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Research Article

A Novel Model of Set Pair Analysis Coupled with Extenics for Evaluation of Surrounding Rock Stability

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The evaluation of surrounding rock stability is a complex problem involving numerous uncertainty factors. Here, based on set pair analysis (SPA) coupled with extenics, a novel model, considering incompatibility, certainty, and uncertainty of evaluation indicators, was presented to analyze the surrounding rock stability. In this model, extension set was first utilized to describe the actual problem of surrounding rock stability. Then, the connectional membership degree of the set pair was introduced to compare the measured values with classification standards from three aspects embracing identity, discrepancy, and contrary. Also, according to identity-discrepancy-contrary (IDC) analysis in the universe of the extension set, the connection numbers were proposed to specify the connectional membership degree of an evaluation indicator to each class. Combined with the weights of evaluation indicators, integrated connectional membership degrees were calculated to determine their classes of rock stability. Finally, a case study and comparison with variable fuzzy set method, triangular fuzzy number method, and basic quality (BQ) grading method were performed to confirm the validity and reliability of the proposed model. The results show that this model can effectively and quantitatively express the differences within a group, transformation of different groups, and uncertainty of complex indicators as a whole.

1. Introduction

Urbanization development and economic growth result in the rapid expansion of transport systems and hydropower and energy construction projects. To ensure the safety of large-scale construction projects and provide useful quantitative information, the rational classification of surrounding rock stability has become increasingly complex because more and more parameters have to be considered. At the same time, it is also a critical engineering question for the crisis management of hazards. Therefore, scholars and engineers pay much attention to this issue.

Rock mass classification schemes have been developing for over 140 years since Ritter tried to formalize an empirical approach to tunnel design in 1879 [1]. The first practicable method of rock load classification system was put forward by Terzaghi [2]. And stand-up time classification method proposed by Lauffer [3] contributed to the development and application of the New Austrian Tunneling method in soft or poor rock mass. To provide a quantitative estimate of rock mass quality from drill hole core, Deere et al. [4] introduced a rock quality designation (RQD) index method, but the influences of joint tightness, orientation, continuity, and infilling were neglected before. Then, some quantitative classification methods including rock structure rating (RSR) system [5], Q-system [6, 7], and rock mass rating (RMR) method [8–11], were proposed on the basis of case histories. All of these systems have limitations, but they are valuable tools if applied properly with care. Because of ease and versatility of use, RMR and Q system methods have been widely accepted at present. In addition, although other numerous organizations have suggested different guidelines for surrounding rock classification, there was little consistency among these guidelines, and they had not found a wide variety of practical applications. In China, basic quality (BQ) grading standard in engineering rock mass classification (GB 50218-94) [12] is often used to evaluate the surrounding rock stability. However, surrounding rock stability involves various uncertainties and other variable and fuzzy factors [13, 14]. With the increasing complexity of construction environment, conventional methods may result in trouble in decision-making due to the uncertainty and changeable features of indicators. And these approaches cannot describe the relativity of the system assessment from the view of information utilization [15]. Thus, conventional methods have some inadequacies and cannot meet the needs of practical engineering. There is an urgent need for development and perfection on the surrounding rock stability rating which can take into account fuzzy variation, incompatibility, and uncertainty of evaluation indicators.

As mentioned above, previous classification schemes for rock mass only considered single type of uncertainty (e.g., fuzziness, variability). However, for problems in real rock engineering, the uncertainty of evaluation indicators shows incompatibility, complexity, and diversity, and they are often fused with each other. Various improved methods, such as the fuzzy sets method [16, 17], the fuzzy analytical hierarchy process [18], the distance discriminate analysis method [19, 20], the support vector machine method [21, 22], the automated rock classification system [23], and the cloud approach [24], have been proposed to overcome those problems. Overall, whereas considerable research has been performed to improve the techniques for analyzing the surrounding rock stability, rock quality evaluation is still not well resolved nowadays. Besides, those analytical methods are mainly dependent on indicators of single type of uncertainty. Conventional methods consider the degree that an object has certain nature as being unchangeable because they lay particular emphasis on the static status on the nature of objects. They are not characterized by addressing the issue of incompatibility and dynamic uncertainty. The true state of surrounding rock stability in specified conditions is changeable, and truthfulness or falseness of the classification may be different in degree. Thus, changes of the nature have to be taken into consideration in the quantitative analysis of the surrounding rock stability. Unfortunately, uncertainty and incompatibility of evaluation indicators cannot be well described by conventional methods. To develop a rational classification method for the engineering practice, uncertainty and incompatibility nature of evaluation indicators of surrounding rock stability should be deeply investigated.

The main objective of this work is to treat the evaluation of surrounding rock stability as a decision-making problem of uncertainty and incompatibility. A novel method using set pair analysis coupled with extenics is introduced to analyze the above issue. This method can effectively and quantitatively take account of certainty and uncertainty, compatibility and incompatibility of indicators as a whole, and make the evaluation results apply in practical engineering more accurately.

2. Theory and Methodology

The newly emerging methodologies of extenics and SPA show their advantages in dealing with contradictory and uncertainty problems [25–30]. They can provide a novel coupling method to know and analyze stability of surrounding rock from a new perspective.

2.1. Introduction of Extension Theory. In practice, things are solvable through the transformation between quality and quantity of the study objects. The Chinese famous ancient story “Cao Chong weighed the elephant” is just a good example. By transformation, Cao turned an elephant into stones and succeeded in weighing the elephant. However, the classic cantor set and fuzzy set both describe mainly static properties of matters and cannot take care of incompatible problems. The extenics proposed by Cai [25] helps people break away from the shackles of traditional fields and reflects the inherent relations of research objects with symbols.

In extenics, the objective world is a world of elements. Matter-element and extension set are basic concepts of extenics. Relation and transformation of quality and quantity can be considered together in the extension theory. Their definitions are illustrated as follows.

An ordered triple $R = (N, C, V)$ is a fundamental element to describe a matter and is called matter-element. In matter-element, R, N, and V stand for the name of an object and the measure N of the nature C, respectively. The extensibility of matter-element strongly provides a tool to solve contradictory problems and presents reversed and conjugate thinking modes [25–28].

Let $\mathbf{U}$ be a universe of discourse and let k be mapping of $\mathbf{U}$ to the real field; there is a real number $k(x_0) \in (-\infty, +\infty)$ corresponding to an element $x_0 \in \mathbf{U}$. Then, call $A = \{(x_0, y) \mid x_0 \in \mathbf{U}, y = k(x_0) \in (-\infty, +\infty)\}$ as an extension set on the discourse universe $\mathbf{U}$. Here, $y = k(x_0)$ is the dependent function of A, and $k(x_0)$ is the dependent degree of x_0 to A. The mathematical model of dependent function is written as follows:

$$\begin{aligned}
k(x_0) &= \frac{\rho(x_0, X_0)}{\rho(x_0, X) - \rho(x_0, X_0)}, \\
&\qquad\qquad \rho(x_0, X_0) \neq \rho(x_0, X), \\
\rho(x_0, X_0) &= \left| x_0 - \frac{m+n}{2} \right| - \frac{n-m}{2}, \\
\rho(x_0, X) &= \left| x_0 - \frac{p+q}{2} \right| - \frac{q-p}{2},
\end{aligned} \tag{1}$$

where $\rho(x_0, X)$ and $\rho(x_0, X_0)$ are the extension distances between point x_0 and intervals X and X_0, respectively. m and n are limits of interval X_0. p and q are limits of interval X. $\rho(x_0, X) - \rho(x_0, X_0)$ denotes the place value of point x_0 about the nest of the interval composed of intervals X and X_0, which are used to describe the relation between x_0 and two intervals.

The positive field, zero, and negative field of an extension set are just right to express transformations between “yes” and “no” and depict quantitative and qualitative changes. Zero point is a criticality of “Both A and B.” Suppose certain attribute of the study object is divided into K real intervals; the universe of extension set is presented as showed in Figure 1. In Figure 1, F_k is the limit of the class k ($k = 1, 2, \ldots, K$). The intervals $[F_k, F_{k+1})$ and $[F_1, F_{K+1}]$ denote the intervals X_0 and X, respectively.

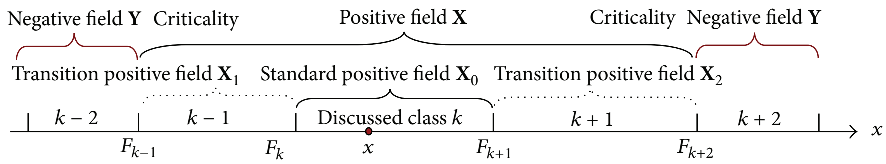

Figure 1: The universe of extension set.

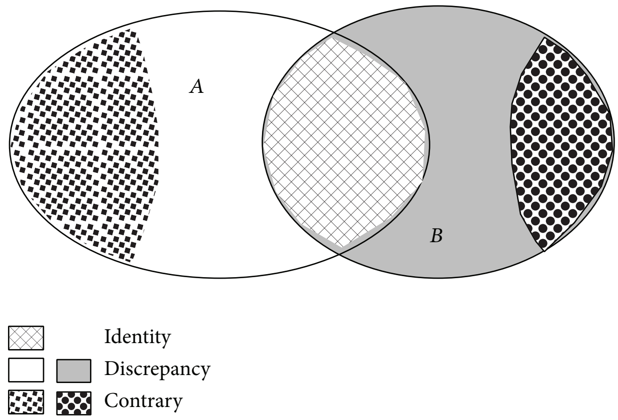

Figure 2: Sketch of identity, discrepancy, and contrary of set pair.

As mentioned above, extension distance achieves a quantitative description of the internal differences within a group or class. The extension set makes dynamic classification and recognition more suitable to people's thinking modes and actual situation. The extenics provides a new formalized and quantified tool for contradictory problems encountered in practice and enriches the uncertainty analysis theory.

2.2. Brief of SPA Theory. SPA theory is a novel analytical method for systematic problems of uncertainty [28–32]. Both the dialectical law of the unity of opposites in philosophy and principle of universal connection are given consideration to depict uncertainty problems. Set pair and connection number are pillars of the SPA theory. Connection number is used to express the interaction and transformation of attributes in multiple scales for a set pair consisting of two related sets. The fundamental procedure of a set pair analysis is to quantitatively express uncertainty problem from identity, discrepancy, and contrary aspects (Figure 2). And the definition of the connection number is given as follows.

Letting a set pair $H = (A, B)$ consisting of set A and set B, there are total attributes N, identical terms S, and contrary attributes P in the H, respectively. So residual attributes $F = N-S-P$ are discrepancy attributes. Uncertainty relation of the two sets can be represented by a connection number of three elements. The Equation of the connection number is written as follows:

$$\mu_{(A,B)} = \frac{S}{N} + \frac{F}{N} i + \frac{P}{N} j = a + bi + cj, \tag{2}$$

where $\mu_{(A,B)}$ is the connection number. a, b, and c are identity degree, discrepancy degree, and contrary degree, respectively, and $a + b + c = 1$. i is the discrepancy coefficient within $[-1, 1]$. j is the contrary coefficient and generally specified as -1. This analytical method is therefore of advantage for the representation of certainty and uncertainty in a unified way and can present transformation of discrepancy.

For problems of evaluation and risk analysis, much attention is usually paid to the relation between a point and a standard interval. The identity-discrepancy-contrary relationships of the discussed point relative to given interval are presented as showed in Figure 3. Namely, when the point x locates in the given standard set $[F_k, F_{k+1})$, it is considered as an identity relationship. Connection number is given as 1 when we do not take account of internal differences in a group. When x is in the separated interval, it is defined as a contrary relationship, and $\mu_{(A,B)} = -1$. When x is in the interval next to the given interval, the relationship is of uncertainty and called discrepancy, $\mu \in [-1, 1]$. However, the connection number in $[0, 1]$ sometimes may not describe the transformation of study objects [28–30]. Here, the connectional membership degree is introduced to expand the connection number in order to satisfy the laws of the unity of opposites and the transformation process of quality and quantity. The mathematical model is written as

$$\mu(x,k) = \begin{cases} -\left|\dfrac{x - F_k}{M_{k-1} - M_k}\right|, & F_{k-1} \le x < F_k \\[2ex] \dfrac{2(x - F_k)}{F_{k+1} - F_k}, & F_k \le x < \dfrac{F_k + F_{k+1}}{2} \\[2ex] \dfrac{2(F_{k+1} - x)}{F_{k+1} - M_k}, & \dfrac{F_k + F_{k+1}}{2} \le x < F_{k+1} \\[2ex] -\left|\dfrac{x - F_{k+1}}{F_{k+1} - F_{k+2}}\right|, & F_{k+1} \le x < F_{k+2} \\[2ex] -1, & \text{other}, \end{cases} \tag{3}$$

where $\mu(x, k)$ is a connectional membership degree and $\mu(x, k) \in [-1, 1]$. M_{k-1}, M_k, M_{k+1}, and M_{k+2} are limits of intervals, respectively.

The connection membership degree displays IDC relations in SPA, but it differs from the fuzzy membership degree. Its value is of wide range, which overcomes the defects of the traditional concept "all are the same within the class." Namely, it can point out differences in degree that point qualities for the given interval; meanwhile, even when points were found in the same group, reflect the difference and transformation of different intervals. It also describes the transformation of certainty and uncertainty. In general, the SPA can express the dynamic change of objects embodied in the certainty of "Either A or B" and uncertainty of "Both A and B."

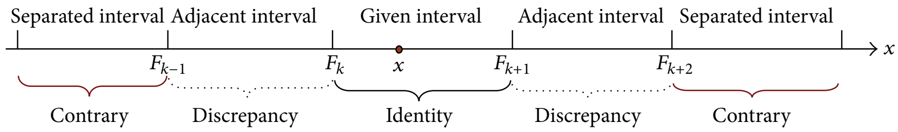

FIGURE 3: IDC relations between the point and intervals.

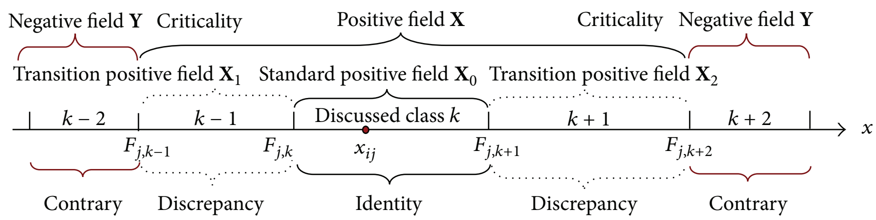

FIGURE 4: Relationship between the IDC of set pair and the universe of extension set.

2.3. Extension Set and IDC Analysis. As we all know, the analysis of cantor set, fuzzy set, and rough set always regards the attributes of objects as invariable from the static perspective. The traditional set theory is incapable of revealing the dialectical transformation from quantitative and qualitative aspects. Unfortunately, in fact, the membership to a certain attribute is changeable. As noted above, the dependent degree in the extension theory was introduced to quantify the change in quality and quantity of objects. Extenics provides a new way for microanalysis of set pair discrepancy. Hence, extenics and SPA are of advantage in depicting the development from "all are the same in the same group" to a quantitative differences in the same group. To couple SPA with extenics, corresponding theoretical basis will be further analyzed as below.

Uncertainty and certainty of interaction and transformation can be treated dialectically by IDC analysis as a whole in SPA. This demonstrates the close relation and similarity between the outlook on uncertainty transformation of extenics and that of SPA. Namely, the measured data may locate in positive field, zero, or negative field of an extension set, which is similar to IDC relationships in SPA. Figure 4 exhibits relationship between the IDC relationship of set pair and the universe of extension set.

Based on the SPA theory, when the measured point locates in the discussed standard positive field, it is defined as identity. And the discrepancy and contrary are defined when measured point locates in the transition positive field and negative field, respectively. To properly specify the discrepancy degree is the key point during the SPA. Unfortunately, up to date, this problem is still controversial. Herein, coupling SPA with extenics, it can make full use of extension transformation and calculation and provides a new idea to overcome the specification of discrepancy coefficient and attempts to establish a valid connection for the two uncertainty analytical theories.

3. Development of a Model of SPA Coupled with Extenics

3.1. Evaluation Procedures. The evaluation procedure with the model of SPA coupled with extenics is as follows.

First, identify evaluation indexes and classification standard, and establish matter-elements based on the measured values of samples.

Then, set up the corresponding set pairs, and conduct IDC analysis of extension universe between the evaluations of samples and standard sets.

Next, based on SPA analysis of extension universe, determine the function of connectional membership degree. Combine indicator weights to calculate evaluation samples' connectional membership degree to each class.

Finally, specify class of samples according to integrated value of connectional membership degree.

The concrete flowchart of the evaluation procedure is shown in Figure 5.

3.2. Development of Evaluation Model. Evaluation of surrounding rock stability is a complex system issue since it links to changes and interaction of evaluation indicators. Besides, evaluation indicators are of contradiction and coexistence of qualitative and quantitative changes and fuzzy characteristic [33]. Here, a new attempt is made to couple SPA and extenics with the aim of analyzing certainty and uncertainty, compatible and incompatible problems from an integral and unified perspective. Assuming $N_1, N_2, \ldots, N_k, \ldots, N_K$ ($k = 1, 2, \ldots, K$) is an evaluation class vector, C_j is jth ($j = 1, 2, \ldots, m$) nature and v_{ik} is the measure of C_i. Then the matter-element model for the class N_k is

$$R_k = \left(N_k, C_j, V_k\right) = \begin{bmatrix} N_k & C_1 & V_{1k} \\ & C_2 & V_{2k} \\ & \vdots & \vdots \\ & C_m & V_{mk} \end{bmatrix}, \quad (4)$$

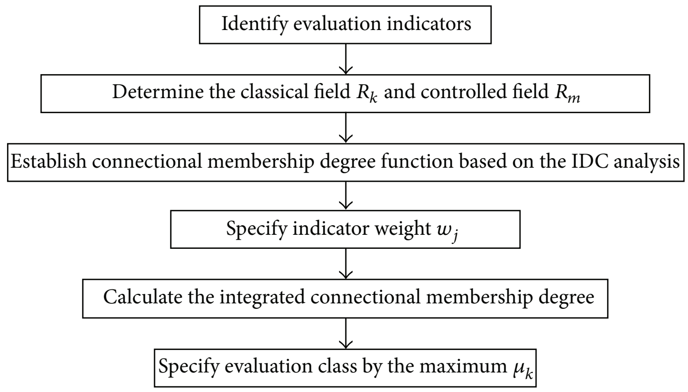

Figure 5: Evaluation flowchart based on the model of SPA coupled with extenics.

where $V_k = \langle a_{jk}, b_{jk}\rangle$ is the range of nature C_i for the evaluation class N_k. And the corresponding expression of matter-element model by means of the classical field and controlled field is depicted in formulas (5) and (6), respectively. Consider

$$R_K = \begin{bmatrix} N & N_1 & N_2 & \cdots & N_K \\ C & V_1 & V_2 & \cdots & V_K \end{bmatrix} = \begin{bmatrix} N & N_1 & N_2 & \cdots & N_K \\ C_1 & \langle a_{11}, b_{11}\rangle & \langle a_{12}, b_{12}\rangle & \cdots & \langle a_{1K}, b_{1K}\rangle \\ C_2 & \langle a_{21}, b_{21}\rangle & \langle a_{22}, b_{22}\rangle & \cdots & \langle a_{2K}, b_{2K}\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_m & \langle a_{m1}, b_{m1}\rangle & \langle a_{m2}, b_{m2}\rangle & \cdots & \langle a_{mK}, b_{mK}\rangle \end{bmatrix}, \quad (5)$$

$$R_m = \begin{bmatrix} N & C_1 & V_{1d} \\ & C_2 & V_{2d} \\ & \vdots & \vdots \\ & C_m & V_{md} \end{bmatrix} = \begin{bmatrix} N & C_1 & \langle a_{1d}, b_{1d}\rangle \\ & C_2 & \langle a_{2d}, b_{2d}\rangle \\ & \vdots & \vdots \\ & C_m & \langle a_{md}, b_{md}\rangle \end{bmatrix}, \quad (6)$$

where N is the class and $\langle a_{1d}, b_{1d}\rangle$ is the total range of nature C_i under a certain condition.

Similarly, matter-element model R_P $(p = 1, 2, 3, \ldots, P)$ of all evaluation samples is given as follows:

$$R_P = \begin{bmatrix} P & p_1 & p_2 & \cdots & p_P \\ C_1 & v_{11} & v_{12} & \cdots & v_{1P} \\ C_2 & v_{21} & v_{22} & \cdots & v_{2P} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_m & v_{m1} & v_{m2} & \cdots & v_{mP} \end{bmatrix}. \quad (7)$$

According to the above analysis, IDC relation can be applied to describe the universe of extension set. If the measured value x_{ij} is in the given interval of class k $(k = 1, 2, \ldots, K)$, which consists of limits of indicator j, the relation between x_{ij} and k is called identity. And the discussed interval of class k is treated as a standard positive field of an extension set, $\mathbf{X}_0 = \langle F_{j,k}, F_{j,k-1}\rangle$; the corresponding computation model of connectional membership degree is

$$\mu_k(x_{ij}) = \begin{cases} \dfrac{2(F_{j,k+1} - x_{ij})}{F_{j,k+1} - F_{j,k}}, & x_{ij} \ge \dfrac{F_{j,k} + F_{j,k+1}}{2} \\ \dfrac{2(x_{ij} - F_{j,k})}{F_{j,k+1} - F_{j,k}}, & x_{ij} < \dfrac{F_{j,k} + F_{j,k+1}}{2}, \end{cases} \quad (8)$$

where $F_{j,k}$ and $F_{j,k+1}$ are limits of the class and $\mu_k(x_{ij})$ is the connectional membership degree of index j for sample i to the class k.

When index j locates in the class $k - 1$ $(k > 2)$ or $k + 1$ and its value still belongs to transition positive field $\mathbf{X}_1 = \langle F_{j,k-1}, F_{j,k}\rangle$ or $\mathbf{X}_2 = \langle F_{j,k+1}, F_{j,k+2}\rangle$, it is defined as discrepancy. And connectional membership degree is calculated by formulas (9), (10), and (11). Consider

$$\mu_k(x_{ij}) = \begin{cases} \dfrac{\rho(x_{ij}, X_0)}{\rho(x_{ij}, X) - \rho(x_{ij}, X_0)} & \rho(x_{ij}, X_0) \ne \rho(x_{ij}, X) \\ -\rho(x_{ij}, X_0) & \rho(x_{ij}, X_0) = \rho(x_{ij}, X), \ x_{ij} \in X_0 \\ -1 & \rho(x_{ij}, X_0) = \rho(x_{ij}, X), \ x_{ij} \notin X_0, \ x_{ij} \in X \end{cases} \quad (9)$$

$$\rho\left(x_{ij}, X_0\right) = \left|x_{ij} - \frac{F_{j,k} + F_{j,k+1}}{2}\right| - \frac{F_{j,k+1} - F_{j,k}}{2} \tag{10}$$

$$\rho\left(x_{ij}, X\right) = \left|x_{ij} - \frac{F_{j,k-1} + F_{j,k+2}}{2}\right| - \frac{F_{j,k+2} - F_{j,k-1}}{2}, \tag{11}$$

where $F_{j,k-1}$ and $F_{j,k+2}$ are limits of the class. $\rho(x_{ij}, \mathbf{X})$ and $\rho(x_{ij}, \mathbf{X}_0)$ are the distances of x_{ij} (the measured value of the sample i for index j) to extension positive field and standard positive field, respectively. In formula (11), while $k = 1$ and $k = K$, get $F_{j,k-1} = F_{j,k}$ and $F_{j,k+2} = F_{j,K}$, respectively.

When index j is not in the extension set positive field $X = \langle F_{j,k-1}, F_{j,k+2}\rangle$ but in extension set negative field $\mathbf{Y}$ including the intervals of classes $k-2$ and $k+2$ $(k > 2)$, this relation can be specified as contrary, and connectional membership degree is

$$\mu_k\left(x_{ij}\right) = -1. \tag{12}$$

Combining the evaluation index weights, the integrated connectional membership degrees can be calculated by the following formula:

$$\mu_k = \sum_{j=1}^{M} w_j \mu_k\left(x_{ij}\right), \tag{13}$$

where w_j are weights of indicators.

Finally, the class is determined by the maximum integrated connectional membership degree. The criteria can be given as

$$k = \max\left\{\mu_1, \mu_2, \ldots, \mu_m\right\}, \tag{14}$$

where k is the evaluated class.

4. Case Study

4.1. Evaluation Indicator System Identification of Surrounding Rock Stability. Classification of surrounding rock stability is substantially dependent on the indicator selection. Generally, the selection process of evaluation indicators is primarily from geological and engineering aspects including rock mass characteristics, underground water, the state of stress, and joint plane. Herein, based on the design codes in China, previous research, and literature reviews, we selected rock mass quality, uniaxial compressive strength, integrality degree of rock mass, groundwater percolation capacity, and joint as the evaluation indicators. Rock mass quality, uniaxial compressive strength, and integrality degree of rock mass all reflect rock mass characteristics. The joints condition reveals geological structure effect on rock mass stability, and groundwater percolation capacity indicates activity of underground water. Here, stability of surrounding rock is divided into five classes named as very good (I), good (II), fair (III), poor, (IV) and very poor (V) by the orders of stability from the highest to the lowest.

To confirm the reliability and the validity of this presented model, data of the references [16, 34] were used to conduct a case study. The classification standard and the indicator values measured from the samples are listed in Tables 1 and 2.

4.2. Model Validation and Discussion. According to Tables 1 and 2, the extension model of the surrounding rock stability and corresponding control field R_m can be written as

$$R_K = \begin{bmatrix} N & N_1 & N_2 & N_3 & N_4 & N_5 \\ C_1 & \langle 0, 0.10\rangle & \langle 0.10, 0.25\rangle & \langle 0.25, 0.40\rangle & \langle 0.40, 0.60\rangle & \langle 0.60, 1.00\rangle \\ C_2 & \langle 200, 300\rangle & \langle 100, 200\rangle & \langle 50, 100\rangle & \langle 25, 50\rangle & \langle 0, 25\rangle \\ C_3 & \langle 0.75, 1.00\rangle & \langle 0.55, 0.75\rangle & \langle 0.30, 0.55\rangle & \langle 0.15, 0.30\rangle & \langle 0.0, 0.15\rangle \\ C_4 & \langle 0, 5\rangle & \langle 5, 10\rangle & \langle 10, 25\rangle & \langle 25, 125\rangle & \langle 125, 250\rangle \\ C_5 & \langle 9, 10\rangle & \langle 7, 9\rangle & \langle 4, 7\rangle & \langle 2, 4\rangle & \langle 0, 2\rangle \end{bmatrix}, \tag{15}$$

$$R_m = \begin{bmatrix} N & C_1 & \langle 0, 1.00\rangle \\ & C_2 & \langle 0, 300\rangle \\ & C_3 & \langle 0, 1.00\rangle \\ & C_4 & \langle 0, 250\rangle \\ & C_5 & \langle 0, 10\rangle \end{bmatrix}. \tag{16}$$

Table 1: Classification standard to surrounding rock stability.

Stability class	Rock mass quality	Uniaxial compressive strength (MPa)	Integrality degree of rock mass	Groundwater percolation capacity (L/(min·10 m))	Joint
Very good I	0–0.10	200–300	0.75–1.00	0–5	9-10
Good II	0.10–0.25	100–200	0.55–0.75	5–10	7–9
Fair II	0.25–0.40	50–100	0.30–0.55	10–25	4–7
Poor IV	0.40–0.60	25–50	0.15–0.30	25–125	2–4
Very poor V	0.60–1.00	0–25	0–0.15	125–250	0–2

Table 2: Measured values of evaluation indicators.

Samples	Rock mass quality	Uniaxial compressive strength (MPa)	Integrality degree of rock mass	Groundwater percolation capacity (L/(min·10 m))	Joint
1	0.12	185.5	0.89	6	8
2	0.27	176.4	0.80	8	7
3	0.08	158.2	0.94	6	7
4	0.04	201.1	0.97	5	9
5	0.24	181.9	0.92	9	8

Table 3: Results of case study and comparison.

Samples	Integrated connectional membership degree					Proposed model	Basic quality grading method	Variable fuzzy set method [34]	Triangular fuzzy number method [16]
	μ_1	μ_2	μ_3	μ_4	μ_5				
1	−0.0844	0.3383	−0.7903	−1.0000	−1.0000	II	II	II	II
2	−0.5669	0.1899	−0.5298	−0.9679	−1.0000	II	II	II	II
3	−0.1752	0.0859	−0.8778	−1.0000	−1.0000	II	II	II	II
4	0.2320	−0.2744	−1.0000	−1.0000	−1.0000	I	I	I	I
5	−0.4351	0.3030	−0.4721	−1.0000	−1.0000	II	II	II	II

And the evaluated matter-element model R_P will be

$$R_P = \begin{bmatrix} P & p_1 & p_2 & p_3 & p_4 & p_5 \\ C_1 & 0.12 & 0.27 & 0.08 & 0.04 & 0.24 \\ C_2 & 185.5 & 176.4 & 158.2 & 201.1 & 181.9 \\ C_3 & 0.89 & 0.80 & 0.94 & 0.97 & 0.92 \\ C_4 & 6 & 8 & 6 & 5 & 9 \\ C_5 & 8 & 7 & 7 & 9 & 8 \end{bmatrix}. \quad (17)$$

The weights of the indexes were from the literature [34]. The discussed coupling method was utilized to conduct classification of surrounding rock stability. Results of the evaluation samples were listed in Table 3. The results evaluated from triangular fuzzy number method [16], basic quality method, and variable fuzzy set method [34] were also taken to confirm the validity and reliability of the proposed method.

It was found from Table 3 that the results determined were in a good agreement with the original data assessed by the BQ grading standards and other methods. It suggests that this proposed model presents the validity in surrounding rock stability evaluation, reveals uncertainty during the evaluation process, and lessens judgmental bias. This method may offer a more quantitative measure of surrounding rock stability because it can depict the degree that an object has certain nature and the change of truthfulness or falseness of position.

5. Conclusions

The classification of rock mass stability is often used in rock engineering and design; meanwhile, the surrounding rock instabilities are major hazards for human activities, causing injuries or fatalities, property damage, and maintenance costs. So it has received wide attention from engineers and scholars. However, previous studies of surrounding rock stability on position and reasoning are performed from a static perspective. It is obvious that those methods cannot depict the changeable and uncertainty features of various overlapping variable and uncertainty factors. This work presents a novel method of SPA coupled with extenics to analyze surrounding rock stability. Some conclusions are drawn as follows.

(1) The results show that this novel method is reliable and acceptable for evaluating the surrounding rock stability and convenient for practical applications. This proposed model may be an alternative effective method for the classification of other problems.

(2) The idea of connectional membership degree brings to light transformation among classes and differences in degree within the class from the transformational perspective and overcomes the drawback of fixed nature expressed by conventional methods. It can be viewed as concise and quantitative expressions of certainty and uncertainty of surrounding rock stability in a unified way.

(3) This coupling method provides a new idea to specify discrepancy coefficient for SPA and sets up a valid bridge connecting the two uncertainty analytical theories, SPA and extenics. And it enables us to express variability of surrounding rock stability due to uncertainty or incompatibility of evaluation indicators.

(4) Although our examination method has provided a useful basis for introducing dynamic uncertainty analysis from three aspects embracing identity, discrepancy, and contrary, into surrounding rock stability, the effects of other evaluation indicators on the prediction are required to clarify in future, and applications in practice will still involve a considerable amount of work both in the laboratory and in the field.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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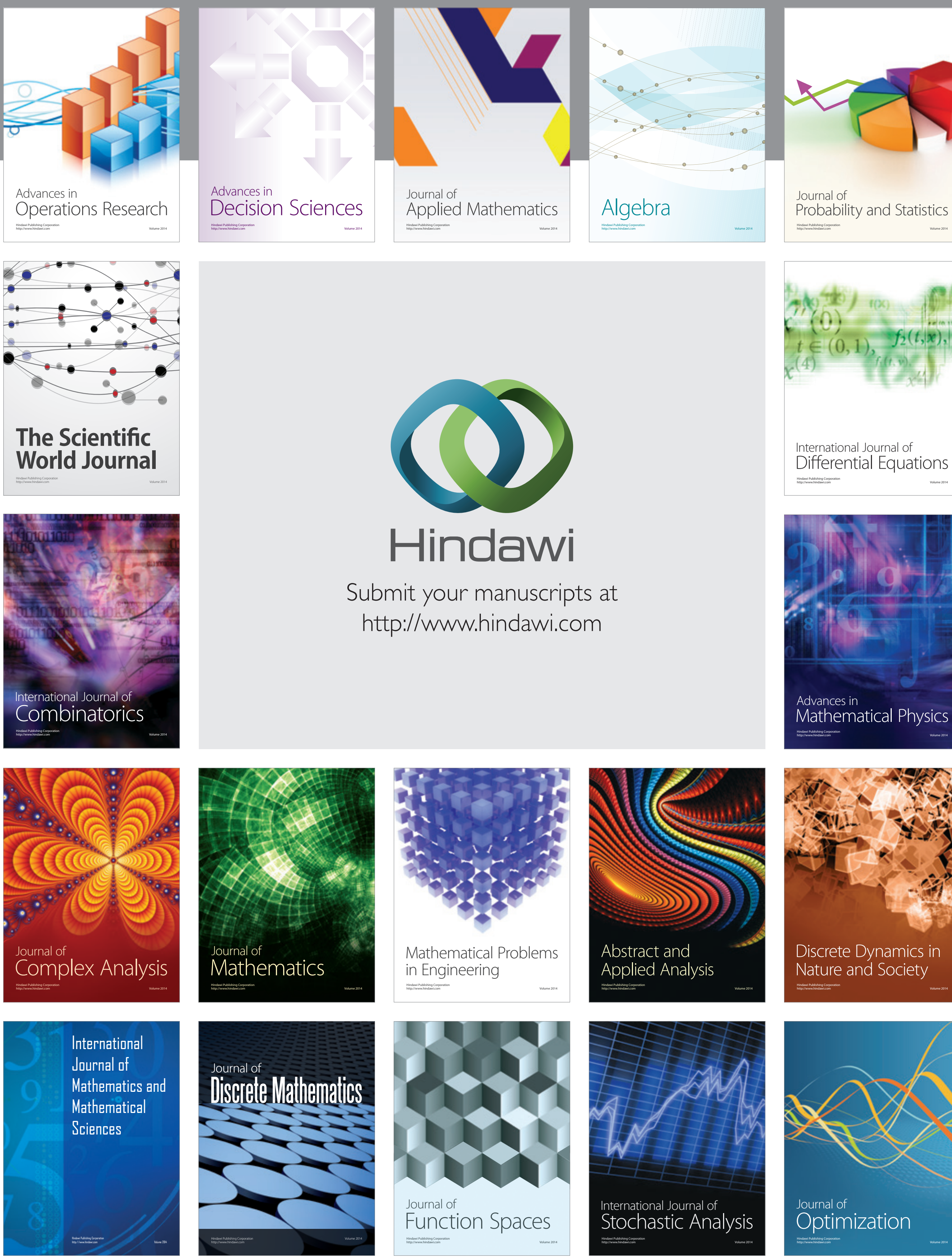
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