Research Article

Controlled Passage through Resonance for Flexible Vibration Units

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The problem of controlled passage through resonance zone for mechanical systems with several degrees of freedom is studied. Control algorithm design is based on speed-gradient method and estimate for the frequency of the slow motion near resonance (Blekhman frequency). The simulation results for two-rotor flexible vibration units illustrating efficiency of the proposed algorithms and fractal dependence of the passage time on the initial conditions are presented. The novelty of the results is in demonstration of good behavior of the closed loop system if flexibility is taken into account.

1. Introduction

A typical problem for control of vibration units is passing through resonance at startup mode of vibroactuators, in the case of the operating modes belonging to a postresonance zone. Such a problem arises in the case when the power of a motor is not sufficient for passage through resonance zone due to Sommerfeld effect [1]. Dynamics of near resonance behavior are nonlinear and very complicated. Their analysis attracts attention of researchers for about 50 years [2–4].

Perhaps the first approach to the problem of controlled passage through resonance zone was the so-called “double start” method due to Gortinskii et al. [5]. The method is based on the insertion of time relay into motor control circuit for repeatedly switching on and off motor at precalculated time instants. Basically, this and other feedforward (nonfeedback) methods are characterized by difficulties in calculation of switching instants of a motor and sensitivity to inaccuracies of model and to interferences. A prospective approach to the problem is based on feedback control. Feedback control algorithms for passing through resonance zone of mechanical systems were considered in [6–8]. In [6] an optimal control

algorithm for passage an unbalanced rotor through critical speed was proposed. For the same problem several control methods are evaluated, and the necessary number of dampers and their optimal location were determined in [7]. In [8] a nonlinear controller reducing resonance effects during despin of a dual-spin spacecraft was designed. A method of vibration suppression for rotating shafts passing through resonances by switching shafts stiffness was proposed in [9]. In [10] the dynamics of passage through resonance of a vibrating system with two degrees of freedom was examined. However, the early algorithms did not have enough robustness with respect to uncertainties and were hard to design.

For practical implementation of control system, it is important to develop reasonably simple passing through resonance zone control algorithms, which have such robustness property: keeping high quality of the controlled system (vibration unit) under variation of parameters and external conditions. Perhaps the first such a simple controller was proposed in [11] based on the speed-gradient method previously used for control of nonlinear oscillatory systems [12]. A number of speed-gradient algorithms for passage through resonance in 2-DOF systems were proposed in [13].

The approach of [11, 13] was applied to two-rotor vibration setup in [14].

This work is dedicated to further extension of the results of [14]. Problem statement for control of passage through resonance zone for mechanical systems with several degrees of freedom is adopted from [13]. The control algorithms based on the speed-gradient method for two-rotor vibration units with flexible cardan shafts are described. The simulation results illustrating efficiency and robustness of the proposed algorithms are presented. Previous results [14] are related to two-rotor vibration units with rigid cardan shafts.

2. Problem Statement and Approach to Solution

To describe the dynamics of a mechanical system and to carry out the control algorithm synthesis, it is easier to use standard Euler-Lagrange form, leading to the following model of controlled system dynamics with n degrees of freedom:

$$A(q)\ddot{q} + C(q, \dot{q}) + G(q) = Bu, \quad (1)$$

where $u = u(t)$ is m -dimensional input vector, representing elastic forces depending, in turn, on controlling forces; $q = q(t)$ is n -vectors of generalized coordinates; $A(q)$ is $n \times n$ - inertia matrix; $C(q, \dot{q})$ is the n -vector of Coriolis and centrifugal forces; $G(q)$ is the n -vector of gravity forces; B is the $n \times m$ control matrix. For synthesis of control algorithms, it is often convenient to use equations in Hamiltonian form:

$$\dot{q} = \left(\frac{\partial H}{\partial p} \right)^T, \quad \dot{p} = - \left(\frac{\partial H}{\partial q} \right)^T + Bu, \quad (2)$$

where $p = p(t)$ is the generalized momenta vector and $H = H(p, q)$ denotes the Hamiltonian function (total energy of the system):

$$H(p, q) = \frac{1}{2} p^T [A(q)]^{-1} p + \Pi(q), \quad (3)$$

where $\Pi(q)$ is the potential energy. For design an idea of speed-gradient energy control is used and introduced in [12]. It is based on an auxiliary control goal: approach of free energy of the system to a surface of the given energy level:

$$H(p(t), q(t)) \longrightarrow H^* \quad \text{when } t \longrightarrow \infty. \quad (4)$$

Introducing the objective function

$$Q(p, q) = \frac{1}{2} [H(p, q) - H^*]^2, \quad (5)$$

the goal (4) is reformulated as

$$Q(q(t), p(t)) \longrightarrow 0 \quad \text{when } t \longrightarrow \infty. \quad (6)$$

Control algorithm for passing through resonant frequencies for an unbalanced rotor is based on the speed-gradient method [10], which allows us to synthesize control algorithms for significantly nonlinear objects. At the same time objective functional $Q(p, q)$ is chosen on the basis of total energy of a

mechanical system excluding friction losses, because in this case total energy is invariant for a mechanical system that is required in the speed-gradient method in the version of [12].

It is assumed that in dynamics model of the system two subsystems are allocated: carrier and rotating body, such that the total energy is represented in the following form:

$$\begin{aligned} H(p(t), q(t)) \\ = H_1(p(t), q(t)) + H_2(p(t), q(t)) + H_{12}(p(t), q(t)), \end{aligned} \quad (7)$$

where H_1 is the energy of rotating subsystem, H_2 is the energy of a carrier subsystem, and H_{12} is the energy of interaction.

The solution of [6, 8] adopted in this paper is based upon usage of the speed gradient algorithms [12] and a motion separation into fast and slow components, which occurs near resonance zone [15]. Quantitative analysis of slow "pendular-like" movements $\Psi(t)$ was performed by Blekhman et al. in [15] where the frequency of an "internal pendulum" was evaluated. It will be further called the "Blekhman frequency." An approach of [15] is briefly described below for completeness.

In [15] the following system with inertial excitation of oscillations was considered:

$$\begin{aligned} I\ddot{\varphi} &= L(\varphi) - R(\varphi) + m\epsilon\ddot{x} \sin \varphi, \\ m_1\ddot{x} + \beta\dot{x} + cx &= m\epsilon(\varphi^2 \cos \varphi + \ddot{\varphi} \sin \varphi), \end{aligned} \quad (8)$$

where φ is the rotation angle of a rotor, x is a platform deflection, M is the mass of a platform, m is the rotor mass, I is the inertia moment of a rotor, $m_1 = M + m$ is the system mass, c is the coefficient of axial stiffness, β is the damping coefficient of the spring, ϵ is the eccentricity of rotors, $L(\varphi)$ is the rotation torque of a motor (static characteristics), and $R(\varphi)$ is the torque of resistance forces. Assuming that frequency of rotation ω varies slowly in terms of

$$\frac{\dot{\omega}}{\omega} \ll \omega \quad (9)$$

and using the method of direct separation of motions [1], Blekhman et al. derived the equation of slow "pendular-like" oscillations in the following form [15]:

$$\ddot{\Psi} + 2n_1\dot{\Psi} + b \sin \Psi - \rho^2 \sin^2 \frac{\Psi}{2} = 0, \quad (10)$$

where Ψ is slow variable: "addition" to rotation frequency, $2n_1 = k/I$, $k = k_L + k_R$ ($k_L > 0$, $k_R > 0$) is the "total damping coefficient," obtained by linearization of expressions for $L(\varphi)$ and $R(\varphi)$ near value $\dot{\varphi} = \omega$:

$$L(\omega + \dot{\Psi}) = L(\omega) - k_L \dot{\Psi}, \quad R(\omega + \dot{\Psi}) = R(\omega) - k_R \dot{\Psi},$$

$$b = \frac{(m\epsilon\omega^2)^2}{2m_1I} \frac{\omega^{*2} - \omega^2}{(\omega^{*2} - \omega^2)^2 + 4n^2\omega^2} = \frac{1}{2} \frac{m_1}{I} (\omega^{*2} - \omega^2) A^2;$$

$$\rho^2 = \frac{(m\epsilon\omega^2)^2}{2m_1I} \frac{2n\omega}{(\omega^{*2} - \omega^2)^2 + 4n^2\omega^2} = \frac{m_1}{I} 2n\omega A^2; \quad (11)$$

FIGURE 1: Two-rotor vibration unit with elastic cardan shafts: (a) general view and (b) schematics-top view ((1) motors, (2) motor supports, (3) frame of the unit, (4) unbalanced rotors, (5) vibrating platform, (6) cardan shafts, (7) rotor bearings, and (8) springs).

$\omega^{*2} = c/m_1$ is a resonance frequency, $2n = \beta/m_1$, and $A = m\varepsilon\omega^2/M\sqrt{(\omega^{*2} - \omega^2)^2 + 4n^2\omega^2}$ is the first approximation to amplitude of oscillations of a platform. If the relation $\dot{\omega}/\omega \ll \omega$ holds in the preresonance zone ($\omega < \omega^*$) then the value

$$\omega_B = \sqrt{b} = A\sqrt{\frac{m_1(\omega^{*2} - \omega^2)}{2I}} \quad (12)$$

appears to be a frequency of small free oscillations of an “internal pendulum” (excluding resistance force), which is called the “Blekhman frequency” thereafter.

As seen before, that frequency vanishes to zero when $\omega = \omega^*$. The necessary condition of the validity of (10) is that the Blekhman frequency ω_B should be significantly less than frequency of rotations ω (usually, $\omega_B/\omega < 1/3$ is typically sufficient).

The idea of control algorithms described below is to extract the slow motion $\Psi(t)$ and to swing it with the aim to increase the energy of the rotating subsystem. To isolate slow motions, low-pass filter is inserted into the energy control algorithms. Particularly, if slow component appears in oscillations of angular velocity of a rotor $\dot{\varphi}$, then the control algorithm proposed in [11] is used:

$$u = -\gamma \operatorname{sign}((H - H^*)\dot{\psi}), \quad T_\psi \dot{\psi} = -\psi + \dot{\varphi}, \quad (13)$$

where $H = H(p, q)$ denotes the Hamiltonian (total energy of the system), $\psi(t)$ is the variable of a filter that is an estimate of the slow motions $\Psi(t)$ satisfying (10), and T_ψ is the time constant of a filter. At low damping, slow motions also fade out slowly, which gives control algorithm an opportunity to create suitable conditions to pass through resonance zone. Thus, the effect of “feedback resonance” [16] is created. After passing the resonance zone it is suggested to turn off the “swinging” and then to switch off control and to apply the constant drive torque. For a proper work of a filter, it should suppress fast oscillations with frequency ω and pass slow oscillations with ω_B frequency, where ω_B is the Blekhman frequency. That is, time constant of a filter T_ψ should be chosen from the inequality

$$T_\psi < \frac{2\pi}{\omega_B}. \quad (14)$$

Algorithms of passing through resonance zone for the two-rotor vibration units are described in [14]. Below we analyze the algorithm of [14] taking into account elasticity of the cardan shafts.

3. Passing through Resonance Control Algorithm of Two-Rotor Vibration Unit

Consider the two-rotor vibration unit in the startup/spin-up mode [17]. The unit consists of two rotors 4 installed on the vibrating platform 5. The rotors are elastically connected with fixed basis 2, and frame 3 by springs 8 (Figure 1).

Since the motors are located on motionless supports and the unbalanced rotors are mounted on the vibrating platform (bearings), the links implemented via cardan shafts should be considered as elastic ones (Figure 2). In this paper, only torsional elasticity will be taken into account.

The block scheme representing the elastic links of drive shafts with unbalanced rotors may be presented in the following form (Figure 3), where M_i , $i = 1, 2$ are the electromechanical torques of motors; ω_i are the angular velocities of motors; $M_{ci} = k_c \cdot \dot{\varphi}_i$ are resistance moments of the rotors mounted on the vibrating platform; M_{ri} are the unbalanced rotor's own moments.

$M_{ri}(t) = mpg \cos(\varphi + \varphi_i)$; $\bar{M}_{pi}$ are the moments, caused by the platform influence:

$$\begin{aligned} \bar{M}_{pi} = & -\ddot{x}_c m p \sin(\varphi + \varphi_i) + \ddot{y}_c m p \cos(\varphi + \varphi_i) \\ & + \ddot{\varphi} (J_i + (-1)^i r m p \cos \varphi_i) + (-1)^i \dot{\varphi}^2 r m p \sin \varphi_i; \end{aligned} \quad (15)$$

c_B , b_B are the stiffness and damping coefficients of the cardan shafts, φ , φ_1 , and φ_2 are angle of the support and rotation angles of the rotors, respectively, measured from the horizontal position, x_c , y_c are the horizontal and vertical displacements of the vibrating platform center O' from its equilibrium position O , $m_i = m$, $i = 1, 2$, and m_n are the masses of the rotors and supporting body, $J_1 = J_2 = J$ are the inertia moments of the rotors, $\rho_i = \rho$, $i = 1, 2$ are the rotor eccentricities, c_{01} , c_{02} are the horizontal and vertical spring stiffness, g is the gravity acceleration, m_0 is the total mass of

FIGURE 2: Schematics of the two-rotor vibration unit with elastic cardan shafts.

FIGURE 3: Block scheme, representing the elastic links of the motors with unbalanced rotors.

the unit, $m_0 = 2m + m_n$, β is the damping coefficient, k_c is the friction coefficient in the bearings, $M_i = u_i(t)$ are the motor torques (controlling variables), and r is the distance from the centers of the rotors to O' . It is assumed that the rotors are identical and the rotor shafts are orthogonal to the motion of the support.

To convert the scheme into the state space equations, accounting elasticity, let us substitute the state variables $z_{1i} = \omega_i$ and z_{2i} as the outputs of corresponding integrators ($i = 1, 2$). Then the dynamics of the cardan shafts taking into account torsional elasticity are described by the differential equations:

$$\begin{aligned} \dot{z}_{1i} &= \frac{1}{J_i} \cdot M_i - \frac{1}{J_i} \cdot M_{ui} = \frac{1}{J_i} \cdot M_i - \frac{1}{J_i} \cdot (z_{2i} + b_B(z_{1i} - \dot{\phi}_i)), \\ \dot{z}_{2i} &= c_B(z_{1i} - \dot{\phi}_i), \quad M_{ui} = z_{2i} + b_B(z_{1i} - \dot{\phi}_i). \end{aligned} \quad (16)$$

Assume that the whole system dynamics may be considered in the vertical plane. Then the equations of dynamics have the following form [8]:

$$\begin{aligned} m_0 \ddot{x}_c - \ddot{\phi} m \rho [\sin(\varphi + \varphi_1) + \sin(\varphi + \varphi_2)] \\ - \ddot{\phi}_1 m \rho \sin(\varphi + \varphi_1) - \ddot{\phi}_2 m \rho \sin(\varphi + \varphi_2) \\ - \dot{\phi}^2 m \rho [\cos(\varphi + \varphi_1) + \cos(\varphi + \varphi_2)] \\ - \dot{\phi}_1^2 m \rho \cos(\varphi + \varphi_1) - \dot{\phi}_2^2 m \rho \cos(\varphi + \varphi_2) \\ - 2\dot{\phi}\dot{\phi}_1 m \rho \cos(\varphi + \varphi_1) - 2\dot{\phi}\dot{\phi}_2 m \rho \cos(\varphi + \varphi_2) \\ + 2c_{01}x_c + \beta\dot{x}_c = 0; \\ m_0 \ddot{y}_c + \ddot{\phi} m \rho [\cos(\varphi + \varphi_1) + \cos(\varphi + \varphi_2)] \\ - \ddot{\phi}_1 m \rho \cos(\varphi + \varphi_1) - \ddot{\phi}_2 m \rho \cos(\varphi + \varphi_2) \\ - \dot{\phi}^2 m \rho [\sin(\varphi + \varphi_1) + \sin(\varphi + \varphi_2)] \\ - \dot{\phi}_1^2 m \rho \sin(\varphi + \varphi_1) - \dot{\phi}_2^2 m \rho \sin(\varphi + \varphi_2) \\ - 2\dot{\phi}\dot{\phi}_1 m \rho \sin(\varphi + \varphi_1) - 2\dot{\phi}\dot{\phi}_2 m \rho \sin(\varphi + \varphi_2) \\ + m_0 g + 2c_{02}y_c + \beta\dot{y}_c = 0; \\ - \ddot{x}_c m \rho [\sin(\varphi + \varphi_1) + \sin(\varphi + \varphi_2)] \\ + \ddot{y}_c m \rho [\cos(\varphi + \varphi_1) + \cos(\varphi + \varphi_2)] \\ + \ddot{\phi} [J + J_1 + J_2 - 2r m \rho (\cos \varphi_1 - \cos \varphi_2)] \\ + \ddot{\phi}_1 (J_1 - r m \rho \cos \varphi_1) + \ddot{\phi}_2 (J_2 + r m \rho \cos \varphi_2) \\ + \dot{\phi}_1^2 r m \rho \sin \varphi_1 - \dot{\phi}_2^2 r m \rho \sin \varphi_2 + 2r m \rho \dot{\phi}\dot{\phi}_1 \sin \varphi_1 \\ - 2r m \rho \dot{\phi}\dot{\phi}_2 \sin \varphi_2 + m \rho g [\cos(\varphi + \varphi_1) + \cos(\varphi + \varphi_2)] \\ + c_{03}\varphi + \beta\dot{\varphi} = 0, \\ - \ddot{x}_c m \rho \sin(\varphi + \varphi_1) + \ddot{y}_c m \rho \cos(\varphi + \varphi_1) \\ + \ddot{\phi} (J_1 - r m \rho \cos \varphi_1) + \ddot{\phi}_1 J_1 - \dot{\phi}^2 r m \rho \sin \varphi_1 \\ + m \rho g \cos(\varphi + \varphi_1) + k_c \dot{\phi}_1 = M_{u1}; \\ - \ddot{x}_c m \rho \sin(\varphi + \varphi_2) + \ddot{y}_c m \rho \cos(\varphi + \varphi_2) \\ + \ddot{\phi} (J_2 + r m \rho \cos \varphi_1) + \ddot{\phi}_2 J_2 + \dot{\phi}^2 r m \rho \sin \varphi_2 \\ + m \rho g \cos(\varphi + \varphi_2) + k_c \dot{\phi}_2 = M_{u2}. \end{aligned} \quad (17)$$

At the low levels of constant control action $u_i(t) \equiv (-1)^i M_0$, $i = 1, 2$ in the near-resonance zone the rotor angle is “captured,” while increase of the control torque leads to passage through resonance zone towards the desired angular velocity. The simulation results for system (16), (17) are shown in Figure 4 with basic system parameters: $J_i = 0.014 \text{ kg}\cdot\text{m}^2$, $m = 1.5 \text{ kg}$, $m_n = 9 \text{ kg}$, $\rho = 0.04 \text{ m}$, $k_c = 0.01 \text{ J/s}$, $\beta = 5 \text{ kg/s}$, $c_{02} = 5300 \text{ N/m}$, $c_{01} = 1300 \text{ N/m}$, and constant torque $M_0 = 0.65 \text{ N}\cdot\text{m}$ (internal curves, “capture”) and $M_0 = 0.66 \text{ N}\cdot\text{m}$

FIGURE 4: Constant drive torque, $u_i(t) \equiv (-1)^i M_0$, $M_0 = 0.65$ N·m (internal curves, “capture”), and $M_0 = 0.66$ N·m (external curves, passing).

(external curves, passage). Simulation of system (17) without elasticity ($M_{ui} = M_i$) and without control ($M_i = u_i = (-1)^i M_0$) has shown that the lowest constant drive torque M_0 providing passing through the resonance is 0.66 N·m (Figure 4).

Like in a number of previous papers the feedback control algorithm $u = U(z)$ where

$$z = [x_c, \dot{x}_c, y_c, \dot{y}_c, \varphi, \dot{\varphi}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2]^T \quad (18)$$

is the state vector of the system that is used. The control goal is passing through resonance zone to achieve the desired level of the velocity of the unbalanced rotors. It is assumed that the level of control is limited and does not allow system to pass through the resonance zone using constant driving torque.

As seen in Figure 4 the rotational motions separate into fast and slow ones, that is, the proposed approach applies. The following modification of control Algorithm (13) proposed in [14] for passing through resonance zone was studied in this paper:

$$u_i = \begin{cases} (-1)^i M_0, & \text{if } \gamma = 1, \\ (-1)^i M_0, & \text{if } \gamma = 0, (H - H^*)(\dot{\varphi}_i - \dot{\psi}_i) > 0, \\ 0, & \text{else,} \end{cases} \quad (19)$$

$$T_\psi \dot{\psi}_i = -\psi_i + \dot{\varphi}_i, \quad i = 1, 2,$$

where $H = T + \Pi$, $\psi_i(t)$ are the variables of the filters, $T_\psi > 0$, $T_\psi = \text{const}$, and H^* are the parameters of the algorithm, and $\gamma(t) = \max_{0 \leq \tau < t} \text{sgn}(H(\tau) - H^*)$, where $\text{sgn}[z] = 1$ with $z > 0$, $\text{sgn}[z] = 0$ with $z \leq 0$. Time constant of the filters T_ψ should satisfy relation (14).

Efficiency of control Algorithm (19) was studied in the MATLAB environment. The relative simulation error does not exceed 5%. Calculations were made with the same values of the basic parameters of the system. Simulation of system (17) without elasticity ($M_{ui} = M_i$) and with control

FIGURE 5: Simulation results for system (17), (19) with $M_0 = 0.45$ N·m.

FIGURE 6: Simulation results for the system with elasticity (16), (17) and with constant drive torque $M_0 = 0.66$ N·m.

Algorithm (19) has shown that passage through the resonance zone occurs for $M_0 \geq 0.36$ N·m. Figure 5 demonstrates the graphs of $\dot{\varphi}_1$ and $\dot{\varphi}_2$ for system with Algorithm (19) for operating value of the torque $M_0 = 0.45$ N·m without elasticity.

Simulation results for the system with elasticity (16), (17) and without control are shown in Figure 6 for $M_0 = 0.66$ N·m. It is seen that the rotor angular velocity achieves the steady state near resonance: $\omega_{\text{res}} = 21.5 \text{ s}^{-1}$, while elastic torque $M_{u1}(t)$ exhibits persisting oscillations with the amplitude exceeding the threshold value for rigid system $M_0 = 0.66$ N·m. It means that passing through resonance for elastic system requires bigger drive torque.

FIGURE 7: $y_c(t)$: vertical motion of the platform for $M_0 = 0.36$ N·m. FIGURE 9: $M_{u1}(t)$: elastic torque for $M_0 = 0.36$ N·m. FIGURE 8: $\dot{\phi}_1(t)$: rotor angular velocity; ψ_1 : value of the filter variable for $M_0 = 0.36$ N·m. FIGURE 10: $\dot{\phi}_1(t) - \omega_1(t)$ difference between velocity of an unbalanced rotor and motor velocity for $M_0 = 0.36$ N·m.

TABLE 1

c_B	t_{res}, s	t_{tr}, s
3000	19.4	24.5
2500	19.42	24.05
2000	18.2	22.0
1500	18.7	23.0
1000	19.4	24.03

Quantitative characteristics of dynamics of the unit with elasticity (16), (17) with control Algorithm (19) for $M_0 = 0.36$ N·m, $c_B = 3000$ (N·m/rad), $b_B = 0.3$ (N·m·s), and $T_\psi = 1.0$ s are presented in Figures 7–10. The above parameter values correspond to the frequency of torsion oscillations of the shafts $\omega \approx 750$ s⁻¹.

It is seen from the pictures in Figures 7–10 that time for passing through resonance is $t_{res} = 19.4$ s, and time of approaching to the desired velocity is $t_{tr} = 24.5$ s. It is also seen that after passing the resonance zone the platform oscillates with steady state amplitude $A = 0.8$ cm, while elastic torque M_{u1} at the near-resonance zone reaches the value of $M_{u1} = 1.2$ N·m. It settles in a range $M_{u1} \leq 0.85$ N·m, and the difference between motor velocity and rotor velocity settles at the level of $|\dot{\phi}_1 - \omega_1| \leq 0.1$ s⁻¹, that is, 0.3% from the steady velocity of the rotor.

In Figures 11 and 12 the plots of $\dot{\phi}_1$ for system (17), (19) (without elasticity, Figure 11) and for system (16), (17), (19) (taking into account the shaft elasticity, Figure 12) are presented. The value of parameter M_0 in Algorithm (19) is different: $M_0 = 0.4$ N·m. As seen from the picture, passing through resonance in the first case takes longer time.

The study of the algorithm efficiency for different values of the elasticity parameters is presented in Tables 1 and 2.

TABLE 2

b_B	t_{res}, s	t_{tr}, s
0.8	19.42	24.0
0.7	19.01	23.5
2000	19.42	24.0
1500	13.06	17.98
1000	19.4	24.5

In Table 1 the times of passing through resonance t_{res} , s and times of achieving the steady state mode t_{tr} , s for different stiffness coefficients c_B are presented. In Table 2 the values of the coefficient b_B for fixed $c_B = 3000$ are given.

As seen from the presented graphs, control algorithm for passing through resonance synthesized without accounting of drive shaft elasticity is efficient if the drive shaft elasticity is taken into account as well. Moreover, as the simulation shows, with the same algorithm parameters, varying the values of the elastic connection can increase the passing through resonance time constant by 1.5–2 times and the time of entering the given velocity is increased by 40%, but control objective is completed in all cases. The computer simulation also shows that using the passing through resonance control algorithm the level of control torque can be reduced by over 55%, in comparison to traditional control with constant torque.

Thus, using control Algorithm (19), the level of control action required for passing through resonance can be significantly reduced.

FIGURE 11: Plots of $\dot{\phi}_1(t)\psi_1$ for $M_0 = 0.4 \text{ N}\cdot\text{m}$. Elasticity is not taken into account.

FIGURE 12: Plots of $\dot{\phi}_1(t)\psi_1$ for $M_0 = 0.4 \text{ N}\cdot\text{m}$. Elasticity is taken into account.

4. Conclusion

In this paper the algorithm for controlled passage through resonance zone proposed in [13, 14] is analyzed taking into account elasticity of the cardan shafts. The simulation results show that the approach for design of simple control algorithms for passing through resonance zone in vibration units based on “swinging” of slow oscillations of an angular velocity of the rotors, proposed in [11, 13] applies to vibration units with elasticity. For constant drive torques passage through resonance requires bigger value of the motor torque, while for controlled torque passage through resonance is achieved with the same value of the motor torque, even for elastic shafts. Even more, for units with flexible cardan shafts it may become more efficient than for rigid units. A possible reason is that additional vibrations caused by elasticity increase amplitude value of rotor angular velocity that may help in passing through resonance.

The design of control algorithm is simple since it has only two tunable parameters. In addition, the value of the parameter H^* (desired energy of the system) is not necessarily related to the working regime of the vibration unit. The value of H^* should meet the only requirement that it should exceed maximum energy of the system in the resonance zone. To achieve the working regime the only parameter to be chosen is the gain γ .

Next step of the research is examination of the closed loop system performance for an ensemble of initial conditions, including asymmetrical ones, like it was done for rigid system [14].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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