \int_{t-\bar{h}}^{t-h_1} \dot{x}^T(s)\bar{S}_1\dot{x}(s)ds \\ & \leq -\begin{bmatrix} x(t-h(t)) - x(t-\bar{h}) \\ x(t-h_1) - x(t-h(t)) \end{bmatrix}^T \\ & \quad \cdot \begin{bmatrix} e^{-2\alpha\bar{h}}\bar{S}_1 & e^{-2\alpha\bar{h}}\bar{M}_1 \\ * & e^{-2\alpha\bar{h}}\bar{S}_1 \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-\bar{h}) \\ x(t-h_1) - x(t-h(t)) \end{bmatrix}. \end{aligned} \quad (27)$$

On the other hand, if $h(t) = \bar{h}$ or $h(t) = h_1$, then $\int_{t-\bar{h}}^{t-h(t)} \dot{x}(s)ds = 0$ or $\int_{t-h_1}^{t-h(t)} \dot{x}(s)ds = 0$, respectively. Hence, inequality (27) holds.

Using Jensen's inequality [19], it is not difficult to arrive at the inequalities

$$\begin{aligned} & -h_1 e^{-2\alpha h_1} \int_{t-h_1}^t \dot{x}^T(s)\bar{S}_3\dot{x}(s)ds \\ & \leq -[x(t) - x(t-h_1)]^T \\ & \quad \cdot e^{-2\alpha h_1}\bar{S}_3[x(t) - x(t-h_1)], \end{aligned} \quad (28)$$

$$\begin{aligned} & -\delta e^{-2\alpha h_2} \int_{t-h_2}^{t-\bar{h}} \dot{x}^T(s)\bar{S}_2\dot{x}(s)ds \\ & \leq -[x(t-\bar{h}) - x(t-h_2)]^T \\ & \quad \cdot e^{-2\alpha h_2}\bar{S}_2[x(t-\bar{h}) - x(t-h_2)]. \end{aligned} \quad (29)$$

It follows now from (6) that

$$\begin{aligned} & 2\dot{x}^T(t)P^{-1}[(A+BK)x(t) + Dx(t-h(t)) \\ & \quad + f(t, x(t), x(t-h(t))) - \dot{x}(t)] = 0. \end{aligned} \quad (30)$$

On the other hand,

$$\begin{aligned} & 2\dot{x}^T(t)P^{-1}f(t, x(t), x(t-h(t))) \\ & \leq \dot{x}^T(t)P^{-1}P^{-1}\dot{x}(t) \\ & \quad + f^T(t, x(t), x(t-h(t)))f(t, x(t), x(t-h(t))) \\ & = \dot{x}^T(t)P^{-1}P^{-1}\dot{x}(t) + \|f(t, x(t), x(t-h(t)))\|^2 \\ & \leq \dot{x}^T(t)P^{-1}P^{-1}\dot{x}(t) + 3a^2x^T(t)x(t) \\ & \quad + 3b^2x^T(t-h(t))x(t-h(t)) + 3\beta^2(t). \end{aligned} \quad (31)$$

Therefore, formulas (19)–(31) imply that

$$\dot{V}(x_t) + 2\alpha V(x_t) \leq \xi^T(t)\Phi_1\xi(t) + 6\beta^2(t), \quad (32)$$

where

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & P^{-1}D & \Phi_{13} & 0 & 0 & (A+BK)^TP^{-1} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & 0 & D^TP^{-1} \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix},$$

$$\begin{aligned}
\Phi_{11} &= P^{-1} (A + BK) + (A + BK)^T P^{-1} + 2\alpha P^{-1} \\
&\quad + P^{-1} P^{-1} + \bar{Q}_1 - e^{-2\alpha h_1} \bar{S}_3 + 6a^2 I, \\
\Phi_{22} &= -(1 - \mu) e^{-2\alpha h_2} \bar{Q}_2 + e^{-2\alpha \bar{h}} \bar{M}_1 + e^{-2\alpha \bar{h}} \bar{M}_1^T \\
&\quad - 2e^{-2\alpha \bar{h}} \bar{S}_1 + 6b^2 I, \\
\Phi_{33} &= e^{-2\alpha h_1} \bar{Q}_2 + e^{-2\alpha h_1} \bar{R}_{11} - e^{-2\alpha h_1} \bar{Q}_1 - e^{-2\alpha \bar{h}} \bar{S}_1 \\
&\quad - e^{-2\alpha h_1} \bar{S}_3, \\
\Phi_{44} &= e^{-2\alpha h_1} \bar{R}_{22} - e^{-2\alpha \bar{h}} \bar{R}_{11} - e^{-2\alpha \bar{h}} \bar{S}_1 - e^{-2\alpha h_2} \bar{S}_2, \\
\Phi_{55} &= -e^{-2\alpha \bar{h}} \bar{R}_{22} - e^{-2\alpha h_2} \bar{S}_2, \\
\Phi_{66} &= h_1^2 \bar{S}_3 + \delta^2 \bar{S}_2 + \delta^2 \bar{S}_1 + P^{-1} P^{-1} - 2P^{-1}, \\
\Phi_{13} &= e^{-2\alpha h_1} \bar{S}_3, \\
\Phi_{23} &= e^{-2\alpha \bar{h}} (\bar{S}_1 - \bar{M}_1), \\
\Phi_{24} &= e^{-2\alpha \bar{h}} (\bar{S}_1 - \bar{M}_1^T), \\
\Phi_{34} &= e^{-2\alpha h_1} \bar{R}_{12} + e^{-2\alpha \bar{h}} \bar{M}_1^T, \\
\Phi_{45} &= -e^{-2\alpha \bar{h}} \bar{R}_{12} + e^{-2\alpha h_2} \bar{S}_2.
\end{aligned} \tag{33}$$

If we pre- and postmultiply Φ_1 by $\text{diag}\{P, P, P, P, P, P\}$ and let

$$K = YP^{-1}, \tag{34}$$

then the condition $\Phi_1 < 0$ is equivalent to condition (11) by using Schur Complement Lemma.

(ii) Assume now that $\bar{h} \leq h(t) \leq h_2$. Applications of Jensen's inequality [19] and Lemma 5 yield

$$\begin{aligned}
& -\delta \int_{t-h_2}^{t-\bar{h}} \dot{x}^T(s) \bar{S}_2 \dot{x}(s) ds \\
&= -\delta \left[\int_{t-h_2}^{t-h(t)} \dot{x}^T(s) \bar{S}_2 \dot{x}(s) ds \right. \\
&\quad \left. + \int_{t-h(t)}^{t-\bar{h}} \dot{x}^T(s) \bar{S}_2 \dot{x}(s) ds \right] \\
&\leq -\frac{\delta}{h_2 - h(t)} \left(\int_{t-h_2}^{t-h(t)} \dot{x}(s) ds \right)^T \\
&\quad \cdot \bar{S}_2 \left(\int_{t-h_2}^{t-h(t)} \dot{x}(s) ds \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\delta}{h(t) - \bar{h}} \left(\int_{t-h(t)}^{t-\bar{h}} \dot{x}(s) ds \right)^T \bar{S}_2 \left(\int_{t-h(t)}^{t-\bar{h}} \dot{x}(s) ds \right) \\
&\leq - \begin{bmatrix} x(t-h(t)) - x(t-h_2) \\ x(t-\bar{h}) - x(t-h(t)) \end{bmatrix}^T \\
&\quad \cdot \begin{bmatrix} \bar{S}_2 & \bar{M}_2 \\ * & \bar{S}_2 \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-h_2) \\ x(t-\bar{h}) - x(t-h(t)) \end{bmatrix},
\end{aligned} \tag{35}$$

and hence

$$\begin{aligned}
& -\delta e^{-2\alpha h_2} \int_{t-h_2}^{t-\bar{h}} \dot{x}^T(s) \bar{S}_2 \dot{x}(s) ds \\
&\leq - \begin{bmatrix} x(t-h(t)) - x(t-h_2) \\ x(t-\bar{h}) - x(t-h(t)) \end{bmatrix}^T \\
&\quad \cdot \begin{bmatrix} e^{-2\alpha h_2} \bar{S}_2 & e^{-2\alpha h_2} \bar{M}_2 \\ * & e^{-2\alpha h_2} \bar{S}_2 \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-h_2) \\ x(t-\bar{h}) - x(t-h(t)) \end{bmatrix}.
\end{aligned} \tag{36}$$

Note that when $h(t) = \bar{h}$ or $h(t) = h_2$, one can obtain $\int_{t-\bar{h}}^{t-h(t)} \dot{x}(s) ds = 0$ or $\int_{t-h(t)}^{t-h_2} \dot{x}(s) ds = 0$, respectively. Therefore, inequality (36) is satisfied.

Using Jensen's inequality [19], we deduce that

$$\begin{aligned}
& -\delta e^{-2\alpha \bar{h}} \int_{t-\bar{h}}^{t-h_1} \dot{x}^T(s) \bar{S}_1 \dot{x}(s) ds \\
&\leq - \begin{bmatrix} x(t-h_1) - x(t-\bar{h}) \end{bmatrix}^T \\
&\quad \cdot e^{-2\alpha \bar{h}} \bar{S}_1 \begin{bmatrix} x(t-h_1) - x(t-\bar{h}) \end{bmatrix}.
\end{aligned} \tag{37}$$

Combining (19)–(25), (28), and (30)–(37), we obtain

$$\dot{V}(x_t) + 2\alpha V(x_t) \leq \xi^T(t) \Phi_2 \xi(t) + 6\beta^2(t), \tag{38}$$

where

$$\begin{aligned}
& \Phi_2 \\
&= \begin{bmatrix} \Phi_{11} & P^{-1}D & \Phi_{13} & 0 & 0 & (A+BK)^T P^{-1} \\ * & \bar{\Phi}_{22} & 0 & \bar{\Phi}_{24} & \bar{\Phi}_{25} & D^T P^{-1} \\ * & * & \Phi_{33} & \bar{\Phi}_{34} & 0 & 0 \\ * & * & * & \Phi_{44} & \bar{\Phi}_{45} & 0 \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}, \\
& \bar{\Phi}_{22} \\
&= -(1 - \mu) e^{-2\alpha h_2} \bar{Q}_2 + e^{-2\alpha h_2} \bar{M}_2 + e^{-2\alpha h_2} \bar{M}_2^T \\
&\quad - 2e^{-2\alpha h_2} \bar{S}_2 + 6b^2 I,
\end{aligned}$$

$$\begin{aligned}
\bar{\Phi}_{24} &= e^{-2\alpha h_2} (\bar{S}_2 - \bar{M}_2), \\
\bar{\Phi}_{25} &= e^{-2\alpha h_2} (\bar{S}_2 - \bar{M}_2^T), \\
\bar{\Phi}_{34} &= e^{-2\alpha h_1} \bar{R}_{12} + e^{-2\alpha \bar{h}} \bar{S}_1, \\
\bar{\Phi}_{45} &= -e^{-2\alpha \bar{h}} \bar{R}_{12} + e^{-2\alpha h_2} \bar{M}_2^T.
\end{aligned} \tag{39}$$

Pre- and postmultiplying Φ_2 by $\text{diag}\{P, P, P, P, P, P\}$ and letting $K = YP^{-1}$, the condition $\Phi_2 < 0$ is equivalent to condition (12) by using Schur Complement Lemma.

From the above discussion, if conditions (11)–(13) are satisfied, then

$$\dot{V}(x_t) + 2\alpha V(x_t) \leq 6\beta^2(t), \quad t \geq 0. \tag{40}$$

By virtue of (40) and the definition of $V(x(t))$,

$$\begin{aligned}
\lambda \|x(t, \phi)\|^2 &\leq V(x(t)) \\
&\leq e^{-2\alpha t} \left(V(x(0)) + 6 \int_0^t \beta^2(s) e^{\int_0^s 2\alpha d\tau} ds \right) \\
&\leq e^{-2\alpha t} \left(\Gamma \|\phi\|^2 + 6 \int_0^{+\infty} \beta^2(s) e^{2\alpha s} ds \right) = \Lambda e^{-2\alpha t}.
\end{aligned} \tag{41}$$

Hence, we have

$$\|x(t, \phi)\| \leq \sqrt{\frac{\Lambda}{\lambda}} e^{-\alpha t}, \quad t \geq 0, \tag{42}$$

which implies that the closed-loop system is α -stable. The proof is completed. $\square$

If there is no perturbation in system (1), that is, $f(t, x(t), x(t-h(t))) = 0$, then system (1) reduces to

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Dx(t-h(t)) + Bu(t), \quad t \geq 0, \\
x(t) &= \phi(t), \quad t \in [-h_2, 0].
\end{aligned} \tag{43}$$

Application of Theorem 6 yields the following result.

Corollary 7. Assume that $\alpha > 0$ and condition (2) is satisfied. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $S_i > 0$ ($i = 1, 2, 3$), $\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$, M_1 , M_2 , and Y such that

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & DP & \Xi_{13} & 0 & 0 & PA^T + Y^T B^T \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & PD^T \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 \\ * & * & * & * & \Xi_{55} & 0 \\ * & * & * & * & * & \Xi_{66} \end{bmatrix} < 0,$$

$$\begin{aligned}
\Xi_2 &= \begin{bmatrix} \Xi_{11} & DP & \Xi_{13} & 0 & 0 & PA^T + Y^T B^T \\ * & \bar{\Xi}_{22} & 0 & \bar{\Xi}_{24} & \bar{\Xi}_{25} & PD^T \\ * & * & \Xi_{33} & \bar{\Xi}_{34} & 0 & 0 \\ * & * & * & \Xi_{44} & \bar{\Xi}_{45} & 0 \\ * & * & * & * & \Xi_{55} & 0 \\ * & * & * & * & * & \Xi_{66} \end{bmatrix} < 0, \\
\begin{bmatrix} S_i & M_i \\ * & S_i \end{bmatrix} &\geq 0 \quad (i = 1, 2),
\end{aligned} \tag{44}$$

where

$$\begin{aligned}
\Xi_{11} &= AP + PA^T + BY + Y^T B^T + 2\alpha P + Q_1 \\
&\quad - e^{-2\alpha h_1} S_3, \\
\Xi_{22} &= -(1-\mu) e^{-2\alpha h_2} Q_2 + e^{-2\alpha \bar{h}} M_1 + e^{-2\alpha \bar{h}} M_1^T \\
&\quad - 2e^{-2\alpha \bar{h}} S_1, \\
\Xi_{33} &= e^{-2\alpha h_1} Q_2 + e^{-2\alpha h_1} R_{11} - e^{-2\alpha h_1} Q_1 - e^{-2\alpha \bar{h}} S_1 \\
&\quad - e^{-2\alpha h_1} S_3, \\
\Xi_{44} &= e^{-2\alpha h_1} R_{22} - e^{-2\alpha \bar{h}} R_{11} - e^{-2\alpha \bar{h}} S_1 - e^{-2\alpha h_2} S_2, \\
\Xi_{55} &= -e^{-2\alpha \bar{h}} R_{22} - e^{-2\alpha h_2} S_2, \\
\Xi_{66} &= h_1^2 S_3 + \delta^2 S_2 + \delta^2 S_1 - 2P, \\
\bar{\Xi}_{13} &= e^{-2\alpha h_1} S_3, \\
\bar{\Xi}_{23} &= e^{-2\alpha \bar{h}} (S_1 - M_1), \\
\bar{\Xi}_{24} &= e^{-2\alpha \bar{h}} (S_1 - M_1^T), \\
\bar{\Xi}_{34} &= e^{-2\alpha h_1} R_{12} + e^{-2\alpha \bar{h}} M_1^T, \\
\bar{\Xi}_{45} &= -e^{-2\alpha \bar{h}} R_{12} + e^{-2\alpha h_2} S_2, \\
\bar{\bar{\Xi}}_{22} &= -(1-\mu) e^{-2\alpha h_2} Q_2 + e^{-2\alpha h_2} M_2 + e^{-2\alpha h_2} M_2^T \\
&\quad - 2e^{-2\alpha h_2} S_2, \\
\bar{\bar{\Xi}}_{24} &= e^{-2\alpha h_2} (S_2 - M_2), \\
\bar{\bar{\Xi}}_{25} &= e^{-2\alpha h_2} (S_2 - M_2^T), \\
\bar{\bar{\Xi}}_{34} &= e^{-2\alpha h_1} R_{12} + e^{-2\alpha \bar{h}} S_1, \\
\bar{\bar{\Xi}}_{45} &= -e^{-2\alpha \bar{h}} R_{12} + e^{-2\alpha h_2} M_2^T,
\end{aligned} \tag{45}$$

then system (43) is robustly α -stabilizable, the state feedback control $u(t) = YP^{-1}x(t)$, and the solution $x(t, \phi)$ of the closed-loop system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\Lambda}{\lambda}} e^{-\alpha t} = \sqrt{\frac{\Gamma}{\lambda}} e^{-\alpha t} \|\phi\|, \quad t \geq 0. \tag{46}$$

4. Numerical Examples

In this section, two numerical examples are given to illustrate the effectiveness of the results obtained in this paper.

Example 1. Consider the system with a nonlinear perturbation

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Dx(t-h(t)) \\ &\quad + f(t, x(t), x(t-h(t))) + Bu(t), \\ t &\geq 0,\end{aligned}\quad (47)$$

$$x(t) = [1 \ 1]^T, \quad t \in [-0.4, 0],$$

where

$$\begin{aligned}A &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ D &= \begin{bmatrix} -2 & -0.1 \\ 0 & 1.1 \end{bmatrix},\end{aligned}\quad (48)$$

$h(t) = 0.1 + 0.3\sin^2(5t/3)$, and the nonlinear perturbation satisfies

$$\begin{aligned}\|f(t, x(t), x(t-h(t)))\| \\ \leq 0.1 \|x(t)\| + 0.1 \|x(t-h(t))\| + e^{-0.66t}.\end{aligned}\quad (49)$$

Note that $h_1 = 0.1$, $h_2 = 0.4$, $\mu = 0.5$, and $\beta(t) = e^{-0.66t}$. It is not difficult to check that A and $A + D$ are Hurwitz unstable.

Let $\alpha = 0.16$. Then $\int_0^{+\infty} \beta^2(s) e^{2\alpha s} ds = \int_0^{+\infty} e^{-s} ds = 1 < +\infty$. Using the LMI Toolbox in MATLAB, LMIs (11)–(13) in Theorem 6 are satisfied with

$$\begin{aligned}P &= \begin{bmatrix} 4.4365 & -0.1306 \\ -0.1306 & 4.9389 \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} 0.1510 & 0.4068 \\ 0.4068 & 3.4481 \end{bmatrix}, \\ Q_2 &= \begin{bmatrix} 0.0115 & 0.0066 \\ 0.0066 & 0.2019 \end{bmatrix}, \\ S_1 &= \begin{bmatrix} 62.2011 & -5.1166 \\ -5.1166 & 52.7981 \end{bmatrix}, \\ S_2 &= \begin{bmatrix} 36.2972 & -1.8408 \\ -1.8408 & 31.9872 \end{bmatrix}, \\ S_3 &= \begin{bmatrix} 71.6879 & -4.4063 \\ -4.4063 & 40.8855 \end{bmatrix}, \\ R_{11} &= \begin{bmatrix} 12.6433 & -1.6878 \\ -1.6878 & 21.3645 \end{bmatrix},\end{aligned}$$

$$R_{12} = \begin{bmatrix} -12.6769 & 2.2144 \\ 2.2144 & -18.8952 \end{bmatrix},$$

$$R_{22} = \begin{bmatrix} 13.2723 & -1.3613 \\ -1.3613 & 21.8765 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 11.3018 & -6.7305 \\ -6.7305 & 20.9430 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -14.5051 & -2.3510 \\ -2.3510 & 0.3503 \end{bmatrix},$$

$$Y = [0.2410 \ -6.8850]. \quad (50)$$

Furthermore, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 6.1258e^{-0.16t}, \quad t \geq 0, \quad (51)$$

and the stabilizing feedback control

$$u(t) = [0.0133 \ -1.3937] x(t), \quad t \geq 0. \quad (52)$$

Observe that the results reported in [3, 8, 16] cannot be applied to (47) since the nonlinear perturbation is related with the term $\beta(t) = e^{-0.66t}$.

Example 2. Consider a linear system with an interval time-varying delay

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Dx(t-h(t)) + Bu(t), \quad t \geq 0, \\ x(t) &= \phi(t), \quad \forall t \in [-h_2, 0],\end{aligned}\quad (53)$$

where

$$\begin{aligned}A &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ D &= \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix},\end{aligned}\quad (54)$$

and $h(t) = 0.5 + 1.28\sin^2(25t/64)$. Note that $h_1 = 0.5$, $h_2 = 1.78$, and $\mu = 0.5$. It is easy to check that A and $A + D$ are Hurwitz unstable. Given $\alpha = 0.01$, using the LMI Toolbox in MATLAB, LMIs (44) in Corollary 7 are satisfied with

$$\begin{aligned}P &= \begin{bmatrix} 84.8721 & -32.8554 \\ -32.8554 & 18.7095 \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} 0.0864 & -0.0424 \\ -0.0424 & 11.7837 \end{bmatrix}, \\ Q_2 &= \begin{bmatrix} 0.2099 & 0.2035 \\ 0.2035 & 11.4665 \end{bmatrix},\end{aligned}$$

$$\begin{aligned}
S_1 &= \begin{bmatrix} 109.3732 & -42.3701 \\ -42.3701 & 16.5117 \end{bmatrix}, \\
S_2 &= \begin{bmatrix} 102.7380 & -39.7555 \\ -39.7555 & 15.5155 \end{bmatrix}, \\
S_3 &= \begin{bmatrix} 124.7698 & -48.3825 \\ -48.3825 & 18.8240 \end{bmatrix}, \\
R_{11} &= \begin{bmatrix} 32.4350 & -12.8325 \\ -12.8325 & 5.1838 \end{bmatrix}, \\
R_{12} &= \begin{bmatrix} -2.3196 & 0.9331 \\ 0.9331 & -0.3969 \end{bmatrix}, \\
R_{22} &= \begin{bmatrix} 54.1560 & -21.0105 \\ -21.0105 & 8.2532 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} -22.9733 & 9.4881 \\ 9.4881 & -3.9352 \end{bmatrix}, \\
M_2 &= \begin{bmatrix} -53.6877 & 20.8764 \\ 20.8764 & -8.2092 \end{bmatrix}, \\
Y &= [23.8997 \quad -27.0789].
\end{aligned} \tag{55}$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 9.3399e^{-0.01t} \|\phi\|, \quad t \geq 0, \tag{56}$$

and the stabilizing feedback control

$$u(t) = [-0.8704 \quad -2.9758] x(t), \quad t \geq 0. \tag{57}$$

Figure 1 shows the trajectories of $x_1(t)$ and $x_2(t)$ of the open-loop system with the initial condition $\phi(t) = [30 \ 10]^T$, $t \in [-1.78, 0]$. Figure 2 shows the trajectories of $x_1(t)$ and $x_2(t)$ of the closed-loop system with the state feedback $u(t) = [-0.8704 \quad -2.9758]x(t)$ and the initial condition $\phi(t) = [30 \ 10]^T$, $t \in [-1.78, 0]$.

Letting the lower and upper bounds of the time delay be the same as in Zhang et al. [12], our results also ensure exponential stability with an α -convergence rate as given in Table 1. Note that Zhang et al. [12] discussed asymptotic stability, whereas the controller derived in this paper provides exponential stability for the closed-loop system. Furthermore, the maximum bound for α is better than Thuan et al. [16] by letting h_1 and h_2 be the same as in [16]. For selected h_1 and $\mu = 0.5$, using Corollary 7, one can easily observe that the maximum allowable delay bounds for h_2 are better than those reported in the papers by Zhang et al. [12] and Thuan et al. [16].

FIGURE 1: Open-loop trajectories of $x_1(t)$ and $x_2(t)$.

FIGURE 2: Closed-loop trajectories of $x_1(t)$ and $x_2(t)$.

5. Conclusions

In this paper, exponential stabilization of a class of time-varying delay systems with nonlinear perturbations has been investigated. By using the delay decomposition approach and constructing a novel Lyapunov functional, some new delay-dependent stabilization criteria are obtained in order to ensure closed-loop stability of the system with any prescribed α -convergence rate. Numerical examples are given to illustrate that the results obtained are much less conservative than some existing results in the literature. Exponential stabilization of impulsive switched delay systems with nonlinear perturbations will be further investigated in the future.

TABLE 1: Admissible upper bound h_2 and K for given h_1 with $\mu = 0.5$.

Method	h_1	h_2	K
Zhang et al. [12]	0.5	0.967	$[-0.5290 \quad -2.7166]$
Thuan et al. [16] ($\alpha = 0.314$)	0.5	0.967	$[-0.0967 \quad -1.8338]$
Corollary 7 ($\alpha = 0.353$)	0.5	0.967	$[-0.1662 \quad -2.0605]$
Thuan et al. [16] ($\alpha = 0.01$)	0.5	1.66	$[-0.8765 \quad -2.9873]$
Corollary 7 ($\alpha = 0.01$)	0.5	1.78	$[-0.8704 \quad -2.9758]$
Zhang et al. [12]	1.0	1.114	$[-0.3096 \quad -2.1894]$
Thuan et al. [16] ($\alpha = 0.225$)	1.0	1.114	$[-0.6344 \quad -2.9059]$
Corollary 7 ($\alpha = 0.236$)	1.0	1.114	$[-0.6714 \quad -2.9317]$
Thuan et al. [16] ($\alpha = 0.01$)	1.0	1.672	$[-0.9023 \quad -2.9983]$
Corollary 7 ($\alpha = 0.01$)	1.0	1.765	$[-0.8701 \quad -2.9712]$

Conflict of Interests

The authors declare that they have no conflict of interests.

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