Adaptive Neural Control Based on High Order Integral Chained Differentiator for Morphing Aircraft

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This paper presents an adaptive neural control for the longitudinal dynamics of a morphing aircraft. Based on the functional decomposition, it is reasonable to decompose the longitudinal dynamics into velocity and altitude subsystems. As for the velocity subsystem, the adaptive control is proposed via dynamic inversion method using neural network. To deal with input constraints, the additional compensation system is employed to help engine recover from input saturation rapidly. The highlight is that high order integral chained differentiator is used to estimate the newly defined variables and an adaptive neural controller is designed for the altitude subsystem where only one neural network is employed to approximate the lumped uncertain nonlinearity. The altitude subsystem controller is considerably simpler than the ones based on backstepping. It is proved using Lyapunov stability theory that the proposed control law can ensure that all the tracking error converges to an arbitrarily small neighborhood around zero. Numerical simulation study demonstrates the effectiveness of the proposed strategy, during the morphing process, in spite of some uncertain system nonlinearity.

1. Introduction

With the development of morphing wing technology, the flight performance of an aircraft can be improved according to the current flight conditions [1–3]. The morphing aircraft are the flight vehicles that change their shape to either effectuate a change in mission or provide control authority for maneuvering [4, 5], without the use of discrete control surfaces or seams. Aircraft with morphing capability exhibit the distinct advantages of being able to fulfill multiple types of missions and to perform extreme maneuvers not possible with conventional aircraft [6, 7].

The field of morphing aircraft research is composed of a large array of interdisciplinary studies, including wing structure, actuation systems, aerodynamic modeling, nonrigid dynamics, and flight control [8]. A number of studies have focused on optimization of the actuator locations in the morphing structure units [9–11]. Other relative research work that involves the aeroelastics analysis is presented in [12]. The importance of the inertial forces and moments is studied in [13], with the goal of reducing the dynamics that must be dealt with in the flight control design. A methodology which is suitable for numerical calculation of the dynamic loads for a morphing aircraft is presented in [14]. In [15], linear parameter varying modeling is proposed for a folding wing morphing aircraft during the wing morphing process, whereas the longitudinal dynamic responses are numerically simulated based on the quasi-steady aerodynamic assumption.

Despite significant advances in the development of wing structure, actuation systems, and dynamic model, much work remains to be done to effectively control the morphing aircraft. The control system of a morphing aircraft must be capable of achieving consistent and robust performance meanwhile maintaining stability during large variations in the aircraft geometry, which may severely affect aerodynamic forces, moments of inertia, and center of mass. For the disturbance rejection, a pair of linear controllers is synthesized for a linear input-varying morphing aircraft in [16]. A simple proportional state feedback control integrated with the eigenstructure assignment is proposed for the span-morphing aircraft in [17]. Based on a linear parameter
vvarying model, self-gain scheduled $H_\infty$ controller is designed for the wing transition process in [18]. On the basis of varying linear parameter and classical methodology, a synthesized multiloop controller of a morphing unmanned aerial vehicle is formulated to guarantee a good performance subjected to large-scale geometrical shape changes in [19].

To cope with system uncertainties, adaptive control and neural network control techniques have been used for decades. For a linear morphing aircraft dynamic model, an indirect adaptive control method is designed in [6], which comprises the receding horizon optimal control law coupled with the modified sequential least squares parameter identification. In [20], a single network adaptive critic tracking controller design for a morphing aircraft is studied, wherein the set of initial weights of the neural network is determined by using a linear system model, which requires offline pretraining. Based on the concepts of feedback linearization, in [21], a combination of dynamic inversion and structured model reference adaptive control is used for the control of a morphing air vehicle. Typically a morphing aircraft exhibits highly nonlinear dynamics characteristics. Because of the morphing aircraft’s design and flight condition, it is extremely sensitive to change in physics as well as aerodynamic parameters. Almost all controller designs discussed above are based on linear models. Moreover the input saturation (physical limitation in engine) has not been considered in any work, which usually applies in many practical systems and severely degrades the closed-loop performance [22].

As a powerful nonlinear technique, backstepping control has been used for control system designs with strict-feedback form, extensively. With conventional backstepping, a possible issue is the explosion of complexity. This is caused by the repeating differentiations of certain nonlinear functions. To efficiently handle the system uncertainty in each subsystem, RBFNN with the universal approximation capability is employed in [23, 24]. Since RBFNN is used, we need to take derivatives of those radial basis functions, which will further lead to heavier calculation burden in each step design. Recently, the dynamic surface control was employed to solve this problem and many research results were presented [25, 26]. However, the determination of virtual control terms during the backstepping design requires tedious and complex analysis. More than one neural network is taken for approximation whose complexity increases like the order of the controlled backstepping design.

The motivation of this paper is to present a nonlinear robust adaptive neural controller for the morphing aircraft based on high order integral chained differentiator to achieve stability in the sweeping process where both system uncertainty and input restrictions are considered. The contribution of this paper can be summarized as follows.

Firstly, a nonlinear longitudinal model is derived from a curved-fitted model, with the center of mass position, aerodynamic forces, and the moments of inertia being varied with respect to the morphing parameters. The longitudinal model is then decomposed into altitude and velocity subsystems.

Secondly, the highlight is that the altitude subsystem dynamics is transformed into normal-feedback formulation and a robust adaptive neural controller using HICD is designed where only one neural network is employed to approximate the lumped uncertain system nonlinearity. The controller is considerably simpler than the ones based on backstepping which requires tedious and complex analysis for their virtual control terms. This feature guarantees that the computational burden of the algorithm can be reduced. Moreover the algorithm is convenient for real-time implementation on flight computers. Meanwhile, the adaptive control is proposed for velocity subsystem and an additional compensation system is employed to deal with input constraints, which will help engine recover from input saturation rapidly.

Finally, the Lyapunov synthesis based on stability analysis is used to prove that all the signals in the closed systems are semiglobally uniformly ultimately bounded with tracking error converging to a close neighborhood of origin.

The rest of the paper is organized as follows: Section 2 introduces the model of the morphing aircraft and formulates the normal output-feedback form of the altitude and velocity subsystems of longitudinal dynamics of the morphing aircraft. Section 3 briefly describes the background theory of RBFNN. Section 4 presents the adaptive neural controller design and the stability analysis for altitude and velocity subsystems. The simulation results are presented and discussed in Section 5. Section 6 gives the concluding remarks and future works.

2. Problem Formulation

2.1. Morphing Aircraft Model. The control-oriented model of the longitudinal dynamics of a morphing aircraft considered in this study is based on Seigler [4, 5]. This model comprises five state variables ($V$, $h$, $\alpha$, $\gamma$, and $q$) and two control inputs ($\delta_z$, $T$), where $V$ is the velocity, $h$ is the altitude, $\alpha$ is angle of attack, $\gamma$ is the flight path angle (FPA), and $q$ is the pitch rate; $\delta_z$ and $T$ represent elevator deflection and thrust force, respectively. Consider

$$
\dot{V} = \frac{(-D + T \cos \alpha - mg \sin \gamma + F_{1x})}{m},
$$

$$
\dot{h} = V \sin \gamma,
$$

$$
\dot{\gamma} = \frac{[L + T \sin \alpha - mg \cos \gamma - F_{1z}]}{mV},
$$

$$
\dot{\alpha} = \frac{[-L - T \sin \alpha + mg \cos \gamma + F_{1z}]}{mV} + q,
$$

$$
\dot{q} = \frac{-I_y q}{I_y} + \frac{(-S_y g \cos \theta + M_A + T Z_T + M_{I_y})}{I_y},
$$

$$
F_{1x} = S_x (q \sin \alpha + q^2 \cos \alpha) + 2 S_x q \sin \alpha - \bar{S}_x \cos \alpha,
$$

$$
F_{1z} = F_{1z} = S_x (q \cos \alpha - q^2 \sin \alpha) + 2 S_x q \cos \alpha + \bar{S}_x \sin \alpha,
$$

$$
M_{I_y} = S_x (\dot{V} \sin \alpha + \dot{V} \alpha \cos \alpha - Vq \cos \alpha),
$$

and $A$, $\bar{S}_x$, $S_x$, $M_A$, $T$, $Z_T$, $M_{I_y}$, and $M$ are system parameters.
where \(D\), \(L\), and \(M_A\) represent drag force, lift force, and pitch moment, respectively; \(m\), \(I_y\), and \(g\) denote the mass of aircraft, moment of inertia about pitch axis, and gravity constant; \(F_{ix}, F_{iz}, F_{ikz},\) and \(M_{iy}\) represent inertial force and moment caused by morphing process; \(Z_T\) is the position of engine in the body axis; \(S_x\) denotes the static moment distributed in the body axis of \(x\); the related definitions are given as follows:

\[
S_x (\zeta) \approx [2m_1 r_{1x} + m_3 r_{3x}],
\]

\[
Q = \frac{1}{2 \rho h V^2},
\]

\[
L = C_L (\zeta) Q S_w (\zeta),
\]

\[
D = C_D (\zeta) Q S_w (\zeta),
\]

\[
M_A = C_m (\zeta) Q S_w (\zeta) c_A (\zeta),
\]

\[
C_L (\zeta) = C_{L0} (\zeta) + C_{L\alpha} (\zeta) \alpha + C_{L\delta e} (\zeta) \delta_e 
\approx C_{L0} (\zeta) + C_{L\alpha} (\zeta) \alpha,
\]

\[
C_D (\zeta) = C_{D0} (\zeta) + C_{D\alpha} (\zeta) \alpha + C_{D\alpha2} (\zeta) \alpha^2,
\]

\[
C_m (\zeta) = C_{m0} (\zeta) + C_{m\alpha} (\zeta) \alpha + C_{m\delta e} (\zeta) \delta_e 
+ \frac{C_{m2}}{2V} \sin \alpha,
\]

where \(\zeta\) represents the sweep angle, \(\rho h\) denotes the air density, \(S_w\) is the wing surface, \(c_A\) represents the mean aerodynamic chord, and \(b\) is the wingspan. \(Q\) and \(M_A\) denote the dynamic pressure and pitch moment. \(C_L\), \(C_D\), and \(C_m\) are the total aerodynamic lift force coefficient, drag force coefficient, and pitching moment coefficient, respectively. \(m_1\) and \(m_3\) represent the mass of aircraft’s wing and body. \(r_{1x}\), and \(r_{3x}\) denote the position of aircraft’s wing and body in the aircraft-body coordinate frame.

We assume that the engine model can be expressed as follows [27].

(A) Engine Rate. The dynamics for the engine speed \(n\) is modeled by a first-order linear system with the time constant \(\tau_n\) and the engine speed reference signal \(n_c\) as follows:

\[
\dot{n} = -\frac{n}{\tau_n} + \frac{n_c}{\tau_n},
\]

where \(\f_1 (x_1) = \left(1 / mV\right) (Q S_w C_{L0} - m g \cos \gamma), \)

\[
\f_2 (x_1, x_2) = 0,
\]

\[
\g_1 (x_1) = \frac{Q S_w C_{L\alpha}}{mV},
\]

(B) Thrust Force. The thrust force is generated by the propeller and can be expressed with dimensionless coefficients. The dimensionless thrust coefficient is

\[
C_{FT} (J) = C_{FT1} + C_{FT2} J + C_{FT3} J^2
\]

with the ratio \(J = V_T / D_T m\), where the diameter of the propeller is \(D_T\), the engine speed is \(n\), and the airspeed is \(V_T\). Here we assume that \(V_T = 1\). The thrust force is computed as shown below:

\[
T = \rho ln^2 D_T^2 C_{FT} (J).
\]

Remark 1. It is important to point out that \(r_{1x}, r_{3x}, I_{yy}, c_A, S_w, b, C_L, C_D,\) and \(C_m\) are associated with sweep angle \(\zeta\) in the morphing process. Their functional relationships will be shown later in Section 5.

2.2. System Transformation

(A) Altitude Subsystem. The tracking error of the altitude is defined as \(\tilde{h} = h - h_d\). Furthermore, the altitude command is transformed into the desired flight path angle (FPA). The demand of flight path angle is generated as [22]

\[
y_d = \arcsin \left(\frac{-k_h \tilde{h} - k_d h_d + h_d}{V}\right).
\]

If \(k_h > 0\) and \(k_d > 0\) are chosen appropriately and the FPA is controlled to follow \(y_d\), then the altitude error is regulated to zero exponentially.

Remark 2. Since the control problem considered in this paper only takes into account cruise trajectories and does not consider the aggressive maneuvering, the thrust \(T \sin \alpha\) can be neglected since it is generally much smaller than the lift. In order to transform the altitude subsystem into strict-feedback form, \(F_{ikz}\) in (3) is regarded as an unmodeled term.

Define \(X = [x_1, x_2, x_3]^T\), \(x_1 = y\), \(x_2 = \theta\), \(x_3 = q\). \(\theta = \alpha + \gamma\), \(u = \delta_i\); the strict-feedback forms of equations of the altitude (3)–(5) are rewritten as

\[
\dot{x}_1 = f_1 (x_1) + g_1 (x_1) x_2,
\]

\[
\dot{x}_2 = f_2 (x_1, x_2) + g_2 (x_1, x_2) x_3,
\]

\[
\dot{x}_3 = f_3 (x_1, x_2, x_3) + g_3 (x_1, x_2, x_3) u,
\]

where
f_3(x_1, x_2, x_3) = \frac{QSWc_A \left(C_{m0} + C_{ma}\alpha + C_{mq}q_A/ (2V) \right) - S_\gamma g \cos \theta - I_yq + T_Z T + M_{1y}}{I_y},

\begin{align*}
g_2(x_1, x_2) &= 1, \\
g_3(x_1, x_2, x_3) &= \frac{QSWc_A C_{mx}}{I_y}.
\end{align*}

Assumption 3. \(f_1, f_3, f_V, g_1, g_3, \) and \(g_V\) are unknown smooth functions; we assume that there exist positive constants \(\bar{g}_1, \ \bar{g}_2, \ \bar{g}_V\), and \(\bar{g}_V\) such that \(\bar{g}_1 \geq g_i(t) \geq \bar{g}_2, \ i = 1, 3, \bar{g}_V \geq g_V \geq \bar{g}_V\). There also exist constants \(g_{id}, \ g_{id}\) such that \(g_{id} \geq [g_1], g_{id} \geq [g_3]\). Meanwhile, in this paper, we assume that all the system states can be measured and there is no time-delay in the signal transmission.

Lemma 4 (high order integral chained differentiator [28]). Suppose the function \(\sigma(t)\) and its first \(n - 1\) derivatives are bounded. Consider the following linear system:

\begin{equation}
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= \zeta_3 \\
\vdots \\
\dot{\zeta}_n &= \frac{a_{f1}}{\chi^1} (\zeta_1 - \zeta (t)) - \frac{a_{f2}}{\chi^2} \zeta_2 \cdots - \frac{a_{fn}}{\chi^n} \zeta_n,
\end{aligned}
\end{equation}

where \(\chi\) is a small positive constant and parameters \(a_{f1}\) to \(a_{fn}\) are chosen such that the polynomial\(\chi^n + a_{f1}\chi^{n-1} + \cdots + a_{fn} = 0\) is Hurwitz. Then

\begin{equation}
\lim_{\zeta \to 0} \zeta = \zeta^{(n-1)}(t).
\end{equation}

In the following, we show that original system (12) can be transformed into the normal form with respect to the newly defined state variables. Let \(z_1 = x_1\) and \(z_2 = z_1 = f_1 + g_1 x_2\). The derivative of \(z_2\) with respect to time is formulated as

\begin{equation}
\begin{aligned}
\dot{z}_2 &= \frac{\partial f_1}{\partial x_1} \dot{x}_1 + \frac{\partial g_1}{\partial x_1} \dot{x}_1 x_2 + g_1 \dot{x}_2 \\
&= \left( \frac{\partial f_1}{\partial x_1} + \frac{\partial g_1}{\partial x_1} x_2 \right) (f_1 + g_1 x_2) + g_1 f_2 + g_1 g_2 x_3 \\
&= a_3 (x_1, x_2) + b_3 (x_1, x_2) x_3,
\end{aligned}
\end{equation}

where \(a_3(x_1, x_2) = (\partial f_1/\partial x_1 + (\partial g_1/\partial x_1) x_2)(f_1 + g_1 x_2) + g_1 f_2, \ b_3(x_1, x_2) = g_1 g_2\).

Similarly, let \(z_3 = z_2 = a_2 + b_2 x_3\) and its time derivative is induced by

\begin{equation}
\begin{aligned}
\dot{z}_3 &= \sum_{i=1}^2 \frac{\partial a_2}{\partial x_i} \dot{x}_i + \sum_{i=1}^2 \frac{\partial b_2}{\partial x_i} \dot{x}_1 x_3 + b_2 \dot{x}_3 \\
&= \sum_{i=1}^2 \left( \frac{\partial a_2}{\partial x_i} + \frac{\partial b_2}{\partial x_i} x_3 \right) (f_1 + g_1 x_2) + b_2 (f_3 + g_3 u) \\
&= a_3 (x_1, x_2, x_3) + b_3 (x_1, x_2, x_3) u,
\end{aligned}
\end{equation}

where \(a_3 = \sum_{i=1}^2 (\partial a_2/\partial x_i + (\partial b_2/\partial x_i) x_3)(f_1 + g_1 x_2) + b_2 f_3\) and \(b_3 = g_1 g_2 g_3\).

As a result, strict-feedback system (12) can be described as the following normal output form with respect to the newly defined state variables \(z_1, z_2, z_3\):

\begin{equation}
\begin{aligned}
\dot{z}_1 &= z_2, \\
\dot{z}_2 &= z_3, \\
\dot{z}_3 &= a_3 + b_3 u, \\
y &= z_1 = x_1,
\end{aligned}
\end{equation}

(B) Velocity Subsystem. With the modeling uncertainties and external disturbance existing, the uncertain nonlinear model can be formulated as

\begin{equation}
\begin{aligned}
\dot{V} &= [f_{V0} (X_V) + \Delta f_V] + g_V (X_V) T + d_V \\
&= f_{V0} (X_V) + g_V (X_V) T + \Delta_V,
\end{aligned}
\end{equation}

where \(f_V (X_V) = f_{V0} (X_V) + \Delta f (X_V), g_V = (1/m) \cos \alpha, X_V = [x_1, x_2, x_3, V]. f_{V0} (X_V)\) is the nominal parts of \(f_V(X_V); \Delta f_V\) is the unknown system uncertainties of \(f_V(X_V); d_V(X_V)\) is the external disturbance and \(\Delta_V = \Delta f_V + d_V\) is the lump of system uncertainty.

Remark 5. It should be noted that \(a_3, b_3\) are totally unknown and need to be approached by NN in the subsequent developments. For the newly defined states \(z_1, z_2, z_3\), an HICD will be introduced to estimate them. From Assumption 3, it is also noted that there exist constants \(\bar{b}_3 > 0\) and \(\bar{b}_{3d} > 0\) such that \(b_3 \geq \bar{b}_3 \) and \(b_{3d} > |b_{3d}|\).
3. Neural Networks

In many references of robust adaptive control of uncertain nonlinear systems, the RBFNNs are usually employed as approximate model terms for the unknown nonlinear and continuous function terms using their inherent approximation capabilities [25]. As a class of linearly parameterized NNs, RBFNNs are adopted to approximate the unknown and continuous function \( H(X_m) : R^3 \rightarrow R \) which can be written as follows:

\[
H(X_m) = \bar{w}^T \Phi(X_m) + \varepsilon,
\]

(20)

where \( X_m \in R^3 \) is an input vector of NN, \( \bar{w} \in R^p \) is a weight vector of the NN, \( \Phi(X_m) = [\phi_1(X_m), \phi_2(X_m), \ldots, \phi_p(X_m)]^T \in R^p \) is a basis function, \( \varepsilon \) is the approximation error which satisfies \( |\varepsilon| \leq \varepsilon_{re} \), and \( \varepsilon_{re} \) is a bounded unknown parameter.

In general, an RBFNN can smoothly approximate any continuous function \( H(X_m) \) over the compact \( \Omega_{X_m} \in R^3 \) to any arbitrary accuracy as

\[
H(X_m) = w^*T \Phi(X_m) + \varepsilon^*,
\]

(21)

where \( w^* \) is the optimal weight value and \( \varepsilon^* \) is the smallest approximation error. The Gaussian basis function is written in the form of

\[
\phi_i(X_m) = \exp \left[ - \frac{(X_m - c_i)^T(X_m - c_i)}{m_i^2} \right],
\]

(22)

\[ i = 1, 2, \ldots, p, \]

where \( c_i \) and \( m_i \) are the center and width of the neural cell of the ith hidden layer.

**Remark 6.** There exists an RBFNN in the form of (21) and an optimal parameter vector \( w^* \) such that \( |H(X_m) - w^*T \Phi(X_m)| = |\varepsilon^*| < \varepsilon_{re} \). \( \varepsilon_{re} \) denotes the supremum of the reconstruction error that is inevitably generated. In what follows, the estimation of \( w^* \) is denoted as \( \bar{w} \).

4. Control Design and Stability Analysis

It is easy to note that \( h \) is mainly related to \( \delta_c \) and \( V \) is mainly affected by \( T \). Therefore, the dynamics can be decoupled into altitude and velocity subsystem and we design the altitude and velocity controller separately. The structure of the proposed control scheme is presented in Figure 1.

4.1. Adaptive Neural Controller for Altitude Subsystem. The control objective of system (12) is to design an adaptive neural controller, which makes \( y \rightarrow \bar{y}_d \), and therefore \( h \rightarrow \bar{h}_d \), while keeping all the signals involved bounded.

The following controller design is mainly based on the scheme in [29–31]. Vectors \( Y_d, \bar{E} \) and a filtered tracking error \( s_y \) are then defined as follows:

\[
Y_d = [y_d, \bar{y}_d, \bar{y}_d]^T,
\]

(23)

\[
E = Z - Y_d,
\]

(24)

\[
s_y = \left( \frac{d}{dt} + \lambda \right)^2 E = [\Lambda^T 1] E,
\]

(25)

where \( Z = [z_1, z_2, z_3]^T, \Lambda = [\lambda^2 2\lambda]^T \) with \( \lambda > 0 \).

By employing a high order integral chained differentiator, the estimation of \( Z = [z_1, z_2, z_3]^T \) is acquired as \( \bar{Z} = [\bar{z}_1, \bar{z}_2, \bar{z}_3]^T \). According to the discussion in [28], there exist positive constant \( \varepsilon_\bar{h} \) and \( \varepsilon^* \) such that \( vt > t^* \)

\[
|\bar{Z} - Z| \leq \varepsilon_\bar{h}.
\]

(27)

The estimations of \( E \) and \( s_y \) using (14) are denoted as given below:

\[
\bar{E} = \bar{Z} - Y_d,
\]

(28)

\[
\bar{s}_y = [\Lambda^* 1] \bar{E}.
\]

Based on (25), the derivative of \( s_y \) with respect to time can be expressed as

\[
\dot{s}_y = 0 \Lambda^T E + (y^{(3)} - y_d^{(3)})
\]

(29)

\[ = a_3 + b_3 u - y_d^{(3)} + [0 \Lambda^T] E \]

\[ = a_3 + b_3 u + \bar{v} - [0 \Lambda^T] \bar{E}, \]

where \( \bar{v} = y_d^{(3)} + [0 \Lambda^T] \bar{E}, \bar{E} = E - \bar{E} = \bar{Z} - Z \).

Define

\[
u_{ad}^* = (X_A, \bar{v}) = \frac{(a_3 + \bar{v})}{b_3}.
\]

(30)

\[ u_{ad}^* \] is approximated by RBFNN as

\[
u_{RBF} = \bar{w}_A^T \Phi(X_A),
\]

(31)

\[ X_A = [X^T, \bar{v}], \]

where \( \bar{w}_A \) is the estimation of the optimal parameter vector \( w_A^* = \bar{w}_A - \bar{w}_A \).

Substituting the unknown \( s_y \) with \( \bar{s}_y \), we determine the control input as follows:

\[
u = -k \bar{s}_y - \bar{w}_A^T \Phi(X_A).
\]

(32)

The update law for \( \bar{w}_A \) is determined as

\[
\dot{\bar{w}}_A = \gamma_A \left( \bar{s}_y \Phi(X_A) - \sigma_s(\bar{w}_A) \bar{w}_A \right),
\]

(33)

\[
\sigma_s(\bar{w}_A) = \begin{cases} \frac{e_w}{\varepsilon_w}, & \text{if } |\bar{w}_A| > \varepsilon_w \\ 0, & \text{otherwise,} \end{cases}
\]

(34)
where $\varepsilon_W$, $c_\Phi$ are positive design constants, $\|\Phi(X_A)\| < c_\Phi$, and $\gamma_A$ denotes the positive learning rate.

**Theorem 7.** Consider the adaptive system consisting of (12) under Assumption 3, controller (32) with HICD (14), and adaptive law (33). The filtered error $s_\gamma$ and $\hat{w}_A$ are semiglobally uniformly ultimately bounded.

**Proof.** Consider the Lyapunov function candidate $L = 1/(2b_3)s_\gamma^2 + 1/(2\gamma_A)\hat{w}_A^T\hat{w}_A$. Taking the time derivation of $L$, we get

\[
\dot{L} = \frac{s_\gamma}{b_3} + \frac{b_3 s_\gamma^2}{2b_3^2} + \frac{\hat{w}_A^T\hat{w}_A}{\gamma_A} = \frac{1}{b_3} s_\gamma (a_3 + b_3u + \nu) - \frac{1}{b_3} \gamma A \left[ \begin{array}{c} 0 \ \Lambda^T \end{array} \right] \dot{E} - \frac{1}{b_3} \gamma A \hat{w}_A^T \Phi + \sigma_s (\hat{w}_A) \hat{w}_A^T \hat{w}_A - \sigma_s (\hat{w}_A) \hat{w}_A^T \hat{w}_A - s_\gamma (\hat{w}_A) \hat{w}_A^T \hat{w}_A - s_\gamma (\hat{w}_A) \hat{w}_A^T \hat{w}_A
\]

\[
\leq - \frac{1}{b_3} \gamma A \left[ \begin{array}{c} 0 \ \Lambda^T \end{array} \right] \dot{E} + \frac{1}{b_3} \gamma A \hat{w}_A^T \Phi + \sigma_s (\hat{w}_A) \hat{w}_A^T \hat{w}_A - s_\gamma (\hat{w}_A) \hat{w}_A^T \hat{w}_A - \sigma_s (\hat{w}_A) \hat{w}_A^T \hat{w}_A
\]

\[
\geq \|\hat{w}_A\|^2 - \|\hat{w}_A^*\|^2
\]

Considering the following facts,

\[
\hat{w}_A^T \left[ \begin{array}{c} 0 \ \Lambda^T \end{array} \right] \dot{E} \leq \frac{1}{8} c_\Phi k_s \|\hat{w}_A\|^2 + \frac{2}{k_2} c_\Phi \|\hat{w}_A\|^2 \|\hat{w}_A^T \dot{E}\|^2
\]

\[
\geq \frac{1}{8} c_\Phi k_s \|\hat{w}_A\|^2 + \frac{2}{k_2} c_\Phi \|\hat{w}_A\|^2 \|\dot{E}\|^2
\]

\[
2\|\hat{w}_A\|^2 = \|\hat{w}_A\|^2 + \|\hat{w}_A\|^2 - \|\hat{w}_A^*\|^2 \geq \|\hat{w}_A\|^2 - \|\hat{w}_A^*\|^2
\]

we have

\[
L \leq - \left( k - \frac{b_3}{2b_3^2} - \frac{1}{2} C_1 \right) s_\gamma^2 - \frac{1}{8} c_\Phi k_s \|\hat{w}_A\|^2 + \frac{2}{k_2} c_\Phi \|\hat{w}_A\|^2 \|\dot{E}\|^2 + \frac{1}{8} c_\Phi k_s \|\hat{w}_A\|^2 + \frac{2}{k_2} c_\Phi \|\hat{w}_A\|^2 \|\dot{E}\|^2
\]

\[
+ \frac{1}{8} c_\Phi k_s \|\hat{w}_A\|^2 + \frac{1}{8} \leq -\rho L + C,
\]
where $c_{\lambda 1} = \| \Lambda^T \|_1$, $c_{\lambda 2} = \| 0 \Lambda^T \|_1$, $\mu_1 = [ \Lambda^T \|_E$, $|\sigma(\hat{w})| \leq c_{\phi}/\epsilon_w, k_5 > 0$, $\| \Phi \| \leq c_{\Phi}$, $|k_5 \epsilon_w + c_{\lambda 2} \epsilon_w/|\hat{w}_3| = C_1$, $\rho$ and $C$ are given by

$$\rho := \min \left\{ \left( k - \frac{b_3}{2\hat{w}_3} - \frac{1}{2} C_1 \right), \left( \frac{1}{2} \frac{c_1}{\epsilon_w} - \frac{1}{8} c_2 k_j \right) \right\},$$

(38)

To ensure the closed-loop stability, the corresponding design parameters should be chosen such that $k - b_3/(2\hat{w}_3) - (1/2) C_1 > 0$ and $(1/2)(c_2/\epsilon_w) - (1/8) c_2 k_j > 0$.

According to (37), we have $0 \leq L \leq C/\rho + [L(0) - C/\rho]e^{-\phi}$. From (37), we can know that $L$ is convergent; that is, $\lim_{t\to\infty} L = C/\rho$. It can be shown that the filtered signal $\hat{s}_i$ and $\hat{w}_A$ are semiglobally uniformly bounded. \hfill $\Box$

Remark 8. (1) The switching function $\sigma_i(\hat{w}_A)$ is adopted so that the RBFNN can learn the retained information, which is based on a novel $\sigma$ switching scheme. The adopted switching scheme prevents the loss of information, if $\epsilon_w$ is chosen sufficiently large such that $\epsilon_w > |\hat{w}_A|$ while guaranteeing the boundness of $|\hat{w}_A|$.

(2) It should be noted that, in this paper, only one RBFNN is employed to approximate the lumped uncertain nonlinear function in the altitude subsystem which highlights the simplicity of our proposed controller. However, at least two RBFNNs need to be used in the backstepping scheme, in [25], which require large computational burden. It is also demonstrated that control law and stability analysis is considerably simpler than the previous backstepping-based algorithms.

4.2. Adaptive Controller for Velocity Subsystem. Define

$$\bar{V} = V - V_d.$$  

(39)

Its time derivative is

$$\dot{\bar{V}} = \bar{V} - \dot{V}_d = f_{V_0} + g_V \dot{V} + \Delta_V - \dot{V}_d.$$  

(40)

By employing an RBFNN $\hat{w}_V^T \Phi_V(X_{V_1})$ to approximate unknown uncertainty $\Delta_V$, we have

$$\Delta_V = T_d = \left[ -k_{pV} \bar{V} - k_{1V} \int_0^t (\bar{V} - \dot{V}_e) \, dt - f_{V_0} + \bar{w}_V^T \Phi_V (X_{V_1}) + \dot{V}_d \right]$$  

(41)

where $X_{V_1} = [V, \dot{V}, \bar{V}]$ and $k_{pV}, k_{1V}$ are the positive design parameters; $\bar{V}$ is the compensatory term which will be defined as follows. $T_d$ represents the desired thrust force.

Equations (9) and (10) are rearranged so as to solve $n_d$ in the following equation:

$$n_d^2 \left( C_{FTT1} \rho_1 D^3 \right) + n_d \left( C_{FTT2} \rho_1 D^3 \right) V - T_d = 0.$$  

(42)

In order to solve (42) at each sampling time, $V$ is assumed to be constant during the sampling period. Then

$$n_d = \left( \frac{c_{\lambda 1} V + c_{\lambda 2} V^2 + c_3 T_d}{c_{\lambda 4}} \right),$$  

(43)

$$n_c = \left\{ \begin{array}{ll} n_{\text{max}}, & n_d \geq n_{\text{max}} \\ n_{\text{d}}, & n_d \leq n_{\text{max}} \end{array} \right.$$  

(44)

where $c_{\lambda 1} = C_{FTT3} \rho_1 D^3/\pi$, $c_{\lambda 2} = C_{FTT2} - 4C_{FTT1} \rho_1 D^3/\pi$, $c_3 = 4C_{FTT1} \rho_1 D^3$, and $c_{\lambda 4} = 2C_{FTT1} \rho_1 D^3$ are the intermediate variables. $n_c$ is the actual engine speed, $n_{\text{max}}$ is the upper limit of $n_d$.

Define

$$\bar{V}_e = \bar{V} - V_e,$$  

(45)

$$\dot{V}_e = -k_{pV} V_e + g_V (T - T_d),$$  

(46)

$$V_e(0) = 0.$$  

The update law of $\hat{w}_V$ is determined as

$$\dot{\hat{w}}_V = \eta_V \left( \bar{V}_e \Phi_V (X_{V_1}) - \sigma_V \hat{w}_V \right),$$  

(47)

where $\sigma_V$ is a positive design constant and $\hat{w}_V = \bar{w}_V - w_1^*$. (46) indicates the auxiliary system used to compensate the engine speed saturation.

The derivatives of $\bar{V}$ and $\bar{V}_e$ with respect to time, $\dot{\bar{V}}$ and $\dot{\bar{V}}_e$, can be expressed as

$$\dot{\bar{V}} = \bar{V} - \dot{V}_d = f_{V_0} + g_V \dot{V} + \bar{w}_V^T \Phi_V + \epsilon_V - \dot{V}_d$$

$$= f_{V_0} + g_V \dot{V} + g_V (T - T_d) + \bar{w}_V^T \Phi_V - \bar{w}_V^T \Phi_V$$

$$\dot{\bar{V}}_e = -k_{pV} \bar{V}_e - k_{1V} \int_0^t (\bar{V}_e) \, dt + g_V (T - T_d)$$

(48)

$$+ \bar{w}_V^T \Phi_V + \epsilon_V.$$

Theorem 9. Consider the adaptive system comprising (19), velocity subsystem controller (41) with adaptive law (47), and auxiliary system (46). $\bar{V}_e$ and $\hat{w}_V$ are semiglobally uniformly bounded.

Proof. Consider the Lyapunov candidate function

$$L_V(t) = \frac{1}{2} \bar{V}_e^2 + \frac{k_{1V}}{2} \int_0^t \bar{V}_e^2 \, dt + \frac{1}{2\eta_V} \bar{w}_V^T \bar{w}_V.$$  

(49)
Its time derivative is
\[
L_V = \dot{\bar{V}}_e + k_{IV} \int_0^t \bar{V}_e d\tau + \frac{\ddot{\bar{V}}_e}{\eta_V}
\]
\[
= \bar{V}_e \left[ -k_{pV} \bar{V} - k_{IV} \int_0^t (\bar{V}_e \Phi - \sigma_V \bar{w}_V) \right] + k_{IV} \bar{V}_e dt + \frac{\ddot{\bar{V}}_e}{\eta_V}
\]
\[
= -k_{pV} \bar{V}_e^2 + \bar{V}_e \dot{\bar{w}}_V - \sigma_V \bar{w}_V \bar{V}_e.
\]
(50)

Considering the following fact,
\[
2\ddot{\bar{V}}_e \bar{w}_V = \|\bar{w}_V\|^2 + \|\bar{V}_e\|^2 - \|\bar{w}_V\|^2 \geq \|\bar{V}_e\|^2 - \|\bar{w}_V\|^2
\]
\[
\dot{\bar{V}}_e \bar{V}_e \leq \frac{1}{2} (\bar{V}_e^2 + \bar{w}_V^2),
\]
we have the following inequality:
\[
\dot{L}_V \leq - \left( k_{pV} - \frac{1}{2} \right) \bar{V}_e^2 - \frac{1}{2} \sigma_V \|\bar{w}_V\|^2
\]
\[
+ \frac{1}{2} \left( \sigma_V \|\bar{w}_V\|^2 \right) \leq -\rho_V L_V + C_V,
\]
(52)

where \(\rho_V\) and \(C_V\) are given by \(\rho_V := \min \{ (k_{pV} - 1/2), \sigma_V/2 \}\) and \(C_V := \{1/2\sigma_V^2 + \sigma_V/2 \|w_V\|^2 \}\).

To ensure the closed-loop stability, the corresponding design parameters \(k_{pV}, \sigma_V\) should be chosen such that \(k_{pV} - 1/2 > 0, \sigma_V > 0\). According to (52), it can be shown that the signals \(\bar{V}_e\) and \(\bar{w}_V\) are semiglobally uniformly bounded.

Remark 10. In this section, the dynamic inversion control based on RBFNN is proposed for velocity subsystem with input saturation constraints. To handle the input saturation, auxiliary design system (46) is introduced to analyze the effect of saturation constraint and the auxiliary variable \(V_e\) is used to design the adaptive law. It is apparent that the constrained control \(T\) produced by the designed control command \(T_d\) can guarantee the closed-loop system’s stability.

5. Numerical Simulation

In this section, the performance of the developed control strategy applied to the longitudinal model of the morphing aircraft is verified by means of simulations. The aircraft model parameters are shown in Table 1. Neural network \(\bar{w}_V^T \Phi(X_A)\) with input vector \(X_A = [x_1, x_2, x_3, \ldots]^T\) contains 50 nodes with centers \(c_i\) \((i = 1 \cdots 50)\) evenly spaced in \([-15^\circ, 15^\circ] \times [-15^\circ, 15^\circ] \times [-15^\circ, 15^\circ] \times [-15^\circ, 15^\circ]\) and widths \(m_{ij}\) \((i = 1 \cdots 50)\) of 1; neural network \(\bar{w}_V^T \Phi(X_V)|_{V}\) with input vector \(X_V = [V, V_d, \lambda, T]^T\) contains 10 nodes with centers \(c_j\) \((i = 1 \cdots 10)\) evenly spaced in \([10, 50] \times [10, 50] \times [10, 50]\) and widths \(m_{ij}\) \((i = 1 \cdots 10)\) of 5. The initial condition is set as \(X_0 = [y_0, \theta, \phi, \psi, \dot{V}_0, \ddot{V}_0] = [0, 0.99512, 0, 1.0000, 30, 300]s\), \(\omega_{\phi}(0) = 0\), and \(\omega_{\psi}(0) = 0\). Control and HICD parameters are set as \(k_{th} = 0.5, k_{f} = 0.01, k = 0.025, \gamma_A = 0.02, \xi_w = 10,\)

### Table 1: Morphing aircraft parameters for different configurations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(\zeta = 0^\circ)</th>
<th>(\zeta = 30^\circ)</th>
<th>(\zeta = 45^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S/(m^2))</td>
<td>1.6040</td>
<td>1.168</td>
<td>0.958</td>
</tr>
<tr>
<td>(c_s/(m))</td>
<td>0.4874</td>
<td>0.411</td>
<td>0.416</td>
</tr>
<tr>
<td>(b/(m))</td>
<td>3.3494</td>
<td>2.981</td>
<td>2.503</td>
</tr>
<tr>
<td>(I_f/(kg\cdot m^2))</td>
<td>6.4929</td>
<td>7.882</td>
<td>8.606</td>
</tr>
</tbody>
</table>

\(c_p = 20,\) and \(\lambda = 5; k_{pV} = 5, k_{f} = 10, \eta_V = 10,\) and \(\sigma_V = 0.01;\)
\(a_{1f} = 10, a_{1f} = 10, a_{1f} = 10,\) and \(\chi = 0.04.\) Reference commands are smoothened via several second-order filters shown in (53) below. The engine speed saturation \(n_{max}\) which is set at 4900 RPM is deliberately tightened to explore the capability of the designed controller in adhering to the limits.

Consider
\[
\frac{h_d}{h_{d0}} = \frac{0.64}{s^2 + 1.6s + 0.64},
\]
\[
\frac{V_d}{V_{d0}} = \frac{1}{s^2 + 2s + 1},
\]
\[
\frac{\dot{c}_d}{\dot{c}_{d0}} = \frac{1}{s^2 + 4s + 1},
\]
\[
\frac{\dot{\zeta}_d}{\dot{\zeta}_{d0}} = \frac{1}{s^2 + 4s + 1}.
\]

Choosing \(\zeta = 0^\circ, 5^\circ, \ldots, 45^\circ\) as the 10 reference points, the longitudinal aerodynamic parameters for different variation configurations can be computed through computational fluid dynamics (CFD). Then the aerodynamic parameters of the morphing aircraft during wing-transforming process can be linearly interpolated by those of static configurations with the help of MATLAB:

\[
C_{L0} = 0.0042\zeta^3 - 0.1374\zeta^2 - 0.0516\zeta + 0.2291,
\]
\[
c_A = 0.2054\zeta^2 - 0.2520\zeta + 0.4874,
\]
\[
C_{L\alpha} = -1.1264\zeta^3 - 0.4351\zeta^2 + 0.3816 + 4.592,
\]
\[
b = -1.4599\zeta^2 + 0.0644\zeta + 3.3494,
\]
\[
C_{L\alpha} = -0.0024\zeta^3 - 0.0045\zeta^2 + 0.0022\zeta + 0.021,
\]
\[
C_{D\alpha} = -0.0310\zeta^2 - 0.0458\zeta + 0.109,
\]
\[
C_{D\alpha} = -1.2990\zeta^4 + 1.8282\zeta^3 - 0.7039\zeta^2 - 0.0258\zeta + 1.097,
\]
\[
S = -0.8271\zeta + 1.6040,
\]
\[
C_{max} = 9.6542\zeta^3 - 6.5395\zeta^2 - 6.1887\zeta - 1.5909,
\]
\[
C_{max} = 0.4392\zeta^2 - 0.4462\zeta^2 - 0.0365,
\]
\[
C_{max} = -0.1624\zeta^2 - 0.9376\zeta - 0.7889,
\]
\[
I_y = -4.9021\zeta^2 + 6.5774\zeta^3 + 0.5500\zeta + 6.4929,
\]
\[
C_{max} = 41.4537\zeta^2 - 50.4868\zeta^2 - 9.7741\zeta - 10.673.
Due to the complex nonlinear aerodynamic of the morphing aircraft, the aerodynamics is not modeled precisely, the same as it appears in the actual flight conditions. Thus it is significant for the controller to have the ability to provide stability in spite of modeling errors due to unmodeled dynamics and plant parameter variations. To demonstrate the robustness of the proposed control scheme, 20% aerodynamic uncertainties are taken into account. The following two scenario simulations are employed to test the performance of the proposed controller in handling with aerodynamic uncertainty and input constraints compared with backstepping controller designed in the altitude subsystem.

Scenario 1. (A) The altitude $h_d$ and velocity $V_d$ reference commands are generated to make the aircraft climb from 1000 m to 1050 m and accelerate from 30 m/s to 40 m/s in 20 s, where the engine speed saturation is not considered. The simulation results of the tracking output are shown in Figures 2 and 3 (“NN” denotes the simulation results based on adaptive NN controller in this paper and “backstepping” represents the backstepping method in [25]). It can be observed that the system outputs $h$ and $V$ on the basis of NN and backstepping follow the desired trajectory of $h_d$ and $V_d$ well. The altitude tracking error of NN is smaller than the one based on backstepping. These simulation results show that good tracking performance can be obtained under the proposed adaptive NN control.

(B) To illustrate the effectiveness of the proposed adaptive NN control further, the sweep reference signal taking place at 30 s is generated to make the aircraft sweep from 0° to 45° at the rate of 9°/s. The simulation results are shown in Figures 4–7. It is clear that the velocity is almost constant, during the sweeping process, and the altitude which decreases about 0.32 m based on adaptive NN which is better than backstepping method decreases about 1.75 m. They can both
converge within 20 s after the wing finishes sweeping. Since the wing area decreases after it sweeps, the angle of attack will increase to achieve a new trim point. In addition, the changes in elevator deflection and thrust are both within acceptable ranges. It can be concluded that the adaptive neural controller, in this paper, can accommodate different wing shapes that result in drastically changing plant dynamic and guarantee the flight more steady compared with backstepping method.

Scenario 2 (engine speed saturation). To illustrate the effectiveness of the auxiliary system, the reference commands are similar to Scenario 1(A), and the engine speed saturation \( n_{\text{max}} \) is set at 4900 RPM. The simulation results are shown in Figures 8 and 9. Due to engine speed saturation, it is obvious to observe that the velocity tracking errors are different between the used \( V_1 \) with 0.5 m/s to the maximum) and unused \( V_2 \) with 0.8 m/s to the maximum) additional system. As shown in Figure 9, the engine speed recovers from saturation in 9 s for \( V_1 \) which is better than \( V_2 \) which recovers in 15 s. These simulation results show that good tracking performance can be obtained under the proposed additional system.

6. Conclusions and Future Works

A robust adaptive neural controller based on high order integral chained differentiator is developed for the nonlinear longitudinal model of a morphing aircraft, where aerodynamic uncertainty and engine input constraint are taken into
The performance of the presented method is verified by simulations, from which we can deduce that the good performance has been ensured.

For future work, we will analyze how minimal parameter learning technique can be implemented on morphing aircraft in order to reduce the computation burden further. Also it is important to do research on theoretical analysis deeply for the system with input nonlinearity and time-delay where it is still an open problem for this scheme.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References


