Research Article

New Delay-Dependent Exponential Stability Criteria for Neural Networks with Mixed Time-Varying Delays

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This study is concerned with the problem of new delay-dependent exponential stability criteria for neural networks (NNs) with mixed time-varying delays via introducing a novel integral inequality approach. Specifically, first, by taking fully the relationship between the terms in the Leibniz-Newton formula into account, several improved delay-dependent exponential stability criteria are obtained in terms of linear matrix inequalities (LMIs). Second, together with some effective mathematical techniques and a convex optimization approach, less conservative conditions are derived by constructing an appropriate Lyapunov-Krasovskii functional (LKF). Third, the proposed methods include the least numbers of decision variables while keeping the validity of the obtained results. Finally, three numerical examples with simulations are presented to illustrate the validity and advantages of the theoretical results.

1. Introduction

Over the course of the past decade, neural networks have become an important area of research and attracted increasing attention due to their extensive applications in many practical systems, such as power systems [1], pattern recognition [2], signal detection [3], landmark recognition [4], and other scientific areas [5–7].

Moreover, it is inevitable to introduce time delay into the signals transmitted among neurons because the processes of transcription and translation are not instantaneous. However, it is a well-known fact that time delay as a source of instability and poor performance usually appears in many dynamical systems, for instance, Cohen-Grossberg neural networks, cellular neural networks, BAM neural network, chaotic neural networks, H_∞ filtering, and nonlinear systems [8–53]. Therefore, stability analysis for neural networks with delays has been an attractive subject of research in recent years [8–42, 50–53].

Furthermore, neural networks (NNs) often have a spatial nature due to the presence of many parallel pathways of a variety of axon sizes and lengths. Thus, in order to have

a more accurate model, a distributed delay over a certain time of duration needs to be included in NNs such that the distant past has less influence compared to the recent behavior of the state. Therefore, there has been a growing interest in the study of neural networks with discrete and distributed delays during the past two decades. To date, some results on delay-dependent exponential stability for neural networks with mixed time-varying delays have been reported in [18–23, 34–36]. In [18], the authors considered the global asymptotic stability for a class of delayed cellular neural networks with mixed time-varying delays by using LMIs approach, Lyapunov theory, and Leibniz-Newton formula. However, the activation functions in [18] were assumed to be monotonically nondecreasing. In [19], several delay-dependent sufficient conditions are obtained to guarantee the global asymptotic and exponential stability of the addressed neural networks by employing appropriate LKF and linear matrix inequality (LMI) technique. In [20], an exponential stability criterion is proposed by constructing an augmented LKF, where the discrete delay $d(t)$ must be differentiable. In [22, 23], some improved delay-dependent stability criteria are derived in terms of linear matrix inequalities by dividing

the discrete delay interval into multiple segments. Different from [35], the appropriate LKF not only divides the discrete delay interval $[0, d]$ into two ones $[0, d/2]$ and $[d/2, d]$, but also divides the discrete delay interval $[0, d]$ into three ones $[0, d(t)/2]$, $[d(t)/2, d(t)]$, and $[d(t), d]$. Although this approach seems to be effective for achieving less conservative conditions, it can increase the larger numbers of computed variables. Hence, there exists great room for further improvement. To the best of our knowledge, it is of a great significance for the current research to find a more effective approach to get rid of the strict constraint and obtain less conservative conditions.

Motivated by the above discussion, combining effective mathematical techniques and a convex optimization approach, we choose a more general type of LKF to study the delay-dependent exponential stability criteria for neural networks (NNs) with mixed time-varying delays in the paper. Some improved delay-dependent stability conditions derived benefit mainly from using firstly a new integral inequality approach, which is proved to be less conservative than the celebrated Jensen's inequality and showed having a great potential efficient in practice. Both theoretical and numerical comparisons have been provided to show the effectiveness and efficiency of the proposed method. Besides, the main merit of this method lies in containing the least numbers of decision variables while keeping the validity of the obtained results. Finally, the stability criteria obtained turn out to be less conservative than some recently reported ones via three numerical examples.

Notation. Notations used in this paper are fairly standard: $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, and $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ dimensional matrices; I is the identity matrix of appropriate dimensions, and A_T is the matrix transposition of the matrix A . By $X > 0$ (resp., $X \geq 0$), for $X \in \mathbb{R}^{n \times n}$, we mean that the matrix X is real symmetric positive definite (resp., positive semidefinite); $\text{diag}\{r_1, r_2, \dots, r_n\}$ denotes block diagonal matrix with diagonal elements r_i , $i = 1, 2, \dots, n$, and the symbol $*$ represents the elements below the main diagonal of a symmetric matrix; $\vec{A}$ is defined as $\vec{A} = A + A^T$.

2. Preliminaries

Consider the following neural networks with mixed time-varying delays:

$$\begin{aligned} \dot{u}(t) = & -A_0 u(t) + A_1 g(u(t)) + A_2 g(u(t-h(t))) \\ & + A_3 \int_{t-r}^t g(u(s)) ds + I, \end{aligned} \quad (1)$$

where $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T \in \mathbb{R}^n$ is the neural state vector and $g(u(t)) = [g_1(u_1(t)), g_2(u_2(t)), \dots, g_n(u_n(t))]^T \in \mathbb{R}^n$ is the neuron activation function; $I = [I_1, I_2, \dots, I_n]^T \in \mathbb{R}^n$ is an external constant input vector, $A_0 = \text{diag}\{a_{01}, a_{02}, \dots, a_{0n}\} > 0$, and A_1, A_2 , and A_3 are the constant matrices of appropriate dimensions.

Assumption A. The time-varying delay $h(t)$ is continuous and differential function satisfying

$$\begin{aligned} 0 & \leq h(t) \leq h, \\ \dot{h}(t) & \leq h_D < 1. \end{aligned} \quad (2)$$

Assumption B. For the constants l_i^- and l_i^+ the bounded activation function $g_i(\cdot)$ in (1) satisfies the following condition:

$$\begin{aligned} l_i^- & \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq l_i^+, \\ \forall \alpha, \beta & \in \mathbb{R}, \alpha \neq \beta, i = 1, 2, \dots, n. \end{aligned} \quad (3)$$

We denote $L^+ = \text{diag}\{l_1^+, l_2^+, \dots, l_n^+\}$, $L^- = \text{diag}\{l_1^-, l_2^-, \dots, l_n^-\}$, $L_1 = \text{diag}\{l_1^+ l_1^-, l_2^+ l_2^-, \dots, l_n^+ l_n^-\}$, $L_2 = \text{diag}\{(l_1^+ + l_1^-)/2, (l_2^+ + l_2^-)/2, \dots, (l_n^+ + l_n^-)/2\}$, and $L = \text{diag}\{\max\{|l_1^+|, |l_1^-|\}, \dots, \max\{|l_n^+|, |l_n^-|\}\} = \text{diag}\{l_1, \dots, l_n\}$.

Under Assumption B, by using Brouwer's fixed-point theorem [25], it can be easily proven that there exists one equilibrium point for system (1). Assuming that $u^* = [u_1^*, \dots, u_n^*]^T$ is an equilibrium point of system (1). For convenience, we firstly shift the equilibrium point u^* to the origin by letting $z(t) = u(t) - u^*$ and $f(z(t)) = g(u(t)) - g(u^*)$, and then system (1) can be converted to

$$\begin{aligned} \dot{z}(t) = & -A_0 z(t) + A_1 f(z(t)) + A_2 f(z(t-h(t))) \\ & + A_3 \int_{t-r}^t f(z(s)) ds, \end{aligned} \quad (4)$$

where $f(x(t)) = [f_1(z_1), f_2(z_2), \dots, f_n(z_n)]^T$. It is easy to check that the function $f_i(\cdot)$ satisfies $f_i(0) = 0$, and

$$l_i^- \leq \frac{f_i(\alpha)}{\alpha} \leq l_i^+, \quad \forall \alpha \in \mathbb{R}, \alpha \neq 0, i = 1, 2, \dots, n. \quad (5)$$

Due to the influence of external factors, $\int_{t-r}^t f(z(s)) ds$ cannot express the actual state of the accurate information. Therefore, by translating r to function $r(t)$ ($0 \leq r(t) \leq r$), we have

$$\begin{aligned} \dot{z}(t) = & -A_0 z(t) + A_1 f(z(t)) + A_2 f(z(t-h(t))) \\ & + A_3 \int_{t-r(t)}^t f(z(s)) ds. \end{aligned} \quad (6)$$

In the paper, we will attempt to formulate some practically computable criteria to check the global exponential stability of system (6). The following lemmas are useful in deriving the criteria.

Definition 1 (see [22]). The equilibrium point 0 of system (6) is said to be globally exponentially stable, if there exist scalars $k > 0$ and $\beta > 0$ such that

$$\|z(t)\| \leq \beta e^{-kt} \sup_{-h \leq s \leq 0} \|z(s)\|, \quad \forall t > 0. \quad (7)$$

Lemma 2 (see [22]). *The following inequalities are true:*

$$\begin{aligned} 0 &\leq \int_0^{z_i(t)} (f_i(s) - l_i^- s) ds \leq (f_i(z_i(t)) - l_i^- z_i(t)) z_i(t), \\ 0 &\leq \int_0^{z_i(t)} (l_i^+ s - f_i(s)) ds \leq (l_i^+ z_i(t) - f_i(z_i(t))) z_i(t). \end{aligned} \quad (8)$$

Lemma 3 (see [24]). *For any positive definite matrix $\begin{bmatrix} S & G \\ G^T & S \end{bmatrix} > 0$ and $r \geq 0$, $0 \leq r(t) \leq r$, the following inequalities hold:*

$$\begin{aligned} &-r \int_{t-r}^t \alpha^T(s) S \alpha(s) ds \\ &\leq - \begin{bmatrix} \int_{t-r(t)}^t \alpha(s) ds \\ \int_{t-r}^{t-r(t)} \alpha(s) ds \end{bmatrix}^T \begin{bmatrix} S & G \\ G^T & S \end{bmatrix} \begin{bmatrix} \int_{t-r(t)}^t \alpha(s) ds \\ \int_{t-r}^{t-r(t)} \alpha(s) ds \end{bmatrix}. \end{aligned} \quad (9)$$

Lemma 4 (see [54]). *For any constant matrix $R \in \mathbb{R}^{n \times n}$, $R = R^T > 0$, a scalar function $h := h(t) > 0$, and a vector-valued function $\dot{z} : [-h, 0] \rightarrow \mathbb{R}^n$ such that the following integrations are well:*

$$\begin{aligned} &-h \int_{t-h}^t \dot{z}^T(s) R \dot{z}(s) ds \\ &\leq \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}. \end{aligned} \quad (10)$$

Lemma 5 (see [17]). *For any positive semidefnite matrices,*

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12}^T & R_{22} & R_{23} \\ R_{13}^T & R_{23}^T & R_{33} \end{bmatrix} \geq 0, \quad (11)$$

the integral inequality holds as follows:

$$\begin{aligned} &\int_{t-h}^t \dot{z}^T(s) R_{33} \dot{z}(s) ds \\ &\leq \int_{t-h}^t \begin{bmatrix} z(t) \\ z(t-h) \\ \dot{z}(s) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12}^T & R_{22} & R_{23} \\ R_{13}^T & R_{23}^T & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h) \\ \dot{z}(s) \end{bmatrix} ds. \end{aligned} \quad (12)$$

3. Main Results

In this section we will give sufficient conditions under which system (6) is globally exponentially stable.

Theorem 6. *For given scalars $r > 0$, $h > 0$, and $1 > h_D$, the origin system (6) with the neuron activation function $f(z(t))$ satisfying condition (5) and the time-varying delay $h(t)$ satisfying (2) is globally exponentially stable with the exponential convergence rate index k if there exist $P > 0$, $Z_i > 0$ ($i = 1, 2$), $R_i > 0$ ($i = 1, 2, 3, 4$), diagonal matrices $H_1 = \text{diag}\{h_{11}, h_{12}, \dots, h_{1n}\} > 0$, $H_2 = \text{diag}\{h_{21}, h_{22}, \dots, h_{2n}\} > 0$, $H_3 = \text{diag}\{h_{31}, h_{32}, \dots, h_{3n}\} > 0$, $H_4 = \text{diag}\{h_{41}, h_{42}, \dots, h_{4n}\} > 0$, $R = \text{diag}\{r_1, r_2, \dots, r_n\} > 0$, $G = \text{diag}\{g_1, g_2, \dots, g_n\} > 0$, any matrices T_i ($i = 1, 2, 3, 4$), and*

$$\begin{aligned} &\begin{bmatrix} S & M \\ M^T & S \end{bmatrix} > 0, \\ &X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} > 0, \\ &Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} > 0, \end{aligned} \quad (13)$$

$$R_3 + (1 - h_D) R_4 - X_{33} > 0,$$

$$R_3 - Y_{33} > 0$$

such that the following symmetric linear matrix inequality holds:

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & \Psi_{15} & T_2 A_2 - A_0 T_4^T & 0 & T_2 A_3 & 0 \\ * & \Psi_{22} & 0 & 0 & \Psi_{25} & T_1 A_2 - T_4^T & 0 & T_1 A_3 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & 0 & H_3 L_2 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & H_4 L_2 & 0 & 0 \\ * & * & * & * & \Psi_{55} & T_3 A_2 + A_1^T T_4^T & 0 & T_3 A_3 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & T_4 A_3 & 0 \\ * & * & * & * & * & * & \Psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -e^{-2kr} S & -e^{-2kr} M \\ * & * & * & * & * & * & * & * & -e^{-2kr} S \end{bmatrix} \leq 0, \quad (14)$$

where

$$\begin{aligned}
\Psi_{11} &= 2kP + 4k(L^+G - L^-R) + R_1 + R_2 \\
&\quad + e^{-2kh} \left(hX_{11} + \overrightarrow{X_{13}} \right) \\
&\quad - \frac{e^{-2kh}}{h} (R_3 + (1 - h_D) R_4 - X_{33}) - H_2 L_1 \\
&\quad - \overrightarrow{T_2 A_0}, \\
\Psi_{12} &= P + L^+G - L^-R - A_0 T_1^T - T_2, \\
\Psi_{13} &= e^{-2kh} (hX_{12} - X_{13} + X_{23}^T) \\
&\quad - \frac{e^{-2kh}}{h} (R_3 + (1 - h_D) R_4 - X_{33}), \\
\Psi_{15} &= 2k(R - G) + H_2 L_2 + T_2 A_1 - A_0 T_3^T, \\
\Psi_{22} &= hR_3 + hR_4 - \overrightarrow{T_1}, \\
\Psi_{25} &= R - G + T_1 A_1 - T_3, \\
\Psi_{33} &= -(1 - h_D) R_2 \\
&\quad - \frac{e^{-2kh}}{h} (R_3 + (1 - h_D) R_4 - X_{33} + R_3 - Y_{33}) \\
&\quad + e^{-2kh} \left(hX_{22} - \overrightarrow{X_{23}} + hY_{11} + \overrightarrow{Y_{13}} \right) - H_3 L_1, \\
\Psi_{34} &= \frac{e^{-2kh}}{h} (R_3 - Y_{33}) + e^{-2kh} (hY_{12} - Y_{13} + Y_{23}^T), \\
\Psi_{44} &= -R_1 - \frac{e^{-2kh}}{h} (R_3 - Y_{33}) + e^{-2kh} \left(hY_{22} - \overrightarrow{Y_{23}} \right) \\
&\quad - H_4 L_1, \\
\Psi_{55} &= Z_1 + Z_3 + r^2 S - H_1 - H_2 + \overrightarrow{T_3 A_1}, \\
\Psi_{66} &= -(1 - h_D) e^{-2kh} Z_2 - H_3 + \overrightarrow{T_4 A_2}, \\
\Psi_{77} &= -e^{-2kh} Z_2 - H_4.
\end{aligned} \tag{15}$$

Proof. Consider an augmentation of LKF for system (6) as follows:

$$\begin{aligned}
V(z_t) &= V_1(z_t) + V_2(z_t) + V_3(z_t) + V_4(z_t) + V_5(z_t) \\
&\quad + V_6(z_t),
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
V_1(z_t) &= e^{2kt} z^T(t) P z(t), \\
V_2(z_t) &= 2e^{2kt} \left[\sum_{i=1}^n r_i \int_0^{z_i(t)} (f_i(s) - l_i^- s) ds \right. \\
&\quad \left. + \sum_{i=1}^n g_i \int_0^{z_i(t)} (l_i^+ s - f_i(s)) ds \right],
\end{aligned}$$

$$\begin{aligned}
V_3(z_t) &= \int_{t-h}^t e^{2ks} z^T(s) R_1 z(s) ds \\
&\quad + \int_{t-h(t)}^t e^{2ks} z^T(s) R_2 z(s) ds, \\
V_4(z_t) &= \int_{-h}^0 \int_{t+\theta}^t e^{2ks} z^T(s) R_3 \dot{z}(s) ds d\theta \\
&\quad + \int_{-h(t)}^0 \int_{t+\theta}^t e^{2ks} \dot{z}^T(s) R_4 \dot{z}(s) ds d\theta, \\
V_5(z_t) &= \int_{t-h}^t e^{2ks} f^T(z(s)) Z_1 f(z(s)) ds \\
&\quad + \int_{t-h(t)}^t e^{2ks} f^T(z(s)) Z_2 f(z(s)) ds, \\
V_6(z_t) &= r \int_{-r}^0 \int_{t+\theta}^t e^{2ks} f^T(z(s)) S f(z(s)) ds d\theta.
\end{aligned} \tag{17}$$

The time derivative of $V(z_t)$ along with the trajectory of system (6) is given as

$$\begin{aligned}
\dot{V}(z_t) &= \dot{V}_1(z_t) + \dot{V}_2(z_t) + \dot{V}_3(z_t) + \dot{V}_4(z_t) + \dot{V}_5(z_t) \\
&\quad + \dot{V}_6(z_t),
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\dot{V}_1(z_t) &= 2ke^{2kt} z^T(t) P z(t) + 2e^{2kt} z^T(t) P \dot{z}(t), \\
\dot{V}_2(z_t) &= 4ke^{2kt} \left[(f^T(z(t)) - z^T(t) L^-) R z(t) \right. \\
&\quad \left. + (z^T(t) L^+ - f^T(z(t))) G z(t) \right] \\
&\quad + 2e^{2kt} \left[f^T(z(t)) (R - G) + z^T(t) (L^+ G - L^- R) \right] \\
&\quad \cdot \dot{z}(t), \\
\dot{V}_3(z_t) &= e^{2kt} z^T(t) (R_1 + R_2) z(t) - e^{2kt} z^T(t-h) \\
&\quad \cdot R_1 z(t-h) - (1 - \dot{h}(t)) e^{2kt} z^T(t-h(t)) R_2 z(t-h(t)) \\
&\quad \leq e^{2kt} z^T(t) (R_1 + R_2) z(t) - e^{2kt} z^T(t-h) \\
&\quad \cdot R_1 z(t-h) - (1 - h_D) e^{2kt} z^T(t-h(t)) R_2 z(t-h(t)),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_4(z_t) &= e^{2kt} \dot{z}^T(t) (hR_3 + h(t)R_4) \dot{z}(t) \\
&\quad - \int_{t-h}^t e^{2ks} \dot{z}^T(s) R_3 \dot{z}(s) ds - (1-h(t)) \\
&\quad \cdot \int_{t-h(t)}^t e^{2ks} \dot{z}^T(s) R_5 \dot{z}(s) ds \leq e^{2kt} h \dot{z}^T(t) (R_3 + R_4) \\
&\quad \cdot \dot{z}(t) - \int_{t-h(t)}^t e^{2ks} \dot{z}^T(s) (R_3 + (1-h_D)R_4 - X_{33}) \\
&\quad \cdot \dot{z}(s) ds - \int_{t-h}^{t-h(t)} e^{2ks} \dot{z}^T(s) (R_3 - Y_{33}) \dot{z}(s) ds \\
&\quad - \int_{t-h(t)}^t e^{2ks} \dot{z}^T(s) X_{33} \dot{z}(s) ds - \int_{t-h}^{t-h(t)} e^{2ks} \dot{z}^T(s) \\
&\quad \cdot Y_{33} \dot{z}(s) ds \leq e^{2kt} h \dot{z}^T(t) (R_3 + R_4) \dot{z}(t) \\
&\quad - e^{2k(t-h)} \int_{t-h(t)}^t \dot{z}^T(s) (R_3 + (1-h_D)R_4 - X_{33}) \\
&\quad \cdot \dot{z}(s) ds - e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s) (R_3 - Y_{33}) \dot{z}(s) ds \\
&\quad - e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s) X_{33} \dot{z}(s) ds \\
&\quad - e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s) Y_{33} \dot{z}(s) ds.
\end{aligned} \tag{19}$$

Using Lemma 4, we can have

$$\begin{aligned}
&- e^{2k(t-h)} \int_{t-h(t)}^t \dot{z}^T(s) (R_3 + (1-h_D)R_4 - X_{33}) \dot{z}(s) ds \\
&\leq - \frac{e^{2k(t-h)}}{h} \begin{bmatrix} z(t) \\ z(t-h(t)) \end{bmatrix}^T \begin{bmatrix} (R_3 + (1-h_D)R_4 - X_{33}) & -(R_3 + (1-h_D)R_4 - X_{33}) \\ -(R_3 + (1-h_D)R_4 - X_{33}) & (R_3 + (1-h_D)R_4 - X_{33}) \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h(t)) \end{bmatrix} \\
&= - \frac{e^{2k(t-h)}}{h} z^T(t) (R_3 + (1-h_D)R_4 - X_{33}) z(t) + 2 \frac{e^{2k(t-h)}}{h} z^T(t) (R_3 + (1-h_D)R_4 - X_{33}) z(t-h(t)) \\
&\quad - \frac{e^{2k(t-h)}}{h} z^T(t-h(t)) (R_3 + (1-h_D)R_4 - X_{33}) z(t-h(t)), \\
&- e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s) (R_3 - Y_{33}) \dot{z}(s) ds \leq - \frac{e^{2k(t-h)}}{h} \begin{bmatrix} z(t-h(t)) \\ z(t-h) \end{bmatrix}^T \begin{bmatrix} (R_3 - Y_{33}) & -(R_3 - Y_{33}) \\ -(R_3 - Y_{33}) & (R_3 - Y_{33}) \end{bmatrix} \begin{bmatrix} z(t-h(t)) \\ z(t-h) \end{bmatrix} \\
&= - \frac{e^{2k(t-h)}}{h} z^T(t-h(t)) (R_3 - Y_{33}) z(t-h(t)) + 2 \frac{e^{2k(t-h)}}{h} z^T(t-h(t)) (R_3 - Y_{33}) z(t-h) \\
&\quad - \frac{e^{2k(t-h)}}{h} z^T(t-h) (R_3 - Y_{33}) z(t-h).
\end{aligned} \tag{20}$$

Using Lemma 5, we may get

$$\begin{aligned}
&- e^{2k(t-h)} \int_{t-h(t)}^t \dot{z}^T(s) X_{33} \dot{z}(s) ds \\
&\leq e^{2k(t-h)} \int_{t-h(t)}^t \begin{bmatrix} z(t) \\ z(t-h(t)) \\ \dot{z}(s) \end{bmatrix}^T \\
&\quad \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h(t)) \\ \dot{z}(s) \end{bmatrix} ds \\
&= e^{2k(t-h)} z^T(t) (h(t)X_{11} + X_{13}^T + X_{13}) z(t) \\
&\quad + 2e^{2k(t-h)} z^T(t) (h(t)X_{12} - X_{13} + X_{23}^T) z(t) \\
&\quad - h(t) + e^{2k(t-h)} z^T(t-h(t)) (h(t)X_{22} - X_{23}^T \\
&\quad - X_{23}) z(t-h(t)) \leq e^{2k(t-h)} z^T(t) (hX_{11} + X_{13}^T \\
&\quad + X_{13}) z(t) + 2e^{2k(t-h)} z^T(t) (hX_{12} - X_{13} + X_{23}^T) \\
&\quad \cdot z(t-h(t)) + e^{2k(t-h)} z^T(t-h(t)) (hX_{22} - X_{23}^T \\
&\quad - X_{23}) z(t-h(t)), \\
&- e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s) Y_{33} \dot{z}(s) ds \\
&\leq e^{2k(t-h)} \int_{t-h}^{t-h(t)} \begin{bmatrix} z(t-h(t)) \\ z(t-h) \\ \dot{z}(s) \end{bmatrix}^T
\end{aligned}$$

$$\begin{aligned}
& \cdot \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & 0 \end{bmatrix} \begin{bmatrix} z(t-h(t)) \\ z(t-h) \\ \dot{z}(s) \end{bmatrix} ds = e^{2k(t-h)} z^T(t) \\
& - h(t)) \left((h-h(t)) Y_{11} + Y_{13}^T + Y_{13} \right) z(t-h(t)) \\
& + 2e^{2k(t-h)} z^T(t-h(t)) \left((h-h(t)) Y_{12} - Y_{13} + Y_{23}^T \right) \\
& \cdot z(t-h) + e^{2k(t-h)} z^T(t-h) \left((h-h(t)) Y_{22} - Y_{23}^T \right. \\
& \left. - Y_{23} \right) z(t-h) \leq e^{2k(t-h)} z^T(t-h(t)) \left(hY_{11} + Y_{13}^T \right. \\
& \left. + Y_{13} \right) z(t-h(t)) + 2e^{2k(t-h)} z^T(t-h(t)) \left(hY_{12} \right. \\
& \left. - Y_{13} + Y_{23}^T \right) z(t-h) + e^{2k(t-h)} z^T(t-h) \left(hY_{22} \right. \\
& \left. - Y_{23}^T - Y_{23} \right) z(t-h), \\
\dot{V}_5(z_t) & \leq e^{2kt} f^T(z(t)) (Z_1 + Z_2) f(z(t)) \\
& - e^{2k(t-h)} f^T(z(t-h)) Z_1 f(z(t-h)) - (1-h_D) \\
& \cdot e^{2k(t-h)} f^T(z(t-h(t))) Z_2 f(z(t-h(t))).
\end{aligned} \tag{21}$$

By Lemma 3, we can obtain

$$\begin{aligned}
\dot{V}_6(z_t) & \leq r^2 e^{2kt} f^T(z(t)) S f(z(t)) \\
& - r \int_{t-r}^t e^{2ks} f^T(z(s)) S f(z(s)) ds \\
& \leq r^2 e^{2kt} f^T(z(t)) S f(z(t)) \\
& - e^{2k(t-r)} \left[\int_{t-r(t)}^t f(z(s)) ds \right]^T \\
& \left[\int_{t-r}^{t-r(t)} f(z(s)) ds \right] \\
& \cdot \begin{bmatrix} S & M \\ M^T & S \end{bmatrix} \begin{bmatrix} \int_{t-r(t)}^t f(z(s)) ds \\ \int_{t-r}^{t-r(t)} f(z(s)) ds \end{bmatrix}.
\end{aligned} \tag{22}$$

From (5), for any $n \times n$ diagonal matrices $H_i > 0$ ($i = 1, 2, \dots, 4$), the following inequality holds:

$$\begin{aligned}
0 & \leq e^{2kt} \left[z^T(t) L H_1 L z(t) - f^T(z(t)) H_1 f(z(t)) \right] \\
& + e^{2kt} \left[-z^T(t) H_2 L_1 z(t) + 2z^T(t) H_2 L_2 f(z(t)) \right. \\
& \left. - f^T(z(t)) H_2 f(z(t)) \right] \\
& + e^{2kt} \left[-z^T(t-h(t)) H_3 L_1 z(t-h(t)) \right. \\
& \left. + 2z^T(t-h(t)) H_3 L_2 f(z(t-h(t))) \right. \\
& \left. - f^T(z(t-h(t))) H_3 f(z(t-h(t))) \right] \\
& + e^{2kt} \left[-z^T(t-h) H_4 L_1 z(t-h) \right. \\
& \left. + 2z^T(t-h) H_4 L_2 f(z(t-h)) \right. \\
& \left. - f^T(z(t-h)) H_4 f(z(t-h)) \right].
\end{aligned} \tag{23}$$

Furthermore, for arbitrary matrices T_1, T_2, T_3 , and T_4 with appropriate dimensions, we have

$$\begin{aligned}
0 & = 2 \left[\dot{z}^T(t) T_1 + z^T(t) T_2 + f^T(z(t)) T_3 \right. \\
& \left. + f^T(z(t-h(t))) T_4 \right] \cdot \left[-\dot{z}(t) - A_0 z(t) \right. \\
& \left. + A_1 f(z(t)) + A_2 f(z(t-h(t))) \right. \\
& \left. + A_3 \int_{t-r(t)}^t f(z(s)) ds \right].
\end{aligned} \tag{24}$$

The combination of (19)–(24) gives

$$V(x_t) \leq \xi^T(t) \Psi \xi(t), \tag{25}$$

where $\xi(t) = [z(t) \ \dot{z}(t) \ z(t-h(t)) \ z(t-h) \ f(z(t)) \ f(z(t-h(t))) \ f(z(t-h)) \ \int_{t-r(t)}^t f(z(s)) ds \ \int_{t-r}^{t-r(t)} f(z(s)) ds]$.

From (14), we know that $\dot{V}(z_t) < 0$, which means the asymptotically stability of system (5). This completes the proof.

Furthermore, setting $d = \max\{h, r\}$, we can have

$$\begin{aligned}
V_1(z(0)) & \leq \lambda_{\max}(P) \|z(s)\|^2 \leq \lambda_{\max}(P) \\
& \cdot \sup_{-d \leq s \leq 0} \|z(s)\|^2, \\
V_2(z(0)) & \leq 2 \left[f^T(z(0)) - L^+ z(0) \right]^T R z(0) \\
& + 2 \left[L^+ z(0) - f(z(0)) \right]^T G z(0) \\
& \leq 2 \lambda_{\max}(L^+ - L^-) (\lambda_{\max}(R) + \lambda_{\max}(G)) \\
& \cdot \sup_{-d \leq s \leq 0} \|z(s)\|^2, \\
V_3(z(0)) & \leq h \left[\lambda_{\max}(R_1) + \lambda_{\max}(R_2) \right] \sup_{-d \leq s \leq 0} \|z(s)\|^2, \\
V_4(z(0)) & \leq \lambda_{\max}(R_3) \int_{-h}^0 \int_{t+\theta}^t z^T(s) \dot{z}(s) ds d\theta \\
& + \lambda_{\max}(R_4) \int_{-h(t)}^0 \int_{t+\theta}^t \dot{z}^T(s) \dot{z}(s) ds d\theta.
\end{aligned} \tag{26}$$

It is easy to have

$$\begin{aligned}
\dot{z}^T(s) \dot{z}(s) & = \left[-A_0 z(s) + A_1 f(z(s)) \right. \\
& \left. + A_2 f(z(s-h(s))) + A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta \right]^T \\
& \cdot \left[-A_0 z(s) + A_1 f(z(s)) + A_2 f(z(s-h(s))) \right. \\
& \left. + A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta \right] = z^T(s) A_0^T A_0 z(s) \\
& + f^T(z(s)) A_1^T A_1 f(z(s)) + f^T(z(s-h(s))) \\
& \cdot A_2^T A_2 f(z(s-h(s)))
\end{aligned}$$

$$\begin{aligned}
& + \int_{s-r(s)}^s f^T(z(\theta)) d\theta A_3^T A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta \\
& - 2z^T(s) A_0^T A_1 f(z(s)) - 2z^T(s) \\
& \cdot A_0^T A_2 f(z(s-h(s))) - 2z^T(s) \\
& \cdot A_0^T A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta + 2f^T(z(s)) \\
& \cdot A_1^T A_2 f(z(s-h(s))) + 2f^T(z(s)) \\
& \cdot A_1^T A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta + f^T(z(s-h(s))) \\
& \cdot A_2^T A_3 \int_{s-r(s)}^s f(z(\theta)) d\theta.
\end{aligned} \tag{27}$$

According to $2a^T b \leq a^T X a + b^T X^{-1} b$ with $X > 0$,

$$\begin{aligned}
\dot{z}^T(s) \dot{z}(s) & \leq 4 \left[\lambda_{\max}(A_0^T A_0) \right. \\
& + \lambda_{\max}^2(L) \lambda_{\max}(A_1^T A_1) + \lambda_{\max}^2(L) \lambda_{\max}(A_2^T A_2) \\
& + r^2 \lambda_{\max}^2(L) \lambda_{\max}(A_3^T A_3) \left. \right] \sup_{-d \leq s \leq 0} \|z(s)\|^2, \\
V_5(z(0)) & \leq h \lambda_{\max}^2(L) (\lambda_{\max}(Z_1) + \lambda_{\max}(Z_2)) \\
& \cdot \sup_{-d \leq s \leq 0} \|z(s)\|^2, \\
V_6(z(0)) & \leq r \lambda_{\max}(S) \\
& \cdot \int_{-r}^0 \int_{t+\theta}^t f^T(z(s)) f(z(s)) ds d\theta \leq \frac{r^3}{2} \lambda_{\max}^2(L) \\
& \cdot \lambda_{\max}(S) \sup_{-d \leq s \leq 0} \|z(s)\|^2.
\end{aligned} \tag{28}$$

Thus according to (26)–(28), there exists a positive constant γ such that

$$V(z(0)) < \gamma \sup_{-d \leq s \leq 0} \|z(s)\|^2, \tag{29}$$

where

$$\begin{aligned}
\gamma & = \lambda_{\max}(P) + 2\lambda_{\max}(L^+ - L^-) (\lambda_{\max}(R) \\
& + \lambda_{\max}(G)) + h [\lambda_{\max}(R_1) + \lambda_{\max}(R_2)] \\
& + 2h^2 [(\lambda_{\max}(R_3) + \lambda_{\max}(R_4))] [\lambda_{\max}(A_0^T A_0) \\
& + \lambda_{\max}^2(L) \lambda_{\max}(A_1^T A_1) + \lambda_{\max}^2(L) \lambda_{\max}(A_2^T A_2) \\
& + r^2 \lambda_{\max}^2(L) \lambda_{\max}(A_3^T A_3)] + h \lambda_{\max}^2(L) \\
& \cdot (\lambda_{\max}(Z_1) + \lambda_{\max}(Z_2)) + \frac{r^3}{2} \\
& \cdot \lambda_{\max}^2(L) \lambda_{\max}(S).
\end{aligned} \tag{30}$$

On the other hand, we have

$$V(z(t)) \geq e^{2kt} \lambda_{\min}(P) \|z(t)\|^2. \tag{31}$$

Therefore

$$\|z(t)\| \geq \sqrt{\frac{\gamma}{\lambda_{\min}(P)}} e^{-kt} \sup_{-d \leq s \leq 0} \|z(s)\|. \tag{32}$$

Then, from Definition 1, system (6) is exponentially stable with convergence rate k , and the proof is completed. $\square$

Remark 7. In the paper, the reduced conservatism of Theorem 6 benefits primarily from a new integral inequality, which is proved to be less conservative than the celebrated Jensen's inequality, and takes fully the relationship between the terms in the Leibniz-Newton formula within the framework of LMIs into account. In order to lower the conservatism of stability criteria, we further deal with the integral terms of $e^{2k(t-h)} \int_{t-h(t)}^t \dot{z}^T(s)(R_3 + (1-h_D)R_4 - X_{33})\dot{z}(s)ds$ and $e^{2k(t-h)} \int_{t-h}^{t-h(t)} \dot{z}^T(s)(R_3 - Y_{33})\dot{z}(s)ds$ via Lemma 4. Different from that of [17], this kind of processing method can reduce ulteriorly the conservatism of stability criteria.

Remark 8. As a matter of fact, Theorem 6 gives a stability criterion for system (6) with $h(t)$ satisfying $0 \leq h(t) \leq h$, $0 \leq \dot{h}(t) \leq h_D$, where h_D is given constant. In many cases, h_D is unknown. Considering this situation, a rate-independent corollary for the delay $h(t)$ satisfying $0 \leq h(t) \leq h$ is derived by setting $R_2 = 0$, $R_4 = 0$, and $Z_2 = 0$ in the proof of Theorem 6.

Theorem 9. For given scalars $0 < r$ and $h > 0$, the origin of system (6) with the neuron activation function $f(z(t))$ satisfying condition (5) is globally exponentially stable with the exponential convergence rate index k if there exist $P > 0$, $Z_1 > 0$, $R_i^T = R_i > 0$ ($i = 1, 3$), diagonal matrices $H_1 = \text{diag}\{h_{11}, h_{12}, \dots, h_{1n}\} > 0$, $H_2 = \text{diag}\{h_{21}, h_{22}, \dots, h_{2n}\} > 0$, $H_3 = \text{diag}\{h_{31}, h_{32}, \dots, h_{3n}\} > 0$, $H_4 = \text{diag}\{h_{41}, h_{42}, \dots, h_{4n}\} > 0$, $R = \text{diag}\{r_1, r_2, \dots, r_n\} > 0$, $G = \text{diag}\{g_1, g_2, \dots, g_n\} > 0$, any matrices T_i ($i = 1, 2, \dots, 4$), and

$$\begin{aligned}
& \begin{bmatrix} S & M \\ M^T & S \end{bmatrix} > 0, \\
& X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} > 0, \\
& Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} > 0,
\end{aligned} \tag{33}$$

$$R_3 - X_{33} > 0,$$

$$R_3 - Y_{33} > 0$$

such that the following symmetric linear matrix inequality holds:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & \Phi_{15} & T_2 A_2 - A_0 T_4^T & 0 & T_2 A_3 & 0 \\ * & \Phi_{22} & 0 & 0 & \Phi_{25} & T_1 A_2 - T_4^T & 0 & T_1 A_3 & 0 \\ * & * & \Phi_{33} & \Phi_{34} & 0 & H_3 L_2 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & H_4 L_2 & 0 & \\ * & * & * & * & \Phi_{55} & T_3 A_2 + A_1^T T_4^T & 0 & T_3 A_3 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & T_4 A_3 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -e^{-2kr} S & -e^{-2kr} M \\ * & * & * & * & * & * & * & * & -e^{-2kr} S \end{bmatrix} \leq 0, \quad (34)$$

where

$$\begin{aligned} \Phi_{11} &= 2kP + 4k(L^+ G - L^- R) + R_1 \\ &\quad + e^{-2kh} \left(hX_{11} + \overrightarrow{X_{13}} \right) - \frac{e^{-2kh}}{h} (R_3 - X_{33}) \\ &\quad - H_2 L_1 - \overrightarrow{T_2 A_0}, \\ \Phi_{12} &= P + L^+ G - L^- R - A_0 T_1^T - T_2, \\ \Phi_{13} &= e^{-2kh} \left(hX_{12} - X_{13} + X_{23}^T \right) - \frac{e^{-2kh}}{h} (R_3 - X_{33}), \\ \Phi_{15} &= 2k(R - G) + H_2 L_2 + T_2 A_1 - A_0 T_3^T, \\ \Phi_{22} &= hR_3 - \overrightarrow{T_1}, \\ \Phi_{25} &= R - G + T_1 A_1 - T_3, \\ \Phi_{33} &= -(1 - h_D) R_2 - \frac{e^{-2kh}}{h} (2R_3 - X_{33} - Y_{33}) \\ &\quad + e^{-2kh} \left(hX_{22} - \overrightarrow{X_{23}} + hY_{11} + \overrightarrow{Y_{13}} \right) \\ &\quad - H_3 L_1, \\ \Phi_{34} &= \frac{e^{-2kh}}{h} (R_3 - Y_{33}) + e^{-2kh} \left(hY_{12} - Y_{13} + Y_{23}^T \right), \\ \Phi_{44} &= -R_1 - \frac{e^{-2kh}}{h} (R_3 - Y_{33}) + e^{-2kh} \left(hY_{22} - \overrightarrow{Y_{23}} \right) \\ &\quad - H_4 L_1, \\ \Phi_{55} &= Z_1 + r^2 S - H_1 - H_2 + \overrightarrow{T_3 A_1}, \\ \Phi_{66} &= -H_3 + \overrightarrow{T_4 A_2}, \\ \Phi_{77} &= -e^{-2kh} Z_2 - H_4. \end{aligned} \quad (35)$$

The other procedure is straight forward from the proof of Theorem 6, so we omit it.

Remark 10. In the paper, we make full use of the relationship between $\int_{t-r(t)}^t f(z(s)) ds$ and $\int_{t-r}^{t-r(t)} f(z(s)) ds$, which can reduce the conservatism of stability criteria once again. However, these useful terms of $\int_{t-r(t)}^t f^T(z(s)) ds M \int_{t-r}^{t-r(t)} f(z(s)) ds$ and $\int_{t-r}^{t-r(t)} f^T(z(s)) ds S \int_{t-r}^{t-r(t)} f(z(s)) ds$ were always ignored in [18–20], which may lead to considerable conservatism to certain extent.

Remark 11. Due to constructing a simple type of Lyapunov-Krasovskii functional and taking full advantage of effective mathematical techniques, the conservatism of improved delay-dependent stability criteria obtained is reduced to a great degree in this study. Compared with those in previous articles [22, 23], we employ a few free variables and do not use a delay decomposition method and add some zero terms, that is, not only dividing the discrete delay interval $[0, d]$ into two ones $[0, d/2]$ and $[d/2, d]$, but also dividing the discrete delay interval $[0, d]$ into three ones $[0, d(t)/2]$, $[d(t)/2, d(t)]$, and $[d(t), d]$ and adding the following equalities:

$$\begin{aligned} &2e^{2k(t-d/2)} \zeta_1^T(t) U \left[z \left(t - \frac{d(t)}{2} \right) - z \left(t - \frac{d}{2} \right) \right. \\ &\quad \left. - \int_{t-d/2}^{t-d(t)/2} \dot{z}(s) ds \right] = 0, \\ &2e^{2k(t-d/2)} \zeta_1^T(t) V \left[z \left(t - \frac{d}{4} \right) - z \left(t - \frac{d(t)}{2} \right) \right. \\ &\quad \left. - \int_{t-d/2}^{t-d/4} \dot{z}(s) ds \right] = 0, \\ &2e^{2k(t-d/2)} \zeta_1^T(t) H \left[z(t) - z \left(t - \frac{d}{4} \right) \right] \end{aligned}$$

$$\begin{aligned}
& - \int_{t-d/4}^t \dot{z}(s) ds \Big] = 0, \\
& 2e^{2k(t-d)} \zeta_1^T(t) W \Big[z(t-d(t)) - z(t-d) \\
& - \int_{t-d}^{t-d(t)} \dot{z}(s) ds \Big] = 0, \\
& 2e^{2k(t-d)} \zeta_1^T(t) M \Big[z\left(t - \frac{d}{2}\right) - z(t-d(t)) \\
& - \int_{t-d(t)}^{t-d/2} \dot{z}(s) ds \Big] = 0, \\
& 2e^{2k(t-d)} \Big[\frac{d}{2} \zeta_1^T(t) Z \zeta_1(t) - \int_{t-d}^{t-d(t)} \zeta_1^T(t) Z \zeta_1(t) ds \\
& - \int_{t-d(t)}^{t-d/2} \zeta_1^T(t) Z \zeta_1(t) ds \Big] = 0, \\
& 2e^{2k(t-d/2)} \Big[\frac{d}{2} \zeta_1^T(t) Y \zeta_1(t) - \int_{t-d/2}^{t-d(t)/2} \zeta_1^T(t) Y \zeta_1(t) ds \\
& - \int_{t-d(t)/2}^{t-d/4} \zeta_1^T(t) Y \zeta_1(t) ds \\
& - \int_{t-d/4}^t \zeta_1^T(t) Y \zeta_1(t) ds \Big] = 0.
\end{aligned} \tag{36}$$

By using this method, the conservatism of the obtained stability condition in [22, 23] is reduced to some degree. However, the computing complexity is also improved since more variables are involved. Besides, we provided a comparison of the numbers of the variables involved in [22, 23] and our paper in Table 1. From Table 1, it is clear to see that the number of decision variables in our paper is much less than those in [22, 23]. Thus, it also expounds validity and applicability of the proposed method.

Remark 12. In many actual applications, maximum allowable time-delay upper bounds h are of interest. In Theorems 6 and 9, with a fixed h_D and r , k can be obtained through following optimization procedure:

$$\begin{aligned}
& \text{Maximize } h, \\
& \text{Subject to } (14) \text{ or } (34).
\end{aligned} \tag{37}$$

Besides, maximum allowable time-delay upper bounds k obtained are very valuable. In Theorems 6 and 9, with a fixed h_D and r , h can be also acquired through following optimization procedure:

$$\begin{aligned}
& \text{Maximize } k, \\
& \text{Subject to } (14) \text{ or } (34).
\end{aligned} \tag{38}$$

Inequalities (37) and (38) are a convex optimization problem and can be obtained efficiently by using the MATLABMI Toolbox.

TABLE 1: Comparison of the numbers of the involved variables.

Method/number of decision variables	[22]	[23]	This paper
Theorem 6	$41n^2 + 14n$	$52n^2 + 18n$	$14n^2 + 13n$
Theorem 9	$\frac{79}{2}n^2 + \frac{25}{2}n$	$\frac{101}{2}n^2 + \frac{33}{2}n$	$\frac{25}{2}n^2 + \frac{23}{2}n$

TABLE 2: Allowable upper bounds of k for k for Case A in Example 1.

Method	[20]	[21]	[24]	[23]	[22]	Theorem 6
$h = 0.5, r = 0.2, h_D = 0$	0.46	0.58	0.56	0.67	0.80	0.88
$h = 0.5, r = 0.2, h_D = 0.5$	0.21	0.35	0.35	0.45	0.50	0.65
$h = 0.6, r = 0.2, h_D = 0.5$	0.06	0.20	0.33	0.29	0.34	0.45
$h = 0.8, r = 0.2, h_D = 0.5$	0.00	0.05	0.10	0.11	0.14	0.21

4. Numerical Examples

In this section, three examples are given to demonstrate the feasibility and effectiveness of the main results derived above.

Example 1. Consider a delayed neural network in (6) with parameters as follows:

$$\begin{aligned}
A_0 &= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 1.2 & -0.8 & 0.6 \\ 0.5 & -1.5 & 0.7 \\ -0.8 & -1.2 & -1.4 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -1.4 & 0.9 & 0.5 \\ -0.6 & 1.2 & 0.8 \\ 0.5 & -0.7 & 1.1 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 1.8 & 0.7 & -0.8 \\ 1.8 & 0.7 & -0.8 \\ -0.4 & -0.6 & 1.2 \end{bmatrix}.
\end{aligned} \tag{39}$$

Case A (set $L^- = \text{diag}\{-1.2, 0, -2.4\}$, $L^+ = \text{diag}\{0, 1.4, 0\}$). For different h and h_D , the allowable upper bounds of the exponential convergence rate index k calculated by Theorem 6 in this paper and Theorem 1 in [20–24] are listed in Table 2. According to Table 2, this example is given to indicate significant improvements over some existing results.

Besides, for the parameters listed above, let $r = 0.2$, $h = 0.5$, $h_D = 0.5$, and $k = 0.65$. Then we can obtain the following feasible parameters by Theorem 6 in our paper. Due to

the limitation of the length of this paper, we only provide a part of the feasible solutions here as follows:

$$\begin{aligned}
 P &= \begin{bmatrix} 18.3158 & -9.1756 & 8.2601 \\ -9.1756 & 15.4632 & -3.5161 \\ 8.2601 & -3.5161 & 12.6661 \end{bmatrix}, \\
 Z_1 &= \begin{bmatrix} 11.6082 & 0.1957 & -0.2876 \\ 0.1957 & 11.0661 & -0.9006 \\ -0.2876 & -0.9006 & 11.3939 \end{bmatrix}, \\
 Z_2 &= \begin{bmatrix} 19.9317 & -6.4931 & -3.0095 \\ -6.4931 & 21.7679 & -1.9799 \\ -3.0095 & -1.9799 & 30.9316 \end{bmatrix}, \\
 R_1 &= \begin{bmatrix} 14.8956 & -0.4754 & 1.0203 \\ -0.4754 & 14.9458 & 0.3653 \\ 1.0203 & 0.3653 & 17.8456 \end{bmatrix}, \\
 R_2 &= \begin{bmatrix} 20.5413 & 2.7925 & 0.1017 \\ 2.7925 & 21.4962 & 1.6793 \\ 0.1017 & 1.6793 & 25.1932 \end{bmatrix}, \\
 R_3 &= \begin{bmatrix} 5.6240 & -3.0814 & 2.1045 \\ -3.0814 & 5.4295 & -1.2819 \\ 2.1045 & -1.2819 & 5.1175 \end{bmatrix}, \\
 R_4 &= \begin{bmatrix} 1.9845 & -1.3207 & 0.9292 \\ -1.3207 & 1.8417 & -0.6156 \\ 0.9292 & -0.6156 & 1.6339 \end{bmatrix}, \\
 T_1 &= \begin{bmatrix} 4.0278 & -1.9174 & 0.9670 \\ -1.6145 & 4.2443 & -0.2659 \\ 1.1456 & -1.1396 & 3.9531 \end{bmatrix}, \\
 T_2 &= \begin{bmatrix} 9.0432 & -0.7086 & 0.7264 \\ -0.8280 & 11.0177 & 0.5688 \\ 0.5356 & -0.9775 & 10.0672 \end{bmatrix}, \\
 T_3 &= \begin{bmatrix} -2.0541 & 0.5035 & -1.2211 \\ -1.2754 & 2.2408 & -1.7175 \\ 1.0163 & 1.5246 & -3.3814 \end{bmatrix}, \\
 X_{11} &= \begin{bmatrix} 13.8117 & -0.5993 & 0.7694 \\ -0.5993 & 13.5660 & 0.1550 \\ 0.7694 & 0.1550 & 14.4814 \end{bmatrix}, \\
 X_{12} &= \begin{bmatrix} -1.2696 & 0.2376 & 0.0313 \\ 0.2855 & -1.2632 & 0.2238 \\ -0.1326 & 0.1963 & -0.6930 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
 X_{13} &= \begin{bmatrix} 0.1994 & -0.4629 & 0.4020 \\ -0.3868 & -0.0134 & -0.1262 \\ 0.4274 & -0.2264 & 0.0949 \end{bmatrix}, \\
 X_{22} &= \begin{bmatrix} 11.6577 & -0.9613 & 0.8217 \\ -0.9613 & 11.2995 & -0.0487 \\ 0.8217 & -0.0487 & 11.7214 \end{bmatrix}, \\
 X_{23} &= \begin{bmatrix} 0.6172 & 0.1352 & -0.1692 \\ -0.0092 & 0.8882 & 0.0132 \\ 0.0119 & 0.0008 & 0.9916 \end{bmatrix}, \\
 X_{33} &= \begin{bmatrix} 3.3075 & -1.9174 & 1.3511 \\ -1.9174 & 3.0607 & -0.8479 \\ 1.3511 & -0.8479 & 2.8448 \end{bmatrix}, \\
 Y_{11} &= \begin{bmatrix} 11.6312 & -0.9820 & 0.8535 \\ -0.9820 & 11.2608 & -0.0736 \\ 0.8535 & -0.0736 & 11.6236 \end{bmatrix}, \\
 Y_{12} &= \begin{bmatrix} -1.3825 & 0.2516 & -0.0664 \\ 0.2417 & -1.5193 & 0.1776 \\ -0.0436 & 0.1741 & -1.3352 \end{bmatrix}, \\
 Y_{13} &= \begin{bmatrix} -0.5087 & -0.1303 & 0.1473 \\ -0.0614 & -0.6698 & -0.0360 \\ 0.1100 & -0.0658 & -0.6497 \end{bmatrix}, \\
 Y_{22} &= \begin{bmatrix} 13.8730 & -0.4984 & 0.5534 \\ -0.4984 & 13.6908 & 0.0897 \\ 0.5534 & 0.0897 & 14.0093 \end{bmatrix}, \\
 Y_{23} &= \begin{bmatrix} -0.1907 & 0.3597 & -0.2902 \\ 0.3244 & -0.1140 & 0.1213 \\ -0.2874 & 0.1574 & -0.1037 \end{bmatrix}, \\
 Y_{33} &= \begin{bmatrix} 2.8308 & -1.5833 & 1.1085 \\ -1.5833 & 2.6778 & -0.6906 \\ 1.1085 & -0.6906 & 2.5054 \end{bmatrix}.
 \end{aligned}$$

(40)

Case B (set $L^- = \text{diag}\{0, 0, 0\}$, $L^+ = \text{diag}\{0.5, 0.5, 0.5\}$). When $k = 0.5$ and $r = 1$, the maximum allowable upper bounds h obtained from Theorem 6 for different values h_D are listed in Table 3. From Table 3, it can be seen clearly that our results have less conservatism over the results acquired by [20, 22, 34–36] and illustrate the effectiveness and the advantage of the proposed methods. The active functions are chosen as $f_1(z(s)) = 0.25(|z(s) + 1| - |z(s) - 1|)$, $f_2(z(s)) = 0.25(|z(s) + 1| - |z(s) - 1|)$, and $f_3(z(s)) = 0.25(|z(s) + 1| - |z(s) - 1|)$. Moreover, the globally exponentially stable with the initial value $[-2.5, -0.5, 2.5]$ is shown in Figures 1–3.

TABLE 3: Maximum allowable time delay upper bounds h for Case B in Example 1.

Method	[20]	[34]	[35]	[36]	[22]	Theorem 6
$h_D = 0$	2.2931	3.3294	5.8415	7.2820	9.6503	9.8356
$h_D = 0.5$	2.1010	2.6363	4.0130	5.8958	8.0394	8.2175
$h_D = 0.8$	1.7939	1.7200	3.6574	4.0847	5.8974	6.0561

FIGURE 1: State trajectories of $z(t)$ in the plane for $h = 9.8356$ in Example 1.

Example 2. Consider a delayed neural network in (6) with parameters as follows:

$$\begin{aligned}
 A_0 &= \begin{bmatrix} 2.3 & 0 & 0 \\ 0 & 3.4 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, \\
 A_1 &= \begin{bmatrix} 0.9 & -1.5 & 0.1 \\ -1.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.8 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} 0.8 & 0.6 & 0.2 \\ 0.5 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.5 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}.
 \end{aligned} \tag{41}$$

Let $L^- = \text{diag}\{0, 0, 0\}$ and let $L^+ = \text{diag}\{0.2, 0.2, 0.2\}$.

Case A. For $h = r$ the corresponding upper bounds of h for unknown h_D obtained by Theorem 9 and the results in [18–20] are listed in Table 4. According to Table 4, this example

TABLE 4: Maximum allowable time delay upper bounds $h = r$ in Example 2.

Method	[19]	[18]	[20]	Theorem 9
$h = r$	1.833	3.597	5.068	5.220

TABLE 5: Maximum allowable time delay upper bounds r for different values k in Example 2.

Method	0.1	0.3	0.5	0.8
[22]	9.082	4.774	3.313	2.217
Theorem 9	9.235	4.963	3.518	2.363

TABLE 6: Comparison of the maximum delay h between different methods for various h_D in Example 3.

h_D	0	0.2	0.5	0.7
[24]	3.377	3.336	3.268	3.264
Theorem 6	3.518	3.427	3.358	3.386

shows that the stability criterion in this paper gives much less conservative results than those in the previous literatures.

The active functions are set as $f_1(z(s)) = 0.1(|z(s) + 1| - |z(s) - 1|)$, $f_2(z(s)) = 0.1(|z(s) + 1| - |z(s) - 1|)$, and $f_3(z(s)) = 0.1(|z(s) + 1| - |z(s) - 1|)$; the globally exponentially stable with the initial value $[-0.4, -0.2, 0.5]$ is shown in Figures 4–6.

Case B. For $h = 1$, the corresponding upper bounds of r for unknown h_D obtained by Theorem 9 are listed in Table 5. From Table 5, it is shown clearly that our results have significant improvement over the existing results.

Example 3. Consider a delayed neural network in (6) with parameters as follows:

$$\begin{aligned}
 A_0 &= \begin{bmatrix} 3.99 & 0 \\ 0 & 2.99 \end{bmatrix}, \\
 A_1 &= \begin{bmatrix} 1.188 & 0.09 \\ 0.09 & 1.188 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} 0.009 & 0.14 \\ 0.05 & 0.09 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} 0.45 & -0.2 \\ 0.3 & 0.42 \end{bmatrix}.
 \end{aligned} \tag{42}$$

Let $L^- = \text{diag}\{0, 0\}$ and let $L^+ = \text{diag}\{1, 1\}$.

When $k = 0.5$ and $r = 0.1$, the corresponding upper bounds of h for different h_D obtained by Theorem 6 are listed in Table 6. The active functions are chosen as $f_1(z(s)) = 0.5(|z(s) + 1| - |z(s) - 1|)$ and $f_2(z(s)) = 0.5(|z(s) + 1| - |z(s) - 1|)$; the globally exponentially stable with the initial value $[-0.6, 0.8]$ is given in Figures 7–9. Therefore, we can say that the results in this paper are less conservative than those in [24].

FIGURE 2: Phase trajectory of model of $z(t)$ in the plane for $h = 9.8356$ in Example 1. FIGURE 3: Phase trajectory of model of $z(t)$ in the space for $h = 9.8356$ in Example 1. FIGURE 4: State trajectories of $z(t)$ in the plane for $h = 5.220$ in Example 2.

FIGURE 5: Phase trajectory of model of $z(t)$ in the plane for $h = 5.220$ in Example 2. FIGURE 6: State trajectories of $z(t)$ on the space for $h = 5.220$ in Example 2. FIGURE 7: State trajectories of $z(t)$ in the plane for $h = 3.518$ in Example 3.

5. Conclusions

In this paper, the delay-dependent exponential stability problem for NNs with mixed time-varying delays has been investigated. By using a new integral inequality approach to express the relationship between the terms in the Leibniz-Newton formula within the framework of LMIs for the first

time, several less conservative delay-dependent exponential stability criteria are obtained. Moreover, combining effective mathematical techniques and a convex optimization approach, new delay-dependent exponential stability conditions are derived by constructing a proper LKF. Finally, three numerical examples are given to illustrate the feasibility and effectiveness of the proposed methods. The foregoing

FIGURE 8: Phase trajectory of model of $z(t)$ in the plane for $h = 3.518$ in Example 3.

FIGURE 9: State trajectories of $z(t)$ on the space for $h = 3.518$ in Example 3.

methods have the potential to be useful for the further study of neural network. Meanwhile, it is expected that these approaches can be further applied to other delayed systems.

Disclaimer

The authors promise that this paper has not been published and will not be simultaneously submitted or published elsewhere.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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