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Research Article

Credit Derivatives Pricing Model for Fuzzy Financial Market

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With various categories of fuzziness in the market, the factors that influence credit derivatives pricing include not only the characteristic of randomness but also nonrandom fuzziness. Thus, it is necessary to bring fuzziness into the process of credit derivatives pricing. Based on fuzzy process theory, this paper first brings fuzziness into credit derivatives pricing, discusses some pricing formulas of credit derivatives, and puts forward a One-Factor Fuzzy Copula function which builds a foundation for portfolio credit products pricing. Some numerical calculating samples are presented as well.

1. Introduction

Credit derivatives is a general term for a series of financial engineering technologies which strips, transfers, and hedges credit risks from basic assets. Its major feature is stripping credit risks from other financial risks with an approach of transferring. With a history of more than 20 years, the credit derivatives have made themselves the mainstream products of over-the-counter (OTC) market exchanges with an explosive growth in its market size. Hence, pricing of credit derivatives became a major concern of both the current financial theoretical circle and the practical area.

With a general view of the literatures available about credit derivatives pricing, their common characteristic can be summarized as that all the pricing processes are built on the base of the theory of random probability. However, financial product pricing is a process of mathematical modeling from social reality; market environment that its existence and development rely on is under a lot of fuzziness; and the human thoughts that describe and judge the pricing model are also fuzzy. Just like credit derivatives, the pricing modeling must be affected by fuzziness produced by the characteristics of OTC exchange (such as nonstandardization of products and the short of strict management system). Thus, with a lot of fuzziness of the market, the factors influencing financial products pricing have not only the characteristic of randomness but also nonrandom fuzziness. Therefore, it is necessary to bring fuzziness into the process of credit derivatives pricing.

Fuzzy theory is a powerful tool to deal with all kinds of fuzziness. Researches on it offer new theoretical foundation for financial product pricing. It is a beneficial and necessary supplement to traditional financial product pricing methods. Different from randomness, fuzziness is another kind of uncertainty in the reality. To depict the nonrandom characteristics of fuzziness, Zadeh [1] put forward the fuzzy set theory based on the concept of membership function. After that, the fuzzy set theory is widely used in various fields. In order to measure a fuzzy event, B. Liu and Y.-K. Liu [2] introduced the concept of credibility measure, and, in order to further deal with the dynamic of fuzzy event over time, Liu [3] founded a fuzzy process, a differential formula, and a fuzzy integral; the related literature can also be seen in You [4], Peng [5], and so forth. Different from random financial mathematics, Liu [3] assumed that the stock price follows geometric Liu process rather than a geometric Brownian motion and offered a new European option pricing model based on his stock price model. Following that, Qin and Li [6] worked out a European call and put option pricing formula based on the condition of fuzzy financial markets; Peng [7] also derived an American option pricing formula based on the condition of fuzzy financial markets; Qin and Gao [8] presented a fractional Liu process in the application of option pricing. The latest development about fuzzy theory can be seen also from Jiao and Yao [9], Ji and Zhou [10], Liu et al. [11], and so forth. Considering the fuzzy uncertainty, Wu and Zhuang [12] proposed a reduced-form intensity-based model under fuzzy environments and presented some applications of the methodology for pricing defaultable bonds and credit default swap; the model results change into a closed interval. However, the pricing issues about credit derivatives pricing based on fuzzy theory have not been studied. This paper first brings fuzzy process into the model of credit derivatives pricing in expectation to match credit derivatives pricing model and real financial market better.

This paper will review some preliminary knowledge of fuzzy process in Section 2. Some credit derivatives pricing models are derived in Section 3 and a One-Factor Fuzzy Copula function is proposed in Section 4, respectively. Finally, a brief summary is given in Section 5.

2. Preliminary

A fuzzy process is a sequence of fuzzy variables indexed by time or space, which was defined by Liu [3]. In this section, we will recall some useful definitions and properties about fuzzy process.

Definition 1 (Liu [3]). Given an index set T and a credibility space (Θ, P, Cr), then a fuzzy process is a function from $T \times (\Theta, P, \mathrm{Cr})$ to the set of real numbers. In other words, a fuzzy process $X(t, \theta)$ is a two-variable function. For convenience, we use the symbol X_t instead of the longer notation $X(t, \theta)$.

Definition 2 (Liu [3]). A fuzzy process X_t is said to have independent increments if

$$X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \ldots, X_{t_k} - X_{t_{k-1}} \tag{1}$$

are independent fuzzy variables for any times $t_0 < t_1 < \cdots < t_k$. A fuzzy process X_t is said to have stationary increments if, for any given time $t > 0$, the increments $X_{s+t} - X_s$ are identically distributed fuzzy variables for all $s > 0$.

Definition 3 (Liu [3]). A fuzzy process C_t is said to be a Liu process (or, namely, C process) if

(i) $C_0 = 0$,

(ii) C_t has stationary and independent increments,

(iii) every increment $C_{s+t} - C_s$ is a normally distributed fuzzy variable with expected value et and variance $\sigma^2 t^2$, whose membership function is

$$\mu(x) = 2\left(1 + \exp\left(\frac{\pi |x - et|}{\sqrt{6}\sigma t}\right)\right)^{-1}, \quad x \in R. \tag{2}$$

The parameters e and σ are called the drift and diffusion coefficients, respectively. The Liu process is said to be standard if $e = 0$ and $\sigma = 1$.

Definition 4 (Liu [3]). Let C_t be a standard Liu process; then the fuzzy process

$$G_t = \exp(et + \sigma C_t) \tag{3}$$

is called a geometric Liu process, or an exponential Liu process sometimes. The geometric Liu process is expected to model asset values in a fuzzy environment; Li and Qin [13] have derived that G_t is of a lognormal membership function:

$$\mu(x) = 2\left(1 + \exp\left(\frac{\pi |\ln x - et|}{\sqrt{6}\sigma t}\right)\right)^{-1}, \quad x \geq 0. \tag{4}$$

Definition 5 (Liu [14]). The credibility distribution $\Phi : R \to [0, 1]$ of a fuzzy variable ξ is defined by $\Phi(x) = \mathrm{Cr}\{\theta \in \Theta \mid \xi(\theta) \leq x\}$.

That is, $\Phi(x)$ is the credibility in which fuzzy variable ξ takes a value less than or equal to x. If the fuzzy variable ξ is given by a membership function μ, then its credibility distribution is determined by

$$\Phi(x) = \frac{1}{2}\left(\sup_{y \leq x} \mu(y) + 1 - \sup_{y > x} \mu(y)\right), \quad \forall x \in R. \tag{5}$$

Definition 6 (Liu [2]). Let ξ be a fuzzy variable. If at least one of the two integrals is finite in the following formula, then the expected value of ξ can be calculated as

$$E(\xi) = \int_0^{\infty} \mathrm{Cr}\{\xi \geq r\}\, dr - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leq r\}\, dr. \tag{6}$$

Let ξ be a fuzzy variable whose credibility density function ϕ exists. Liu [14] proved that $E(\xi) = \int_{-\infty}^{+\infty} x\phi(x) dx$, provided that the Lebesgue integral is finite. If $\lim_{x \to -\infty} \Phi(x) = 0$ and $\lim_{x \to +\infty} \Phi(x) = 1$, then $E(\xi) = \int_{-\infty}^{+\infty} x\, d\Phi(x)$, provided that the Lebesgue-Stieltjes integral is finite.

Definition 7 (Liu [3]). Let X_t be a fuzzy process and let C_t be a standard Liu process. For any partition of closed interval $[a, b]$ with $a = t_1 < t_2 < \cdots < t_k = b$, the interval length can be expressed as $\Delta = \max_{1 \leq i \leq k} |t_{i+1} - t_i|$. Then the fuzzy integral of X_t with respect to C_t is

$$\int_a^b X_t dC_t = \lim_{\Delta \to 0} \sum_{i=1}^{k} X_{t_i} \left(C_{t_{i+1}} - C_{t_i}\right), \tag{7}$$

provided that the limit exists almost surely and is a fuzzy variable.

Definition 8 (Liu [3]). Suppose C_t is a standard Liu process, and f and g are two given functions. Then

$$dX_t = f(t, X_t)\, dt + g(t, X_t)\, dC_t \tag{8}$$

is called a fuzzy differential equation. A solution is a fuzzy process X_t that satisfies (8) identically in t.

Let X_t be the bond price and Y_t the stock price. Assume that stock price Y_t follows a geometric Liu process (Liu [15]). Then Liu's stock model is written as below:

$$\begin{aligned} dX_t &= rX_t dt, \\ dY_t &= eY_t dt + \sigma Y_t dC_t, \end{aligned} \tag{9}$$

where r is the riskless interest rate, e is stock drift, σ is stock diffusion, and C_t is a standard Liu process. This is the fuzzy stock model for fuzzy financial market and is used to simulate future bond stocks or financial derivatives.

3. Defaultable Bond and CDS Pricing

3.1. Defaultable Bond Pricing. The defaultable bond is a contract that pays full face value at maturity date, as long as the bond's issuer is not default. If a default occurs during the validity period of the contract, then the recovery is paid to the bond's holder, and the contract is ended. Let the face value of zero-coupon defaultable bond is 1 unit and the maturity date is T. The interest rate and recovery rate are r and δ. Take advantage of Liu's stock model; we have the following results.

In order to get this defaultable bond price of Liu's stock model, we need to derive the bond's default distribution function firstly. If Y_t is the price of the underlying defaultable bond, then it is easy to solve (9) in which $Y_t = Y_0 \exp(et + \sigma C_t)$. Without loss of generality, we assume that the default threshold K is a positive real number $K \in R^+$. Then, based on Merton's structural model [16], the bond's default distribution function is derived as follows:

(i) If $0 < K < Y_0$, we have

$$\begin{aligned}\Phi_t(K) &= P(\tau \le t) = \mathrm{Cr}\{Y_t \le K\} \\ &= \mathrm{Cr}\{Y_0 \exp(et + \sigma C_t) \le K\} \\ &= \mathrm{Cr}\left\{\exp(et + \sigma C_t) \le \frac{K}{Y_0}\right\} \\ &= \frac{1}{2}\left(\sup_{y \le K/Y_0} \mu(y) + 1 - \sup_{y > K/Y_0} \mu(y)\right) \\ &= \left(1 + \exp\left(\frac{\pi(et - \ln(K/Y_0))}{\sqrt{6}\sigma t}\right)\right)^{-1},\end{aligned} \tag{10}$$

where τ represents the bond's default time.

(ii) It is obvious that $\Phi_t(K) = 1$, when $K \ge Y_0$ (the initial bond price is lower than the default threshold).

Theorem 9. *Under an equivalent martingale measure Q, the present fuzzy value of a defaultable bond Y_t is given as*

$$\begin{aligned}Y_t &= \exp(-r(T-t)) \\ &\quad \cdot \left(1 + (\delta - 1)\left(1 + \exp\left(\frac{\pi e}{\sqrt{6}\sigma}\right)\right)^{-1}\right).\end{aligned} \tag{11}$$

Table 1: Fuzzy defaultable bond.

T	0.5	1	3	5	10
Y_0	0.8483	0.8273	0.7486	0.6773	0.5275

Proof. By the definition of expected value of fuzzy variable, we have

$$\begin{aligned}Y_t &= E^Q\left[\exp(-r(T-t))\, 1_{\{\tau > T\}} + \exp(-r(T-t)) \right. \\ &\quad \left. \cdot\, \delta 1_{\{\tau \le T\}}\right] = E^Q\left[\exp(-r(T-t))\left(1 - 1_{\{\tau \le T\}}\right)\right. \\ &\quad \left. + \exp(-r(T-t))\, \delta 1_{\{\tau \le T\}}\right] = E^Q\left[\exp(-r(T-t))\right. \\ &\quad \left. + \exp(-r(T-t))(\delta - 1)\, 1_{\{\tau \le T\}}\right] = \exp(-r(T-t)) \\ &\quad + \int_{-\infty}^{+\infty} \exp(-r(T-t))\, d\left((\delta - 1)\Phi_T(K)\right) \\ &= \exp(-r(T-t)) + (\delta - 1) \\ &\quad \cdot \int_0^{Y_0} \exp(-r(T-t))\, d\Phi_T(K) \\ &= \exp(-r(T-t))\left(1 + (\delta - 1)\left(1 \right.\right. \\ &\quad \left.\left. + \exp\left(\frac{\pi e}{\sqrt{6}\sigma}\right)\right)^{-1}\right),\end{aligned} \tag{12}$$

where $1_{\{\tau \le T\}}$ is an indicator function; that is, if the bond is default then the function value is 1; otherwise the function value is 0. □

Example 10. Suppose that the riskless interest rate r is 5% per annum, the drift e is 0.25, the diffusion σ is 0.25, and the recovery is 0.4. Then, we can calculate the defaultable bond price that expires in different years (see Table 1).

From Table 1, it can be known that these results are the portrayal of reality with the time increase; the more the fuzziness of the financial markets becomes intense, the more the people's ability to predict the future becomes weak and fuzzy, so the present value of defaultable bond becomes lower with time increasing.

3.2. CDS Pricing. A credit default swap (CDS) is a financial swap agreement that the seller of the CDS will compensate the buyer in the event of a loan default. The buyer of the CDS makes a series of payments (the CDS "fee" or "spread") to the seller and, in exchange, receives a payoff if the loan defaults. Let N be the notional amount, δ the recovery rate, s the credit spread, and $T_1 < T_2 < \cdots < T_n$ the sequence payment dates, with $T_n = T$. The CDS running spread is computed such that the fair price of the CDS equals zero at initiation. That is, the present value of the default leg equals the present value of the fixed leg: PV (default leg) = PV (fixed leg). In this case, we derive the following theorem.

Theorem 11. *The present fuzzy credit spread s is given as*

$$s = \frac{(1-\delta)\exp(-r(T-t))}{\Delta T_i \exp\left(\pi e/\sqrt{6}\sigma\right)\sum_{i=1}^{n}\exp(-r(T_i-t))}. \tag{13}$$

Proof. By the definition of expected value of fuzzy variable, we have

$$\begin{aligned}
&PV(\text{default leg}) = E^{Q}[(1-\delta)N\exp(-r(T-t)) \\
&\cdot 1_{\{\tau\le t\}}] = (1-\delta)N\int_0^{Y_0}\exp(-r(T-t))\,d\Phi_t(K) \\
&= (1-\delta)N\exp(-r(T-t))\left(1+\exp\left(\frac{\pi e}{\sqrt{6}\sigma}\right)\right)^{-1}, \\
&PV(\text{fixed leg}) = E^{Q}\left[sN\sum_{i=1}^{n}1_{\{\tau>T_i\}}\Delta T_i\right. \\
&\left.\cdot\exp(-r(T_i-t))\right] = E^{Q}\left[sN\Delta T_i\sum_{i=1}^{n}\left(1-1_{\{\tau\le T_i\}}\right)\right. \\
&\left.\cdot\exp(-r(T_i-t))\right] = sN\Delta T_i\sum_{i=1}^{n}\left[\exp(-r(T_i-t))\right. \\
&\left.-\int_0^{Y_0}\exp(-r(T_i-t))\,d\Phi_{T_i}(K)\right] \\
&= sN\Delta T_i\sum_{i=1}^{n}\left[\exp(-r(T_i-t))-\exp(-r(T_i-t))\right. \\
&\left.\cdot\left(1+\exp\left(\frac{\pi e}{\sqrt{6}\sigma}\right)\right)^{-1}\right] \\
&= sN\Delta T_i\sum_{i=1}^{n}\left[\exp(-r(T_i-t))\right. \\
&\left.\cdot\left(\frac{\exp\left(\pi e/\sqrt{6}\sigma\right)}{1+\exp\left(\pi e/\sqrt{6}\sigma\right)}\right)\right].
\end{aligned} \tag{14}$$

Then, based on no arbitrage pricing principles, we can prove this theorem. □

In the following, we will simulate Theorem 11 and give a numerical example and then employ the model results to compare with the market data for analysis. The parameters involved in the simulation process are assumed as follows: suppose that the riskless interest rate r is 4.6% per annum, the drift e is 0.01, the diffusion σ is 0.02, and the recovery rate δ is 0.5. Then, through the use of MATLAB R2010a, we can calculate the present running spread that expires in different years and get the following contrast results (see Figure 1).

In Figure 1, the point line (data item 1) represents the numerical results of the model derived from Theorem 11 and the solid line (data item 2) represents the actual market data (the data come from the report of the international clearing bank (BIS)—OTC derivatives statistics at end-December 2014). Through the comparison and analysis we found that

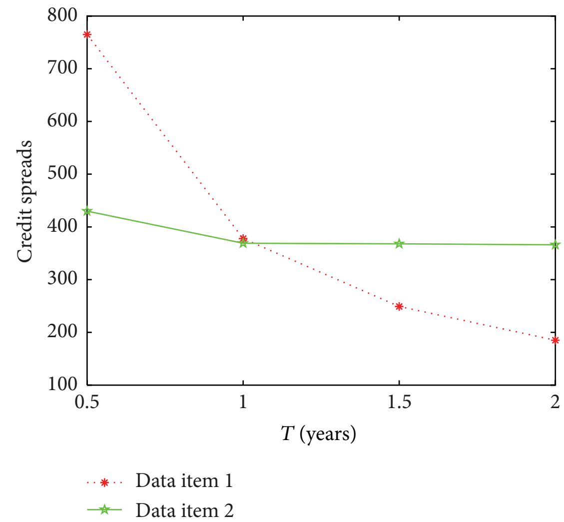

Figure 1: The contrast analysis of model results.

the model results and market data are all gradually decreased with time and they are all falling credit curves; this is in accordance with the current market environment of financial crisis. Meanwhile, in the fuzzy random environment, the model results fell faster than the market data; this shows that the fuzziness of the market environment has increased the pessimism of investors' expectations in the future.

4. One-Factor Fuzzy Copula Function

The key point of single credit derivative pricing is the abovementioned default time modeling with just one variable needed. As for portfolio credit products, its earnings and the default correlation in the investment portfolio are of strong sensibility. Thus, to confirm the default correlation among the assets, which is the correlation of default risks, is the core part of portfolio credit derivatives pricing. Moreover, One-Factor Gaussian Copula function in portfolio credit asset pricing model has particular advantages in establishing united default probability distribution function—the model is simple, convenient to calculate, and easy to be extended. Motivated by the definition of Li's One-Factor Gaussian Copula model [17], One-Factor Fuzzy Copula function can be put forward as follows.

Definition 12. Suppose that the factors influencing company asset value can be divided into two parts, system factors and idiosyncrasy factors; and both of the two kinds of factors are fuzzy processes and obey the standard Liu process C_t, and the asset value of company V_i (latent variable) can be expressed as the following One-Factor Fuzzy Copula model:

$$V_i = \sqrt{\rho_i}Y + \sqrt{1-\rho_i}Z_i, \quad i = 1, 2, \ldots, n. \tag{15}$$

In the above model, system factor Y represents macroeconomic situation, and idiosyncrasy factor Z_i represents the factor that is only effective for company asset i, and they are mutually independent. The marginal distribution of the company asset value V_i is independent of the condition that factor Y is known. $\sqrt{\rho_i}$ is the correlation coefficient between the company asset and system factors. And on the condition that system factors Y and idiosyncrasy factors Z_i's all obey the standard Liu process C_t, the company asset V_i will also obey the standard Liu process.

According to the structural model by Merton [16], when the company asset V_i is under a certain default threshold value $K^i \in R^+$, a company default event will take place. Through Copula function, the condition default probability of single company asset can be expressed as follows:

$$\begin{aligned}\Phi_t\left(K^i\right) &= p_t^{i|Y} = P\left(V_i \le K^i \mid Y\right) \\ &= P\left(\sqrt{\rho_i}Y + \sqrt{1-\rho_i}Z_i \le K^i \mid Y\right) \\ &= P\left(Z_i \le \frac{K^i - \sqrt{\rho_i}Y}{\sqrt{1-\rho_i}} \mid Y\right) \\ &= \left(1 + \exp\left(\frac{\pi}{\sqrt{6}t}\frac{\left(K^i - \sqrt{\rho_i}Y\right)}{\sqrt{1-\rho_i}}\right)\right)^{-1}.\end{aligned} \tag{16}$$

Theorem 13. *On the assumed condition of One-Factor Fuzzy Copula model, the united default probability for every company is*

$$\begin{aligned}p &= P\left(V_1 \le K^1, V_2 \le K^2, \ldots, V_n \le K^n\right) \\ &= \int_{-\infty}^{+\infty}\prod_{i=1}^{n} p_t^{i|Y} d\Phi\left(y\right),\end{aligned} \tag{17}$$

where $p_t^{i|Y}$ is the expression of formula (16).

Proof. On the condition that the system factor Y is known, we can put forward, independently of the expectation-oriented quality and company default,

$$\begin{aligned}p &= P\left(V_1 \le K^1, V_2 \le K^2, \ldots, V_n \le K^n\right) \\ &= E\left(1_{\{V_1\le K^1, V_2\le K^2,\ldots,V_n\le K^n\}}\right) \\ &= E\left(E\left(1_{\{V_1\le K^1, V_2\le K^2,\ldots,V_n\le K^n\}}\right) \mid Y = y\right) \\ &= E\left(E\left(1_{\{V_1\le K^1\}} \mid Y = y\right)\cdots E\left(1_{\{V_n\le K^n\}} \mid Y = y\right)\right) \\ &= E\left(\prod_{i=1}^{n} p_t^{i|Y}\right) = \int_{-\infty}^{+\infty}\prod_{i=1}^{n} p_t^{i|Y} d\Phi\left(y\right).\end{aligned} \tag{18}$$

□

According to the conclusion of Theorem 13, portfolio credit products can be priced, for example, ITRAXX, CDX series, and *collateralized debt obligations* (CDO). As for the idea that the detailed pricing process is similar to CDS pricing, it will not be discussed in this paper.

5. Conclusion

In this paper, we have investigated the credit derivatives pricing problems for fuzzy financial market. Defaultable bond and CDS price formulas have been calculated for Liu's stock model, and a One-factor Fuzzy Copula function has been proposed. Some numerical examples of these formulas have been studied. Through the comparison and analysis we can find that the model results and market data are all gradually decreased with time, and they are all falling credit curves; this is in accordance with the current market environment of financial crisis. Meanwhile, in the fuzzy random environment, the model results fell faster than the market data; this shows that the fuzziness of the market environment has increased the pessimism of investors' expectations in the future. Therefore, in the management of credit risk, we should taking into account both the market fuzziness and randomness.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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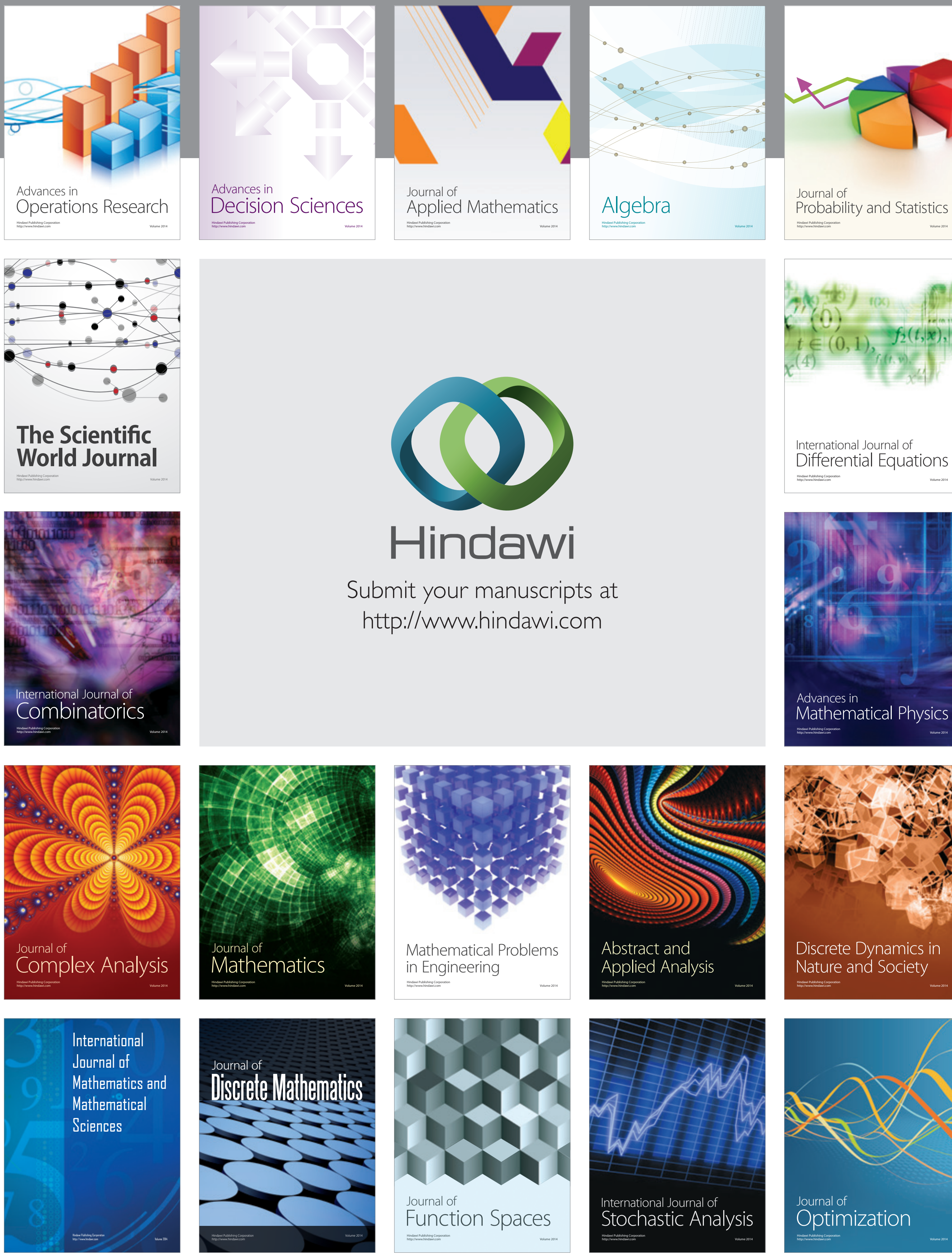

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