Research Article

Symmetry Analysis and Conservation Laws of a Generalized Two-Dimensional Nonlinear KP-MEW Equation

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Lie symmetry analysis is performed on a generalized two-dimensional nonlinear Kadomtsev-Petviashvili-modified equal width equation. The symmetries and adjoint representations for this equation are given and an optimal system of one-dimensional subalgebras is derived. The similarity reductions and exact solutions with the aid of (G'/G) -expansion method are obtained based on the optimal systems of one-dimensional subalgebras. Finally conservation laws are constructed by using the multiplier method.

1. Introduction

Nonlinear evolution equations (NLEEs) have been widely used to describe natural phenomena of science and engineering. Therefore it is very important to find exact solutions of NLEEs. However, this is not an easy task. During the past few decades various integration techniques have been developed by the researchers to solve these NLEEs. Some of the well-known techniques used in the literature are the inverse scattering transform method [1], the homogeneous balance method [2], the Bäcklund transformation [3], the Weierstrass elliptic function expansion method [4], the Darboux transformation [5], the ansatz method [6, 7], Hirota's bilinear method [8], the (G'/G) -expansion method [9], the Jacobi elliptic function expansion method [10, 11], the variable separation approach [12], the sine-cosine method [13], the trifunction method [14, 15], the F-expansion method [16], the exp-function method [17], the multiple exp-function method [18], and the Lie symmetry method [19–25].

The purpose of this paper is to study one such NLEE, namely, the generalized two-dimensional nonlinear Kadomtsev-Petviashvili-modified equal width (KP-MEW) equation [26] that is given by

$$(u_t + \alpha(u^n)_x + \beta u_{xxt})_x + \gamma u_{yy} = 0. \quad (1)$$

Here, in (1) α , β , γ and $n > 1$ are real valued constants. The solutions of (1) have been studied in various aspects. See, for example, the recent papers [26–28]. Wazwaz [26] used the tanh method and the sine-cosine method, for finding solitary waves and periodic solutions. Saha [27] used the theory of bifurcations of planar dynamical systems to prove the existence of smooth and nonsmooth travelling wave solutions. Wei et al. [28] used the qualitative theory of differential equations and obtained peakon, compacton, cuspons, loop soliton solutions, and smooth soliton solutions.

In this paper we obtain symmetry reductions of (1) using Lie group analysis [19–24] and based on the optimal systems of one-dimensional subalgebras. Furthermore, the (G'/G) -expansion method is employed to obtain some exact solutions of (1). In addition to this conservation laws will be derived for (1) using the multiplier method [29].

2. Symmetry Reductions and Exact Solutions of (1)

The vector field of the form

$$X = \xi^1 \frac{\partial}{\partial x} + \xi^2 \frac{\partial}{\partial y} + \xi^3 \frac{\partial}{\partial t} + \eta \frac{\partial}{\partial u}, \quad (2)$$

where ξ^i , $i = 1, 2, 3$, and η depend on x , y , t , and u , is a Lie point symmetry of (1) if

$$\text{pr}^{(4)}X \left[(u_t + \alpha(u^n)_x + \beta u_{xxt})_x + \gamma u_{yy} \right] = 0 \quad (3)$$

whenever $(u_t + \alpha(u^n)_x + \beta u_{xxt})_x + \gamma u_{yy} = 0$. Here $\text{pr}^{(4)}X$ [20] denotes the fourth prolongation of X . Expanding (3) and splitting on the derivatives of u , we obtain an overdetermined system of linear partial differential equations. Solving this system one obtains the following four Lie point symmetries:

$$\begin{aligned} X_1 &= \frac{\partial}{\partial x}, \\ X_2 &= \frac{\partial}{\partial y}, \\ X_3 &= \frac{\partial}{\partial t}, \\ X_4 &= \gamma(1-n) \frac{\partial}{\partial y} + 2t(1-n) \frac{\partial}{\partial t} + 2u \frac{\partial}{\partial u}. \end{aligned} \quad (4)$$

2.1. One-Dimensional Optimal System of Subalgebras. We now calculate the optimal system of one-dimensional subalgebras for (1) and use it to find the optimal system of group-invariant solutions for (1). We follow the method given in [20]. Recall that the adjoint transformations are given by

$$\begin{aligned} \text{Ad}(\exp(\epsilon X_i)) X_j &= X_j - \epsilon [X_i, X_j] \\ &\quad + \frac{1}{2} \epsilon^2 [X_i, [X_i, X_j]] - \dots, \end{aligned} \quad (5)$$

where $[X_i, X_j]$ is the commutator defined by

$$[X_i, X_j] = X_i X_j - X_j X_i. \quad (6)$$

We present the commutator table of the Lie symmetries and the adjoint representations of the symmetry group of (1) on its Lie algebra in Tables 1 and 2, respectively. These two tables are then used to construct the optimal system of one-dimensional subalgebras for (1). As a result, after some calculations, one can obtain an optimal system of one-dimensional subalgebras given by $\{aX_1 + bX_2 + cX_3, dX_1 + X_4\}$, where $a, d \in \mathbb{R}$, $b, c = 0, \pm 1$.

2.2. Symmetry Reductions and Exact Solutions of (1). In this subsection we use the optimal system of one-dimensional subalgebras calculated above to obtain symmetry reductions and exact solutions of the KP-MEW equation.

Case 1. Consider the following: $aX_1 + bX_2 + cX_3$; $a \in \mathbb{R}$, $b, c = \pm 1$.

The symmetry $aX_1 + bX_2 + cX_3$ gives rise to the following three invariants:

$$f = \frac{bt - cy}{b}, \quad g = \frac{bx + ay}{b}, \quad \phi = u. \quad (7)$$

TABLE 1: Commutator table of the Lie algebra of equation (1).

	X_1	X_2	X_3	X_4
X_1	0	0	0	0
X_2	0	0	0	$(1-n)X_2$
X_3	0	0	0	$(2-2n)X_3$
X_4	0	$-(1-n)X_2$	$-(2-2n)X_3$	0

TABLE 2: Adjoint table of the Lie algebra of equation (1).

Ad	X_1	X_2	X_3	X_4
X_1	X_1	X_2	X_3	X_4
X_2	X_1	X_2	X_3	$X_4 - \epsilon(1-n)X_2$
X_3	X_1	X_2	X_3	$X_4 - \epsilon(2-2n)X_3$
X_4	X_1	$e^{\epsilon(1-n)}X_2$	$e^{\epsilon(2-2n)}X_3$	X_4

Now treating ϕ as the new dependent variable and f and g as new independent variables, the KP-MEW equation (1) transforms to

$$\begin{aligned} (2ac\gamma + b^2) \phi_{fg} + a^2 \gamma \phi_{gg} + \alpha n(n-b^2) \phi^{n-2} \phi_g^2 \\ + b^2 \alpha n \phi^{n-1} \phi_{gg} + b^2 \beta \phi_{fgg} + c^2 \gamma \phi_{ff} = 0, \end{aligned} \quad (8)$$

which is a nonlinear PDE in two independent variables. We now use the Lie point symmetries of (8) and transform it to an ordinary differential equation (ODE). Equation (8) has the two translational symmetries; namely,

$$\Gamma_1 = \frac{\partial}{\partial f}, \quad \Gamma_2 = \frac{\partial}{\partial g}. \quad (9)$$

The combination $\Gamma_1 + \Gamma_2$ of the two symmetries Γ_1 and Γ_2 yields the two invariants

$$z = f - g, \quad F = \phi, \quad (10)$$

which gives rise to a group-invariant solution $F = F(z)$. Consequently using these invariants, (8) is transformed into the fourth-order nonlinear ODE:

$$\begin{aligned} \gamma(a+c)^2 F'' + \alpha n b^2 F^{n-1} F'' - b^2 F'' \\ + \alpha n b^2 (n-1) F^{n-2} F'^2 - \beta b^2 F'''' = 0. \end{aligned} \quad (11)$$

Integrating the above equation twice and taking the constants of integration to be zero we obtain a second-order ODE:

$$(\gamma(a+c)^2 - c^2) F + \alpha b^2 F^n - \beta b^2 F'' = 0. \quad (12)$$

Multiplying (12) by F' , integrating once and taking the constant of integration to be zero, we obtain the first-order ODE:

$$\frac{1}{2} (\gamma(a+c)^2 - b^2) F^2 + \frac{\alpha b^2}{n+1} F^{n+1} - \frac{1}{2} \beta b^2 F'^2 = 0. \quad (13)$$

One can integrate the above equation by separating the variables. After integrating and reverting back to the original variables, we obtain the following group-invariant solutions

of the KP-MEW equation (1) for arbitrary values of n in the following form:

$$u(x, y, t) = \left(\frac{(n+1)(b^2 - \gamma(a+c)^2)}{2\beta b^2} \right)^{1/(n-1)} \text{sech}^{2/(n-1)}[Q], \quad (14)$$

where

$$Q = \frac{\sqrt{\gamma(a+c)^2 + b^2}}{2\sqrt{2}} \left(\frac{\sqrt{2}(1-n)}{c\sqrt{\beta}} z + (n-1)C \right), \quad (15)$$

$$z = t - x - \frac{(a+c)}{b}y,$$

and C is a constant of integration. By taking $n = 2$, $\alpha = 1/2$, $\beta = 1$, $\gamma = 1$, $a = 1$, $b = 1$, $c = 1$, $t = 0$, and $C = 1$ in (14), the profile of the solution is given in Figure 1.

Case 2. Consider the following: $dX_1 + X_4$.

The symmetry $dX_1 + X_4$ gives rise to the three invariants:

$$f = \frac{t}{y^2}, \quad g = \frac{d \ln y + nx - x}{n-1}, \quad \phi = uy^{2/(n-1)}. \quad (16)$$

By treating ϕ as the new dependent variable and f and g as new independent variables, the KP-MEW equation (1) transforms to

$$\begin{aligned} & \beta n^2 \phi_{fggg} - 2\beta n \phi_{fggg} + 2d\gamma \phi_{gg} + 2n\gamma \phi + 4f^2 \gamma \phi_{ff} \\ & - 2f\gamma \phi_f - 3a_1 \gamma \phi_g + \phi_{fg} - \alpha n \phi^{n-2} \phi_g^2 \\ & + \alpha n^4 \phi^{n-2} \phi_g^2 + 3\alpha n^2 \phi^{n-2} \phi_g^2 + 6f n^2 \gamma \phi_f \\ & - 4f n \gamma \phi_f - 3\alpha n^3 \phi^{n-2} \phi_g^2 + \alpha n \phi^{n-1} \phi_{gg} \\ & + \alpha n^3 \phi^{n-1} \phi_{gg} - 8f^2 n \gamma \phi_{ff} + 4f^2 n^2 \gamma \phi_{ff} \\ & + 4df \gamma \phi_{fg} - 2\alpha n^2 \phi^{n-1} \phi_{gg} + 2\gamma \phi + \beta \phi_{fggg} \\ & + f n^2 \phi_g - 2f n \phi_g - 4df n \gamma \phi_{fg} - df n \gamma \phi_g = 0. \end{aligned} \quad (17)$$

Equation (17) has a single Lie point symmetry; namely,

$$\Gamma_1 = \frac{\partial}{\partial g}, \quad (18)$$

and this symmetry yields the two invariants

$$r = f, \quad F = \phi, \quad (19)$$

which gives rise to a group-invariant solution $F = F(r)$ and consequently, using these invariants, (17) is then transformed to a second-order Cauchy-Euler ODE:

$$\begin{aligned} & (2n^2 r^2 - 4nr^2 + 2r^2) F'' + (3n^2 r - 2nr - r) F' \\ & + (1+n) F = 0. \end{aligned} \quad (20)$$

FIGURE 1: Profile of solution (14).

Now solving this equation and reverting back to the original variables, we obtain the following solution of the KP-MEW equation (1):

$$u(x, y, t) = \frac{1}{y^{2/(n-1)}} \left(C_1 r^{-(n+1)/2(n-1)} + C_2 r^{-1/(n-1)} \right), \quad (21)$$

where $r = t/y^2$ and C_1 and C_2 are constants of integration.

3. (G'/G) -Expansion Method

In this section we use the (G'/G) -expansion method [9, 30] to obtain a few exact solutions of the KP-MEW equation (1) for $n = 2$ and $n = 3$.

Let us consider the solutions of (11) in the form

$$F(z) = \sum_{i=0}^M \mathcal{A}_i \left(\frac{G'(z)}{G(z)} \right)^i, \quad (22)$$

where $G(z)$ satisfies

$$G'' + \lambda G' + \mu G = 0 \quad (23)$$

and λ and μ are constants. The homogeneous balance method between the highest order derivative and highest order nonlinear term appearing in (11) determines the value of M and $\mathcal{A}_0, \dots, \mathcal{A}_M$ are constants to be determined.

Consider $n = 2$. Application of the balancing procedure to fourth-order ODE (11) yields $M = 2$, so the solution of (11) is of the form

$$F(z) = \mathcal{A}_0 + \mathcal{A}_1 \left(\frac{G'(z)}{G(z)} \right) + \mathcal{A}_2 \left(\frac{G'(z)}{G(z)} \right)^2. \quad (24)$$

Substituting (23) and (24) into (11) leads to an overdetermined system of algebraic equations. Solving this system of algebraic equations with the aid of Maple, we obtain

$$\begin{aligned}\mathcal{A}_0 &= \frac{b^2\alpha\lambda^2 A_2 + 8b^2\mu\alpha A_2 - 6\gamma a^2 - 12ac\gamma - 6\gamma c^2 + 6b^2}{12\alpha b^2}, \\ \mathcal{A}_1 &= \frac{6\beta\lambda}{\alpha}, \\ \mathcal{A}_2 &= \frac{6\beta}{\alpha}.\end{aligned}\quad (25)$$

Now using the general solution of (23) in (24), we have the following three types of travelling wave solutions of the KP-MEW equation (1).

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution:

$$\begin{aligned}u(x, y, t) &= \mathcal{A}_0 + \mathcal{A}_1 \left(-\frac{\lambda}{2} + \delta_1 \frac{C_1 \sinh(\delta_1 z) + C_2 \cosh(\delta_1 z)}{C_1 \cosh(\delta_1 z) + C_2 \sinh(\delta_1 z)} \right) \\ &\quad + \mathcal{A}_2 \left(-\frac{\lambda}{2} + \delta_1 \frac{C_1 \sinh(\delta_1 z) + C_2 \cosh(\delta_1 z)}{C_1 \cosh(\delta_1 z) + C_2 \sinh(\delta_1 z)} \right)^2,\end{aligned}\quad (26)$$

where $z = t - x - ((a+c)/b)y$, $\delta_1 = (1/2)\sqrt{\lambda^2 - 4\mu}$, and C_1 and C_2 are arbitrary constants.

The profile of the solution (26) is given in Figure 2.

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function solution:

$$\begin{aligned}u(x, y, t) &= \mathcal{A}_0 + \mathcal{A}_1 \left(-\frac{\lambda}{2} + \delta_2 \frac{-C_1 \sin(\delta_2 z) + C_2 \cos(\delta_2 z)}{C_1 \cos(\delta_2 z) + C_2 \sin(\delta_2 z)} \right) \\ &\quad + \mathcal{A}_2 \left(-\frac{\lambda}{2} + \delta_2 \frac{-C_1 \sin(\delta_2 z) + C_2 \cos(\delta_2 z)}{C_1 \cos(\delta_2 z) + C_2 \sin(\delta_2 z)} \right)^2,\end{aligned}\quad (27)$$

where $z = t - x - ((a+c)/b)y$, $\delta_2 = (1/2)\sqrt{4\mu - \lambda^2}$, and C_1 and C_2 are arbitrary constants.

The profile of the solution (27) is given in Figure 3.

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solution:

$$\begin{aligned}u(x, y, t) &= \mathcal{A}_0 + \mathcal{A}_1 \left(-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 z} \right) \\ &\quad + \mathcal{A}_2 \left(-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 z} \right)^2,\end{aligned}\quad (28)$$

where $z = t - x - ((a+c)/b)y$ and C_1 and C_2 are arbitrary constants.

The profile of the solution (28) is given in Figure 4.

FIGURE 2: Profile of solution (26).

FIGURE 3: Profile of solution (27).

FIGURE 4: Profile of solution (28).

Consider $n = 3$. Again the application of the balancing procedure to fourth-order ODE yields $M = 1$, so the solution of (11) is of the form

$$F(z) = \mathcal{A}_0 + \mathcal{A}_1 \left(\frac{G'(z)}{G(z)} \right). \quad (29)$$

Solving this system of algebraic equations with the aid of Maple, we obtain

$$\begin{aligned} \beta &= \frac{\alpha}{2} \mathcal{A}_1^2, \\ \mathcal{A}_0 &= \frac{\lambda \sqrt{-a^2\gamma - 2ac\gamma + b^2 - c^2\gamma}}{\sqrt{\alpha b^2 \lambda^2 - 4\alpha b^2 \mu}}, \\ \mathcal{A}_1 &= \frac{2\sqrt{-a^2\gamma - 2ac\gamma + b^2 - c^2\gamma}}{\sqrt{\alpha b^2 \lambda^2 - 4\alpha b^2 \mu}}. \end{aligned} \quad (30)$$

Now using the general solution of (23) in (29), we have the following two types of travelling wave solutions of the KP-MEW equation (1).

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution:

$$\begin{aligned} u(x, y, t) &= \mathcal{A}_0 + \mathcal{A}_1 \left(-\frac{\lambda}{2} + \delta_1 \frac{C_1 \sinh(\delta_1 z) + C_2 \cosh(\delta_1 z)}{C_1 \cosh(\delta_1 z) + C_2 \sinh(\delta_1 z)} \right), \end{aligned} \quad (31)$$

where $z = t - x - ((a+c)/b)y$, $\delta_1 = (1/2)\sqrt{\lambda^2 - 4\mu}$, and C_1 and C_2 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function solution:

$$\begin{aligned} u(x, y, t) &= \mathcal{A}_0 + \mathcal{A}_1 \left(-\frac{\lambda}{2} + \delta_2 \frac{-C_1 \sin(\delta_2 z) + C_2 \cos(\delta_2 z)}{C_1 \cos(\delta_2 z) + C_2 \sin(\delta_2 z)} \right), \end{aligned} \quad (32)$$

where $z = t - x - ((a+c)/b)y$, $\delta_2 = (1/2)\sqrt{4\mu - \lambda^2}$, and C_1 and C_2 are arbitrary constants.

4. Conservation Laws of (1)

In this section we construct conservation laws for (1). The multiplier method [29, 30] will be used.

The zeroth-order multiplier $\Lambda(t, x, y, u)$ for (1) is given by

$$\begin{aligned} \Lambda &= -\frac{y^3}{6\gamma} f_1'(t) + xyf_1(t) - \frac{y^2}{2\gamma} f_2'(t) \\ &\quad + xf_2(t) + yf_3(t) + f_4(t), \end{aligned} \quad (33)$$

where $f_1(t)$, $f_2(t)$, $f_3(t)$, and $f_4(t)$ are arbitrary functions of t . Corresponding to the above multiplier we have the following conserved vectors of (1):

$$\begin{aligned} T_1^t &= \frac{1}{24\gamma} \left\{ -12\gamma y f_1(t) u - 6\beta \gamma y f_1(t) u_{xx} \right. \\ &\quad \left. + 6\beta \gamma x y f_1(t) u_{xxx} + 12\gamma x y f_1(t) u_x \right. \\ &\quad \left. - \beta y^3 f_1'(t) u_{xxx} - 2y^3 f_1'(t) u_x \right\}, \\ T_1^x &= -\frac{y}{24\gamma u} \left\{ 4\alpha n y^2 f_1'(t) u_x u^n - 24\alpha \gamma n x f_1(t) u_x u^n \right. \\ &\quad \left. - \beta y^2 f_1''(t) u_{xx} u + 3\beta y^2 f_1'(t) u_{txx} u \right. \\ &\quad \left. + 2y^2 f_1'(t) u_t u - 12\beta \gamma f_1'(t) u_x u \right. \\ &\quad \left. + 6\beta \gamma x f_1'(t) u_{xx} u + 12\beta \gamma f_1(t) u_{tx} u \right. \\ &\quad \left. - 18\beta \gamma x f_1(t) u_{txx} u - 12\gamma x f_1(t) u_t u \right. \\ &\quad \left. + 24\alpha \gamma f_1(t) u^{n+1} - 2y^2 f_1''(t) u^2 \right. \\ &\quad \left. + 12\gamma x f_1'(t) u^2 \right\}, \\ T_1^y &= \frac{1}{6} \left\{ 3y^2 f_1'(t) u - 6\gamma x f_1(t) u \right. \\ &\quad \left. + 6\gamma x y f_1(t) u_y - y^3 f_1'(t) u_y \right\}; \\ T_2^t &= \frac{1}{8\gamma} \left\{ -4\gamma f_2(t) u - 2\beta \gamma f_2(t) u_{xx} + 2\beta \gamma x f_2(t) u_{xxx} \right. \\ &\quad \left. + 4\gamma x f_2(t) u_x - \beta y^2 f_2'(t) u_{xxx} - 2y^2 f_2'(t) u_x \right\}, \\ T_2^x &= -\frac{1}{8\gamma u} \left\{ 4\alpha n y^2 f_2'(t) u_x u^n - 8\alpha \gamma n x f_2(t) u_x u^n \right. \\ &\quad \left. - \beta y^2 f_2''(t) u_{xx} u + 3\beta y^2 f_2'(t) u_{txx} u \right. \\ &\quad \left. + 2y^2 f_2'(t) u_t u - 4\beta \gamma f_2'(t) u_x u \right. \\ &\quad \left. + 2\beta \gamma x f_2'(t) u_{xx} u + 4\beta \gamma f_2(t) u_{tx} u \right. \\ &\quad \left. - 6\beta \gamma x f_2(t) u_{txx} u - 4\gamma x f_2(t) u_t u \right. \\ &\quad \left. + 8\alpha \gamma f_2(t) u^{n+1} - 2y^2 f_2''(t) u^2 \right. \\ &\quad \left. + 4\gamma x f_2'(t) u^2 \right\}, \\ T_2^y &= \frac{1}{2} \left\{ 2y f_2'(t) u + 2\gamma x f_2(t) u_y - y^2 f_2'(t) u_y \right\}; \\ T_3^t &= \frac{1}{4} \left\{ \beta y f_3(t) u_{xxx} + 2y f_3(t) u_x \right\}, \\ T_3^x &= -\frac{y}{4u} \left\{ -4\alpha n f_3(t) u_x u^n + \beta f_3'(t) u_{xx} u - 3\beta f_3(t) u_{txx} u \right. \\ &\quad \left. - 2f_3(t) u_t u + 2f_3'(t) u^2 \right\}, \\ T_3^y &= \gamma y f_3(t) u_y - \gamma f_3(t) u; \end{aligned}$$

$$\begin{aligned}
T_4^t &= \frac{1}{4} \{ \beta f_4(t) u_{xxx} + 2f_4(t) u_x \}, \\
T_4^x &= -\frac{1}{4u} \{ -4\alpha n f_4(t) u_x u^n + \beta f_4'(t) u_{xx} u - 3\beta f_4(t) u_{txx} u \\
&\quad - 2f_4(t) u_t u + 2f_4'(t) u^2 \}, \\
T_4^y &= \gamma f_4(t) u_y.
\end{aligned} \tag{34}$$

Remark. The presence of the arbitrary functions in the multiplier leads to a family of infinitely many conservation laws for (1).

5. Concluding Remarks

In this paper we obtained the solutions of a generalized two-dimensional nonlinear Kadomtsev-Petviashvili-modified equal width equation by employing the Lie group analysis, the optimal systems of one-dimensional subalgebras, and the (G'/G) -expansion method. The solutions obtained are solitary waves and nontopological solitons. The conservation laws for the underlying equation were also derived by using the multiplier method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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