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Research Article

Optimization Model and Algorithm Design for Airline Fleet Planning in a Multiairline Competitive Environment

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This paper presents a multiobjective mathematical programming model to optimize airline fleet size and structure with consideration of several critical factors severely affecting the fleet planning process. The main purpose of this paper is to reveal how multiairline competitive behaviors impact airline fleet size and structure by enhancing the existing route-based fleet planning model with consideration of the interaction between market share and flight frequency and also by applying the concept of equilibrium optimum to design heuristic algorithm for solving the model. Through case study and comparison, the heuristic algorithm is proved to be effective. By using the algorithm presented in this paper, the fleet operational profit is significantly increased compared with the use of the existing route-based model. Sensitivity analysis suggests that the fleet size and structure are more sensitive to the increase of fare price than to the increase of passenger demand.

1. Introduction

The fleet planning decision-making process is considered to be one of the most problematic issues for airline industry. An over-large fleet size would cause an airline unnecessary expense since the increasing capital assets account for a large proportion of the airline operational costs. On the contrary, an underestimated fleet size would also result in a great number of passengers overflowing to other market competitors. Moreover, considering the profit margin of the airline industry around the world continuously pressured by a long-term exposure to a high-cost and low-fare environment, the irrational fleet composition would necessarily deteriorate the airline's operation. Therefore, airlines may have to develop a more practical fleet planning approach to meet passenger demand with lower costs and more controllable risks at a strategic level.

The aim of airline fleet planning is to determine the fleet size and structure for a given operational environment, including network characteristics, flight schedule, and mean fare levels. Macro-fleet planning is considered to be one of the most popular approaches around the world, where network-based passenger demand within a future area is used to estimate the needed number of aircraft of different types for a given candidate aircraft type set. However, the oversimplifying macro-approach is hardly to reflect the adaptability of a specific type of aircraft flying on route; for example, aircraft of common types without modification to propulsion and oxygen system are forbidden to fly on plateau routes. In addition, the economic feature is also beyond the consideration scope of macro-fleet planning approach, for example, the passenger-spilling problem in single aisle aircraft with small seating capacity on heavily traveled routes as well as the vacant seat problem in two-aisle aircraft with big seating capacity flying on less traveled routes.

In order to avoid these disadvantages, more attentions have been paid to the application of micro-fleet planning approaches, where passenger demand on a single route or flight leg is accommodated by different types of aircraft. Then the number of different types of aircraft flying on every route is aggregated to determine the fleet size and structure.

Dynamic fleet management is one of the most important branches in micro-fleet planning approaches, where the fleet assignment technique [1–3] is widely used to optimize

the fleet size and structure under the condition that the future detailed flight schedule has been already presented. In this aspect, Listes and Dekker used time-space network to construct a fleet assignment-based model to determine the fleet composition. They also developed a scenario aggregation-based algorithm to solve the model [4]. Wang and Sun utilized simulated annealing algorithm to solve airline fleet planning problem and discussed a robust airline fleet planning method [5, 6]. However, this kind of approach is based on a given flight schedule, which is hardly simulated due to the uncertainty of airline's future environment. This drawback may result in an unreliable fleet size and structure deprived from the fleet assignment-based approaches.

Therefore, recent relevant studies have focused mainly on route-based fleet planning approaches, in which the best aircraft type or aircraft type mix is assigned to each route to maximize the fleet operational profit (or minimize the fleet-related costs). In this aspect, Schick and Stroup proposed a multiyear fleet planning model with consideration of passenger demand constraints and aircraft balance equations, as well as minimum and maximum flight frequency to minimize the fleet-related-costs [7]. Sun et al. applied a similar model to analyze the Chinese market [8]. Wang et al. presented a new fleet planning approach for those airlines operating in Hub and Spoke network, in which network effects are highlighted [9]. Wei and Hansen discussed the competitive relationship between aircraft size and flight frequency using game theory. It was concluded that the extra landing fee could reduce flight delay and airport congestion [10, 11]. Takebayashi constructed a supply-demand interaction/SDI model for Haneda airport. He held the view that airlines did not always adopt a downsizing aircraft size strategy in response to airport runway expansion [12]. Tsai et al. incorporated the constraint of the European Emissions Trading Scheme into a mixed activity-based fleet planning model. They believed that a self-purchased wide-bodied airplane could benefit from high revenue tone kilometers [13]. Givoni and Rietveld evaluated the impact of environmental factors on the choice of aircraft size. They thought that environmental improvement could benefit from those airlines using large aircraft size [14]. Rosskopf et al. proposed a multiobjective linear programming model to study the trade-off between economic-environment goals. They argued that the environment goal could be achieved by 6% improvement at the cost of 3% deviation from economic optimum [15]. Pai analyzed the main factors affecting the choice of aircraft size and flight frequency [16]. Other relevant studies [17, 18] analyzed some external impacting factors on fleet size and structure (e.g., Brown, 1992; Bahram et al., 1999).

Previous studies on route-based fleet planning approaches investigated the issue mainly depending on airline itself. Few researches have taken into account the impact of other airlines' competitive behaviors on the airline's operation. This paper aims to optimize the fleet size and structure through network-wide allocating different types of aircraft and flight frequency under multiairline competitive behaviors. Efforts are made to construct a multiobjective model to maximize each airline fleet operational profit subject to the available flight frequency offered to each route and air-crew flying hours for each aircraft fleet type. This study formulates the fleet operational profit as evaluating function, including fleet operational costs, penalty cost, and passenger spilling cost. In addition, this paper also develops an effective algorithm to solve the proposed model. The validation and benefits of the model are shown with a case study. Therefore, the main contributions of this paper can be summarized as follows.

(i) We develop a new route-based fleet planning model, which is capable of reflecting the impact of multiairline competitive behaviors on fleet size and structure.

(ii) We devise a heuristic algorithm for our route-based model and show its validation through Monte Carlo simulation.

(iii) Through case study using real airline data, we quantify to analyze the benefits of the model presented in this paper.

(iv) Through sensitivity analysis, we find the important factors impacting airline fleet size and structure.

The remainder of this paper is organized as follows. In the next section, the problem is presented in detail including mathematical modeling in competitive environment. In Section 3, a heuristic algorithm is introduced according to equilibrium optimum theory. For a case with real airline data, the multiobjective function is solved by the proposed algorithm coding with MATLAB software in Section 4. And Section 5 is the conclusion of this paper.

2. Problem Modeling

In this section, the paper illustrates how multiairline competitive behaviors affect the choice of aircraft size and flight frequency. Then we extend the existing route-based approach listed in reference [8] to construct a network-wide allocation model of the choice of aircraft size and flight frequency with multiairline competitive behaviors.

2.1. Problem Description. Traditional viewpoint generally holds that an airline's choice of aircraft size and flight frequency largely depends on the network-wide distribution of the airline's passenger demand. However, few studies [7–9] inversely consider that an airline's choice of aircraft size and flight frequency in a competitive environment is also greatly affecting the airline capability of capturing passenger demand. *S* curve presented by Simpson [19] in the last century is considered as one of most famous functions around the world for evaluating airline market share, which is defined as a ratio of captured demand for an airline to the total demand in the market. It is concluded that airline market share is a function with respect to flight frequency and yields to *S* shape. Recently, Wei and Hansen [20], once again, have confirmed *S* curve function widely existing in airline market. Based on these evidences, *S* curve function is used throughout this paper to analytically express the relationship between airline

market share and flight frequency. The mathematical model can be written as

$$\mathrm{MS}_i = \frac{(N_i)^{\beta}}{\sum_{j=1}^{o} (N_j)^{\beta}}, \quad (1)$$

where MS_i is the market share for airline i on a competitive route. N_j is the number of flights that airline j ($j = 1, 2, \ldots, i, \ldots, o$) provides to the competitive route. β means market share index. Symbol o denotes the total number of airlines on the route.

Previous route-based approaches must predict an airline's demand in advance and then use the proper number of flights provided by different types of aircraft to determine the airline's fleet size and structure while satisfying its predicted demand [7–9]. This process obviously neglects the impact of the choice of aircraft size and flight frequency on the airline captured passenger demand. In order to capture the desired market share, the airline must provide a certain flight frequency. Furthermore, the flight frequency that the airline provides alone could not wholly determine its market share, which is also affected by the flight frequencies provided by other airlines in the same market.

In order to facilitate the description of the problem, we assume that two airlines (named A and B for simplicity) operate on the same route in which 600 total passengers exist. The mean fare level is 1,000 Yuan and the market share index $\beta = 1$. Both of airlines use aircraft type 1 with 50 seats and aircraft type 2 with 100 seats as their candidate aircraft types. Suppose that the unit operational cost of aircraft 1 and aircraft type 2 is 14,000 and 20,000 Yuan, respectively. When airlines A and B provide three flights with aircraft type 2, respectively, they all account for half market share and the corresponding operational profit reaches 240,000 Yuan, respectively. In this situation, when airline A increases the number of flights to six, then the market share of airlines A and B is changed to 67% and 33%, respectively. The corresponding fleet operational profit of airline A reaches 280,000 Yuan, but the operational profit of airline B is reduced to 140,000 Yuan. This evidence shows how one airline's choice of flight frequency impacts other airlines' operational profits. If we use four airplanes with type 1 to replace three airplanes with type 2 for airline A, then the fleet operational profit becomes 30,400 Yuan. This suggests that an airline can also increase the fleet operational profit through optimizing its fleet structure. Therefore, the choice of aircraft size and flight frequency not only affects the airline itself, but also significantly influences other airlines' decision policies in the same market. Using previous fleet planning approaches without consideration of airlines' competitive behaviors, the resulting fleet size and structure would be unreliable. As a result, it is necessary for airlines to develop a more practical fleet planning approach to deal with the multiairline competitive behaviors.

2.2. Assumptions and Limitations

(i) Airlines in competitive environment are rational participants and the objectives of them are all to maximize fleet operational profits.

(ii) Airline market share is affected only by the number of flights the airline offered to the market (route).

(iii) All information, such as fare price, passenger demand, and candidate-aircraft-type-related information, is known during the planning period.

(iv) Critical resources, such as flight frequency and air-crew flying hours, are known and finite during the planning period. Moreover, none of them can be exceeded.

(1) Sets

$I = \{1, 2, \ldots, G\}$: set of airlines in a competitive network environment, indexed by i.

$J_i = \{1, 2, \ldots, H_i\}$: set of routes in a competitive network environment, indexed by j.

$K_i = \{1, 2, \ldots, Q_i\}$: set of candidate aircraft types for airline i in a competitive network environment, indexed by k.

(2) Parameters

p_{ij}: the average fare price on route j for airline i.

c_{ijk}: the fleet related costs per flight for airline i flying on route j with aircraft type k.

s_{ijk}: the available seats offered by airline i flying on route j with aircraft type k.

D_j: the passenger demand on route j.

b_{ijk}: the average flying hours offered by airline i flying on route j with aircraft type k.

F_{ij}: the maximum flight frequency offered by airline i on route j.

T_{ik}: the maximum air-crew flying hours in aircraft fleet type k for airline i.

u_{ik}: the expected aircraft utilization rate of aircraft type k for airline i.

β_j: the market share index for route j.

ε: a torrent term used to reflect the extent to which the planning manager allows the loss of fleet operational profit.

δ_{ijk}: = 1 if candidate aircraft type k is airworthy on route j for airline i, otherwise = 0.

M: a sufficient large positive number.

(3) Decision Variables

x_{ijk}: the number of flights on route j flown by aircraft type k for airline i.

y_{ij}: the spilling number of passengers on route j for airline i.

(4) Functions

z_i: the fleet operational profit of airline i, a function with respect to x_{ijk} and y_{ij}.

MS_{ij}: the market share of airline i on route j.

2.3. A Unified Evaluating Function. In general, previous mathematical models [5–7] for airline fleet planning approaches use fleet operational costs as objective function, which can hardly reflect the impact of route fare levels on fleet size and structure and finally lead to a conservative result. Furthermore, airworthiness and passenger spilling are two important factors that should be sufficiently considered in evaluating each airline's operation. Therefore, a unified evaluating function (called "fleet operational profit" for simplicity) needs to be defined and it should mainly include the following components:

(i) Fleet operational costs: it is the sum of the cost items directly related to the operation on a route with a given aircraft type, including fuel consumption, landing and take-off fees, and gate rental. Besides that, the capital assets related to the depreciation and amortization of different types of aircraft should be also included in these cost items.

(ii) Penalty cost: the cost item used to reflect a route whether a specific aircraft type can fly or not. A sufficient positive number (M) will be added to the route operational costs when airline uses an unqualified aircraft type to fly, otherwise the penalty cost equaling zero.

(iii) Passenger spilling cost: the cost item represents the passenger revenue loss due to an insufficient aircraft capacity.

(iv) Passenger revenue: the revenue item represents the generated revenue from an airline capturing passengers.

2.4. Mathematical Formulation. As is analyzed in Section 2.1, the coupling relationship for an airline itself is not only reflected by the interaction between the choice of aircraft size and flight frequency and the airline capturing passenger demand, but also reflected by other airlines' competitive behaviors. Moreover, this coupling relationship would become more complicated in a network-wide environment because airlines' competitive behaviors suffer more limitations from their critical resources:

(i) Maximum flight frequency on each route: there exists an upper-bound in airline flight frequency on each route due to the airspace structure and management model as well as allocating policy on airport slots over a period of time. The lack of runway slots, especially in congested airports, is regarded as a serious bottleneck for airlines to provide better services. In this context, the limited flight frequency tends to be offered to more profitable markets and the wide-bodied aircraft types with more seating capacity are used so long as the growth of passenger demand can afford their high operational costs. Therefore, the flight frequency is such a critical resource that inevitably affects airline fleet size and structure. The mathematical formulations can be written as

$$\sum_{k\in K_i} x_{ijk} \le F_{ij}, \quad \forall i \in I,\ j \in J_i. \tag{2}$$

Constraints (2) are maximum flight frequency constraints for each airline, ensuring that the number of flights cannot exceed the maximum flight frequency on each route.

(ii) Maximum captured passenger demand on each route: an airline captured demand on a route depends on the number of flights that both the airline provides and other competitive airlines provide. Meanwhile, an airline could also use different mixes of aircraft types to balance its captured demand and the corresponding supply. But in any case, airline enplaned passengers could not exceed the airline captured demand. The mathematical formulations can be written as

$$\sum_{k\in K_i} s_{ijk} x_{ijk} + y_{ij} = \mathrm{MS}_{ij} D_j, \quad \forall i \in I,\ j \in J_i. \tag{3}$$

Constraints (3) are passenger flow conservation constraints for each airline, ensuring the enplaned and spilled passengers on each route equal to the market the airline captures. In addition, MS_{ij} can be mathematically written as

$$\mathrm{MS}_{ij} = \frac{\left(\sum_{k\in K_i} x_{ijk}\right)^{\beta}}{\sum_{i\in I}\left(\sum_{k\in K_i} x_{ijk}\right)^{\beta}}, \quad \forall i \in I,\ j \in J_i, \tag{4}$$

where term $\sum_{k\in K_i} x_{ijk}$ is the number of flights that airline i provides to route j.

(iii) Maximum air-crew flying hours of each aircraft fleet type: to ensure flight safety, the number of air-crew flying hours for each aircraft fleet type is strictly limited by civil regulations (e.g., 100 flying hours per month for one pilot). These limitations make some routes incapable of being flown by the desired aircraft types because the air-crew flying hours have been wholly exhausted during a period of time (e.g., one year). This means no qualified pilots (compliance with regulation requirements) are left to drive the corresponding aircraft to fly these routes. This limitation maybe results in some other substitute aircraft types to fly those routes. Therefore, the maximum air-crew flying hours for each aircraft fleet type necessarily affect the fleet planning process and mathematically can be written as

$$\sum_{j\in J_i} b_{ijk} x_{ijk} \le T_{ik}, \quad \forall i \in I,\ k \in K_i. \tag{5}$$

Constraints (5) are the limitations of available air-crew flying hours, ensuring that the number of flying hours for a specific aircraft fleet type can not exceed the corresponding maximum number.

Constraints (6) are the numeric types and value ranges of decision variables:

$$x_{ijk} \ge 0, \text{int}, y_{ij} \in R, \quad \forall i \in I,\ j \in J_i,\ k \in K_i. \tag{6}$$

Note that decision variables y_{ij} are generally relaxed to a real numeric type because airline market share typically belongs to $[0, 1]$.

Finally, the multiairline objective function maximizing each airline's fleet operational profit can be mathematically written as

$$\max z_i = \sum_{j\in J_i}\sum_{k\in K_i}\left(p_{ij}s_{ijk} - c_{ijk} - M\delta_{ijk}\right)x_{ijk} - \sum_{j\in J_i} p_{ij}\left|y_{ij}\right|, \quad \forall i \in I, \tag{7}$$

where the first term in right hand means the fleet planning profit comprised of (1)–(4) items described in Section 2.3. The second term also in right hand represents the passenger spilling cost when decision variables y_{ij} are positive. If not, an offset item is generated to offset the overcalculated value from the first term due to the surplus of seating capacity unfilled with passengers when decision variables y_{ij} are negative.

3. Algorithm Design

Model (2)–(7) is a nonlinear mix integer programming problem, in which one airline's network-wide distribution of the number of flights with different types of aircraft affects the fleet operational profit of both the airline and other competitors in the same network environment. Meanwhile it tends to be accompanied with other airlines' decision polices changed. Hence, our interests are in the equilibrium optimal solution [21, 22] to the multiobjective function. In this section, we introduce a heuristic algorithm to solve the proposed model.

3.1. Description of Equilibrium Optimum. Consider n airlines in the game of network-wide distribution of aircraft size and flight frequency, with N and i representing the set of airlines and a particular airline, respectively, $i \in N$. Let X_i be the distributing policy set of airline i and let ϕ_i be the fleet operational profit of airline i. One policy set $(x_1, x_2, \ldots, x_n)$ can be constructed when each airline selects its own distributing policy x_i $(x_i \in X_i)$. We define $(x_i' \mid X) \equiv (x_1, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_n)$ as a distributing policy set with airline i adopting policy x_i' while other competitors remain distributing policy x_j $(j = 1, \ldots, i-1, i+1, \ldots, n)$ unchanged. Then the equilibrium optimal solution X^* could be achieved when each airline's fleet operational profit is all maximized in the game, or mathematically written as

$$\phi_i\left(X^*\right) = \max_{(x_i|X^*)\in X} \phi_i\left(x_i' \mid X\right), \quad \forall i. \tag{8}$$

We introduce formulation (9) as a function so as to easily solve the equilibrium optimal solution:

$$\varphi\left(X, x'\right) = \sum_{i\in I}\left(\phi_i\left(x_i' \mid X\right) - \phi_i\left(X\right)\right). \tag{9}$$

Function $\varphi(X, x')$ is a sum of airlines' increased profits when each airline i changes its distributing policy from x_i to x_i'. If policy x_i is the best solution to airline i, then equation $x_i = x_i'$ holds. Otherwise, the policy x_i is replaced by x_i'. Therefore, equilibrium optimal solution X^* will be found when function $\varphi(X, x')$ equals zero, or mathematically written as

$$Z\left(X^*\right) = \arg\max_{x'\in X} \varphi\left(X, x'\right). \tag{10}$$

3.2. Optimization Procedure. Therefore, the model presented in this paper must be iteratively solved since the solution to one airline is based on other airlines' given solutions according to formulations (8)–(10). In order to obtain the equilibrium optimum solution, each airline is selected in order of increasing airline number. For each airline, the most profitable mix of aircraft type and route is selected one by one. Then the algorithm determines the corresponding number of flights to ensure that the increased fleet operational profit is always maximized so long as the remaining number of flights on the route and the remaining air-crew flying hours of the aircraft fleet type are still positive. After the algorithm visiting all of feasible mixes, the current solution is determined. Then, according to a certain principle, both current and last solutions are used to update the next iterative solution until the increased fleet operational profit for each airline can not be improved any more, which suggests the equilibrium optimum solution is obtained. This process can be depicted as the flow chart shown in Figure 1.

Algorithm Procedure

Step 1. Initialize the maximum iteration number (*max_iteration_nums*), the number of airlines (*airline_nums*). Set iterative count $t = 0$, airline number $i = 0$, and decision variables $x_{ijk}^t = 0, \forall i, j, k$.

Step 2. If $t < max\_iteration\_nums$, then use formulations (2)–(7) to calculate airline fleet operational profit $profit_i^t, \forall i$, and otherwise go to Step 10.

Step 3. Set airline number $i = i + 1$. If $i < airline\_nums$, then initialize the set of assigning mixes of aircraft types and routes by formulation (11) and otherwise go to Step 9.

$$type\_route\_set_i = \left\{(j,k) \mid \left(p_{ij}s_{ijk} - c_{ijk}\right) > 0,\ j \in J_i,\ k \in K_i\right\}. \tag{11}$$

Step 4. Select the mix of aircraft type and route with maximum fleet operational profit for airline i from the set $type\_route\_set_i$ using the following formulation:

$$\left(j^*, k^*\right) = \left\{(j,k) \mid \max_{(j,k)\in type\_route\_set_i}\left\{p_{ij}s_{ijk} - c_{ijk}\right\}\right\}. \tag{12}$$

Step 5. Calculate current maximum flight frequency $avail\_flight\_nums_{ij^*}$ for airline i using formulation (13) with aircraft type k flying on route j:

$$avail\_flight\_nums_{ij^*} = \min\left\{F_{ij^*}, \left\lfloor \frac{T_{ik^*}}{b_{ij^*k^*}} \right\rfloor\right\}. \tag{13}$$

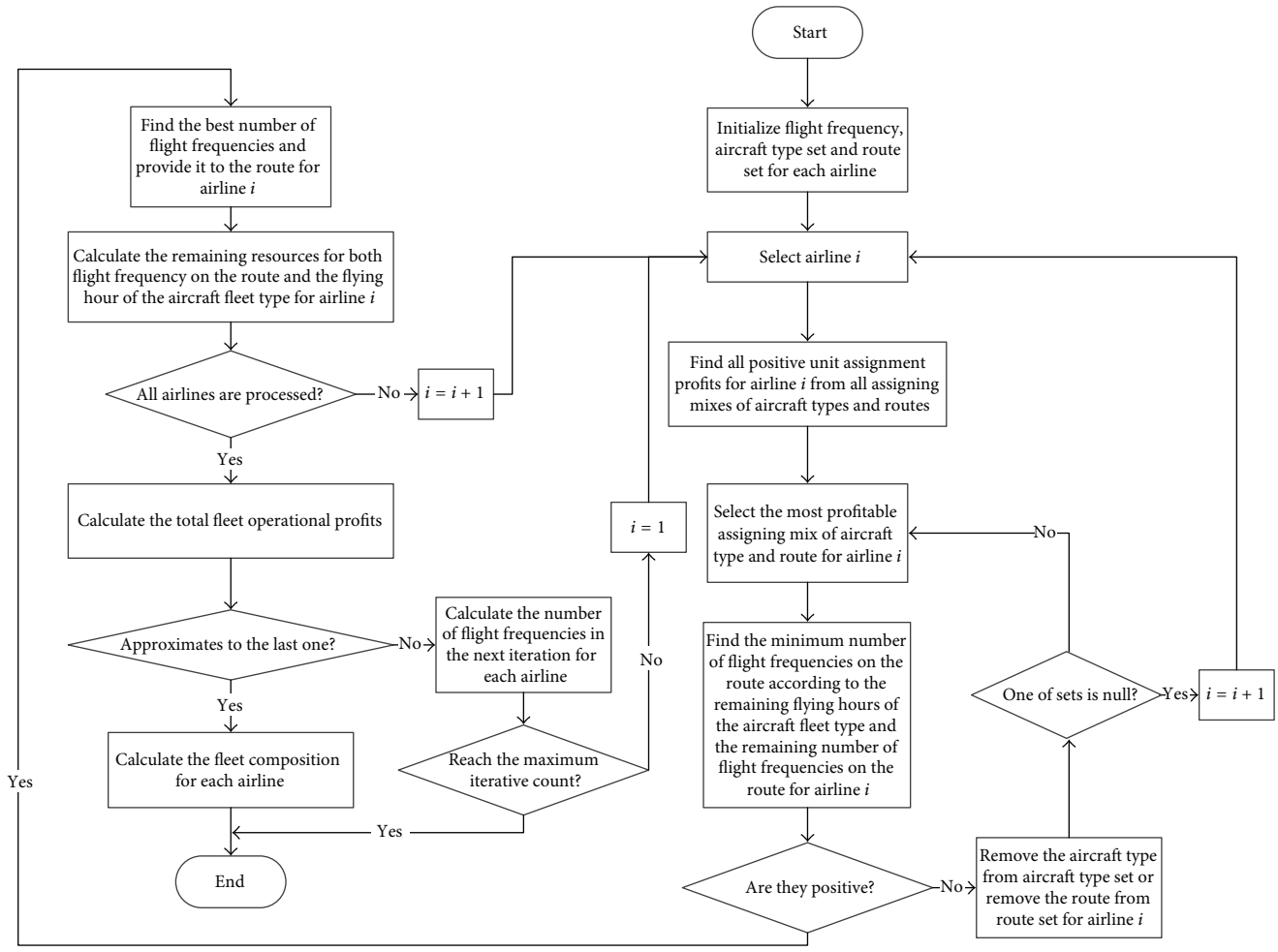

FIGURE 1: Flow chart of optimization procedure.

Step 6. If $avail\_flight\_nums_{ij^*} > 0$, then calculate the revenue loss $y^t_{ij^*}$ using formulation (14) and the fleet operational profit for airline i operating on route j^* using formulation (15), respectively, so as to find such $x^t_{ij^*k^*}$ that ensure the condition $\{z^t_{ij^*}(x^t_{ij^*k^*}) \geq z^t_{ij^*}(x^t_{ij^*k^*} - 1)\} \wedge \{z^t_{ij^*}(x^t_{ij^*k^*}) \geq z^t_{ij^*}(x^t_{ij^*k^*} + 1)\}, x^t_{ij^*k^*} \in \{0, \ldots, avail\_flight\_nums_{ij^*}\}$ holds and go to Step 8. Otherwise go to Step 7:

$$y^t_{ij^*} = \mathrm{MS}^t_{ij^*} D_{j^*} - \sum_{k \in K_i} s_{ij^*k} x^t_{ij^*k}, \tag{14}$$

$$\begin{aligned} z^t_{ij^*}\left(x^t_{ij^*k^*}\right) &= \sum_{k \in K_i} \left(p_{ij^*} s_{ik} - c_{ij^*k}\right) x^t_{ij^*k} \\ &\quad - p_{ij^*} \left| \mathrm{MS}^t_{ij^*} D_{j^*} - \sum_{k \in K_i} s_{ij^*k} x^t_{ij^*k} \right|, \end{aligned} \tag{15}$$

where $\mathrm{MS}^t_{ij^*} = (\sum_{k \in K_i} x^t_{ij^*k})^{\beta_{j^*}} / ((\sum_{k \in K_i} x^t_{ij^*k})^{\beta_{j^*}} + \sum_{i' \in I \setminus \{i\}} (\sum_{k \in K_{i'}} x^{(t-1)}_{i'j^*k})^{\beta_{j^*}})$.

Step 7. If $\lfloor T_{ik^*} / b_{ij^*k^*} \rfloor \leq 0$, then set $route\_type\_set_i = route\_type\_set_i \setminus \{(j^*, k^*)\}$. Otherwise set $route\_type\_set_i = route\_type\_set_i \setminus \{(j^*, k) \mid k \in K_i\}$. If $route\_type\_set_i = \{\}$, calculate the fleet operational profit $profit^t_i$ for airline i according to current decision variables x^t_{ijk} $(\forall j, k)$ and go to Step 3. Otherwise go to Step 4.

Step 8. Set $F_{ij^*} = F_{ij^*} - x^t_{ij^*k^*}$, $T_{ik^*} = T_{ik^*} - b_{ij^*k^*} x^t_{ij^*k^*}$ and $route\_type\_set_i = route\_type\_set_i \setminus \{(j^*, k^*)\}$. If $route\_type\_set = \{\}$, then go to Step 3. Otherwise go to Step 4.

Step 9. If $\mathrm{sum}_{i \in I}(profit^t_i - profit^{(t-1)}_i) \leq \varepsilon$, then go to Step 10. Otherwise update decision variables x^t_{ijk} $(\forall i, j, k)$ for all airlines in competitive environment according to formulation (16), set iterative count $t = t + 1$, and go to Step 2:

$$x^{(t+1)}_{ijk} = \left\lceil \alpha x^t_{ijk} + (1 - \alpha) x^{(t-1)}_{ijk} \right\rceil, \quad 0 \leq \alpha \leq 1, \tag{16}$$

where α is a weighting factor used to reflect the relative importance for current decision variables x^t_{ijk}.

Table 1: Basic information of candidate aircraft types.

Airline number	1	2	3	4	5	6	7	8	9	10	11
Candidate aircraft type	1~7	1~6	1~6	1~6	1~5	1~3	1~2	1~2	1~4	1~2, 4	1~2

Step 10. Calculate the number of different types of aircraft for each airline by term $\lceil \sum_{j \in J_i} x_{ijk}/u_{ik} \rceil$ $(\forall i \in I, k \in K_i)$ with a given utilization rate of each specific aircraft fleet type and terminate.

3.3. Computation Complexity. The algorithm procedure presents the computation complexity in nature. It reveals that the iterative process is terminated when the maximum iterative count (here denoted by M) or the tolerant term is satisfied. For each iteration, each airline needs to calculate the unit assignment profits of all mixes of aircraft types and routes, which largely depend on the number of both candidate aircraft types and the quasi-operating routes in the candidate route set. According to Step 3, the worst situation will happen at the time that all unit assignment profits are positive. It suggests that all candidate aircraft types flying on each route need to be calculated one time, or mathematically the computation complexity can be written as $O(H_iQ_i)$ for airline i and $O(IH_iQ_i)$ for all airlines. Another concern on computation complexity must be given to the selection of the best number of flights provided by different types of aircraft for each airline. After selecting the most profitable mix of aircraft type and route, the best number of flights provided by the selected aircraft type on the route must be found in the interval [0, the current remaining number of flights] according to Step 6. Note that the upper-bound of the interval is determined by the minimum one between the remaining number of flights from the maximum flight frequency on the route and the remaining number of flights converted from the remaining air-crew flying hours of the aircraft fleet type for the given route through using term $\lfloor T_{ij}/b_{ijk} \rfloor$. This also can be mathematically expressed as $\min\{F_{ij}, \lfloor T_{ij}/b_{ijk} \rfloor\}$. Therefore, the worst situation will happen at the time that the minimum value reaches the maximum. It suggests that the range of the interval is extended to the largest. Accordingly, the computation complexity can be mathematically expressed as $O(\min\{F_{ij}, \lfloor T_{ij}/b_{ijk} \rfloor\})$ for each selected unit profit mix and $O(IH_iQ_i \cdot \min\{F_{ij}, \lfloor T_{ij}/b_{ijk} \rfloor\})$ for each airline.

Therefore, the computation complexity of the proposed algorithm can be mathematically expressed as $O(M(IH_iQ_i + IH_iQ_i \cdot \min\{F_{ij}, \lfloor T_{ij}/b_{ijk} \rfloor\}))$ or $O(M(IH_iQ_i \cdot \min\{F_{ij}, \lfloor T_{ij}/b_{ijk} \rfloor\}))$ for simplicity.

4. Case Study

To demonstrate applications of the model, a case study is conducted, based on available data from 11 airlines with 20 routes and 7 candidate aircraft types in China.

4.1. Basic Information. For the sake of simplification, 25 major airports in China are selected from all the cities being served by 11 airlines in 2009. None of the routes are served by all eleven airlines. On each route, there exist four airlines on average, including one monopolized market (route) and one route operated by six airlines. Seven aircraft types are served as candidate aircraft types. Airline number shown in Table 1 represents the number of airlines participating in the competitive environment. The candidate aircraft types present the preliminary choice of aircraft types each airline probably selects in the fleet planning process. The selection of candidate aircraft types mainly depends on airlines' operational data on 20 routes in 2009. More detailed information is listed in Table 1.

Table 2: Basic information of routes and airlines.

Route number	Demand/year	Airline number
1	94046	1, 4, 6, 10
2	26377	1, 2, 4, 6
3	87934	2, 3, 4, 5, 7
4	103394	1, 2, 6, 7
5	65260	1, 2, 5, 6
6	271588	1, 4, 9
7	58920	3, 8
8	103490	3, 8
9	602044	1, 3, 6, 7, 10
10	140488	1, 3, 6, 9
11	63703	2, 4, 6, 9
12	57753	2, 4, 5
13	223	3
14	33106	1, 4, 6, 8, 10
15	55585	1, 2, 4, 5, 6
16	104557	1, 3, 5, 9
17	28042	2, 3, 4, 6, 9
18	9060	1, 3, 8
19	201189	1, 3, 8, 11
20	37931	1, 2, 3, 4, 5, 9

The seating capacity of candidate aircraft types 1~7 is 130, 162, 165, 192, 198, 244, and 298, respectively. The operational scope of each airline is listed in Table 2, in which the second column represents the statistical data of 20 routes in 2009. In addition, we suppose that the market share index $\beta = 2.8$ for all routes and the parameters of the proposed algorithm are $max\_iteration\_nums = 100$, $\alpha = 0.6$, and $\varepsilon = 0$, respectively.

4.2. Results and Discussions. According to the presented solution algorithm shown in Section 3.2, the resulting fleet size and structure for each airline is listed in Table 3. Each row in Table 3 represents the equilibrium optimum number of different types of aircraft each airline owns. The last column in Table 3 shows the resulting fleet operational profit for each airline. Therefore, the total fleet operational profits for

Table 3: Fleet planning scheme in the competitive environment.

Airline number	Fleet planning scheme							Operational profit
	Aircraft type							
	1	2	3	4	5	6	7	
1	0	0	1	5	0	0	0	65,476,930
2	0	0	2	0	0	0	0	13,309,389
3	0	0	1	1	0	0	0	3,599,863
4	0	0	0	1	0	1	0	23,307,034
5	0	0	0	2	0	0	0	12,355,883
6	0	0	7	0	0	0	0	51,897,709
7	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	323,388
9	0	0	0	1	0	0	0	8,898,600
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
Total	1	0	11	10	0	1	0	179,168,800

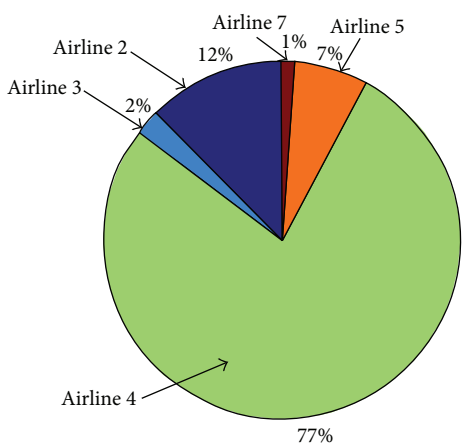

Figure 2: Proportion of flight frequency on route 3.

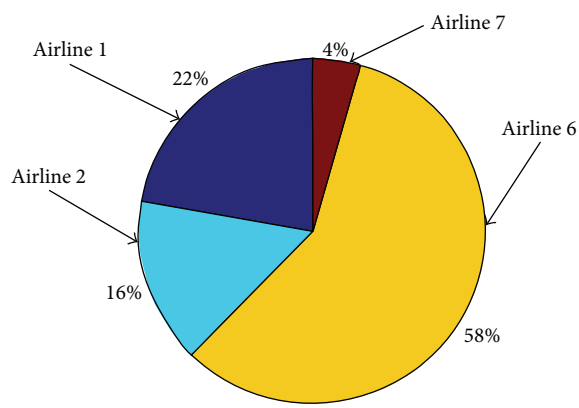

Figure 3: Proportion of flight frequency on route 4.

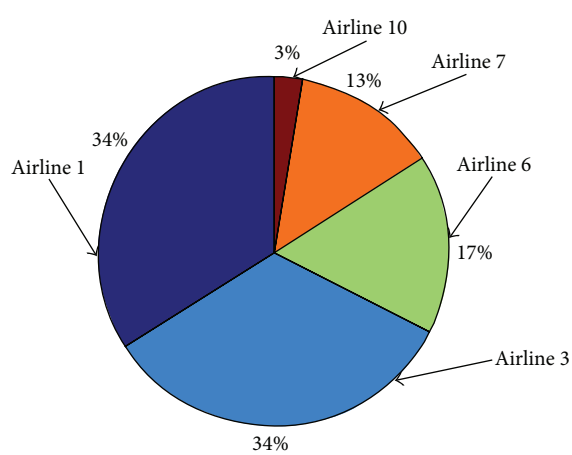

Figure 4: Proportion of flight frequency on route 9.

airlines operating in the competitive environment amount to 179,168,800 Yuan.

As can be seen from Table 3, small-scale airlines are eliminated from the competitive environment. They are unable to seize any profitable markets against the competitors since few critical resources are given to them. Small-scale here refers to the airline with (relative) few routes to be operated and few critical resources used to seize market share. Figure 2 shows that small-scale airline 7 needs to compete with another four airlines on route 3 to seize the market share using yearly available flight frequencies, a proportion of 1% in total on route 3, from which almost no market share can be obtained according to S curve function. It is similar to the cases on route 4 for airline 7 (shown in Figure 3) and routes 9 and 1 for airline 10 (shown in Figures 4 and 5).

Note particularly that the proportion for airline 11 on route 19 shown in Figure 7 reaches 55%, but no flight needs to be offered to the route. The reason is that whatever candidate aircraft types are used to operate on route 19, the unit assignment profit is constantly negative. This is also the case for airline 10 on route 14 (Shown in Figure 6). These results show that small-scale airlines with few critical resources available to be used can hardly carry out operations on routes with intensified competition due to their disadvantage

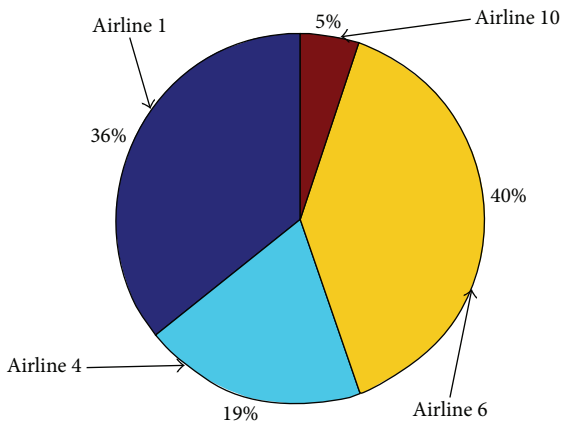

Figure 5: Proportion of flight frequency on route 1.

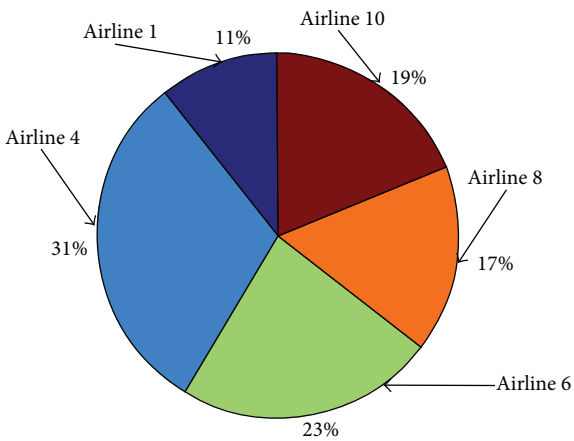

Figure 6: Proportion of flight frequency on route 14.

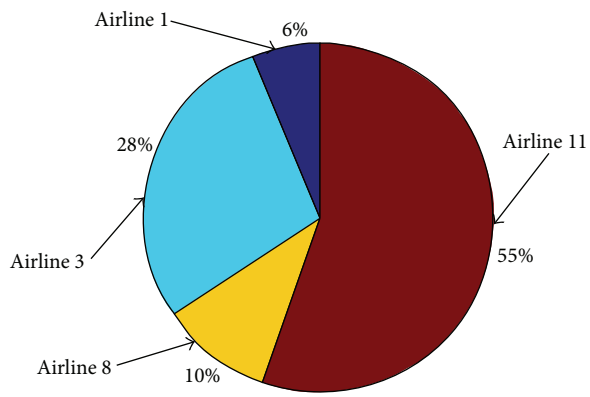

Figure 7: Proportion of flight frequency on route 19.

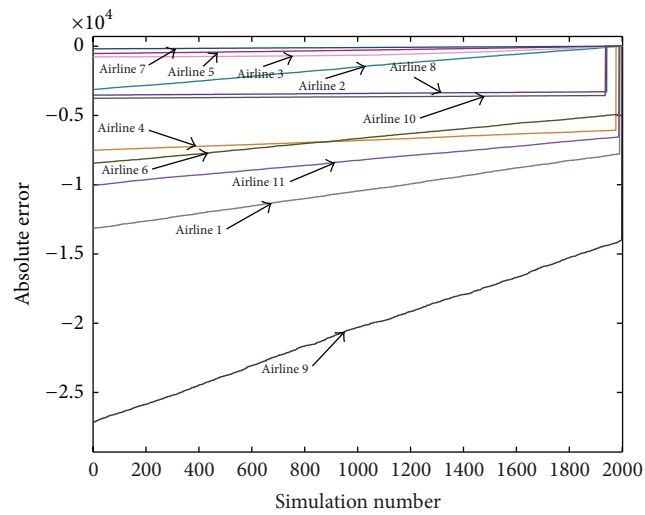

Figure 8: Monte Carlo simulations on absolute error of fleet operational profit.

positions in the aspects of critical resources. In order to enter into these routes, one regular approach for these airlines is to lower their fare levels to ensure their unit assignment profits are changed to the positive values. To some extent, it is just the reason why small-scale airlines need to lower their fare levels when entering into a new competitive market.

In addition, equilibrium optimum solution refers to the fact that one rational participant who changes its decision policy alone can not increase its profit any more so long as other participants keep their decision polices unchanged. In order to illustrate the effectiveness of the solution algorithm presented in Section 3.2, we adopt Monte Carlo simulation to verify the result. Detailed procedure of simulation can be depicted as follows.

(i) We select airline as adjustor in order of increasing airline number and randomly generate one decision policy of the selected airline while keeping the decision policies of other unselected airlines unchanged.

(ii) Based on (i), we simulate 2000 times for each adjustor in order using Monte Carlo simulation and calculate the corresponding fleet operational profit for each time.

(iii) We then use corresponding equilibrium optimum value to minus each simulated fleet operational profit so as to formulate the absolute error of fleet operational profit for each simulation.

(iv) We draw these points in Figure 8 in order of increasing absolute error for each adjustor.

For each airline, Figure 8 shows that all of absolute errors are all below zero. It suggests that there is no one decision policy that could increase one airline fleet operational profit alone when all the decision policies of other airlines remain unchanged and the solution deprived from presented algorithm is just the equilibrium optimum.

In order to illustrate the benefits of the approach presented in this paper (called "competitive model" for simplicity), we compare it with the model presented in reference [8] (called "traditional model" for simplicity), in which the route-based approach optimizing airline fleet size and structure is based on the assumption that the airline's passenger demand on each route has been already given. However, an airline predicted passenger demand largely depends on the methods

Table 4: Comparisons to the traditional model on related indices.

Airline number	Competitive model			Traditional model								
	Operational profit/ten thousand	Load factor	Spilling rate	Operational profit/ten thousand			Load factor			Spilling rate		
				H	M	L	H	M	L	H	M	L
1	6547.69	0.90	0.05	−67050.61	−72526.87	−82963.03	0.78	0.59	0.51	0.76	0.72	0.71
2	1330.94	0.59	0.00	−1952.80	−5397.65	−8850.21	0.37	0.30	0.22	0.31	0.02	0.00
3	359.99	0.61	0.00	−45501.85	−49305.16	−54436.66	0.59	0.51	0.40	0.88	0.78	0.78
4	2330.70	0.76	0.00	−25092.23	−36008.55	−39799.08	0.57	0.29	0.23	0.78	0.61	0.58
5	1235.59	0.82	0.00	−915.41	−2212.67	−4155.67	0.55	0.40	0.36	0.57	0.01	0.00
6	5189.77	0.85	0.05	−1036.96	−3936.72	−13159.13	0.72	0.59	0.51	0.48	0.18	0.16
7	0.00	0.00	0.00	−447.18	−447.18	−447.18	0.01	0.01	0.01	0.00	0.00	0.00
8	32.34	0.66	0.00	−6350.43	−6545.28	−9329.96	0.90	0.75	0.57	0.84	0.64	0.63
9	889.86	1.00	0.00	−5315.25	−6174.76	−22315.95	0.88	0.85	0.70	0.77	0.42	0.40
10	0.00	0.00	0.00	−50781.93	−51437.41	−51461.72	0.77	0.44	0.42	0.98	0.96	0.96
11	0.00	0.00	0.00	−5462.05	−5462.05	−5462.05	1.00	1.00	1.00	0.71	0.71	0.71

Note: H and L represent the highest and lowest values in all simulations, respectively. M is the average value of all simulating results.

and assumptions that the airline uses. It means that the different methods and assumptions would present different predicting results. In order to cover all possible predicted passenger demand for each airline in a competitive environment, we assume passenger demand predicted by each airline ranges from 0 to total passenger demand on route and yield to uniform distribution. We use Monte Carlo simulation to generate passenger demand first for each airline (served as predicted passenger demand) and obtain the number of flights of different types of aircraft using the traditional model. These resulting values of decision variables are then put into the competitive model to calculate the actual fleet operational profit and the values of related indices (all listed in the column of "Traditional model") while the decision policies of other airlines remain unchanged during the airline being processed. The whole simulating experiment conducts 10,000 times for each airline and the results are listed in the last nine columns of Table 4, in which load factor is defined as a ratio of the captured passenger demand to the total passenger demand. Spilling rate is also a fraction which is referred to as a ratio of the spilling number of passenger demand to the total passenger demand. As can be seen from Table 4, the fleet operational profit for each airline is dramatically decreased by using the traditional model. None of fleet operational profits resulting from the competitive model are lower than the ones from the traditional model. All the results listed in Table 4 are similar to each other. Row 1 (airline 1) in Table 4 can be considered as a representative result for these comparisons and shows that the resulting fleet operational profit and load factor deprived from competitive model are all much better than the corresponding values resulting from the traditional model. The spilling rate in row 1 also reveals that the competitive model (0.05) outperforms the traditional one (0.71~0.76). Note particularly that all fleet operational profits of airlines 7, 10, and 11 become negative in case of any one quasi-route these airlines operate on or any one candidate aircraft type these airlines are used to operate with. This shows again that the results of airlines 7, 10, and 11 deprived from the model presented in this paper are the equilibrium optimum. Therefore, it could be concluded that the relationship between flight frequency and market share is a significant factor that must be considered into airline fleet planning process. The competitive model could better reflect the realistic environment and therefore the results are more consistent with realities.

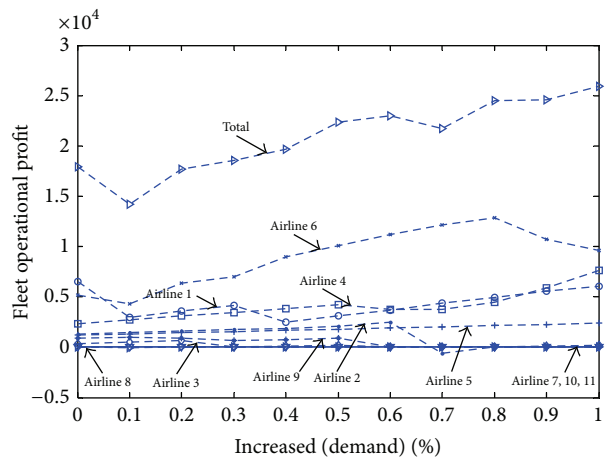

Figure 9: Impact of passenger demand on fleet operational profit.

4.3. Sensitivity Analysis. Passenger demand and fare price in competitive environment are the most important parameters that compel airlines to change their decision policies so as to (relatively) maximize their fleet operational profits. To analyze these impacts, we consider data in case study as benchmark and increase the proportion of passenger demand and fare price within interval [0, 1], respectively. Then the solution algorithm shown in Section 3.2 is used again to solve the competitive model for each time increase of passenger demand and fare price. The total fleet operational profit shown in Figure 9 is roughly positive proportion to

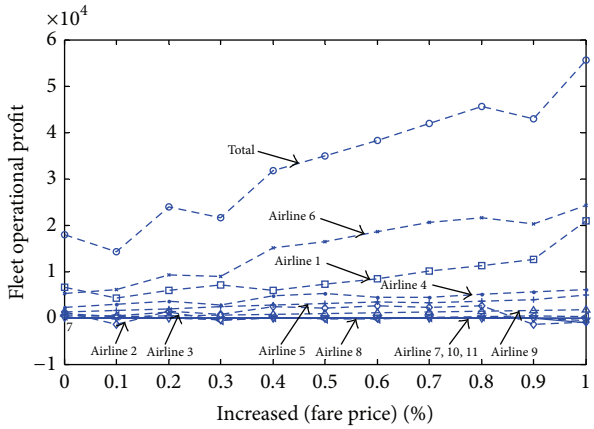

FIGURE 10: Impact of fare price on fleet operational profit.

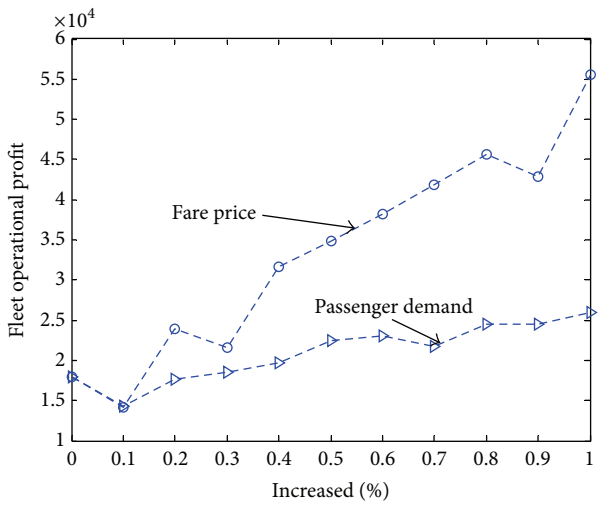

FIGURE 11: Comparison of passenger demand and fare price on fleet operational profit.

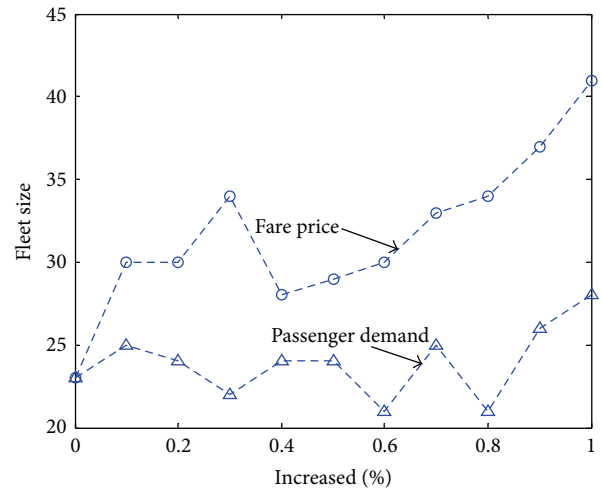

FIGURE 12: Comparison of passenger demand and fare price on fleet size.

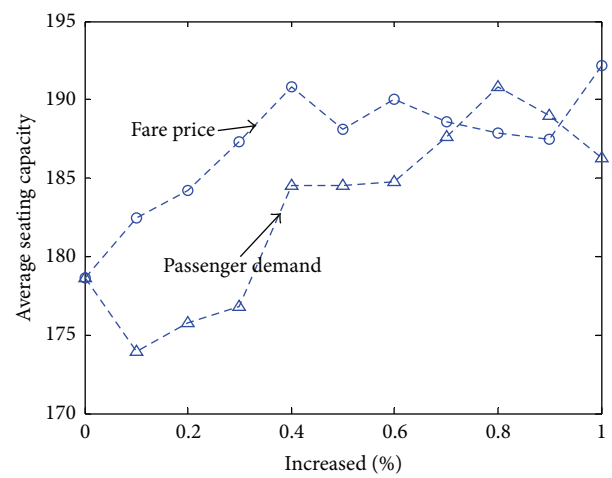

FIGURE 13: Comparison of passenger demand and fare price on average seating capacity.

passenger demand. For each airline, the fleet operational profit also shows positive proportion to passenger demand. In particular, airlines 7, 10, and 11 are still eliminated from competitive environment despite the increasing passenger demand. The reason is that the unit assignment profit of aircraft type and route is unchanged yet. In addition, Figure 10 reveals similar laws in terms of fleet operational profit for the increase of fare price.

However, as shown in Figure 11, its increasing rate is much faster than the rate in the increase of passenger demand at the aggregate level. This suggests that the fleet operational profit is more sensitive to the increase of fare price than to the increase of passenger demand for the whole competitive environment. The reason is that airlines could obtain the extra enhanced revenue without needing to provide any flight in the increased fare environment. This suggests that airlines can obtain additional profits without incurring any additional (or few) costs in maintaining or acquiring more market share. On the contrary, the rapid growth of passenger demand in the competitive environment cannot make airlines directly acquire more market share. The only way to acquire the desired market share is to provide more flights on the market (route), which means more related costs incurred and parts of fleet operational profits are offset. As a result, the competitive model is more sensitive to fare price than to passenger demand.

Furthermore, in order to illustrate the sensitivity of fleet size and structure, we record each of corresponding data on fleet size and structure in Figures 12 and 13. Fleet size refers to the total number of different types of aircraft in the whole environment. On the basis of fleet size, fleet structure is defined as the average seating capacity per aircraft.

As can be seen from Figures 12 and 13, both fleet size and average seating capacity are all roughly positive proportion to passenger demand and fare price at the aggregate level. They also perform more sensitivity to the increase of fare price than to the increase of passenger demand. This is similar to the conclusion deprived from Figures 9, 10, and

11. The results show that fare price, rather than passenger demand, is the major factor contributing to the increase of fleet size and average seating capacity. This seems to be contradiction with the traditional viewpoint, in which fleet size increases with the growth of passenger demand and average seating capacity is up-sized only when the available flight frequency on route cannot meet passenger demand any more. Figures 12 and 13, contrarily, show that fleet size and average seat capacity are all more sensitive to fare price. It could be explained that the increase of fare price means more direct fleet operational profits (without any additional costs incurred) airlines could be obtained. Based on them, airlines could afford more costs incurred from using either larger aircraft types or more aircraft numbers in pursuit of maximizing airlines' fleet operational profits.

However, the increase of passenger demand just only means the possibility for airlines to capture more passengers. To capture their desired passengers, airlines unnecessarily need to provide corresponding flights due to the vacant seats which may exist on some flights, or airlines are actually unwilling to provide additional flights to capture these passengers since the costs incurred cannot be offset by the generated revenue. These are the reasons that result in the slow growth of flight frequency and less sensitivity of fleet size and average seating capacity to the increase of passenger demand.

5. Conclusions

Motivated by the concern that the assumption of airline captured demand on each route in route network to be presented first is too simplistic to reflect the true complexity of the relationship between flight frequency and market share, we develop a new route-based fleet planning approach in which we (i) incorporate S curve function into airline fleet planning process, (ii) construct a multiobjective function maximizing each airline fleet operational profit under the constraints that the number of flights offered to each route and air-crew flying hours obtained from each aircraft fleet type must not exceed the corresponding maximum number, respectively, and (iii) devise a practical solution procedure to solve the presented model. Through Monte Carlo simulation, the solution procedure presented in this paper is proved to be effective. Case studies show that no one airline can increase fleet operational profit by changing its distribution of aircraft size and flight frequency alone while the distributing policies of other airlines remain unchanged. Sensitivity analysis suggests that the fare is the main changing factor to airline fleet size and structure as opposed to the passenger demand.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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