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Research Article

Deteriorating Inventory Model for Chilled Food

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With many aspects that affect inventory policy, product perishability is a critical aspect of inventory policy. Most goods will deteriorate during storage and their original value will decline or be lost. Therefore, deterioration should be taken into account in inventory practice. Chilled food products are very common consumer goods that are, in fact, perishable. If the chilled food quality declines over time customers are less likely to buy it. The value the chilled food retains is, however, closely dependent on its quality. From the vendor's point of view, quantifying quality and remaining value should be a critical business issue. In consequence, we combined the traditional deterioration model and quality prediction model to develop a new deteriorating inventory model for chilled food products. This new model quantifies food quality and remaining value. The model we propose uses real deterioration rate data, and we regard deterioration rate as temperature-dependent. We provide a numerical example to illustrate the solution. Our model demonstrates that high storage temperatures reduce profits and force shorter order cycles.

1. Introduction

We know that inventory policy may affect supply chain performance from Beheshti's [1] research. If inventory policy is not appropriate, it will burden enterprises with high costs. Hence, many enterprises would like to find out the most suitable inventory policy. Inventory policy researchers have included numerous different conditions into their models. These different conditions frequently include stochastic demand/lead time, lead time reduction, product quality, deterioration, and the learning curve (reviewed in [2, 3]). The aim of including such conditions is to make models more realistic and complete. In general, most physical goods will deteriorate during storage and customers are less likely to buy deteriorated goods such as food, drugs, and chemicals. The value of goods will decline or even be lost. Vendors therefore have to reduce the price of deteriorated goods in order to sell them. The economic loss this represents necessitates, we believe, the more detailed consideration of deterioration. Glock et al. [3] also indicated that deterioration may affect the productivity of an inventory system. Chilled food has become common in real life these years [4]. Herbon et al. [5] indicated that the value of chilled food has much to do with its quality. From the vendor's point of view, quantifying quality and remaining value should be a critical business issue. We therefore desired to develop a model of deteriorating inventory for chilled food. We also reviewed the literature in order to identify currently neglected research factors.

2. Literature Review

2.1. Early Deterioration Inventory Model. Deterioration is defined as decay, damage, spoilage, or perishability and its effect cannot be disregarded in inventory models [6]. Goyal and Giri [7] differentiated between two types of deterioration, perishability, and decay, where perishability refers to products with a fixed shelf-life and decay to products with unlimited shelf-life. Now we would like to review some early researches that have considered deterioration in their inventory model. Ghare and Schrader [8] were the first to consider deterioration in their research. They indicated that inventories are depleted not only by demand but also by decay. They proposed an exponential decay model to illustrate how deterioration works in a normal inventory model. Many scholars continued using the Ghare and Schrader's exponential decay model to establish further deteriorating inventory models. Shah and Jaiswal [9] developed a periodic review inventory model with deteriorating items. They considered both constant and variable deterioration rates. Sachan [10] established a deteriorating inventory model with time-dependent demand but he only considered a constant deterioration. Kim and Hwang [11] assumed deterioration rate to be time-proportional. They also showed that there was a substantial impact of deterioration on the optimal procurement policy. Datta and Pal [12] discussed a finite-horizon model and special sales of deteriorating items, while allowing shortage. The items began to deteriorate at a constant rate. Pal et al. [13] proposed a deterministic inventory model that assumed that demand rate was stock-dependent. Benkherouf [14] presented an optimal procedure to find out the solutions of an inventory model with deteriorating goods and demand rates that decreased over a known horizon. Benkherouf [15] extended his research in 1995; he assumed demand rates increased over time in the same model. Wee [16] noticed that there is less attention in the literature to price-dependent demand and varying deterioration rate. Therefore, he studied and established a model that included these factors and provided an algorithm for obtaining the optimal profit.

In the above-mentioned studies, deterioration rate is regarded as a constant, varying, or time-proportional value and the demand rate as stock-dependent, time-dependent, or price-dependent. With this type of deterioration inventory model becoming increasingly mature, researchers began to elaborate new deterioration inventory models.

2.2. Lately Deterioration Inventory Model. Mukhopadhyay et al. [6] established such a deteriorating model in which deterioration rate is taken to be time-proportional and a power law form of the price-dependence of demand is considered. Hou [17] studied optimal production run length for deteriorating products by using Markov chains. Mandal et al. [18] constrained deteriorating items; they assumed the storage space was limited and demand rate for items was finite. Adel and Balkhi [19] studied the effect of learning and forgetting on optimal lot size based on a deteriorating inventory model with time-varying demand. Feng et al. [20] built a deteriorating model with price-dependent demand. They took two commodities as examples to find out the joint optimal pricing strategies in their model. Sung and Gong [21] indicated that deteriorating items caused defects during the production process. These defective items have to be reworked immediately and so incur a cost. The model determines the final expected total cost due to the reworking. Barketau et al. [22] proposed a similar model to Sung and Gong's [21], but they did not assume immediate reworking but allowed the defective items to continue deteriorating during the time they wait for reworking. The authors' objective was then to determine the optimal batch size for minimizing the total cost. Nakhai and Jafari [23] emphasized the perishability of food, chemicals, and medicines made them particularly sensitive in deteriorating inventory models. They chose medicines for their research object and obtained the optimum cost of the drug supply chain.

Chen et al. [24] pointed out in-transit deterioration was seldom considered. They therefore proposed a deteriorating model under permissible delay in payments and used different deterioration rates during in-transit and on-hand periods. Molana et al. [25] also assumed products deteriorate during transportation and storage. Giri and Maiti [26] studied a supply chain model with deteriorating products. With the in-control state and out-of-control state during the production run, they considered that a coordinated policy is more beneficial than policies obtained separately from the buyer's and the vendor's perspectives. Sarkar [27] developed a deteriorating model under permissible delay in payments and assumed the demand rate and deterioration rate were time-varying. Panda et al. [28] thought that perishable products were an extremely important part of inventory management. They investigated the effect of price discounts and proved that dynamic pricing was better than static pricing. Kim et al. [29] studied a two-stage supply chain where returnable transport items (RTIs) were used to ship finished products from the supplier to the buyer. They indicated that the supply chain can influence both the risk of RTI stockouts at the supplier and the deterioration rate by changing the value of the return lot size of RTIs.

We discovered that price-dependent demand was a frequent basic assumption. Along with a basic assumption of demand rate or deterioration rate, researchers have developed many new deterioration models in order to make them complete. Some authors discussed storage limits and some reworking and defective items. Other authors have discussed the perishable goods of specific industries or pricing policy. We also compared the relevant works to our work in Table 1.

2.3. Perishable Products. Inspired by Nakhai and Jafari [23] we noticed that specific perishable products such as food, drugs, or chemicals are more sensitive in deteriorating model than other goods. Perishable products are those that worsen in quality over time and loose value. Common perishable goods include foods, medicines, plants, and agricultural products. Dairy products, fashion products, blood, fruits, and vegetables are examples of time and temperature-sensitive perishable products that can rot or spoil easily. In general, perishable goods decay rapidly if not refrigerated, or if some other preservation technique is not employed.

Among these perishable products, food or food products are very common consumer goods in real life. In recent years, more and more families have replaced home meals with chilled food because chilled food is more convenient than home meals. Chilled food also saves a lot of preparation and cooking time [4]. Chilled food has been incorporated in many deteriorating inventory models [32–36]. Chilled food is therefore a very appropriate perishable product for inclusion in deteriorating inventory model.

Herbon et al. [5] used time-temperature indicators (TTI) for keeping track of the quality of perishable products. They claimed that the price of perishable products and their remaining value has much to do with their quality and this caused the vendor to pay attention to maintaining quality. It is therefore of interest to be able to quantify the quality of food and its remaining value. We also compared the relevant works to our work in Table 2.

Table 1: Overview of deteriorating model.

Authors	Deterioration rate	Demand rate
Kim and Hwang [11]	Time-dependent	Constant
Wee [16]	Constant	Price-dependent
Mukhopadhyay et al. [6]	Time-dependent	Price-dependent
Hou [17]	Time-dependent	Price-dependent
Feng et al.[20]	Time-dependent	Price-dependent
Nakhai and Jafari [23]	Time-dependent	Constant
Sarkar [27]	Time-dependent	Time-dependent
Giri and Maiti [26]	In-control and out-of-control	Price-dependent
Iao and Hsiao [30]	Temperature-dependent	Constant
Qin et al. (2014) [31]	Temperature-dependent	Price-dependent
*Our work	Temperature-dependent	Price-dependent

Table 2: Overview of perishable goods works.

Authors	Claims
Bogataj et al. [37]	Appropriate control over the transportation time and storage temperature can keep the product on the required level of quality and quantity at the final delivery.
Osvald & Stirn [38]	They developed a vehicle routing algorithm for distributing perishable goods; to obtain minimum distribution cost, their algorithm planned a schedule to reduce distribution time.
Chen et al. [39]	They established a nonlinear mathematical model to consider production scheduling and vehicle routing for perishable goods; their model could determine the time to start producing and vehicle routes simultaneously.
Nakhai and Jafari [23]	Perishable goods were quite sensitive in time and temperature.
Xu & Wang [40]	Suitable facility and precise temperature controlling are two major factors to keep perishable goods.
Aiello et al. [41]	They implied that supply chain organization and operative characteristics have a significant influence on perishable goods, ensuring that the suitable temperature conditions in warehouse were important.
*Our work	We should focus on time and temperature if we consider perishable goods in our works. We therefore built an inventory model quantifying chilled food quality and remaining value.

2.4. Predictive Quality Model. For food quality, observing the growth of microorganisms is a highly reliable approach to defining food quality and, further, determining food safety. If microorganisms abundance exceeds the standard value, the food can be defined as inedible [42–45].

To observe and predict the growth of microorganisms, we learned some predictive quality models in predictive microbiology such as modified Gompertz, Baranyi, Rosso, and Gompertz models [46]. In 1993, Buchanan indicated that function graph of the Gompertz model (see Appendix A) closely fitted the growth curve of microorganisms. Researchers therefore prefer the Gompertz model to other predictive quality models. Afterwards Gibson et al. [47] advanced the Gompertz model to develop a predictive model of microorganisms. Many researchers continued using their model to predict the microorganism counts in their researches. Linton et al. [48] established the nonlinear survival curve for *Listeria monocytogenes* by extending the Gompertz model. Huang [49] described the growth of *Clostridium perfringens* on cooked beef by using the Gompertz model. Chowdhury et al. [50] predicted the growth of *Pediococcus acidilactici* by using logistic model and Gompertz model and also compared the predictive ability of these two models. They regarded the Gompertz model as appropriate model for predicting food quality. We also compared the relevant works to our work in Table 3.

Table 3: Overview of predictive quality model.

Authors	Predictive quality model
Gibson et al. [47]	Gompertz model
Buchanan et al. [51]	Gompertz model
Linton et al. [48]	Gompertz model
Huang [49]	Gompertz model
Chowdhury et al. [50]	Logistic model and Gompertz model
*Our work	Gompertz model

Qin et al. [31] formulated a model of the pricing and lot-sizing problem for food products where the quality and physical quantity deteriorated simultaneously. They indicated that temperature has the most profound effect on the deterioration rate. Thus, deterioration rate should vary with temperature when considering food products as the research object.

Based on the discussion above, we developed a deteriorating model for chilled food including the Gompertz model. This predictive model describes the growth rate of microorganisms over time. We also regard deterioration rate as temperature-dependent. We take pork sandwich, a kind of chilled food, as research object in our proposed model. Our new model treats pork sandwich deterioration and uses real deterioration rate data to illustrate the solutions [30].

3. Assumptions and Notations

The following assumptions and notations are used to formulate the problem and model.

3.1. Notations

p is selling price per unit item, a decision variable.

K is the duration of each cycle, a decision variable.

$d(p)$ is the price-dependent demand rate, equal to $\alpha p^{-\beta}$.

c is purchase cost per unit item.

S is ordering cost per cycle.

h is inventory holding cost per unit per unit time.

θ is deterioration rate.

A is initial bacterial count of food.

G is growth bacterial count of food.

$N(k)$ is end bacterial count of food at time k, $0 \le k \le K$ [30].

w is the remaining ratio of value after deterioration per unit item.

M is time at which maximum growth rate occurs [30].

t is storage temperature of product.

$I(k)$ is inventory level at time k, $0 \le k \le K$.

$Z(K)$ is the loss in stock due to deterioration in time $[0, k]$.

$C(K, p)$ is the total cost per cycle.

$\overline{C}(K, p)$ is the total cost per unit time.

$P(K, p)$ is the total profit per unit time.

3.2. Assumptions

(1) A single product and a single vendor are assumed.

(2) We take the pork sandwiches as products in this research.

(3) *Pseudomonas* spp. is the observed microorganisms on pork sandwich [30].

(4) The deterioration rate of pork sandwiches θ is different with temperature t, $\theta = 0.03e^{0.084t}$ [30].

(5) The time at maximum growth rate of pork sandwiches M is different with temperature t, $M = 96.332e^{-0.127t}$ [30].

(6) Shortages are not allowed.

(7) Lead time is assumed to be 0.

(8) The time horizon is infinite.

(9) The price-dependent demand rate is equal to $\alpha p^{-\beta}$, where α is scale parameter and β is the shape parameter of the demand curve, $\alpha > 0$, $\beta > 0$.

(10) Once the *Pseudomonas* spp. exceed the 7 logCFU/g, most meat products will be inedible [30, 42, 52–57]; we can set w to be equal to $7-N(k)/7$; hence the value of w will decrease if $N(k)$ increases.

4. Modeling and Solution

According to research by Huang and Wang [58] and Mukhopadhyay et al. [6], the inventory level $I(k)$ can be described by the differential function:

$$\frac{dI(k)}{dk} = \theta I(k) - d(p), \quad 0 \le k \le K, \tag{1}$$

with the boundary condition

$$I(K) = 0. \tag{2}$$

And the solution of (1) is

$$I(k) = \frac{d(p)}{\theta}\left[e^{\theta(K-k)} - 1\right], \quad 0 \le k \le K. \tag{3}$$

$I(k)$ is the inventory level at any time k ($k \ge 0$), $I(0)$ represents the initial inventory level, and $I(K)$ represents the end inventory level. Now letting $Z(K)$ be the loss in stock due to deterioration in time $[0, k]$, with $I(K) = 0$, the loss in stock at the end can be expressed as (see Appendix B)

$$Z(K) = \frac{d(p)}{\theta}\left[e^{\theta K} - 1\right] - d(p)K. \tag{4}$$

$Z(K)$ means the difference between the inventory levels with and without deterioration at the end and the quantity ordered Q_k in each cycle obtained in the following function:

$$Q_K = Z(K) + d(p)K = \frac{d(p)}{\theta}\left[e^{\theta K} - 1\right]. \tag{5}$$

We may know the Q_K equals $I(0)$ from (3) and (5), after determining the inventory level $I(k)$ as discussed above, and letting $C(K, p)$ be the total cost per cycle, we have

$$\begin{aligned} C(K, p) &= S + cQ_k + h\int_0^K I(k)\,dk \\ &= S + \frac{cd(p)\left(e^{\theta K} - 1\right)}{\theta} + \frac{hd(p)\left(e^{\theta K} - \theta K - 1\right)}{\theta^2}. \end{aligned} \tag{6}$$

And the total cost per unit time is

$$\begin{aligned} \overline{C}(K, p) &= \frac{C(K, p)}{K} \\ &= \frac{S}{K} + \frac{cd(p)\left(e^{\theta K} - 1\right)}{\theta K} + \frac{hd(p)\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}. \end{aligned} \tag{7}$$

As far as selling the product is concerned, we applied the predictive quality model [30, 47] to determine the remaining value of pork sandwiches after deterioration:

$$N(k) = A + Ge^{-e^{-\theta(k-M)}}, \quad w = \frac{7 - N(k)}{7}, \quad 0 < w < 1. \tag{8}$$

And the remaining value of every pork sandwich after deterioration will be as follows:

$$wpd(p) = pd(p)\frac{7 - N(k)}{7}. \tag{9}$$

Now let $P(K, p)$ be the profit per unit time and obtain the profit function as follows:

$$\begin{aligned} P(K, p) &= wpd(p) - \overline{C}(K, p) \\ &= wpd(p) - \frac{S}{K} - \frac{cd(p)\left(e^{\theta K} - 1\right)}{\theta K} \\ &\quad - \frac{hd(p)\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}. \end{aligned} \tag{10}$$

Replacing $d(p)$ with $\alpha p^{-\beta}$, we have

$$\begin{aligned} P(K, p) &= w\alpha p^{1-\beta} - \frac{S}{K} \\ &\quad - \alpha p^{-\beta}\left(\frac{c\left(e^{\theta K} - 1\right)}{\theta K} - \frac{h\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}\right). \end{aligned} \tag{11}$$

To maximize the profit function $P(K, p)$, there must be a concave function for $K > 0$ and $p > 0$, as already proved in Appendix C. Now take the first partial derivation of $P(K, p)$ with respect to p. We have

$$\begin{aligned} \frac{\partial P(K, p)}{\partial p} &= \frac{1}{7}\left(7 - A - Ge^{-e^{-\theta(K-M)}}\right)(1 - \beta)\alpha p^{-\beta} \\ &\quad + \alpha\beta p^{-1-\beta}\left(\frac{c\left(e^{\theta K} - 1\right)}{\theta K} - \frac{h\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}\right). \end{aligned} \tag{12}$$

Let (12) = 0, and it can be solved to obtain the optimal price p^* by the following function:

$$p^* = -\frac{\alpha\beta\left(1 + \theta K^2/6\right)(c + hK/2)}{\alpha(\beta - 1)\left(A/7 + G/7e^{1/e^{\theta(K-M)}} - 1\right)}. \tag{13}$$

Algorithm 1.

Step 1. Set K to be 0.5, 1, 2, 3, 4, 5, 6, and 7, and set t to be 0°C to solve (13).

Step 2. Determine p value by different K under t = 0°C.

Step 3. Substitute determined p value into (11) to obtain corresponding $P(K, p)$.

Step 4. We can determine optimal value of $^*P(K, p)$ under t = 0°C.

Step 5. Reset t to be 0°C, and repeat Step 1 to Step 4 to find out the optimal value of $^*P(K, p)$ under t = 7°C.

Step 6. Follow Step 1 to Step 5, to determine the optimal value $^*P(K, p)$ under different t = 0°C, 7°C, 16°C, and 25°C.

5. Numerical Example

To illustrate the preceding model, we present an example in this section and consider the following data (provided by a top 3 enterprise that produce chilled food in Taiwan).

Relevant Parameters. Consider

α = 160000000;

β = 3.21;

c = \$40 per unit item;

S = \$250 per order cycle;

h = \$1.5 per unit per unit time;

A = 3.13 logCFU/g;

G = 6.87 logCFU/g.

First, we solved (13) by different temperature t, and the solution results are in Table 4. With the results in Table 4, we have observed that p increased in t and K; in other words, higher storage temperature and/or longer order cycle will cause higher pricing.

Second, we substituted these p values into (11) to obtain optimal value of total profit $P(K, p)$ under different temperature t (Figure 1). The solution results are shown in Table 5 and Figure 2.

Summarizing the results in Tables 4 and 5, as storage temperature t = 0°C, the optimal value of p, K, and $P(K, p)$ was 112.33, 2, and 685.32; as storage temperature t = 7°C, the optimal value of p, K, and $P(K, p)$ was 115.16, 2, and 641.23; as storage temperature t = 16°C, the optimal value of p, K, and $P(K, p)$ was 130.17, 1, and 425.11; as storage temperature t = 25°C, the optimal value of p, K, and $P(K, p)$ was 154.95, 1, and 61.41. We summarized the above results into a concise table as follows.

As shown in Table 6, best profit occurred at t = 0°C and value of p and K was 112.33 and 2; worst profit occurred at t = 25°C and value of p and K was 154.95 and 1. We may conclude that higher temperatures lead to less total profit. If

Table 4: Optimal value of p under different t.

t	K							
	0.5	1	2	3	4	5	6	7
0°C	106.87	108.67	112.33	116.06	119.86	123.75	127.71	131.75
7°C	107.57	110.05	115.16	120.48	126.02	131.81	137.85	144.18
16°C	112.72	118.07	130.17	144.61	162.07	183.52	210.21	243.95
25°C	135.21	154.95	213.65	319.81	540.36	1151.92	6229.64	31980.1

Table 5: Optimal value of total profit $P(K, p)$ under different t.

t	θ	M	t	K							
				0.5	1	2	3	4	5	6	7
0°C	0.03	96.33	0°C	404.56	621.78	685.32*	670.557	639.51	604.25	568.57	533.89
7°C	0.054	39.6	7°C	391.33	597.39	641.23*	609.815	564.7	517.52	471.75	428.513
16°C	0.115	12.63	16°C	278.83	447.17*	425.11	340.392	254.75	179.92	118.92	71.8915
25°C	0.245	4.03	25°C	−51.16	61.41*	3.49	−42.47	−53.66	−49.09	−41.66	−35.75

*Optimal solution of $P(K, p)$ in each t.

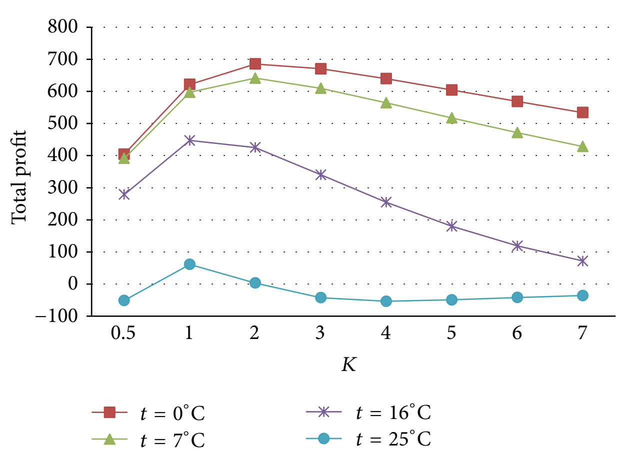

Figure 1: Optimal value of total profit $P(K, p)$ under different t.

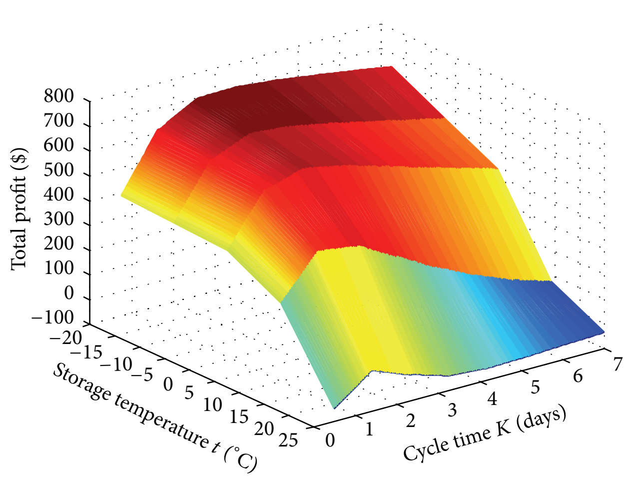

Figure 2: Optimal value of total profit $P(K, p)$ under different t.

Table 6: Optimal value of p, K, and $P(K, p)$ under different t.

	t = 0°C	t = 7°C	t = 16°C	t = 25°C
$^*p$	112.33	115.16	130.17	154.95
$^*K$	2	2	1	1
$^*P(K, p)$	685.32	641.23	425.11	61.41

*Optimal value.

buyers are unable to maintain low storage temperatures, they must shorten the order cycle if they are to obtain optimal total profit.

6. Conclusions

In this research, we discussed the issue of deteriorating inventory model. After reviewing Glock's [2] research, we discovered that deterioration is always popular in inventory issues. Product perishability is a critical aspect of inventory policy, most goods will deteriorate during the storage period, and the value of goods will decline or even be lost. So the deterioration should be taken into account in inventory practices.

We noticed that food products are those whose quality worsens the most over time. Tracking the quality of perishable products such as food products, chemicals, or medicines is an essential approach to observe their remaining value [5]. It is interesting to know how to quantize quality and remaining value; hence we would like to apply predictive quality model to determine quality and remaining value.

In the proposed model, we took pork sandwiches as our research object. First, we used the Gompertz model to determine the quality of pork sandwiches after deterioration has started. The Gompertz model closely fits the growth curve of microorganisms on food products and has been widely used in relevant researches. We also set w value to represent remaining value of pork sandwiches. Second, we combined traditional deteriorating inventory model with Gompertz model to develop a new model in Section 4. Finally, we illustrated an example to operate our proposed model with different storage temperatures for products.

Our proposed model was able to quantify quality and remaining value of pork sandwiches. It could help vendors to assess the whole system profit under different storage temperature. We discovered that higher storage temperatures lead to less profit and shorten the order cycle. These are main contributions of this paper.

Finally, it is our hope that this work will encourage future works in this area and related area. And we will improve our further research in more real-world complexities. In real life, storage temperature will change depending on the environment, refrigeration technology, refrigeration device, and similar factors. In other words, temperature fluctuation during storage is common. We may apply fuzzy theory to simulate temperature fluctuation.

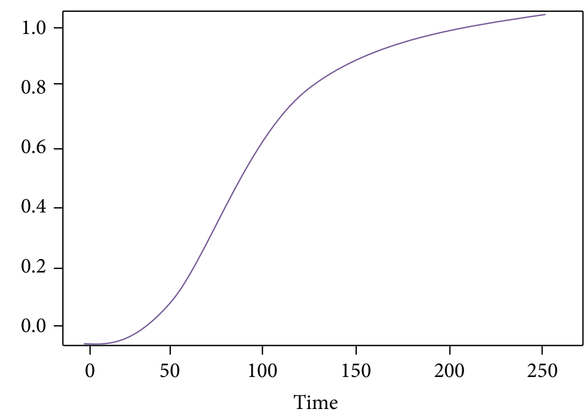

Figure 3: Gompertz function example.

Appendices

A. Gompertz Function

See Figure 3.

B. Inventory Level with and without Decay

The inventory level at any time k $(0 \le k \le K)$ is

$$I(k) = \frac{d(p)}{\theta}\left[e^{\theta(K-k)} - 1\right]. \tag{B.1}$$

And the initial inventory level is

$$I(0) = \frac{d(p)}{\theta}\left[e^{\theta K} - 1\right]. \tag{B.2}$$

Let $\bar{I}(k)$ be the inventory level at any time k $(0 \le k \le K)$ with no decay; we have

$$\bar{I}(k) = I(0) - kd(p). \tag{B.3}$$

The loss in stock at the end is equal to $\bar{I}(k) - I(k)$; then

$$\begin{aligned} Z(k) &= \bar{I}(k) - I(k) \\ &= I(0) - kd(p) - \frac{d(p)}{\theta}\left[e^{\theta(K-k)} - 1\right]. \end{aligned} \tag{B.4}$$

According to (B.2) and (B.4), we have

$$\begin{aligned} Z(k) &= \frac{d(p)}{\theta}\left[e^{\theta K} - e^{\theta(K-k)}\right] - kd(p), \\ Z(K) &= \frac{d(p)}{\theta}\left[e^{\theta K} - 1\right] - Kd(p). \end{aligned} \tag{B.5}$$

C. Hessian Matrix

We need a concavity condition of the profit function $P(K, p)$ to confirm the existence of a unique point of maximum for $P(K, p)$; we apply Hessian matrix as follows:

$$\begin{aligned} &P(K, p) \\ &\quad = w\alpha p^{1-\beta} - \frac{S}{K} \\ &\qquad - \alpha p^{-\beta}\left(\frac{c\left(e^{\theta K} - 1\right)}{\theta K} - \frac{h\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}\right), \end{aligned} \tag{C.1}$$

$$\begin{aligned} &\frac{\partial P(K, p)}{\partial p} \\ &\quad = \frac{1}{7}\left(7 - A - Ge^{-e^{-\theta(K-M)}}\right)(1-\beta)\alpha p^{-\beta} \\ &\qquad + \alpha\beta p^{-1-\beta}\left(\frac{c\left(e^{\theta K} - 1\right)}{\theta K} - \frac{h\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K}\right), \end{aligned} \tag{C.2}$$

$$\begin{aligned} &\frac{\partial P(K, p)}{\partial K} \\ &\quad = \frac{S}{K^2} - \frac{1}{7}Ge^{-e^{-\theta(K-M)} - \theta(K-M)}\theta\alpha p^{1-\beta} \\ &\qquad - \alpha p^{-\beta}\left(\frac{ce^{\theta K}}{K} - \frac{c\left(e^{\theta K} - 1\right)}{\theta K^2}\right. \\ &\qquad\qquad - \frac{h\left(e^{\theta K} - \theta K - 1\right)}{\theta^2 K^2} \\ &\qquad\qquad \left. + \frac{h\left(e^{\theta K}\theta - \theta\right)}{\theta^2 K}\right), \end{aligned} \tag{C.3}$$

$$
\begin{aligned}
&\frac{\partial^2 P(K,p)}{\partial p^2} \\
&\quad = \frac{1}{7}\left(7 - A - Ge^{-e^{-\theta(K-M)}}\right)(1-\beta)\beta\alpha p^{1-\beta} \\
&\qquad + (-\beta-1)\beta\alpha p^{-2-\beta} \\
&\qquad \cdot \left(\frac{c\left(e^{\theta K}-1\right)}{\theta K} + \frac{h\left(e^{\theta K}-\theta K-1\right)}{\theta^2 K}\right),
\end{aligned} \tag{C.4}
$$

$$
\begin{aligned}
&\frac{\partial^2 P(K,p)}{\partial K^2} \\
&\quad = -\frac{2S}{K^3} - \frac{1}{7}Ge^{-e^{-\theta(K-M)}-\theta(K-M)}\theta\alpha p^{1-\beta} \\
&\qquad \cdot \left(-1 + e^{-\theta(K-M)}\right)\alpha p^{1-\beta}\theta^2 \\
&\qquad - \frac{p^{-\beta}\alpha(h+c\theta)\left(-2 + e^{\theta K}\left(2 - 2\theta K + \theta^2 K^2\right)\right)}{\theta^2 K^3},
\end{aligned} \tag{C.5}
$$

$$
\begin{aligned}
&\frac{\partial^2 P(K,p)}{\partial p \partial K} \\
&\quad = \frac{\partial^2 P(K,p)}{\partial K \partial p} \\
&\quad = \frac{-1}{7}Ge^{-e^{-\theta(K-M)}-\theta(K-M)}(1-\beta)\theta\alpha p^{-\beta} \\
&\qquad + p^{-1-\beta}\alpha\beta\left(\frac{ce^{\theta K}}{K} - \frac{c\left(e^{\theta K}-1\right)}{\theta K^2}\right. \\
&\qquad\qquad + \frac{h\left(-\theta + e^{\theta K}\theta\right)}{\theta^2 K} \\
&\qquad\qquad \left. - \frac{h\left(e^{\theta K}-\theta K-1\right)}{\theta^2 K^2}\right).
\end{aligned} \tag{C.6}
$$

Now the Hessian matrix of $P(K,p)$ will be

$$
\begin{bmatrix}
\dfrac{\partial^2 P(K,p)}{\partial K^2} & \dfrac{\partial^2 P(K,p)}{\partial p \partial K} \\
\dfrac{\partial^2 P(K,p)}{\partial p \partial K} & \dfrac{\partial^2 P(K,p)}{\partial p^2}
\end{bmatrix}. \tag{C.7}
$$

And the function $P(K,p)$ will be concave if

$$
H_{11} = \frac{\partial^2 P(K,p)}{\partial K^2} < 0. \tag{C.8}
$$

We need to prove $e^{\theta K}(2 - 2\theta K + \theta^2 K^2) > 2$ and (C.5) will be < 0, if θ, K are arbitrary positive value and θ, K must be positive value. We therefore have $e^{\theta K}(2 - 2\theta K + \theta^2 K^2) > 2$ and $H_{11} < 0$:

$$
H_{22} = \frac{\partial^2 P(K,p)}{\partial p^2}\frac{\partial^2 P(K,p)}{\partial K^2} - \left(\frac{\partial^2 P(K,p)}{\partial p \partial K}\right)^2 > 0. \tag{C.9}
$$

Using (C.4)–(C.6) and (C.5)-(C.6), we can obtain

$$
\begin{aligned}
&\frac{1}{7}\left(7 - A - Ge^{-e^{-\theta(K-M)}}\right)(1-\beta)\beta\alpha p^{1-\beta} \\
&\qquad + (-\beta-1)\beta\alpha p^{-2-\beta} \\
&\qquad \cdot \left(\frac{c\left(e^{\theta K}-1\right)}{\theta K} + \frac{h\left(e^{\theta K}-\theta K-1\right)}{\theta^2 K}\right) \\
&\quad > \frac{-1}{7}Ge^{-e^{-\theta(K-M)}-\theta(K-M)}(1-\beta)\theta\alpha p^{-\beta} \\
&\qquad + p^{-1-\beta}\alpha\beta\left(\frac{ce^{\theta K}}{K} - \frac{c\left(e^{\theta K}-1\right)}{\theta K^2} + \frac{h\left(-\theta + e^{\theta K}\theta\right)}{\theta^2 K}\right. \\
&\qquad\qquad \left. - \frac{h\left(e^{\theta K}-\theta K-1\right)}{\theta^2 K^2}\right), \\
&- \frac{2S}{K^3} - \frac{1}{7}Ge^{-e^{-\theta(K-M)}-\theta(K-M)}\theta\alpha p^{1-\beta}\left(-1 + e^{-\theta(K-M)}\right) \\
&\qquad \cdot \alpha p^{1-\beta}\theta^2 \\
&\qquad - \frac{p^{-\beta}\alpha(h+c\theta)\left(-2 + e^{\theta K}\left(2 - 2\theta K + \theta^2 K^2\right)\right)}{\theta^2 K^3} \\
&\quad > \frac{-1}{7}Ge^{-e^{-\theta(K-M)}-\theta(K-M)}(1-\beta)\theta\alpha p^{-\beta} \\
&\qquad + p^{-1-\beta}\alpha\beta\left(\frac{ce^{\theta K}}{K} - \frac{c\left(e^{\theta K}-1\right)}{\theta K^2} + \frac{h\left(-\theta + e^{\theta K}\theta\right)}{\theta^2 K}\right. \\
&\qquad\qquad \left. - \frac{h\left(e^{\theta K}-\theta K-1\right)}{\theta^2 K^2}\right).
\end{aligned} \tag{C.10}
$$

Hence H_{22} must be > 0; with $H_{11} < 0$ and $H_{22} > 0$, there exists a unique point of maximum for $P(K,p)$.

Conflict of Interests

The authors declare that they have no conflict of interests.

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