A novel variable structure compensation PID control, VSCPID in short, is proposed for trajectory tracking of asymmetrical hydraulic cylinder systems. This new control method improves the system robustness by adding a variable structure compensation term to the conventional PID control. The variable structure term is designed according to sliding mode control method and therefore could compensate the disturbance and uncertainty. Meanwhile, the proposed control method avoids the requirements for exact knowledge of the systems associated with equivalent control value in SMC that means the controller is simple and easy to design. The stability analysis of this approach is conducted with Lyapunov function, and the global stability condition applied to choose control parameters is provided. Simulation results show the VSCPID control can achieve good tracking performances and high robustness compared with the other control methods under the uncertainties and varying load conditions.

1. Introduction

Traditional PID control has been widely used in many industrial applications because of its simple form and clear physical meaning. However, in the control of nonlinear systems such as asymmetrical hydraulic cylinder, classical PID control theory cannot deal with the robustness problems stemming from uncertainties and disturbances well [1].

In recent years, with the development of the theory of nonlinear control, the sliding mode control (SMC) has attracted substantial interest and investigation. In SMC, a sliding mode surface is designed based on the system state equation, and then the system state trajectories are controlled to converge to the surface; once these trajectories reach and keep in the sliding mode surface, the system will be adaptable and invariant to the uncertainties and disturbances [2–4]. The robustness feature makes the SMC applied in many fields characterized by nonlinear and uncertainty such as control of mobile robot [5], vehicles [6, 7], robotic manipulators [8], and exoskeletons [9]. In particular, considering the asymmetrical hydraulic cylinder trajectory tracking system, the asymmetrical structures of cylinder result in the nonlinear variation of system parameters, so the SMC could improve the tracking performance and robustness predominantly [10]. It needs to calculate the equivalent control value associated with the exact knowledge of the system parameters during the SMC design. Unfortunately, in most cases, these parameters cannot be fully obtained (at least not accurately obtained) and thus the design of controller is difficult. One solution is to avoid the equivalent control value by using intelligent control [11, 12], but the cost is the complex controller constructions and impracticability.

Since we want to take the advantages of the simplicity of PID control and the robustness of SMC, the natural way is to combine the two control methods together. Stepanenko et al. [13] designed a PID-like sliding surface including an integral term in the sliding surface expression, and the result showed this design could provide faster response and filter out the steady state tracking error. It should be noted that the equivalent control value is ignored in this paper, which may limit the final control performance. Some other ways such as neural network and state observation were used to estimate the system parameters and then get the equivalent control value [14, 15]. Obviously, the estimation part led to the complex controller constructions.
Jafarov et al. [16, 17] combined the PID control and SMC further to get the variable structure PID control (VSPID). The VSPID used PID algorithm to replace the variable structure coefficient in the SMC, so the coefficient could adapt the tracking errors and a good tracking performance was achieved. The same control method had been proved to be effective for level process [18] and servo motor [19]. Su [20] proposed a nonlinear PD plus method directly; this method changed the linear errors terms of conventional PD control to be nonsmooth but continuous exponential errors. And the similar nonlinear PD control method was also used and analyzed in the parallel manipulator [21]. For these VSPID and nonlinear PID control methods, the potential problem was that the sole nonlinear control could not guarantee the optimal performance.

Zhang et al. [22] proposed a single neuron PID and sliding mode parallel compound control strategy. The most main characteristic was that this controller included both linear and nonlinear part. Then, PID-SMC method was carried out in the manipulator trajectory tracking; it has the features of linear control provided by PD control and nonlinear control contributed by SMC. The linear control was used to stabilize the controlled system and SMC was used to compensate the disturbance and uncertainty [23]. A linear observer was added into the similar method to improve the accuracy of linear control part. Compared to the VSPID, these methods could overcome the disadvantages related to the lack of equivalent control value to some extent.

In this paper, a novel variable structure compensation PID control, VSCPID in short, is proposed for trajectory tracking of asymmetrical hydraulic cylinder systems. Firstly, the valve controlled asymmetrical hydraulic cylinder model is established to get the state space equations which will be the mathematical foundation of variable structure compensation design. Secondly, the VSCPID method is proposed via the analysis of traditional SMC method. The proposed method employs the tuned PID control instead of the SMC’s equivalent control to avoid the requirement of accurate system parameters, and the variable structure part based on sliding mode surface is retained as the compensation of PID control to attenuate the effects of disturbances and uncertainties. Therefore, this approach has the characteristics of model-free, simple structures and easy realizing. Furthermore, the convergence and stability of the VSPID method are proved by Lyapunov function, and the guidelines for the selection of control parameters are provided. Finally, the simulation results show that the proposed control approach could achieve better tracking performances and higher robustness than the traditional methods such as PID control and VSPID control.

2. Mathematical Model

The general valve controlled asymmetrical cylinder system is showed in Figure 1. \( p_1 \) is oil source pressure; \( p_0 \) is return pressure and often considered as zero; \( x_s \) is the spool displacement of servo valve; \( Q_1 \) and \( Q_2 \) are the fluxes of nonrod chamber and rod chamber, respectively; the oil pressure, piston area, and bulk are represented by \( p_1, A_1 \), and \( V_1 \) for the nonrod chamber as well as \( p_2, A_2 \), and \( V_2 \) for the rod chamber. The load is presumed to consist of inertia \( M \), damping \( B \), and stiffness \( K \), and \( y \) means the piston displacement. The disturbance is defined as \( F_L \).

The motion in Figure 1 is extending of piston rod under \( x_s > 0 \). In this condition, the pressure oil enters the nonrod chamber of cylinder through the left valve cavity to push the piston rod out. The load driving force is the push force \( p_1A_1 \) in nonrod chamber minus the resistance force \( p_2A_2 \) in rod chamber, and the load motion velocity is determined by \( Q_L \). Conversely, the pressure oil enters the rod chamber of cylinder through the right valve cavity to pull the piston rod back when \( x_s < 0 \). The force imposed on the load is \( p_2A_2 \) minus \( p_1A_1 \) and the load motion velocity is determined by \( Q_2 \).

Obviously, the main difference between nonrod chamber piston area and rod chamber piston area leads to the nonlinear variation of driving force during the reciprocating motions. In other words, the nonlinear characteristic of this system is due to the asymmetry of system structure. Though there are some nonlinear factors in the servo valve, it used to be linearized with the condition of working near the equilibrium point to simplify the system model [24].

Define the load flow and load pressure as follows:

\[
\begin{align*}
\text{when } x_s > 0: & \quad p_L = p_1 - np_2, \left( n = \frac{A_2}{A_1} \right) \quad Q_L = Q_1, \\
\text{when } x_s < 0: & \quad p_L = p_2 - \frac{p_1}{n}, \left( n = \frac{A_2}{A_1} \right) \quad Q_L = Q_2.
\end{align*}
\]

When \( x_s > 0 \) and ignoring the leak, there are valve flow equation, load flow equation, and equilibrium equation as follows:

\[
\begin{align*}
Q_L &= K_q x_s - K_c p_L, \\
Q_L &= A_1 \ddot{y} + \frac{V_1}{p_c} \dot{p}_L, \\
A_1 p_L &= M \ddot{y} + B \dot{y} + Ky + F_L.
\end{align*}
\]
where \( K_q = \frac{\partial Q_L}{\partial x} = C_d \omega \sqrt{2(p_1 - p_L)/\rho(1 + n^3)} \) and
\( K_c = -\frac{\partial Q_L}{\partial p_L} = (1/2)C_d \omega x \sqrt{2/\rho(1 + n^3)} \) are the flow coefficient and flow-pressure coefficient of the servo valve, respectively; \( \beta_e \) is the modulus of elasticity and \( V_1 = V_1/(1 + n^3) \) is the equivalent volume.
Let \( u = x_1, x_1 = y, x_2 = \dot{y}, \) and \( x_3 = \ddot{y} \); the system (2) could be described as the form of state equations \((u > 0)\)

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -\gamma K_c K_x_1 - (\gamma K_c B + \gamma A_x^2 + \frac{K_c}{M}) x_2 \\
&\quad - \left(\gamma K_c M + \frac{B}{M}\right) x_3 + \gamma A_x K_c u + d + f,
\end{align*}
\]  

where \( y = \beta_e/MV_f, d = -\gamma K_c F_L - \dot{F}_L/M \) means the disturbances, such as friction, impact force, and load variation. \( f \) is the additional term which represents the uncertainties; for example, the oil source pressure is not stable and the modulus of elasticity changes along the variation of oil temperature. Considering the trajectory tracking; if we define the error as \( e_i = x_i - x_{id} \) \((i = 1, 2, 3)\) and \( x_{id} = \dot{x}_{id} \) is the desired trajectory, then we have

\[
\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
\dot{e}_3 &= \dot{x}_3 - \dot{x}_{id}.
\end{align*}
\]  

The state space equations shown as (4) will be the mathematical foundation of the variable structure compensation design and stability analysis.

Besides, the system state equations when \( u < 0 \) can be obtained in the same way. Compared to the condition of \( u > 0 \), the whole structure is similar but the parameters change greatly.

**3. The Design of Variable Structure Compensation PID Controller**

According to state equations (4), the sliding mode surface is chosen to be

\[
s = \lambda_1 e_1 + \lambda_2 e_2 + e_3,
\]  

where \( \lambda_1 \) and \( \lambda_2 \) are the positive constant.

And the derivative of sliding mode surface is

\[
\dot{s} = \lambda_1 \dot{e}_1 + \lambda_2 \dot{e}_2 + \dot{e}_3 = \lambda_1 e_2 + \lambda_2 e_3 + \dot{x}_3 - \dot{x}_{id}.
\]  

In general SMC, the controller is usually designed as

\[
u = u_{eq} + u_r,
\]  

where \( u_{eq} \) is called equivalent control value and can be calculated via setting \( \dot{s} \) as zero, which guarantees that the final system state will stabilize on the sliding surface. \( u_r \) is the discontinuous control value to compensate the disturbances and uncertainties, which guarantees that the system will converge to the sliding surface.

The equivalent control value requires the exact system parameters and a lot of math operations. However, it is difficult to acquire all the accurate parameters and, consequently, there always exist the deviations for the design of equivalent control value. If we can tolerate the deviations aiming at a simple controller structure, the PID control will be a reasonable approximation since the tuned PID control provides the stability and certain error range. After that, the variable structure term, which is used to compensate the disturbances and uncertainties, is added to improve the robustness the same as SMC. The block diagram of variable structure compensation PID control method can be seen in Figure 2.

In the industrial application, PD control is one of the most common methods, and the lack of integral term in sliding mode surface is noticed. So the variable structure compensation PID controller is finally designed as

\[
u = -\left(K_p e_1 + K_d e_1\right) - h \text{sign}(s),
\]  

where \( K_p > 0 \) and \( K_d > 0 \) are the proportional and derivative gains; \( h > 0 \) is the variable structure gain; \( s \) is the sliding mode surface; and \( \text{sign}() \) is the sign function.

**Figure 2:** The block diagram of variable structure compensation PID control method.
4. Stability Analysis

The following assumption is required in the stability analysis.

**Assumption A.** The desired trajectories and disturbances of system (4) are bounded. In other words, there exist positive constant scalars $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, and $\alpha_5$ such that

$$
\|x_{id}\| \leq \alpha_1; \quad \|\dot{x}_{id}\| \leq \alpha_2; \quad \|x_{fd}\| \leq \alpha_3; \quad \|\dot{x}_{fd}\| \leq \alpha_4; \quad \|d + f\| \leq \alpha_5,
$$

where $\|\cdot\| = \sqrt{\langle \cdot \rangle^T \langle \cdot \rangle}$ is the Euclidean norm.

**Remark.** Consider the physical system; it is obvious that the desired trajectories are bounded. And in most cases, disturbances change within a certain range rather than infinite. Therefore, Assumption A is reasonable.

For the system stability, we have the following theorem.

**Theorem 1.** Consider the asymmetrical cylinder system (4) (when $u > 0$) with the variable structure compensation PID control (8); the controlled system will be globally stable, provided that the following conditions are satisfied:

$$
\lambda_1 \leq \rho_2 + \rho_4 K_d,
\lambda_2 < \rho_3,
\rho_4 \lambda_1 \leq \rho_2 \lambda_2, \quad (i = 1, 2, \ldots, n),
$$

where $\rho_1 = \gamma K_c K_i$, $\rho_2 = \gamma K_c B + \gamma \lambda_1^2 + K_c M$, $\rho_3 = \gamma K_c M$, and $\rho_4 = \gamma A_1 K_p$ are the positive constant associated with system parameters.

**Proof.** In general, consider the system (4) as $n$ dimension and the sliding mode surface is redefined as

$$
S = [s_1 \ s_2 \ \cdots \ s_n]^T,
$$

$$
s_i = s_1 e_1 + s_2 e_2 + e_3 \quad (i = 1, 2, \ldots, n).
$$

The control law is

$$
U = -K_p E - K_d \dot{E} - H \text{sign} (S),
$$

where

$$
K_p = \text{diag} \{K_{ip} \ K_{2p} \ \cdots \ K_{np}\},
K_d = \text{diag} \{K_{id} \ K_{2d} \ \cdots \ K_{nd}\},
H = \text{diag} \{h_1 \ h_2 \ \cdots \ h_n\},
E = [e_{11} \ e_{21} \ \cdots \ e_{1n}]^T,
\text{sign} (S) = [\text{sign} (s_1) \ \text{sign} (s_2) \ \cdots \ \text{sign} (s_n)]^T.
$$

Then define a $2n \times 2n$ matrix $\Phi$ as

$$
\Phi = [Q \ P; -P \ G],
$$

where

$$
Q = \text{diag} \{q_1 \ q_2 \ \cdots \ q_n\},
G = \text{diag} \{g_1 \ g_2 \ \cdots \ g_n\},
P = \text{diag} \{p_1 \ p_2 \ \cdots \ p_n\},
q_i = \lambda_{ii} (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}) + \lambda_{i2} (\rho_{i1} + \rho_{i4} K_{ip}),
g_i = \lambda_{i2} (\rho_{i3} - \lambda_{i2}) + (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}),
p_i = \lambda_{ii} (\rho_{i3} - \lambda_{i2}) + (\rho_{i1} + \rho_{i4} K_{ip}).
$$

Obviously, the matrix $\Phi$ is positive definite under conditions (10) according to the definition of positive definite matrix.

Define the following Lyapunov function:

$$
V = \frac{1}{2} \left[ \begin{array}{c} S^T S + [E^T \ E^T] \Phi \begin{bmatrix} E \\ E \end{bmatrix} \end{array} \right].
$$

As the matrix $\Phi$ is positive definite, therefore we conclude that $V$ is a positive definite function. Differentiating $V$, we obtain

$$
\dot{V} = S^T \dot{S} + [E^T \ E^T] \Phi \begin{bmatrix} E \\ E \end{bmatrix}.
$$

Applying (3), (4), (6), and (8), we have

$$
\dot{s}_i = -s_{i3d} - \rho_{i1} x_{i3d} - \rho_{i2} x_{i2d} - \rho_{i3} x_{i3d} - \rho_{i4} x_{i4d} - \rho_{i5} x_{i5d} - (\rho_{i1} + \rho_{i4} K_{ip}) e_{i1} - (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}) e_{i2} - (\rho_{i3} - \lambda_{i2}) e_{i3} - \rho_{i4} h_i \text{sign} (s_i) + d_i + f_i.
$$

Then

$$
s_i \dot{s}_i = s_i [-s_{i3d} - \rho_{i1} x_{i3d} - \rho_{i2} x_{i2d} - \rho_{i3} x_{i3d} - \rho_{i4} x_{i4d} - \rho_{i5} x_{i5d} - \rho_{i4} h_i \text{sign} (s_i) + d_i + f_i]
- \lambda_{i1} (\rho_{i1} + \rho_{i4} K_{ip}) e_{i1}^2 - \lambda_{i1} (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}) e_{i2}^2
- \lambda_{i1} \lambda_{i2} e_{i1} e_{i2} - \lambda_{i2} (\rho_{i1} + \rho_{i4} K_{ip}) e_{i1} e_{i2}
- \lambda_{i2} (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}) e_{i2}^2 - \lambda_{i2} \lambda_{i3} e_{i2} e_{i3}
- \lambda_{i3} (\rho_{i1} + \rho_{i4} K_{ip}) e_{i1} e_{i3} - (\rho_{i2} + \rho_{i4} K_{id} - \lambda_{i1}) e_{i2} e_{i3} - (\rho_{i3} - \lambda_{i2}) e_{i3}^2.
Substituting (18) and (19) into (17),

\[ \dot{V} = \sum_{i=1}^{n} \left[ -x_{i3d} - \rho_{11}x_{i1d} - \rho_{12}x_{i2d} \right] \]

\[ -\rho_{33}x_{i3d} - \rho_{34}h_i \, \text{sign} \left( s_i \right) + d_i + f_i \]

\[ - \sum_{i=1}^{n} \lambda_{i1} \left( \rho_{11} + \rho_{14}k_{ip} \right) e_{i1}^2 - \sum_{i=1}^{n} \left( \rho_{33} - \lambda_{i2} \right) e_{i3}^2 \]

\[ - \sum_{i=1}^{n} \left[ \lambda_{i2} \left( \rho_{12} + \rho_{14}k_{ip} - \lambda_{i1} \right) \right] + \lambda_{i1} \left( \rho_{33} - \lambda_{i2} \right) + \left( \rho_{11} + \rho_{14}k_{ip} \right) \]

\[ e_{i2}^2. \]

(20)

Based on Assumption A

\[ -s_i \rho_{11}x_{i1d} \leq \rho_{11} \| s_i \| \cdot \max \| x_{i1d} \| \leq \rho_{33} \| s_i \| \cdot \alpha_{i1}, \]

\[ s_i \, \text{sign} \left( s_i \right) = |s_i| = \| s_i \|. \]

Equation (20) can be rewritten as

\[ \dot{V} \leq - \sum_{i=1}^{n} \| s_i \| \left( \rho_{33}h_i - \rho_{11}\alpha_{i1} - \rho_{22}\alpha_{i2} - \rho_{33}\alpha_{i3} - \alpha_{i4} - \alpha_{i5} \right) \]

\[ - \sum_{i=1}^{n} \lambda_{i1} \left( \rho_{11} + \rho_{14}k_{ip} \right) e_{i1}^2 - \sum_{i=1}^{n} \left( \rho_{33} - \lambda_{i2} \right) e_{i3}^2 \]

\[ - \sum_{i=1}^{n} \left[ \lambda_{i2} \left( \rho_{12} + \rho_{14}k_{ip} - \lambda_{i1} \right) \right] + \lambda_{i1} \left( \rho_{33} - \lambda_{i2} \right) + \left( \rho_{11} + \rho_{14}k_{ip} \right) \]

\[ e_{i2}^2. \]

(22)

So we can conclude that \( \dot{V} \) is negative definite with conditions (10), and in other words, the system is globally stable.

It is noticed that the stability analysis when \( u < 0 \) is just the same although the system parameters are different. Due to space limitations, the same will not be repeated here.

5. Simulation Verification

Simulations on the asymmetrical hydraulic cylinder trajectory tracking system are conducted within MATLAB environment to illustrate the effectiveness of the proposed controller. The parameters of the tracking system are assumed as follows:

\[ \rho_s = 7 \text{ MPa}; \quad A_1 = 1.96 \times 10^{-3} \text{ m}^2; \]

\[ A_2 = 1.47 \times 10^{-3} \text{ m}^2; \]

\[ V_1 = 2 \times 10^{-4} \text{ m}^3; \quad V_2 = 1.5 \times 10^{-4} \text{ m}^3; \]

\[ \beta_c = 1.4 \times 10^9 \text{ N/m}^2; \]

\[ M = 100 \text{ kg}; \quad B = 340 \text{ N·s/m}; \]

\[ K = 1000 \text{ N/m}; \]

the additional parameters used to calculate \( K_c \) and \( K_d \) are the discharge coefficient of the valve orifices \( C_d = 0.62 \); the gradient of the valve core area \( \omega = 0.008 \text{ m} \); and the oil density \( \rho = 870 \text{ kg/m}^3 \).

The tuned PD parameters are

\[ K_p = 1000; \quad K_d = 80. \]

(24)

And the parameters of variable structure compensation term are

\[ \lambda_1 = 200; \quad \lambda_2 = 1; \quad h = 2. \]

(25)

The traditional PID control and variable structure PID (VSPID) control proposed in [17] are also simulated under the same condition to compare with the variable structure compensation PID (VSCPID) control.

According to [17], the VSPID controller is designed as

\[ u = - \left[ \left( h + K_p \right) |e| + K_d |\dot{e}| \right] \, \text{sign} \left( s \right). \]

(26)

Figures 3 and 4 show the tracking trajectories and errors of three control methods with the condition of static positioning, and the disturbance is presumed to be a step signal acting at 1 sec. Obviously, the effectiveness of PID control is affected by the disturbance and it spends a long time to converge to the desired position. Correspondingly, both of the VSPID and VSCPID put up good inhibitory effect on disturbance. Meanwhile, it should be noticed that the VSPID control has steady state error, and this may be the disadvantage of the sole nonlinear control as we have discussed in the beginning. In addition, the comparison among the errors of three methods shows that, relative to the linear PID control, the nonlinear control methods speed up the convergence process and thus improve the system response time.

Figures 5 and 6 are the dynamic tracking trajectories and errors when the desired trajectory is 1 Hz sinusoidal signal, and the disturbance is also a step signal acting at 1 sec. It can be seen that the PID and VSPID tracking trajectories both have more obvious phase lag than the VSCPID, which means the greater tracking errors. What is more, when the piston rod motions change from extending (time 1 sec–1.2 sec) to contraction (time 1.4 sec–1.6 sec), the tracking errors of PID control and VSPID control begin to jitter rather than keep smooth, and this inconsistency during the reciprocating motions indicates the variable system parameters, which is
Figure 3: Static trajectory tracking performances: (a) PID control; (b) VSPID control; (c) VSCPID control.

Figure 4: Static tracking errors.
due to the asymmetric cylinder structure, lead to worse performance of PID control and VSPID control.

After the comparison and analysis, the advantages of the proposed VSCPID control method can be summarized as follows.

(i) **High precision**: the tracking errors of VSCPID method under static condition and dynamic condition (especially the dynamic condition) are both less than the errors of other two control methods.

(ii) **Fast convergence rate**: the convergence time of VSCPID method is less than or equal to the convergence time of other methods.

(iii) **Good robustness**: not only does the disturbance have little affection on VSCPID, but also the tracking performances keep a good consistency during the reciprocating motions. In other words, the VSCPID control method could attenuate the negative influence of disturbances and uncertainties.

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**Figure 5**: Dynamic trajectory tracking performances: (a) PID control; (b) VSPID control; (c) VSCPID control.
6. Conclusion

The proposed variable structure compensation PID control method in this paper takes the advantages of the simplicity and easy design of PID control and the robustness of SMC to disturbances and uncertainties. The calculating of equivalent control value is avoided by approximate fitting with PID control, which makes the new control method mode-free. Lyapunov function is designed to prove the global stability of tracking system controlled by the proposed method, and meanwhile the guidelines for the selection of control parameters are provided according to the stability theory. Finally, the simulation results with comparisons of three control methods give the conclusion that the new control method improves the system performance and robustness significantly.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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