Research Article

Chaos Synchronization of Two Chaotic Nonlinear Gyros Using Backstepping Design

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We investigate the chaos synchronization of two certain chaotic gyros using the backstepping approach. We design a simple controller and verify the stability of the error system by using proper Lyapunov functions. The designed control laws ensure stable controlled and synchronized states for two chaotic nonlinear gyros. Numerical simulations are implemented for illustration and verification of the effectiveness of the backstepping technique.

1. Introduction

Control and synchronization of chaos as important parts of dynamic systems have great significance to make a good grasp of the fundamentals and basic tools of the science of chaotic dynamics. Synchronization of chaotic systems has become one of the most interesting subjects in chaos theory. Researching chaotic dynamics is the trend of developing technology, since Pecora and Carroll first demonstrated the synchronization of two identical chaotic systems under different initial conditions [1].

The chaos synchronization problem can be defined as two coupled systems conducting coupling evolution in time with given different conditions. In other words, the purpose of synchronization is to use the output of master system to control the slave system, so that the output of slave system achieves asymptotic synchronization with that of master system. Since the pioneering work of synchronization of chaotic systems by Pecora and Carroll, various schemes of synchronization such as backstepping design [2], active control [3], and adaptive control [4] have been successfully employed for chaos synchronization. H_∞ synchronization of general chaotic systems with external disturbance was discussed in [5]. The feedback controller was designed to not only guarantee the asymptotic synchronization of master and slave systems but also control

the effect of disturbance to an H_∞ norm constraint. Additionally, considering the channel noise and parameter mismatch, the technique of robust chaotic H_∞ synchronization for Lur'e systems was applied to the secure communication [6]. In [7, 8], the passive control technique was applied to chaos synchronization and control.

Gyros are a particularly interesting form of nonlinear system and have attributes of great utility to navigational, aeronautical, and space engineering [9], which have been widely used to evaluate synchronization schemes of chaotic systems. Lei et al. [10] extended the findings of Chen [9] and employed active control method to synchronize two identical chaotic gyros with nonlinear damping. Considering the use of two controllers, however, the active control scheme increases the implementation cost and complexity. Yau [11] presented a robust fuzzy sliding mode control scheme for the synchronization of two chaotic nonlinear gyros subject to uncertainties and external disturbances. Moreover, a growing body of researches [12–17] has emerged in order to address the synchronization of two chaotic nonlinear gyros.

In recent years, the backstepping method [17] of chaos synchronization has attracted many researchers' attention because of its advantages. For example, it performs well on the applicability to a variety of chaotic systems whether they contain external excitation or not, it requires only one controller

to realize synchronization between chaotic systems, and there are no derivatives in the controller [2]. Backstepping design is a kind of synthetic technique to the controller, which recursively interlaces the choice of a Lyapunov function with the design of feedback control.

In order to reduce the complexity of synchronization schemes and increase the effectiveness and feasibility of control technique, we adopt the backstepping design technique which uses only one controller without three order terms, to realize the synchronization of two chaotic nonlinear gyros. Thus, the main work in this paper is to apply the backstepping technique that designs a simple controller to the synchronization of two certain chaotic nonlinear gyros. In addition, the superiority of the backstepping design is supported by the theoretical analysis and simulations results.

The rest of the paper is organized as follows. In Section 2, a brief description of the gyro system is introduced. In Section 3, we discuss the design of the backstepping controller and verify the stability of error system by using Lyapunov function. In Section 4, numerical simulations are given for illustration and verification of the effectiveness of the backstepping technique. The conclusion is presented in Section 5.

2. System Description

The symmetric gyroscope mounted on a vibrating base is shown in Figure 1. According to Chen [9], the dynamics of a symmetrical gyro with linear-plus-cubic damping of angle θ can be expressed as

$$\begin{aligned} \ddot{\theta} + \alpha^2 \frac{(1 - \cos \theta)^2}{\sin^3 \theta} - \beta \sin \theta + c_1 \dot{\theta} + c_2 \dot{\theta}^3 \\ = f \sin \omega t \sin \theta, \end{aligned} \quad (1)$$

where $f \sin \omega t$ represents a parametric excitation, $c_1 \dot{\theta}$ and $c_2 \dot{\theta}^3$ are linear and nonlinear damping terms, respectively, and $\alpha^2((1 - \cos \theta)^2 / \sin^3 \theta) - \beta \sin \theta$ is a nonlinear term.

Given $g(\theta) = -\alpha^2((1 - \cos \theta)^2 / \sin^3 \theta) - \beta \sin \theta$ and the states $x_1 = \theta$, $x_2 = \dot{\theta}$, system (1) can be transformed into the following nominal form:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= g(x_1) - c_1 x_2 - c_2 x_2^3 + (\beta + f \sin \omega t) \sin(x_1). \end{aligned} \quad (2)$$

The complex dynamics of (2) has been studied by Chen [9] for the value of f in the range of $32 < f < 36$ and constant values of $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, and $\omega = 2$. Figures 2, 3, and 4 illustrate the irregular motion exhibited by system (2) for $f = 35.5$ and initial conditions of $(x_1, x_2) = (1, -1)$. Figure 5 reveals that the corresponding maximum Lyapunov exponent has a positive value, and thus it can be inferred that the gyro trajectory is in a state of chaotic motion.

Assumption 1. Suppose that there is a constant $l > 0$, and then $\|g(x_1) - g(y_1)\| \leq l\|x_1 - y_1\|$ holds.

FIGURE 1: A schematic diagram of a symmetric gyroscope [9].

FIGURE 2: Time history of chaotic gyro: x_1 versus time t .

FIGURE 3: Time history of chaotic gyro: x_2 versus time t .

FIGURE 4: Phase plane trajectory of chaotic gyro, x_1 versus x_2 .

FIGURE 5: Maximum Lyapunov exponent of gyro trajectory plotted as function of number of drive cycles.

We take gyroscope (2) as the master system. By using the backstepping technique, we design controller $u(t)$ in the following slave system:

$$\begin{aligned}\dot{y}_1 &= y_2, \\ \dot{y}_2 &= g(y_1) - c_1 y_2 - c_2 y_2^3 + (\beta + f \sin \omega t) \sin(y_1) - u(t),\end{aligned}\quad (3)$$

where $u(t)$ is an appropriate control signal.

The goal of the current control problem is to design $u(t)$, so that, for any initial conditions of two systems (2) and (3), the behavior of the slave system converges to that of the master system; that is,

$$\lim_{t \rightarrow \infty} \|x(t) - y(t)\| \rightarrow 0, \quad (4)$$

where $\|\cdot\|$ is the Euclidean norm of a vector.

We define the error states between the master system and slave system as

$$\begin{aligned}e_1 &= x_1 - y_1, \\ e_2 &= x_2 - y_2.\end{aligned}\quad (5)$$

Dynamics equation (6) of these errors can be obtained directly by subtracting (3) from (2):

$$\begin{aligned}\dot{e}_1 &= e_2, \\ \dot{e}_2 &= -c_1 e_2 + g(x_1) - g(y_1) - c_2 (x_2^3 - y_2^3) \\ &\quad + (\beta + f \sin \omega t) (\sin x_1 - \sin y_1) + u(t) \\ &= -c_1 e_2 + g(x_1) - g(x_1 - e_1) - c_2 x_2^3 \\ &\quad + c_2 (x_2 - e_2)^3 \\ &\quad + (\beta + f \sin \omega t) (\sin(x_1) - \sin(x_1 - e_1)) \\ &\quad + u(t).\end{aligned}\quad (6)$$

3. Backstepping Design

Compared with other controllers, we design simple controller $u(t)$ without three order terms.

Based on the backstepping method, error variable ω_2 needs to be defined: $\omega_2 = e_2 - \alpha_1(e_1)$, where $\alpha_1(e_1) = -e_1$, and then we get $\omega_2 = e_2 + e_1$.

When $\omega_2 = e_2 + e_1$, error dynamics equation (6) is depicted as

$$\begin{aligned}\dot{e}_1 &= \omega_2 - e_1, \\ \dot{e}_2 &= -c_1 e_2 + g(x_1) - g(x_1 - e_1) - c_2 x_2^3 + c_2 (x_2 - e_2)^3 \\ &\quad + (\beta + f \sin \omega t) (\sin(x_1) - \sin(x_1 - e_1)) \\ &\quad + u(t).\end{aligned}\quad (7)$$

Considering (e_1, ω_2) subspace given by

$$\begin{aligned}\dot{e}_1 &= \omega_2 - e_1, \\ \dot{\omega}_2 &= e_2 - c_1 e_2 + g(x_1) - g(x_1 - e_1) - c_2 x_2^3 \\ &\quad + (\beta + f \sin \omega t) (\sin(x_1) - \sin(x_1 - e_1)) \\ &\quad + c_2 (x_2 - e_2)^3 + u(t),\end{aligned}\quad (8)$$

we design controller $u(t)$ as follows:

$$\begin{aligned}u(t) &= c_1 e_2 - e_2 \\ &\quad - (\beta + f \sin \omega t) (\sin(x_1) - \sin(x_1 - e_1)) \\ &\quad - \frac{1}{\epsilon_1} l^2 \omega_2^2 - \frac{c_2}{\epsilon_2} \delta^2 \omega_2^2 - c_2 \delta \omega_2^2 - \frac{e_1 e_2}{\omega_2},\end{aligned}\quad (9)$$

FIGURE 6: Time responses of controlled chaotic gyro system: master and slave system outputs are x_1, x_2 (red) and y_1, y_2 (blue), respectively. Note that the control $u(t)$ is activated at $t = 20$ s.

where $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_1 + \varepsilon_2 c_2 < 1$, and ε_1 and ε_2 are constant parameter signals, which are used to adjust the controller in the slave system. From (9), we can see that, by means of the backstepping algorithm for strict feedback nonlinear continuous systems, the controller is easy to realize.

Remark 2. In [17], the authors proposed a controller with three order terms for chaos synchronization in gyros systems, while we have already removed these terms in our controller (see (9)), so that the complexity is reduced.

Theorem 3. Consider the master-slave system given in (2) and (3). The two systems can be globally asymptotically synchronized by control $u(t)$ defined in (9). That is, error dynamical system (6) is globally exponentially stable about the origin.

Proof. Choose the Lyapunov function:

$$V_2(e_1, \omega_2) = V_1(e_1) + \frac{1}{2}\omega_2^2, \quad (10)$$

where $V_1(e_1) = (1/2)e_1^2$.

The derivative of (10) is

$$\begin{aligned} \dot{V}_2 &= e_1 \dot{e}_1 + \omega_2 \dot{\omega}_2 = -e_1^2 + \omega_2 [-c_1 e_2 + g(x_1) - g(y_1) \\ &\quad - c_2 x_2^3 + c_2 y_2^3 + (\beta + f \sin \omega t)(\sin(x_1) - \sin(y_1)) \\ &\quad - e_1 + u(t)] = -e_1^2 + \omega_2 [g(x_1) - g(y_1) \\ &\quad - c_2(x_2^3 - y_2^3) - \frac{1}{\varepsilon_1} l^2 \omega_2^2 - \frac{c_2}{\varepsilon_2} \delta^2 \omega_2^2 - c_2 \delta \omega_2^2]. \end{aligned} \quad (11)$$

From Assumption 1, we obtain

$$\begin{aligned} \omega_2 (g(x_1) - g(x_1 - e_1)) &\leq l \|\omega_2\| \|e_1\| \\ &\leq \frac{1}{\varepsilon_1} l^2 \|\omega_2\|^2 + \varepsilon_1 \|e_1\|^2 = \frac{1}{\varepsilon_1} l^2 \omega_2^2 + \varepsilon_1 e_1^2. \end{aligned} \quad (12)$$

Because x_2 and y_2 are bounded, there is a constant $\delta > 0$ such that $\|y_2^3 - x_2^3\| \leq \delta \|y_2 - x_2\|$, and we get

$$\begin{aligned} \omega_2 (c_2 y_2^3 - c_2 x_2^3) &\leq \|\omega_2\| c_2 \delta \|y_2 - x_2\| \leq \|\omega_2\| c_2 \delta \|e_2\| \\ &= \|\omega_2\| c_2 \delta \|\omega_2 - e_1\| \\ &\leq \|\omega_2\| c_2 \delta (\|\omega_2\| + \|e_1\|) \\ &= c_2 \delta \|\omega_2\|^2 + c_2 \delta \|\omega_2\| \|e_1\| \\ &\leq c_2 \delta \|\omega_2\|^2 + \frac{\delta^2}{\varepsilon_2} c_2 \|\omega_2\|^2 \\ &\quad + \varepsilon_2 c_2 \|e_1\|^2 \\ &= c_2 \delta \omega_2^2 + \frac{c_2}{\varepsilon_2} \delta^2 \omega_2^2 + \varepsilon_2 c_2 e_1^2. \end{aligned} \quad (13)$$

According to analysis above, we get

$$\begin{aligned} \dot{V} &\leq -e_1^2 + \frac{1}{\varepsilon_1} l^2 \omega_2^2 + \varepsilon_1 e_1^2 + c_2 \delta \omega_2^2 + \frac{c_2}{\varepsilon_2} \delta^2 \omega_2^2 + \varepsilon_2 c_2 e_1^2 \\ &\quad - \frac{1}{\varepsilon_1} l^2 \omega_2^2 - c_2 \delta \omega_2^2 - \frac{c_2}{\varepsilon_2} \delta^2 \omega_2^2 \\ &= -e_1^2 + \varepsilon_1 e_1^2 + \varepsilon_2 c_2 e_1^2, \end{aligned} \quad (14)$$

where $\varepsilon_1 + \varepsilon_2 c_2 < 1$, and then $\dot{V} < 0$. Error dynamical system (6) will converge to zero as $t \rightarrow \infty$, while equilibrium $(0, 0)$ remains globally asymptotically stable. In the above analysis, $\dot{V} < 0$ is bounded away from zero for all points except where $e_1 = 0$, $e_2 = 0$. Therefore, the error of synchronization is verified to be asymptotically stable. $\square$

FIGURE 7: Time response of error states. Note that control $u(t)$ is activated at $t = 20$ s.

4. Simulations Results

The parameters of the nonlinear gyros systems are specified as follows: $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, and $f = 35.5$, which is shown in Section 2 and gives rise to a chaotic state. The initial conditions are defined as $x_1(0) = 1$, $x_2(0) = -1$, $y_1(0) = 1.6$, and $y_2(0) = 0.8$. In the proposed design, the controller is determined in accordance with the backstepping control law.

Figure 6 shows the time responses of controlled chaotic gyro synchronization system: master system and slave system outputs are x_1 , x_2 (red) and y_1 , y_2 (blue), respectively, where the control signal is activated at $t = 20$ s. This result demonstrates that the output of slave system (3) achieves asymptotic synchronization with the output of master system (2).

Error states of coupled system are defined as $e_1 = x_1 - y_1$ and $e_2 = x_2 - y_2$ in the backstepping controller. Figures 7 and 8 show the time responses of the error states and control signal $u(t)$, respectively. The results show that the error states are regulated to zero asymptotically following activation of the control signal at $t = 20$ s.

5. Conclusions

Based on Lyapunov stability theory, a controller which employed the backstepping approach has been designed for the synchronization of two chaotic nonlinear gyros. The backstepping technique we have applied allows for flexibility in the controller design and global stability based on the appropriate choice of Lyapunov functions. The simulation results show that the synchronization scheme of backstepping approach is effective and has low complexity. Compared with the existing synchronization schemes, our design avoids the complexity of behavior on the chaos controller and therefore has a lower implementation cost.

FIGURE 8: Variation of control action over time. Note that control $u(t)$ is activated at $t = 20$ s.

In the future, we will improve the design of controller with consideration of more realistic situations, such as the uncertainties and disturbance, to realize the synchronization of two uncertain chaotic nonlinear gyros.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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